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Appendix:
Chapterwise Solved January 2019 JEE Main Questions (All Sets) A.1–A.

1 Coulomb's Law and Electric Field

INTRODUCTION

Humans have known about electrical effects for thousands of years, and it is impossible to say as to when electricity was discovered. However, records show that the attractive properties of amber were known to Greeks as early as 600 BC. When amber is rubbed briskly with a cloth, it attracts and picks up small, light objects such as bits of feather or straw or even thin scraps of metal. The Greek word for amber is elektron, and it is from this root word that we get our word electricity.

Static electricity can be observed in various ways. For example, when clothes are removed from a dryer, they cling together because electric charges are transferred between the clothes. When we brush our hair, both hair and brush become charged. The brush attracts dust or small bits of paper and our hair stand on their roots. All these phenomena result from the forces between charges at rest. This chapter and the next three chapters are devoted to electrostatics, which is the study of charges (i.e., electrons) at rest (i.e., static).

ELECTRIC CHARGE

Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects. It is known that every atom is electrically neutral, containing as many electrons as the number of protons in the nucleus. Charged particles can be created by disturbing the neutrality of an atom. Loss of electrons gives positive charge (as $n_p > n_e$) and gain of electrons gives negative charge (as $n_e > n_p$) to a particle. When an object is negatively charged, it gains electrons and therefore its mass increases negligibly. Similarly, on charging, a body with positive electricity its mass decreases. Change in the mass of the object is equal to $n \times m_e$, where n is the number of electrons transferred and m_e is the mass of electron.

Note: The rate of flow of electric charge is called electric current, i.e., $i = dQ/dt$ or $dQ = idt$. Hence, SI unit of charge is ampere × second = coulomb (C). Smaller SI units are mC, μC, and nC (1 mC = 10^{-3} C, 1 μC = 10^{-6} C, 1 nC = 10^{-9} C). CGS unit of charge is stat coulomb or esu. Dimensional formula of charge is $[Q] = [AT]$.

PROPERTIES OF CHARGE

Charge is transferable: If a charged body is put in contact with an uncharged body, the uncharged body becomes charged due to the transfer of electrons from one body to the other.

Charge is always associated with mass: Charge cannot exist without mass though mass can exist without charge.

Charge is conserved: Charge can neither be created nor be destroyed. Some important observations regarding this law are as follows:

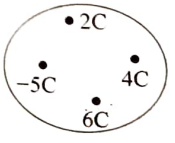
- The net charge remains constant even though it may not be zero.
- If the universe is considered as a whole, conservation of charge means that the net charge of the universe is constant.
- This principle does not prohibit the creation and destruction of charged particles. However, because of charge conservation, charged particles are created and destroyed only in pairs with equal and opposite charges.
- The law is also true for relativistic motion which means that the total electric charge of an isolated system is relativistically invariant.
- On the basis of the evidence on the macroscopic scale, the principle is largely speculative, but there is ample justification for it on the microscopic scale.

Invariance of charge: The numerical value of an elementary charge is independent of velocity. It is proved by the fact that an atom is neutral. The difference in masses on an electron and a proton suggests that electrons move much faster in an atom than in a proton. If the charges were dependent on velocity, the neutrality of atoms would be violated.

Charge produces electric field and magnetic field: A charged particle at rest produces only electric field in the space surrounding it. However, if the charged particle is in unaccelerated motion, it produces both electric and magnetic fields. And if the motion of the charged particle is accelerated, it not only produces electric and magnetic fields but also radiates energy in the space surrounding the charge in the form of electromagnetic waves.

Quantization of charge: When a physical quantity can have only discrete values rather than any value, the quantity is said to be quantized. The smallest charge that can exist in nature is the charge of an electron. If the charge of an electron (= 1.6×10^{-19} C) is taken as elementary unit, i.e., quanta of charge, the charge on any body will be some integral multiple of e, i.e., $Q = \pm ne$ with $n = 1, 2, 3$. Charge on a body can never be $\pm 2/3e$, $\pm 17.2e$, or $\pm 10^{-5}\ e$.

Additivity of charge: Total charge on a body is the algebraic sum of all the charges located anywhere on the body. While adding the charges, their sign must be taken into consideration. For example, if a body has the charges 2C, –5C, 4C, 6C, etc., then the total charge on body is $2 - 5 + 4 + 6 = 7$C. Note that charges are added like real numbers. They have no direction. So charge is a scalar quantity.



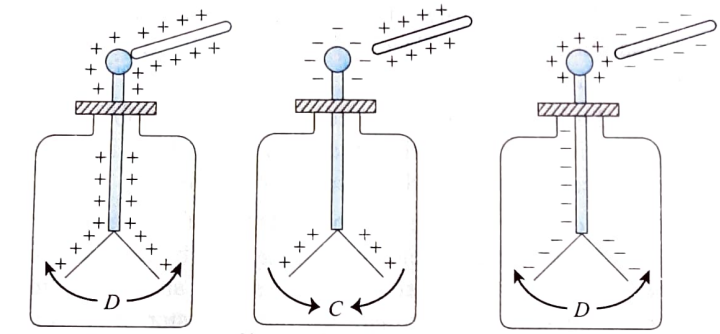
CONDUCTORS AND INSULATORS

In conductors, the valence electrons of the atoms, that is, the electrons in the outermost orbits are loosely bound. As a result, they can be easily removed from the atoms and moved about in the conductor, or they can leave the conductor altogether. That is, the valence electrons are not permanently bound to a particular atom. In insulators, however, even the loosest bound electrons are too tightly bound to be easily removed from their atoms. Thus, charge does not readily move through, nor it can be readily removed from, an insulator.

ELECTROSCOPE

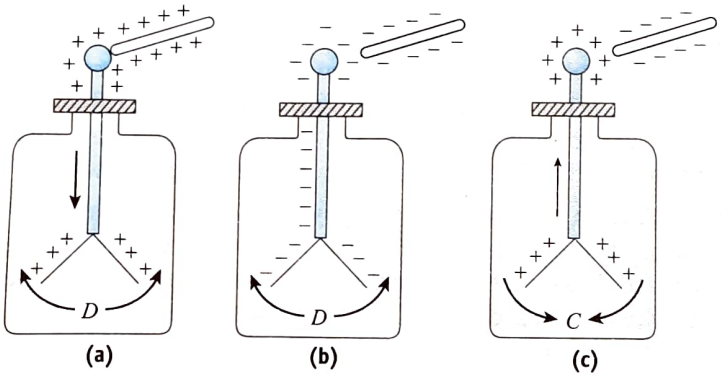
It is a simple apparatus with which the presence of electric charge on a body is detected (see figure). When metal knob is touched with a charged body, some charge is transferred to the gold leaves, which then diverges due to repulsion. The separation gives a rough idea of the amount of charge on the body. If a charged body brought near a charged electroscope the leaves will further diverge. If the charge on body is similar to that on electroscope and will usually converge if opposite. If the induction effect is strong enough, leaves after converging may again diverge.

(1) **Uncharged electroscope**



Charging by conduction (a) | Charging by induction (b) | Charging by induction (c)

(2) **Charged electroscope**



(a) (b) (c)

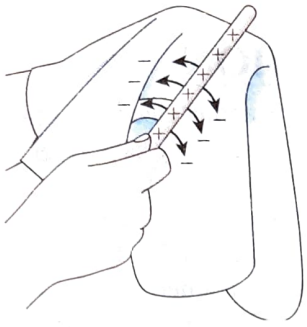
CHARGING OF A BODY

It is possible to give a net charge to an object by different methods, all of which are said to involve electrostatic charging.

CHARGING BY FRICTION

When certain insulators are rubbed with cloth or fur, they become electrically charged by a transfer of charge (i.e., electrons). As the two objects are rubbed together, one object loses electrons while the other gains electrons; that is, there is transfer of electrons from one object to the other. The object that gains electrons becomes negatively charged, while the object that loses electrons has an excess of positive charge. Hence it is positively charged. The transfer of charge is due to the contact between the materials, and the amount of charge transferred depends, as we might expect, on the nature of these materials.

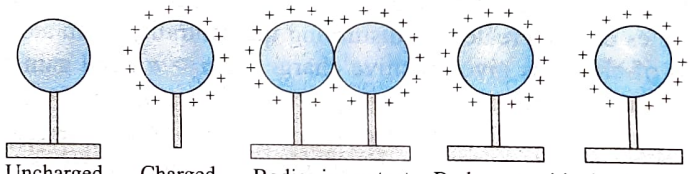
Because of conservation of charge, each electron adds negative charge to the silk and an equal positive charge is left on the glass rod.



If we hold a metallic rod (a conductor) in one hand and rub it with any material, the metallic rod cannot be charged. The reason for this is that the electrons can easily flow through the metal to our hand and then to the ground. However, if the metallic rod has a handle made of wood or glass and is rubbed while holding it by the handle, the rod will get charged. The handle prevents the flow of electrons from the metal rod to the hand, hence to the ground.

CHARGING BY CONDUCTION

Charging by conduction requires contact between two objects (see figure). Take two conductors, one charged and the other uncharged. Bring the conductors in contact with each other. The charge (whether positive/negative) under its own repulsion will spread over both the conductors. Thus, the conductors will be charged with the same sign. This is called charging by conduction (through contact).

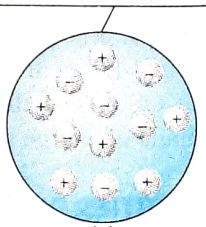


Uncharged (a) | Charged | Bodies in contact (b) | Both are positively charged (c)

CHARGING BY INDUCTION

Induction is a process by which a charged body can be used to create other charged bodies without touching them or losing its own charge. If a charged body is brought near a neutral body, the charged body attracts opposite charge and repels similar charge present on the neutral body. If the neutral body is now earthed, the like charge is neutralized by the flow of charge from earth, leaving unlike charge on the body. Now the earthing and the charging body are removed leaving the initially neutral body charged. The whole process is shown step by step in the given figures.

The neutral sphere has equal numbers of positive and negative charges.



(a)

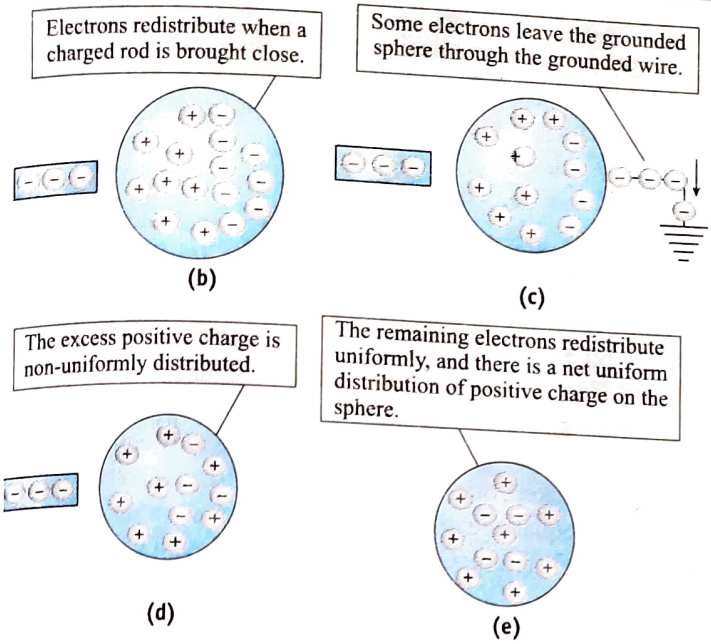


ILLUSTRATION 1.1

A glass rod is rubbed with a silk cloth. The glass rod acquires a charge of $+19.2 \times 10^{-19}$C.

(a) Find the number of electrons lost by glass rod.

(b) Find the negative charge acquired by silk.

(c) Is there transfer of mass from glass to silk?

Given, $m_e = 9 \times 10^{-31}$ kg.

Sol.

(a) The number of electrons lost by glass rod is

$$n = \frac{q}{e} = \frac{19.2 \times 10^{-19}}{1.6 \times 10^{-19}} = 12$$

(b) Charge on silk is -19.2×10^{-19} C.

(c) Since an electron has a finite mass ($m_e = 9 \times 10^{-31}$ kg), there will be transfer of mass from glass rod to silk cloth. Mass transferred $= 12 \times (9 \times 10^{-31}) = 1.08 \times 10^{-29}$ kg.

Note that mass transferred is negligibly small. This is expected because the mass of an electron is extremely small.

ILLUSTRATION 1.2

If an object made of substance A is rubbed with an object made of substance B, then A becomes positively charged and B becomes negatively charged. If, however, an object made of substance A is rubbed against an object made of substance C, then A becomes negatively charged. What will happen if an object made of substance B is rubbed against an object made of substance C?

Sol. When A and B are rubbed, A becomes positively charged and B becomes negatively charged. It means electrons are loosely bound with A in comparison to B. When A and C are rubbed together, A becomes negatively charged and C positively charged. It means electrons are loosely bound with C in comparison to A. Hence, electrons are most loosely bound in C. So if B and C are rubbed together, C will lose electrons and B will receive electrons. Hence, C will become positively charged and B will become negatively charged.

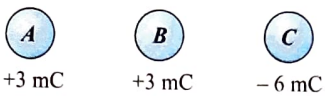
ILLUSTRATION 1.3

Objects A, B, and C are three identical, insulated, spherical conductors. Originally, A and B have charges of +3 mC, whereas C has a charge of −6 mC. Objects A and C are touched and moved apart. Objects B and C are touched before they are moved apart.

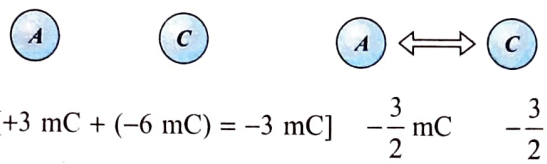
(a) If objects A and B are now held near each other, they will
(a) attract (b) repel (c) have no effect on each other.

(b) If objects A and C are held near each other, they will
(a) attract (b) repel (c) have no effect on each other.

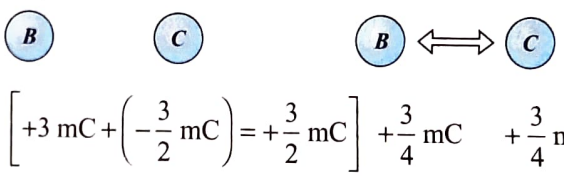
Sol. Initially,



- When the objects A and C are allowed to touch and then moved apart:



- When the objects B and C are allowed to touch and then moved apart:

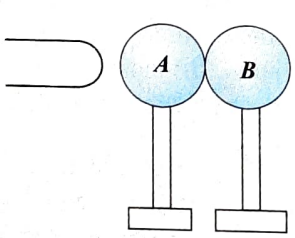


(a) Hence, if A and B are now held near each other, they will attract each other.

(b) If A and C are now held near each other, they will also attract each other.

ILLUSTRATION 1.4

Figure shows that a positively charged rod is brought near two uncharged metal spheres A and B clamped on insulated stands and placed in contact with each other.



(a) What would happen if the rod is removed before the spheres are separated?

(b) Would the induced charges be equal in magnitude even if the spheres had different sizes or different conductors?

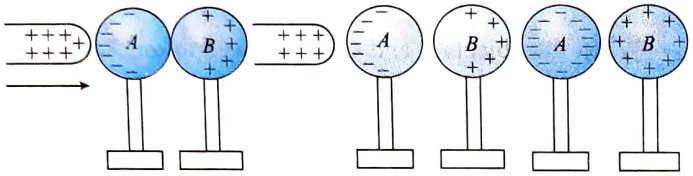
(c) What will happen if the spheres are separated first and then the rod is removed far away?

Sol.

(a) When a positively charged rod is brought near A, the free electrons in the sphere A are attracted toward the rod and moved to the left side of A. This movement leaves unbalanced positive charge on B. If the rod is removed before the spheres are separated, the excess electrons on sphere A would flow back to B. Both the spheres will become uncharged.

(b) Yes, net charge is conserved. Before the rod is brought near A, both A and B were neutral. They will remain so even if they have different sizes or materials.

(c) If the rod is removed after the spheres are separated, then sphere A will have net negative charge and sphere B will have net positive charge of same magnitude as shown in figures.



(a) (b) (c)

ILLUSTRATION 1.5

Calculate the total positive (or negative) charge on a 3.11 g copper penny. Given Avogadro's number = $6.023 \times 10^{23}\ g^{-1}\ mol^{-1}$; for copper, atomic number = 29 and atomic mass = 63.5.

Sol. The number of atoms in the penny is

$$\left[\frac{6.02 \times 10^{23}}{63.5}\right] 3.11 = 0.295 \times 10^{23}$$

Since each copper atom contains 29 protons (and 29 electrons), the total number of positive (or negative) charges on the penny is $n = 29(0.295 \times 10^{23}) = 8.56 \times 10^{23}$.

Total charge (positive or negative) on the penny is

$$q = ne = (8.56 \times 10^{23})(1.6 \times 10^{-19}\ C)$$
$$= 1.37 \times 10^{5}\ C$$

ILLUSTRATION 1.6

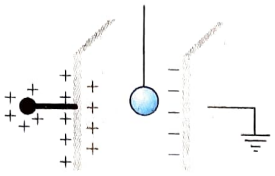
An electron and a proton, initially separated by a distance d in air are released from rest simultaneously. The two particles are free to move. When they collide, are they **(a)** at the midpoint of their initial separation, **(b)** closer to the initial position of the proton, or **(c)** closer to the initial position of the electron?

Sol. Because of Newton's third law, the forces exerted on the electron and the proton are equal in magnitude and opposite in direction. For this reason, it might seem that the particles meet at the midpoint. The masses of the particles, however, are quite different. In fact, as the mass of the proton is about 2000 times greater than the mass of the electron, the proton's acceleration ($a = F/m$) is about 2000 times less than the electron's acceleration. As a result, the particles collide closer to the initial position of the proton. More specifically, they collide at the location of the center of mass of the system, which remains at rest throughout the process because there is no external force.

CONCEPT APPLICATION EXERCISE 1.1

1. **(a)** How many electrons are in 1 C of negative charge?
   (b) Which is the true test of electrification: attraction or repulsion?
   (c) Can a body have a charge of 0.8×10^{-19} C?
2. If only one charge is available, can it be used to obtain a charge many times greater than itself in magnitude?
3. **(a)** Can two bodies having like charges attract each other? (Yes/No)
   (b) Can a charged body attract an uncharged body? (Yes/No)
   (c) Two identical metallic spheres of exactly equal masses are taken, one is given a positive charge q and the other an equal negative charge. Their masses after charging are different. Comment on the statement.
4. A particle has a charge of $+10^{-12}$ C.
   (a) Does it contain more or less number of electrons as compared to the neutral state?
   (b) Calculate the number of electrons transferred to provide this charge.
5. An ebonite rod is rubbed with fur and is found to have a charge of -3.2×10^{-8} C on it.
   (a) Calculate the number of electrons transferred.
   (b) What is the charge on fur after rubbing?
6. The electric charge of macroscopic bodies is actually a surplus or deficiency of electrons. Why not protons?
7. A charged rod attracts bits of dry paper, which after touching the rod often jump away from it rapidly. Explain.
8. A person standing on an insulating stool touches a charged insulated conductor. Will the conductor get completely discharged?
9. Define the following statement: "If there were only one electrically charged particle in the entire universe, the concept of electric charge would be meaningless".
10. How many megacoulombs of positive (or negative) charge are present in 2.0 mol of neutral hydrogen gas.
11. A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.
    (a) Estimate the number of electrons transferred (from which to which)?
    (b) Is there a transfer of mass from wool to polythene?
12. Two identical conducting spheres, one having an initial charge $+Q$ and the other initially uncharged, are brought into contact.
    (a) What is the new charge on each sphere?
    (b) While the spheres are in contact, a positively charged rod is moved close to one sphere, causing a redistribution of the charges on the two spheres, so the charge on the sphere closest to the rod has a charge $-Q$. What is the charge on the other sphere?
13. Two identical conducting spheres are charged by induction and then separated by a large distance; sphere-1 has charge $+Q$ and sphere-2 has charge $-Q$. A third sphere is initially uncharged. If sphere-3 is touched to sphere-1 and separated and then touched to sphere-2 and separated, what is the final charge on each of the three spheres?

14. A table tennis ball covered with a conducting paint is suspended by a silk thread so that it hangs between two metal plates (see figure). One plate is earthed. When the other plate is connected to a high voltage generator, what will happen to the ball.



15. In 1 g of a solid, there are 5×10^{21} atoms. If one electron is removed from each of 0.01% atoms of the solid, find the charge gained by the solid (given that electronic charge is 1.6×10^{-19} C).

ANSWERS

1. (a) 6.25×10^{18} (b) Repulsion (c) No
2. Yes
3. (a) Yes (b) Yes (c) Yes
4. (a) Less (b) 6.25×10^{6}
5. (a) 2×10^{11} (b) 3.2×10^{-8} C **8.** No **10.** 0.358
11. (a) 1.875×10^{12} (b) Yes; 1.7×10^{-18} kg
13. $\frac{Q}{2}, \frac{-Q}{4}, \frac{-Q}{4}$
15. 0.08 C

COULOMB'S LAW

Charles Coulomb measured the magnitude of electric forces between charged objects using the torsion balance. From Coulomb's experiments, we can generalize the properties of the electric force (sometimes called electrostatic force) between two stationary charged particles.

We use the term point charge to refer to a charged particle of zero size. The electrical behavior of electrons and protons is very well described by modeling them as point charges. From experimental observations, Coulomb showed that the electric force between two stationary charged particles:

- is inversely proportional to the square of the separation r between the particles and directed along the line joining them;
- is proportional to the product of the charges q_1 and q_2 on the two particles;
- is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

Thus,

$$F_e = k_e \frac{q_1 q_2}{r^2}$$

where k_e is a constant called the Coulomb constant. The value of the Coulomb constant depends on the choice of units. The SI unit of charge is coulomb (C). The Coulomb constant k_e in SI units has the value $k_e = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$. This constant is also written in the form $k_e = 1/4\pi\varepsilon_0$, where the constant ε_0 (Greek letter epsilon) is known as the permittivity of free space and has the value $\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$. Now we can write

$$F_e = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

Important Points:

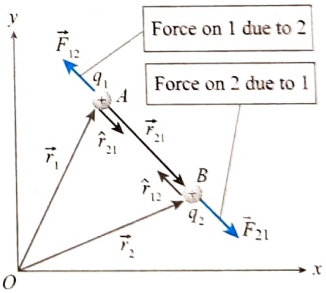
- The permittivity of a given medium is the measure of the fact how strongly a medium is influenced by external electric field. If an externally applied field has stronger polarizing effect on the medium, it has high permittivity.
- If two charges are placed in any medium other than vacuum or air, the force between two charges decreases due to the polarization of the medium. Thus, the resultant force on a charge gets reduced by a factor k known as the dielectric constant of the medium or relative permittivity of the medium. Thus $\varepsilon/\varepsilon_0 = k = \varepsilon_r$, where ε is the permittivity of the medium, ε_0 is the permittivity of vacuum, and ε_r is relative permittivity. Thus,

$$|\vec{F}| = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q_1 q_2}{r^2}$$

  For vacuum, $\varepsilon_r = 1$, and for conductivity medium, $\varepsilon_r = \infty$.
- Coulomb force between two charges is an action-reaction pair, which is conservative in nature and is a central force. It acts along the line joining two point charges.
- Coulomb's law is valid only for point charges. If the size of an object is very small as compared to the separation, then they are considered point charges.
- The force between two point charges is independent of the presence or absence of any other charges. Due to the presence of the surrounding medium the resultant force changes because of polarization of the molecules of the medium.
- Coulomb's law is not valid for distances less than 10^{-15} m.
- Electrostatic forces are comparatively stronger than gravitational forces.
- Coulomb's law is similar to Newton's gravitational law and both obey inverse square law.
- Coulomb's law obeys Newton's third law, i.e., the forces exerted by the two charges on each other are equal and opposite.
- Electrostatic force is a conservative force.

COULOMB'S LAW IN VECTOR FORM

Let q_1 and q_2 be two like charges placed at points A and B, respectively, in vacuum. $\vec{r}_1$ is the position vector of point A, and $\vec{r}_2$ is the position vector of point B. Let $\vec{r}_{21}$ be vector from A to B, then



$$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$$

and $$|\vec{r}_{21}| = r = |\vec{r}_2 - \vec{r}_1|$$

$$\hat{r}_{21} = \frac{\vec{r}_{21}}{r} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

From figure it is clear that $\vec{F}_{21}$ and $\hat{r}_{12}$ are in the same direction, so

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \frac{\vec{r}}{r} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\vec{r}}{r^3}$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3}$$

The above equations give the Coulomb's law in vector form. As we know that charges apply equal and opposite forces on each other, we have $\vec{F}_{12} = -\vec{F}_{21}$

or $$\vec{F}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3}$$

We can also write in terms of unit vector notation:

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

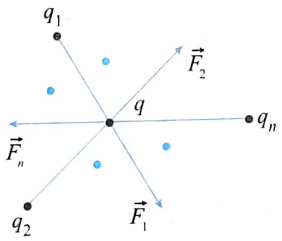
where $\hat{r}_{12}$ is a unit vector directed toward q_1 from q_2. This force of Coulomb's law is illustrated in figure. For three different point charge distributions, we have $\hat{r}_{12} = -\hat{r}_{21}$. So

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_2 q_1}{r^2} (-\hat{r}_{21})$$

$$= -\frac{1}{4\pi\varepsilon_0} \frac{q_2 q_1}{r^2} \hat{r}_{21} = -\vec{F}_{21}$$

SUPERPOSITION PRINCIPLE

The superposition principle enables us to calculate the force acting on a charge due to more than one charge. "According to the superposition principle, the total force on a given charge is the vector sum of all the individual forces exerted by each of the other charges."



$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n$$

Another important point is that the force between two charges remains unaffected due to the presence of the third charge.

COMPARISON BETWEEN COULOMB'S LAW AND NEWTON'S UNIVERSAL LAW OF GRAVITATION

Coulomb's law is the second fundamental force law encountered in the study of physics. It is similar in form to the fundamental force law, i.e., Newton's law of universal gravitation. Recall that the gravitational force between two particles, is proportional to the square of the distance between them, i.e.,

$$F = G\frac{m_1 m_2}{r^2}$$

where $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}. Thus, Coulomb force depends on the charge in the same way that the gravitational force depends on mass, and both forces have the same $1/r^2$ dependence. There are, of course, important differences between Coulomb's law and the gravitational force law:

- There is only one kind of mass, but there are two kinds of charges: positive and negative.
- Gravitational force is always attractive, but electrostatic force may be attractive or repulsive, depending on the sign of the charges.
- The electrostatic force constant k_e is much larger than the gravitational constant G. Two bodies 2.00 m apart, each carrying a charge of magnitude 1.00 C, experience a force of 2.25×10^9 N, or about half a billion pounds. On the other hand, two 1.00 kg bodies 2.00 m apart experience a mutual gravitational force of only 1.67×10^{-11} N.

The large value of the electric force constant means that the interaction of even relatively small charges can produce significant forces.

ILLUSTRATION 1.7

Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of the force between them exactly given by $Q_1 Q_2 / 4\pi\varepsilon_0 r^2$, where r is the distance between their centers?

Sol. As the spheres are brought close to each other, the charge on one will affect the other and as such the distribution of charge is no longer uniform (i.e., the charges are not concentrated at the centers of the spheres). Hence, the electrostatic force between them is not given exactly by $Q_1 Q_2 / 4\pi\varepsilon_0 r^2$; the relation is true only for point charges. In other words, Coulomb's law is applicable only to point charges.

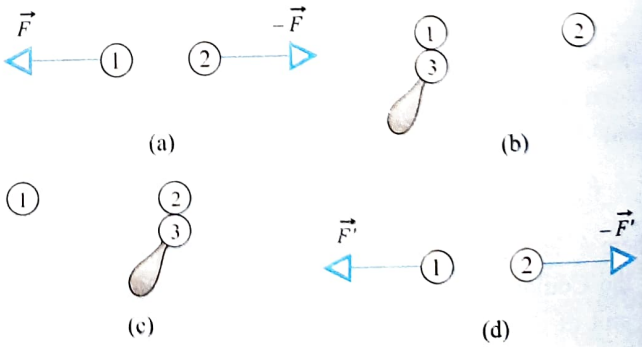
ILLUSTRATION 1.8

Four identical point charges are placed at the corners of a square. A fifth point charge placed at the center of the square experiences zero net force. Is this a stable equilibrium for the fifth charge? Explain.

Sol. No. Only for very special displacements, the electrostatic force acts in a direction that points back toward the equilibrium position. For a general displacement, the electrostatic force does not point toward the equilibrium position, and the fifth charge moves farther from equilibrium, making the system unstable.

ILLUSTRATION 1.9

Two identical conducting spheres 1 and 2 carry equal amounts of charge and are fixed apart at a certain distance larger than their diameters. The spheres repel each other with an electrical force of 88 mN. A third identical sphere 3, having an insulating handle and initially uncharged, is touched first to sphere 1 and then to sphere 2 and finally removed. Find the force between spheres 1 and 2 as shown in Fig. (d).



Sol. Initial force between 1 and 2 is $F = kq^2/r^2 = 88$ mN. Charge on 1 after 3 is touched with 1 is $q/2$. Same charge will be on 3 also. Charge on 2 after 3 is touched with 2 is

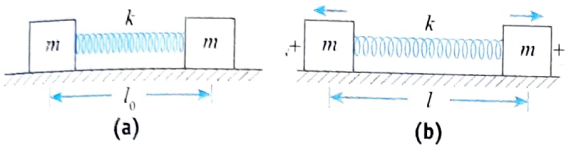
$$\frac{q + (q/2)}{2} = \frac{3q}{4}$$

Now, force between 1 and 2 in situation (d) is

$$F=\frac{k(q/2)(3q/4)}{r^2}=\frac{3kq^2}{8r^2}=\frac{3}{8}\times 88=33\text{ mN}$$

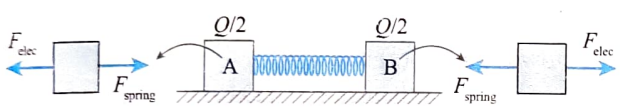
ILLUSTRATION 1.10

Two identical metallic blocks resting on a frictionless horizontal surface are connected by a light metallic spring having a spring constant k and an unstretched length l_0 as shown in the figure. A total charge of Q is slowly placed on the system, causing the spring to stretch to an equilibrium length l as shown in figure. Determine the value of Q, assuming that all the charges reside on the blocks and that the blocks are like point charges.



Sol. The blocks are under two forces: (i) electrostatic force (ii) spring force

Considering FBD of the block at equilibrium: $F_{el}=F_{sp}$



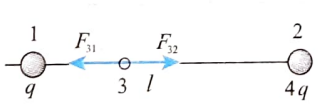
$$\frac{(Q/2)^2}{4\pi\varepsilon_0 l^2}=k(l-l_0)\text{ and }Q^2=16\pi\varepsilon_0 kl^2(l-l_0)$$

which give $Q=\sqrt[4]{\pi\varepsilon_0 kl^2(l-l_0)}$

ILLUSTRATION 1.11

Two free point charges $+q$ and $+4q$ are at distance l apart. A third charge is so placed that the entire system is in equilibrium.

Sol. For the system to be in equilibrium, the net force on each charge should be zero. Hence, the third charge should be negative and it should be placed near q between (1) and (2).



For equilibrium of (3),

$$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{x^2}=\frac{1}{4\pi\varepsilon_0}\frac{Q4q}{(l-x)^2}$$

$$\text{or }\left[\frac{l-x}{x}\right]^2=4\text{ or }\frac{l-x}{x}=2\text{ or }x=\frac{l}{3}$$

For equilibrium of (1),

$$\frac{1}{4\pi\varepsilon_0}\frac{q4q}{l^2}=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{x^2}$$

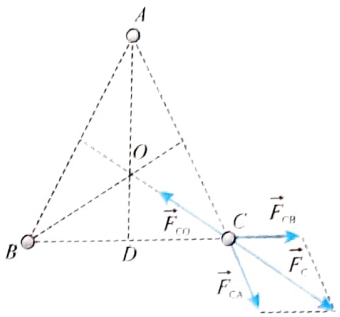
$$\text{or }Q=4q\left(\frac{x}{l}\right)^2=4q\left(\frac{1}{3}\right)^2=\frac{4q}{9}$$

$$\text{or }Q=-\frac{4q}{9}$$

ILLUSTRATION 1.12

Three charges of equal magnitude q are placed at the vertices of an equilateral triangle of side l. How can the system of charges be placed in equilibrium?

Sol. To keep the system in equilibrium, the net force experienced by charges at A, B, and C should be zero. For this, another charge of opposite sign should be placed at the centroid of the triangle. Let this charge be $-Q$.



$$AD=l\cos 30^\circ=\frac{l\sqrt{3}}{2}$$

$$AO=\frac{2}{3}AD=\frac{l}{\sqrt{3}}=CO$$

$$2|\vec{F}_{CA}|\cos 30^\circ=|\vec{F}_{CO}|$$

$$|\vec{F}_{CB}|=|\vec{F}_{CA}|$$

$$2\frac{1}{4\pi\varepsilon}\frac{q^2}{l^2}\frac{\sqrt{3}}{2}=\frac{1}{4\pi\varepsilon}\frac{Qq}{(l/\sqrt{3})^2}\text{ or }Q=\frac{q}{\sqrt{3}}$$

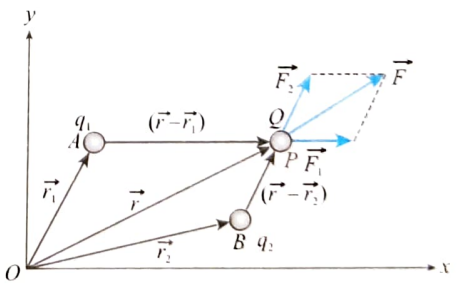
ILLUSTRATION 1.13

A charge Q located at a point $\vec{r}$ is in equilibrium under the combined field of three charges q_1, q_2 and q_3. If the charges q_1 and q_2 are located at points $\vec{r}_1$ and $\vec{r}_2$, respectively, find the direction of the force on Q due to q_3 in terms of q_1, q_2, $\vec{r}_1$, $\vec{r}_2$, and $\vec{r}$.

Sol. From figure,

$$\overrightarrow{OA}+\overrightarrow{AP}=\overrightarrow{OP}\text{ or }\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\vec{r}-\vec{r}_1$$

Similarly, $\overrightarrow{OB}=\overrightarrow{BP}=\overrightarrow{OP}$ or $\overrightarrow{BP}=\overrightarrow{OP}-\overrightarrow{OB}=\vec{r}-\vec{r}_2$



Force on Q due to q_1 is

$$\vec{F}_1=k_e\frac{Qq_1}{r_{AP}^2}\hat{r}_{AP}=k_e\frac{Qq_1(\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^2\,|\vec{r}-\vec{r}_1|}=k_eQq_1\frac{(\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^3}$$

Similarly, force on Q due to q_2 is

$$\vec{F}_2=k_eQq_2\frac{(\vec{r}-\vec{r}_2)}{|\vec{r}-\vec{r}_2|^3}$$

The net force on Q due to charges q_1 and q_2, using the principle of superposition, is

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = k_e Q\left[q_1 \frac{(\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^3} + q_2 \frac{(\vec{r}-\vec{r}_2)}{|\vec{r}-\vec{r}_2|^3}\right]$$

Let the force exerted by q_3 on Q be $\vec{F}_3$.

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 \quad \text{or} \quad \vec{F} + \vec{F}_3 = 0.$$

Hence $\vec{F}_3 = -\vec{F}$. For the charge Q to be in equilibrium,

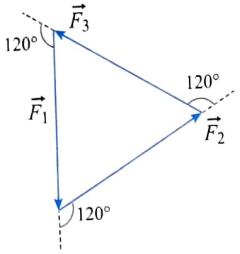
$$\vec{F}_3 = -k_e Q\left[\frac{q_1(\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^3} + \frac{q_2(\vec{r}-\vec{r}_2)}{|\vec{r}-\vec{r}_2|^3}\right]$$

Obviously, $-\vec{F}$ gives the direction of the force on Q due to q_3.

ILLUSTRATION 1.14

Consider three charges q_1, q_2, and q_3, each equal to q, at the vertices of an equilateral triangle of side l. What is the force on a charge Q placed at the centroid of the triangle?

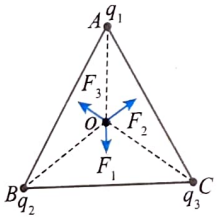
Sol. **Method 1:** The resultant of three equal coplanar vectors acting at a point will be zero if these vectors form a closed polygon (figure). Hence, the vector sum of the forces $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ is zero.



Method 2: The forces acting on the charge Q are

$$\vec{F}_1 = \text{force on } Q \text{ due to } q_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_1}{AO^2}\widehat{AO}$$

$$\vec{F}_2 = \text{force on } Q \text{ due to } q_2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_2}{BO^2}\widehat{BO}$$



$$\vec{F}_3 = \text{force on } Q \text{ due to } q_3 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_3}{CO^2}\widehat{CO}$$

The resultant force is

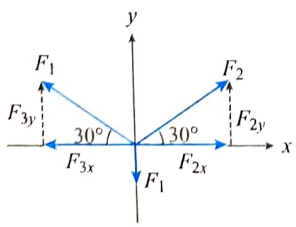
$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qq}{AO^2}(\widehat{AO} + \widehat{BO} + \widehat{CO}) = 0$$

$$\left(\text{as } |q_1| = |q_2| = |q_3| \text{ and } |\overline{AQ}| = |\overline{BQ}| = |\overline{CQ}|\right)$$

Also, $\overline{AO} + \overline{BO} + \overline{CO} = 0$ because these are three equal vectors in a plane making angles of 120° with each other.

Method 3: The resultant force $\sum\vec{F}$ is the vector sum of individual forces, i.e.,



$$\sum\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\therefore \quad \sum F_x = F_{1x} + F_{2x} + F_{3x}$$

$$= 0 + F_2\cos 30° - F_3\cos 30° \quad \text{...(i)}$$

and $$\sum F_y = F_{1y} + F_{2y} + F_{3y}$$

$$= -F_1 + F_2\sin 30° + F_3\sin 30° \quad \text{...(ii)}$$

As $|F_1| = |F_2| = |F_3| = |F|$ (say), Eqs. (i) and (ii) become $\sum F_x = 0$ and $\sum F_y = 0$. Hence, resultant force $\sum\vec{F} = 0$.

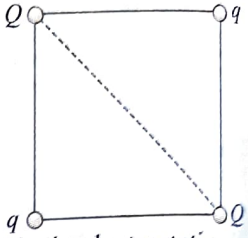
ILLUSTRATION 1.15

Point charges are placed at the vertices of a square of side a as shown in figure. What should be the sign of charge q and magnitude of the ratio $|q/Q|$ so that

(a) net force on each Q is zero?

(b) net force on each q is zero?

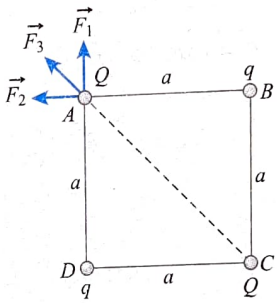
Is it possible that the entire system could be in electrostatic equilibrium?



Sol.

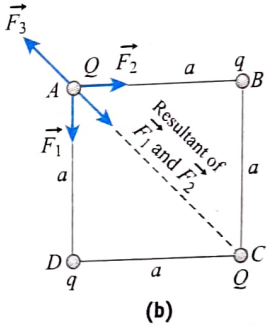
(a) Consider the forces acting on charge Q placed at A [shown in Figs. (a) and (b)].

Case I. Let the charges q and Q be of same sign.



(a)

(q and Q are of same nature. Here, net force cannot be zero.)



(b)

(q and Q are of opposite nature. Here, net force can be zero.)

Here,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2} \quad \text{(force of } q \text{ at } D \text{ on } Q \text{ at } A)$$

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2} \quad \text{(force of } q \text{ at } B \text{ on } Q \text{ at } A)$$

$$F_3 = \frac{1}{4\pi\varepsilon_0}\frac{QQ}{2a^2} \quad \text{(force of } Q \text{ at } C \text{ on } Q \text{ at } A)$$

In Fig. (a), the resultant of forces $\vec{F}_1$ and $\vec{F}_2$ will lie along $\vec{F}_3$ so that the net force on Q cannot be zero. Hence, q and Q have to be of opposite signs.

Case II. Let the charges q and Q be of opposite signs.

In this case, as shown in Fig. (b), resultant of $\vec{F}_1$ and $\vec{F}_2$ will be opposite to $\vec{F}_3$ so that it becomes possible to obtain a condition of zero net force. Let us write $\vec{F}_R = \vec{F}_1 + \vec{F}_2$. So

$$\sqrt{F_1^2 + F_2^2} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}\sqrt{2}$$

The direction of $\vec{F}_R$ will be along AC ($\vec{F}_R$, being the resultant of forces of equal magnitude, bisects the angle between the two). $\vec{F}_R$ and $\vec{F}_3$ are in opposite directions. Theat force on Q can be zero if their magnitudes are also equal, i.e.,

$$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}\sqrt{2}=\frac{1}{4\pi\varepsilon_0}\frac{QQ}{2a^2} \text{ or } \frac{Q}{4\pi\varepsilon_0 a^2}\left(q\sqrt{2}-\frac{Q}{2}\right)=0$$

$$\text{or } q=\frac{Q}{2\sqrt{2}} \qquad \left|\frac{q}{Q}\right|=\frac{1}{2\sqrt{2}} \quad \{Q\neq 0\}$$

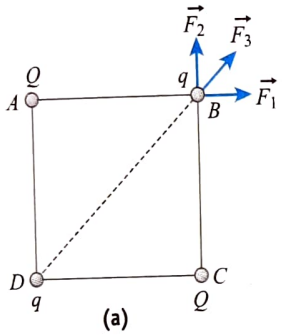
Therefore, the sign of q should be the negative of Q.

(b) Consider now the forces acting on charge q placed at B [see Figs. (a) and (b)]. In a similar manner, as discussed in Fig. (a), for the net force on q to be zero, q and Q have to be of opposite signs. This is also shown in the given figures.

Now, $F_1=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}$ (force of Q at A on q at B)

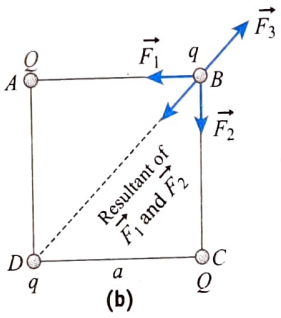
$F_2=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}$ (force of Q at C on q at B)

$F_3=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a^2}$ (force of q at D on q at B)



(a)

(q and Q are of same sign. Here, net force cannot be zero.)



(b)

(q and Q are of opposite sign. Here, net force can be zero.)

From Fig. (b), we can write

$$\vec{F}_R=\vec{F}_1+\vec{F}_2$$

$$\therefore \quad F_R=\sqrt{F_1^2+F_2^2}=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\sqrt{2}$$

The resultant of $\vec{F}_1$ and $\vec{F}_2$, i.e., $\vec{F}_R$, is opposite to $\vec{F}_3$. Net force can become zero if their magnitudes are also equal, i.e.,

$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\sqrt{2}=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a^2} \text{ or } \frac{q}{4\pi\varepsilon_0 a^2}\left[\sqrt{2}Q-\frac{q}{2}\right]=0$$

$$\text{or } Q=\frac{q}{2\sqrt{2}} \quad \text{or} \quad \left|\frac{q}{Q}\right|=2\sqrt{2} \quad \{q\neq 0\}$$

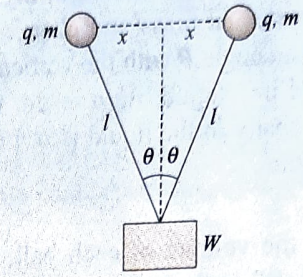
Therefore, the sign of q should be negative of Q.
In this case, we need not to repeat the calculation as the present situation is same as the previous one; we can directly write $|q/Q|=2\sqrt{2}$. The entire system cannot be in equilibrium since both conditions, i.e., $q=-Q/2\sqrt{2}$ and $Q=-q/2\sqrt{2}$ cannot be satisfied together.

ILLUSTRATION 1.16

Two identical He-filled spherical balloons each carrying a charge q are tied to a weight W with strings and float in equilibrium, as shown in figure.

(a) Find the magnitude of q, assuming that the charge on each balloon acts as if it were concentrated at the center.

(b) Find the volume of each balloon. Take the density of He as ρ_{He} and the density of air as ρ_a. Ignore the weight of the unfilled balloons.



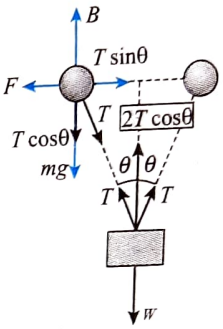
Sol.

(a) $2T\cos\theta=W$, $T\sin\theta=F$ (figure)

or $\frac{\tan\theta}{2}=\frac{F}{W}$ or $F=W\frac{\tan\theta}{2}$

or $\frac{q^2}{4\pi\varepsilon_0(2x)^2}=\frac{W\tan\theta}{2}$

or $q=\sqrt{8W\tan\theta\pi\varepsilon_0 x^2}$



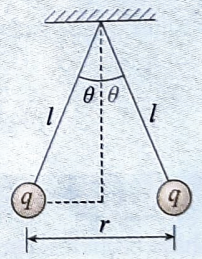
(b) $T\cos\theta+mg=B$

$\Rightarrow \frac{W}{2}+V\rho_{He}g=V\rho_a g$

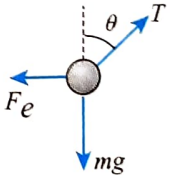
or $V=\frac{W}{2(\rho_a-\rho_{He})g}$

ILLUSTRATION 1.17

Two identical small charged spheres, each having a mass m, hang in equilibrium as shown in figure. The length of each string is l, and the angle made by any string with the vertical is θ. Find the magnitude of the charge on each sphere.



Sol. The forces acting on the sphere are tension in the string, T; force of gravity, mg; repulsive electric force, F_e, as shown in the free-body diagram of the sphere (figure). The sphere is in equilibrium. The forces in the horizontal and vertical directions must separately add up to zero.



$$\sum F_x=T\sin\theta-F_e=0 \quad \text{...(i)}$$

$$\sum F_y=T\cos\theta-mg=0 \quad \text{...(ii)}$$

From Eq. (ii), $T=mg/\cos\theta$. Thus, we can eliminate T from Eq. (i) to obtain

$$F_e=mg\tan\theta \text{ or } \frac{kq^2}{r^2}=mg\tan\theta \quad \text{...(iii)}$$

where $k = \dfrac{1}{4\pi\varepsilon_0}$ and $r = 2l\sin\theta$. Equation (iii) now reduces to

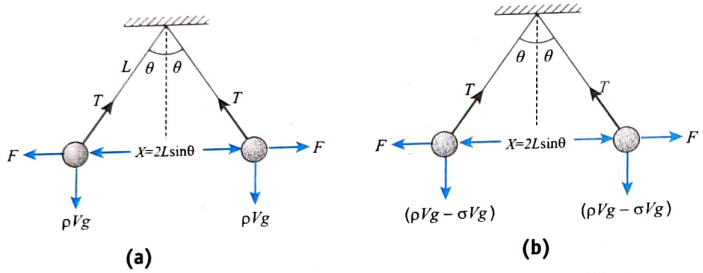
$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2l\sin\theta)^2} = mg\tan\theta \quad \text{or} \quad q = \sqrt{16\pi\varepsilon_0 l^2 mg\tan\theta\sin^2\theta}$$

ILLUSTRATION 1.18

Two identical balls, each having a density ρ, are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium, each string makes an angle θ with the vertical. Now, both the balls are immersed in a liquid. As a result, the angle θ does not change. The density of the liquid is σ. Find the dielectric constant of the liquid.

Sol. Let V be the volume of each ball, then the mass of each ball is $m = \rho V$. When the balls are in air, from the previous problem,

$$F = mg\tan\theta = \rho Vg\tan\theta \qquad ...(i)$$



When the balls are suspended in liquid, the Coulombic force is reduced to $F' = F/K$ and apparent weight = weight – upthrust, i.e.,

$$W' = \rho Vg - \sigma Vg$$

According to the problem, angle θ is unchanged. Therefore,

$$F' = W'\tan\theta = (\rho Vg - \sigma Vg)\tan\theta. \qquad ...(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{F}{F'} = K = \frac{\rho Vg}{\rho Vg - \sigma Vg} = \frac{\rho}{\rho - \sigma}$$

ILLUSTRATION 1.19

Three particles, each of mass m and carrying a charge q each, are suspended from a common point by insulating massless strings, each of length L. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side a, calculate the charge q on each particle. Assume $L >> a$.

Sol. From Fig. (a), for the equilibrium of a particle along a vertical line, we get

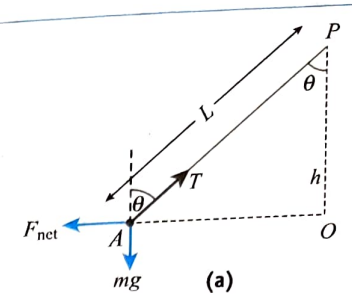
$$T\cos\theta = mg \qquad ...(i)$$

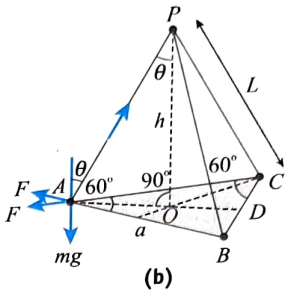
While for equilibrium in the plane of equilateral triangle, we get

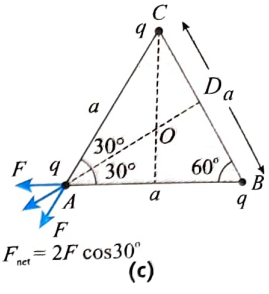
$$T\sin\theta = 2F\cos 30^\circ \qquad ...(ii)$$

So from Eqs. (i) and (ii), we have

$$\tan\theta = \frac{\sqrt{3}F}{mg} \qquad ...(iii)$$







Here, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a^2}$ and $\tan\theta = \dfrac{OA}{OP} = \dfrac{OA}{\sqrt{L^2 - OA^2}}$

Also from Fig. (c), we get

$$OA = \frac{2}{3}AD = \frac{2}{3}a\sin 60^\circ = \frac{a}{\sqrt{3}}$$

So $$\tan\theta = \frac{a/\sqrt{3}}{\sqrt{L^2(a^2/3)}} = \frac{a}{(\sqrt{3})L} \quad (\text{as } L >> a)$$

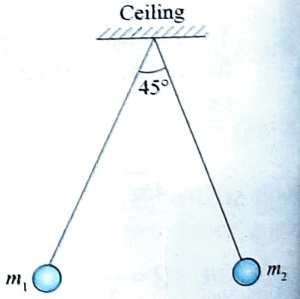
On substituting the above values of F and $\tan\theta$ in Eq. (iii), we get

$$\frac{a}{(\sqrt{3})L} = \frac{\sqrt{3}}{mg}\frac{q^2}{4\pi\varepsilon_0 a^2}$$

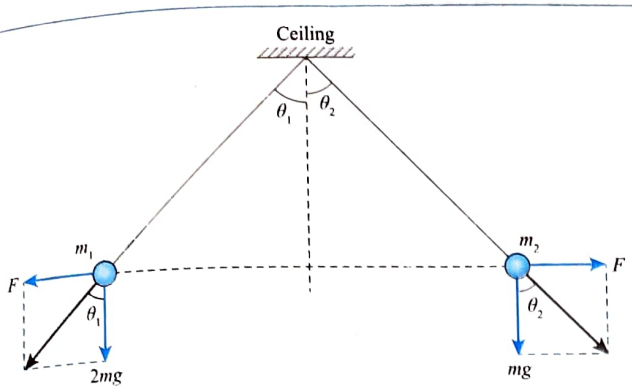
i.e., $$q = \left[\frac{4\pi\varepsilon_0 a^3 mg}{3L}\right]^{1/2}$$

ILLUSTRATION 1.20

Two small charged balls of masses, $m_1 = 2m$ and $m_2 = m$ have unequal positive charge on them. Both balls are suspended by two strings of unequal lengths from a common point such that, in equilibrium, both the balls are on same horizontal level. The angle between the two strings is $\theta = 45^\circ$ in this position. Find the electrostatic force between the balls in this position.



Sol. As masses of the balls are different hence the angle made by strings with vertical will be different. Let the angle made by string with vertical be θ_1 and θ_2 respectively. Three forces are acting on each ball, its weight, tension of string and electrostatic force. Let us draw the FBD of balls



From FBD: $\tan\theta_1 = \dfrac{F}{2mg}$ and $\tan\theta_2 = \dfrac{F}{mg}$ ...(i)

We know, $\tan(\theta_1+\theta_2) = \dfrac{\tan\theta_1 + \tan\theta_2}{1-\tan\theta_1\tan\theta_2}$ ...(ii)

Here $(\theta_1+\theta_2) = 45°$, from (i) and (ii) we have,

$$\tan 45° = \frac{\frac{F}{2mg}+\frac{F}{mg}}{1-\frac{F}{2mg}\cdot\frac{F}{mg}}$$

$$\Rightarrow \quad 2m^2g^2 - F^2 = 3mgF$$

$$\Rightarrow \quad F^2 + 3mgF - 2m^2g^2 = 0$$

$$\therefore \quad F = \frac{-3mg \pm \sqrt{9m^2g^2+8m^2g^2}}{2}$$

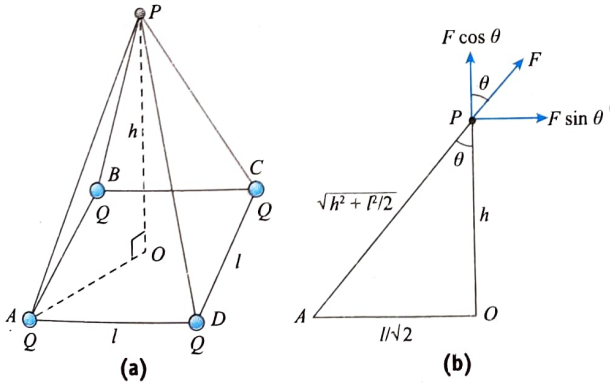
As negative sign of F is not acceptable, Hence,

$$F = \left(\frac{\sqrt{17}-3}{2}\right)mg$$

ILLUSTRATION 1.21

Four identical positive charges each of value Q are arranged at the four corners of a square of side l. A unit positive charge having mass m is placed at P, at a height h above the of center of the square. What should be the value of Q in order to keep the unit charge is in equilibrium?

Sol. Four charges are arranged at the four corners A, B, C and D of the square and the unit positive charge is placed at P, at a height h above the of center of the square. The situation is shown in Fig. (a).



The force experienced by unit positive charge placed at P due to a charge Q at A is given by

$$F = k\frac{(Q\times 1)}{\left(h^2+\frac{l^2}{2}\right)}$$

Similarly, equal forces act on unit positive charge at P due to charge at B, C and D. When these forces are resolved in horizontal and vertical directions, the horizontal components ($F\sin\theta$) cancel each other and the net vertical force is $4F\cos\theta$.

Hence the net upward electric force acting on the unit charge

$$F_{el} = \frac{4kQ}{\left(h^2+\frac{l^2}{2}\right)}\cdot\cos\theta$$

For the equilibrium of unit positive charge placed at P,

Upward electrical force = Weight of unit charge

$$\frac{4kQ}{\left(h^2+\frac{l^2}{2}\right)}\cdot\cos\theta = mg$$

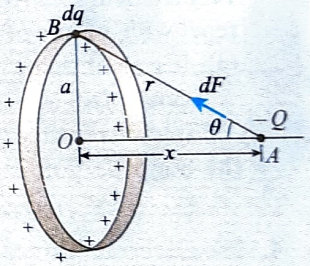
From Fig. (b) $\cos\theta = \dfrac{h}{\sqrt{\left(h^2+\frac{l^2}{2}\right)}}$

or $$\frac{4kQh}{\left(h^2+\frac{l^2}{2}\right)^{3/2}} = mg$$

or $$Q = \frac{4k\ mg}{h}\left(h^2+\frac{l^2}{2}\right)^{3/2}$$

ILLUSTRATION 1.22

A thin fixed ring of radius a has a positive charge q uniformly distributed over it. A particle of mass m, having a negative charge Q, is placed on the axis at a distance of x ($x << a$) from the center of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. Calculate the time period of oscillation.



Sol. The force on the point charge Q due to the element dq of the ring is

$$dF = \frac{1}{4\pi\varepsilon_0}\frac{dqQ}{r^2} \text{ along } AB$$

For every element of the ring, there is a diametrically opposite element. The components of forces along the axis will add up, while those perpendicular to it will cancel each other. Hence, the net force on the charge is $-Q$; negative sign shows that this force will be toward the center of ring.

$$F = \int dF\cos\theta = \cos\theta\int dF = \frac{x}{r}\int\frac{1}{4\pi\varepsilon_0}\left[-\frac{Qdq}{r^2}\right]$$

So $$F = -\frac{1}{4\pi\varepsilon_0}\frac{Qx}{r^3}\int dq = -\frac{1}{4\pi\varepsilon_0}\frac{Qqx}{(a^2+x^2)^{3/2}} \quad ...(i)$$

[as $r = (a^2+x^2)^{1/2}$ and $\int dq = q$]

As the restoring force is not linear, the motion will be oscillatory. However, if $x \ll a$ so that $x^2 \ll a^2$, then

$$F = -\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^3}x = -kx \text{ with } k = \frac{Qq}{4\pi\varepsilon_0 a^3}$$

Thus, the restoring force will become linear and so the motion is simple harmonic with time period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4\pi\varepsilon_0 ma^3}{qQ}}$$

CONCEPT APPLICATION EXERCISE 1.2

1. Figure shows two charge particles on an axis. The charges are free to move.

At one point, however, a third charged particle can be placed such that all three particles are in equilibrium.
   (a) Is that point to the left of the first two particles, to their right, or between them?
   (b) Should the third particle be positively or negatively charged?
   (c) Is the equilibrium stable or unstable?

2. Two small balls having equal positive charge Q (coulomb) on each are suspended by two insulating strings of equal length L (meter) from a hook fixed to a stand. The whole set-up is taken in a satellite into space where there is no gravity (state of weightlessness).
   (a) What is the angle between the two strings?
   (b) What is the tension in each string?

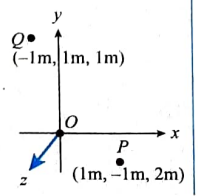
3. Suppose we have a large number of identical particles, very small in size. Any two of them at 10 cm separation repel with a force of 3×10^{-10} N.
   (a) If one of them is at 10 cm from a group (of very small size) of n others, how strongly do you expect it to be repelled?
   (b) Suppose you measure the repulsion and find it 6×10^{-6} N. How many particles were there in the group?

4. The electrostatic force of repulsion between two positively charged ions carrying equal charge is 3.7×10^{-9} N when these are separated by a distance of 5Å. How many electrons are missing from each ion?

5. Two fixed point charges $+4e$ and $+e$ units are separated by a distance a. Where should a third point charge be placed for it to be in equilibrium?

6. Two insulated identically sized charged copper spheres A and B each have charge 6.5×10^{-7} C and their centers separated by a distance of 50 cm. A third sphere of the same size but uncharged is brought in contact with the first, then in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?

7. Find the force on a charge q_1 $(= 20\ \mu C)$ due to the charge of q_2 $(= -10\ \mu C)$ if the positions of the charges are given as P_1 (1, −1, 2) and Q (−1, 1, 1) (see figure).



8. Two particles A and B having charges 8×10^{-6} C and -2×10^{-6} C, respectively, are held fixed with a separation of 20 cm. Where should a third charged particle be placed so that it does not experience a net electric force?

9. A particle of mass m carrying a charge $-q_1$ starts moving around a fixed charge $+q_2$ along a circular path of radius r. Find the time period of revolution T of charge $-q_1$.

10. Two identical conducting small spheres are placed with their centers 0.3 m apart. One is given a charge of 12.0 nC and the other a charge of −18.0 nC.
   (a) Find the electric force exerted by one sphere on the other?
   (b) If the spheres are connected by a conducting wire, find the electric force between the two after they attain equilibrium.

11. Four equal point charges, each of magnitude $+Q$, are to be placed in equilibrium at the corners of a square. What should be the magnitude and sign of the point charge that should be placed at the center of the square to do this job?

12. Two point electric charges of values q and $2q$ are kept at a distance d apart from each other in air. A third charge Q is to be kept along the same line in such a way that the net force acting on q and $2q$ is zero. Find the location of the third charge from charge q.

13. Two identical particles are charged and held at a distance of 1 m from each other. They are found to be attracting each other with a force of 0.027 N. Now, they are connected by a conducting wire so that charge flows between them. When the charge flow stops, they are found to be repelling each other with a force of 0.009 N. Find the initial charge on each particle.

14. A copper atom consists of copper nucleus surrounded by 29 electrons. The atomic weight of copper is 63.5 $gmol^{-1}$. Let us now take two pieces of copper each weighing 10 g. Let one electron from one piece be transferred to another for every 1000 atoms in a piece.
   (a) Find the magnitude of charge appearing on each piece.
   (b) What will be the Coulomb force between the two pieces after the transfer of electrons if they are 10 cm apart? (Avogadro's number is 6×10^{23} mol^{-1}.)

15. Two identical small equally charged conducting balls are suspended from long threads secured at one point. The charges and masses of the balls are such that they are in equilibrium when the distance between them is a (the length of thread $L \gg a$). Then one of the balls is discharged. What will be the distance b $(b \ll L)$ between the balls when equilibrium is restored?

ANSWERS

1. (a) Right (b) Negative (c) Unstable

2. (a) 180° (b) $\frac{k_e Q^2}{4L^2}$

3. (a) $3n \times 10^{-10}$ N (b) 2×10^4

4. 2 **5.** $\frac{2a}{3}$ from $+4e$ **6.** 5.7×10^{-3} N

7. $-0.06(2\hat{i}-2\hat{j}+\hat{k})$N **8.** 0.2 m from B

9. $\sqrt{\dfrac{16\pi^3\varepsilon_0 mr^3}{q_1q_2}}$ **10.** (a) 2.16×10^{-5}N (b) 9×10^{-7}N

11. $\dfrac{-Q(2\sqrt{2}+1)}{4}$ **12.** $\dfrac{d}{(1+\sqrt{2})}$

13. 3 μC and -1μC or -3μC and 1μC

14. (a) 15.12 C (b) 2.05×10^{14}N **15.** $\dfrac{a}{2^{2/3}}$

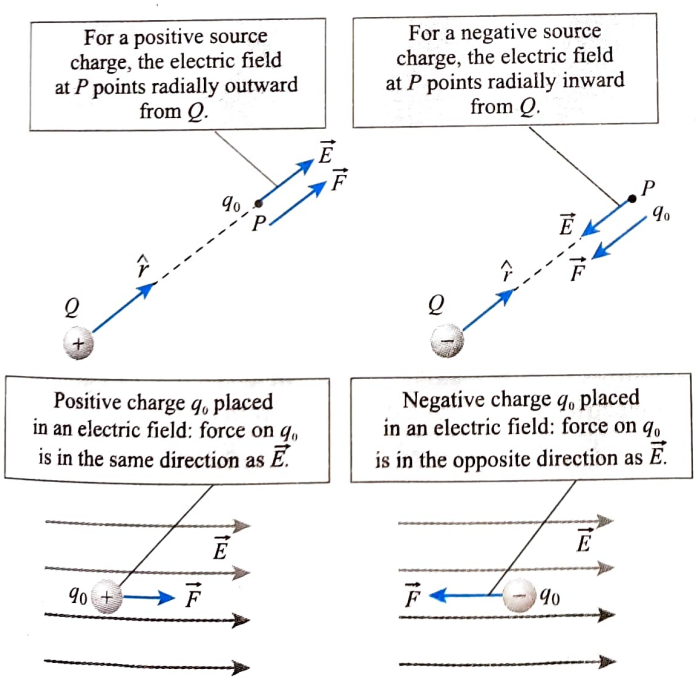
ELECTRIC FIELD

Newton's universal gravitation law and Coulomb's law enable us to calculate the magnitudes as well as the directions of gravitational and electrical forces, respectively. These laws are limited to describing only the interactions of two point masses (or charges). On the contrary, the field approach is a powerful tool to handle distributed and continuous masses (or charges) and their interactions.

In this approach, "we define electric field as the space around a charge in which its influence can be felt by any other charged particle." An electric field is said to exist in the region of space around a charged object, the source charge. When another charged object—the test charge—enters this electric field, an electric force acts on it.

As an example, consider figure, which shows a small positive test charge q_0 placed near a second object carrying a much greater positive charge Q. We define the electric field due to the source charge at the location of the test charge to be the electric force on the test charge per unit charge. To be more specific, the electric field vector $\vec{E}$ at a point in space is defined as the electric force $\vec{F}_e$ acting on a positive test charge q_0 placed at that point divided by the test charge:

$$\vec{E}=\lim_{q_0\to0}\frac{\vec{F}}{q_0}$$



The direction of $\vec{E}$ will be same as that of $\vec{F}$. The magnitude of the test charge is kept small because otherwise it may disturb the original charge distribution and then we will get electric field due to disturbed configuration and not original.

The unit of electric field is NC^{-1} (newton per coulomb). The dimensional formula of electric field is

$$\frac{\text{Force}}{\text{Charge}}=\frac{MLT^{-2}}{\text{ampere}\times\text{time}}=\frac{MLT^{-2}}{AT}=[MLT^{-3}A^{-1}]$$

A POINT CHARGE IN AN ELECTRIC FIELD

What happens if a point charge q_0 is placed at any point in an electric field produced by some other stationary charges? Let this electric field be $\vec{E}$. Charge q_0 will experience a force; let this force be $\vec{F}$. Then the value of electric field at that point must be

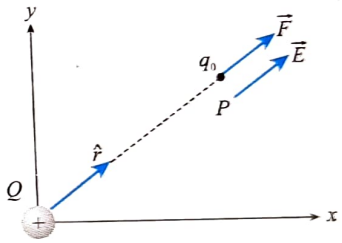
$$\vec{E}=\frac{\vec{F}}{q_0}\quad\text{or}\quad\vec{F}=q_0\vec{E}$$

This is the force on q_0 by E. q_0 has no contribution in $\vec{E}$. A charged particle is not affected due to its own field. It means a charge particle can experience force due to the field produced by other charge particles, but not due to the field produced by itself.

ELECTRIC FIELD DUE TO AN ISOLATED POINT CHARGE

Consider a point charge Q placed at the origin O. To find the electric intensity at a point P, distant r from O, place a test charge q_0 at P. According to Coulomb's law, the force exerted on q_0 by Q is

$$\vec{F}=k_e\frac{Qq_0}{r^2}\hat{r}$$



where $\hat{r}$ is a unit vector from O to P, i.e., from Q to q_0. If $\vec{E}$ is the electric field created by Q at P, then by definition,

$$\vec{E}=\lim_{q_0\to0}\frac{\vec{F}}{q_0}=\lim_{q_0\to0}\frac{k_e\dfrac{Qq_0}{r^2}\hat{r}}{q_0}$$

$$=k_e\frac{Q}{r^2}\hat{r}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}\vec{r}$$

If Q is positive, $\vec{E}$ is directed radially outward from it and if Q is negative, $\vec{E}$ is directed toward it. In magnitude,

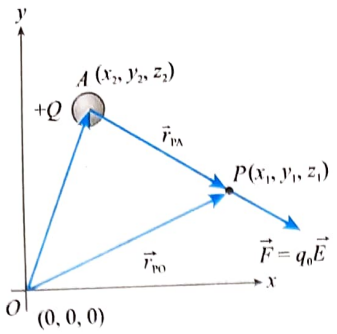
$$E=k_e\frac{Q}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

ELECTRIC FIELD DUE TO A POINT CHARGE IN VECTOR NOTATION

Electric force on test charge $+q_0$ in the direction of position vector $\vec{r}_{PA}$ is given by $\vec{r}_{PA}=\vec{r}_{PO}-\vec{r}_{AO}$

$$\vec{F}_E=\frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{r^3}(\vec{r}_{PA})$$

$$\vec{E}=\frac{\vec{F}_E}{q_0}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}(\vec{r}_{PA})$$



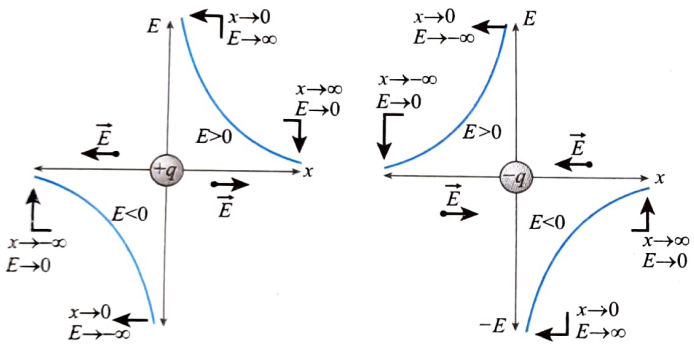
$\vec{E}$ is the electric field intensity vector at position P due to a point charge placed at the position A.

$$\vec{r}_{PA} = \vec{r}_{PO} - \vec{r}_{AO}$$

If $\vec{r}_{PO} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{r}_{AO} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$, then

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q\{(x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j} + (z_1 - z_2)\hat{k}\}}{\{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2\}^{3/2}}$$

Graphical Variation of $\vec{E}$ on X-axis Due to a Point Charge



Electric Field Intensity Due to a Group of Charges

Using the principle of superposition, the net field at point P is given by

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\hat{r}_1 + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}\hat{r}_2 + \cdots + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n^2}\hat{r}_n$$

$$= \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

In terms of position vectors,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$$

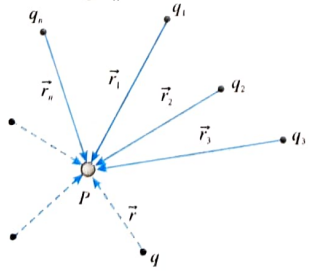


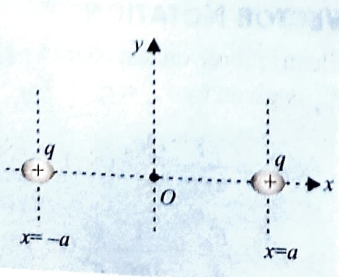
Illustration 1.23

Two point charges each having charge q are placed on the x-axis at $x = -a$ and $x = +a$ as shown in figure.

(a) Plot the variation of electric field ($\vec{E}$) on the x-axis.

(b) Plot the variation of electric field ($\vec{E}$) on the y-axis.

(c) Find the position on the y-axis where electric field is maximum.

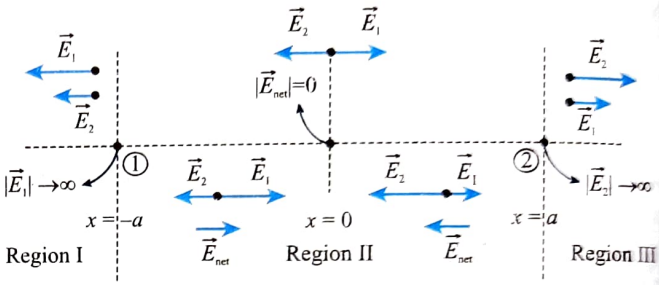


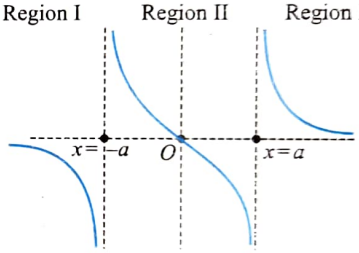
Sol. We can divide the x-axis into three regions:

Region I (left of the charges): The field due to the left charge dominates and acts toward $-x$ direction.

Region II (between the charges): From $-a < x < 0$, the field of left charge dominates and acts towards $+x$ direction. At $x = 0$, resultant electric field is zero. From $0 < x < a$, the field of right charge dominates and acts toward $-x$ direction.

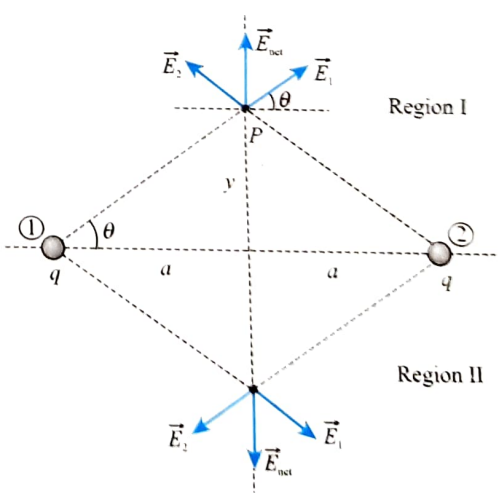
Region III (right of the charges): The field due to the right charge dominates and acts toward $+x$ direction.





Variation of $\vec{E}$ in the x-axis: We can divide the y-axis in two regions:

Region I (above y-axis): Electric field is along $+y$ direction.



Region II (below x-axis): Electric field is along y direction. Net electric field at P at a distance y from origin is

$$E_{net} = |E_1|\sin\theta + |E_2|\sin\theta = 2|E_1|\sin\theta \text{ as } [|\vec{E}_1| = |\vec{E}_2|]$$

$$= 2\frac{1}{4\pi\varepsilon_0}\frac{q}{(a^2 + y^2)}\frac{y}{(a^2 + y^2)^{1/2}}$$

Hence, $E_{net} = \dfrac{2}{4\pi\varepsilon_0}\dfrac{qy}{(a^2 + y^2)^{3/2}}$

For maximum or minimum value of E_{net},

$$\frac{dE_{net}}{dy} = 0 = \frac{q}{2\pi\varepsilon_0}\left[y\left(\frac{3}{2}\right)(a^2+y^2)^{-5/2}(2y) + (a^2+y^2)^{-3/2}(1)\right]$$

or $y = \pm\frac{a}{\sqrt{2}}$

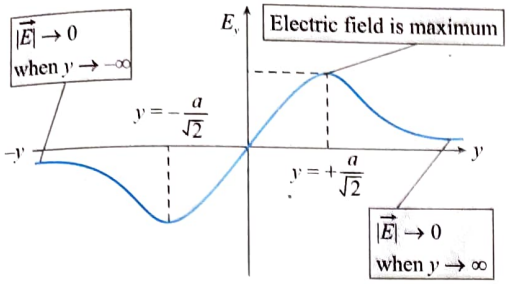


ILLUSTRATION 1.24

Two negative charges of a unit magnitude are fixed at a certain distance apart, and a positive charge q is placed along the straight line joining both the charges. At what position and value of q will the charge q be in equilibrium?

Sol. The charge q can be in equilibrium if it is placed at the middle of negative charges, as the electric field is zero at the middle of negative charges. The charge q can have any value.

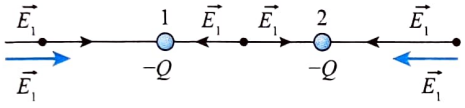


ILLUSTRATION 1.25

Two point charges of $+5 \times 10^{-19}$ C and $+20 \times 10^{-19}$ C are separated by a distance of 2 m. Find the point on the line joining them at which electric field intensity is zero.

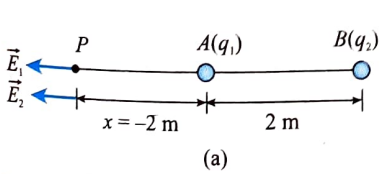
Sol. Here, $q_1 = +5 \times 10^{-19}$ C, $q_2 = +20 \times 10^{-19}$ C. If P is the point [distant x from q_1, as shown in Fig. (b)], where electric field intensity is zero, then $|E_1| = |E_2|$

or $$k_e\frac{q_1}{x^2} = k_e\frac{q_2}{(2-x)^2}$$

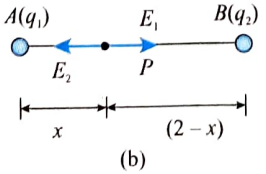
or $$\frac{(2-x)^2}{x^2} = \frac{q_2}{q_1} = \frac{20\times10^{-19}\,\text{C}}{5\times10^{-19}\,\text{C}} = 4$$

or $$\frac{(2-x)}{x} = \pm 2$$

Hence, $x = -2$ m or $x = 2/3$ m. If $x = -2$ m, the point P lies to the left of A. In this case, since $\vec{E}_1$ and $\vec{E}_2$ are in the same direction, the two positive charges produce a resultant electric field $\vec{E}$, where $\vec{E} = \vec{E}_1 + \vec{E}_2$ as shown in Fig. (a). Hence, this is not a feasible situation.



(a)



(b)

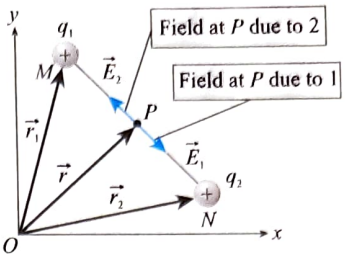
Thus, as shown in Fig. (b), $x = 2/3$ m.

ILLUSTRATION 1.26

The positions of two point charges q_1 and q_2 are $\vec{r}_1$ and $\vec{r}_2$, respectively. Find the position of the point where the net field is zero due to these charges.

Sol. At P, let the net field be zero.

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 = 0 \qquad ...(i)$$



As we know,

$$\vec{E}_1 = \frac{q_1(\vec{r}-\vec{r}_1)}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|^3} \qquad ...(ii)$$

and $$\vec{E}_2 = \frac{q_2(\vec{r}-\vec{r}_2)}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|^3} \qquad ...(iii)$$

Substituting $\vec{E}_1$ and $\vec{E}_2$ from Eqs. (ii) and (iii) in Eq. (i), we have

$$\frac{q_1(\vec{r}-\vec{r}_1)}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|^3} + \frac{q_2(\vec{r}-\vec{r}_2)}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|^3} = 0$$

or $$q_1(\vec{r}-\vec{r}_1) + q_2(\vec{r}-\vec{r}_2)\cdot\frac{|\vec{r}-\vec{r}_1|^3}{|\vec{r}-\vec{r}_2|^3} = 0 \qquad ...(iv)$$

Since $E = 0$ at P,

$$|\vec{E}_1| = |\vec{E}_2|$$

$$\Rightarrow \quad \frac{q_1}{4\pi\varepsilon_0 r_{p1}^2} = \frac{q_2}{4\pi\varepsilon_0 r_{p2}^2}$$

where $\vec{r}_{P_1} = \vec{r} - \vec{r}_1$ and $\vec{r}_{P_2} = \vec{r} - \vec{r}_2$. Therefore,

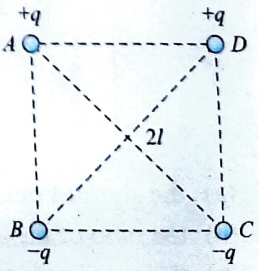
$$\frac{q_1}{|\vec{r}-\vec{r}_1|^2} = \frac{q_2}{|\vec{r}-\vec{r}_2|^2} \text{ or } \frac{|\vec{r}-\vec{r}_1|}{|\vec{r}-\vec{r}_2|} = \frac{\sqrt{q_1}}{\sqrt{q_2}} \qquad ...(v)$$

Substituting the value of $|\vec{r}-\vec{r}_1|/|\vec{r}-\vec{r}_2|$ from Eq. (v) in Eq. (iv), we get

$$q_1(\vec{r}-\vec{r}_1) + q_2(\vec{r}-\vec{r}_2)\left(\frac{q_1}{q_2}\right)^{3/2} = 0 \quad \text{or} \quad \vec{r} = \frac{\vec{r}_1\sqrt{q_2} + \vec{r}_2\sqrt{q_1}}{\sqrt{q_1}+\sqrt{q_2}}$$

ILLUSTRATION 1.27

Point charges q and $-q$ are located at the vertices of a square with diagonals $2l$ as shown in figure. Evaluate the magnitude of the electric field strength at a point located symmetrically with respect to the vertices of the square at a distance x from the centre.

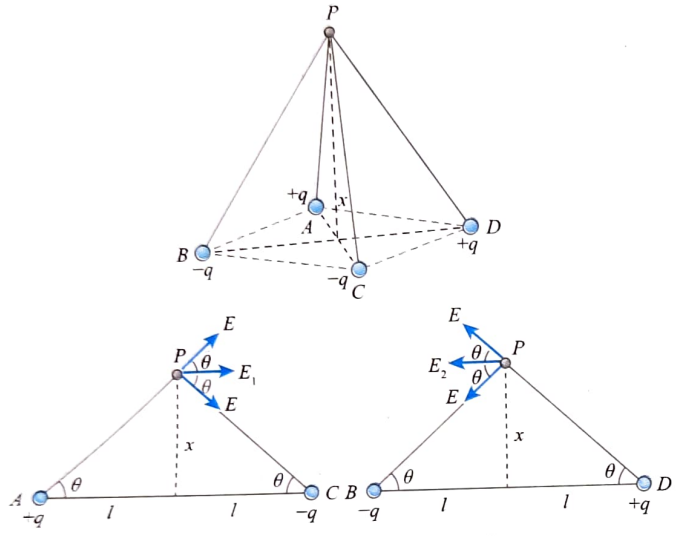


Sol. The electric field at P due to the charge placed on the diagonal AC,

$$E_1 = 2E\cos\theta = 2\cdot k\frac{q}{(x^2+l^2)}\cdot\frac{l}{(x^2+l^2)^{1/2}} = \frac{2kq(l)}{\left[x^2+l^2\right]^{3/2}} \text{ (along } AC)$$

The electric field at P due to the charged placed on the diagonal BD,

$$E_2 = 2E\cos\theta = 2\cdot k\frac{q}{(x^2+l^2)}\cdot\frac{l}{(x^2+l^2)^{1/2}} = \frac{2kql}{(x^2+l^2)^{3/2}} \text{ (along } BD)$$



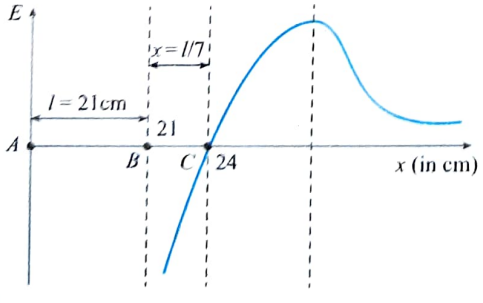
As diagonal intersects at angle 90°, so $E_1 \perp E_2$.

Net electric field, $E_0 = \sqrt{E_1^2+E_2^2} = \dfrac{2\sqrt{2}kql}{(x^2+l^2)^{3/2}}$

$$= \frac{ql}{\sqrt{2}\pi\varepsilon_0(x^2+l^2)^{3/2}}$$

ILLUSTRATION 1.28

Two point charges Q_a and Q_b are positioned at points A and B. The field strength to the right of charge Q_b on line that passes through two charges varies according to a law that is represented schematically in the figure accompanying the problem (without employing a definite scale). The field strength is assumed to be positive if its direction coincides with the positive direction of x-axis. The distance between the charges is $l = 21$ cm. Find



(a) the signs of the charges.

(b) the ratio of the absolute values of charges Q_a and Q_b.

(c) the coordinate x of the point where the field strength is maximum.

Sol.

(a) Electric field near B and to the right of B is along negative x direction, so sign of Q_b should be negative. There is a neutral point at $x = 24$ cm, so sign of Q_a should be opposite of Q_b. Hence Q_a should be positive.

(b) At neutral point (at $x = 24$ cm), net electric field should be zero.

$$k\frac{Q_a}{24^2} = k\frac{Q_b}{3^2} \Rightarrow \left|\frac{Q_a}{Q_b}\right| = \left(\frac{24}{3}\right)^2 = 64$$

(c) Electric field to the right of charges (right side of point C)

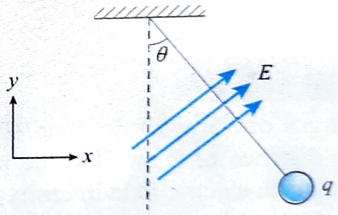
$$E = k\frac{Q_a}{x^2} - k\frac{Q_b}{(x-21)^2}$$

For field to be maximum: $\dfrac{dE}{dx} = 0$

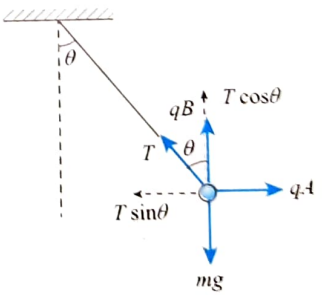
$$\Rightarrow \quad \frac{kQ_a(-2)}{x^3} - \frac{kQ_b(-2)}{(x-21)^3} = 0 \text{ which gives } x = 28 \text{ cm}$$

ILLUSTRATION 1.29

A charged cork ball of mass m is suspended on a light string in the presence of a uniform electric field as shown in figure. When $E = (A\hat{i} + B\hat{j})\text{NC}^{-1}$, where A and B are positive numbers, the ball is in equilibrium at the angle θ. Find the charge on the ball.



Sol. The forces acting on the charge are electric forces, tension due to string and weight of the ball. Considering the FBD of the ball



In vertical direction: $T\cos\theta + qB = mg$

$$\Rightarrow \quad T\cos\theta = mg - qB \quad \ldots(i)$$

In horizontal direction: $T\sin\theta = qA$ $\ldots(ii)$

Dividing (i) and (ii), we get $\tan\theta = \dfrac{qA}{mg - qB}$ $\ldots(iii)$

From (iii), charge on the ball, $q = \dfrac{mg\tan\theta}{A + B\tan\theta}$

ILLUSTRATION 1.30

The bob of a simple pendulum has mass m and carries a charge q on it. Length of the pendulum is l. There is a uniform electric field E in the region. Calculate the time period of small oscillations for the pendulum about its equilibrium position in following cases:

(a) E is vertically down having magnitude $E = \frac{mg}{q}$

(b) E is vertically up having magnitude $E = \frac{2mg}{q}$

(c) E is horizontal having magnitude $E = \frac{mg}{q}$

(d) E has magnitude of $E = \frac{\sqrt{2}mg}{q}$ and is directed upward making an angle of 45° with the horizontal.

Sol. The time period of oscillation of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}}$$

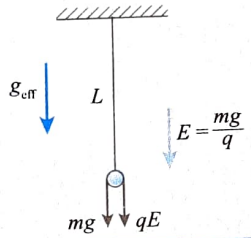
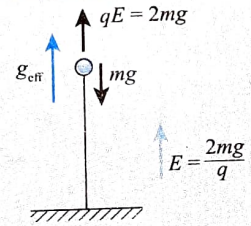
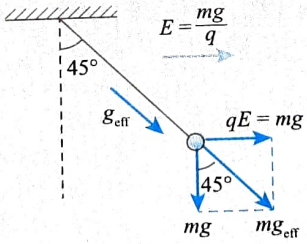
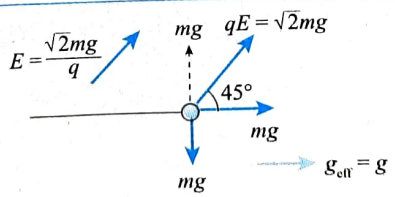
S.No.	Case	Orientation of the pendulum at equilibrium	Effective value of g	Time period
(a)	When electric field is vertically down, and having magnitude $E = \frac{mg}{q}$		$g_{\text{eff}} = \frac{mg + gE}{m} = 2g$	$T = 2\pi\sqrt{\frac{l}{2g}}$
(b)	When electric field is vertically down, and having magnitude $E = \frac{2mg}{q}$	 At equilibrium the pendulum will be vertical up.	$g_{\text{eff}} = \frac{qE - mg}{m} = g$	$T = 2\pi\sqrt{\frac{l}{g}}$
(c)	When electric field is horizontal, and having magnitude $E = \frac{mg}{q}$	 At equilibrium the pendulum will be at angle 45° with vertical.	$g_{\text{eff}} = \frac{\sqrt{(mg)^2 + (mg)^2}}{m} = \sqrt{2}g$	$T = 2\pi\sqrt{\frac{l}{\sqrt{2}g}}$
(d)	When electric field is directed upward making an angle of 45° with the horizontal and having magnitude $E = \frac{\sqrt{2}mg}{q}$	 At equilibrium the pendulum will be horizontal	$g_{\text{eff}} = \frac{mg}{m} = g$	$T = 2\pi\sqrt{\frac{l}{g}}$

ILLUSTRATION 1.31

Electric field $E = -bx + a$ exists in a region parallel to the x-direction (a and b are positive constants). A charge particle having charge q and mass m is released from the origin $x = 0$. Find the acceleration of the particle at the instant its speed becomes zero for the first time after release.

Sol. The force on the charged particle, $F = qE = q(-bx + a)$

$\Rightarrow$ $mA = q(-bx + a)$ [A = acceleration]

$$v\frac{dv}{dx} = -\frac{bq}{m}x + \frac{aq}{m}$$

$$\int_0^v v\,dv = -\frac{bq}{m}\int_0^x x\,dx + \frac{aq}{m}\int_0^x dx$$

$$\frac{v^2}{2} = -\frac{bqx^2}{2m} + \frac{aqx}{m}$$

When speed of the particle becomes zero, $v = 0$ when

$$\frac{bqx^2}{2m} = \frac{aqx}{m} \Rightarrow x = \frac{2a}{b}$$

As the acceleration of the particle is given as

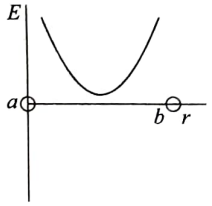
$$A_0 = \frac{q}{m}(-bx + a) \quad \ldots(i)$$

The acceleration at $x = \frac{2a}{b}$

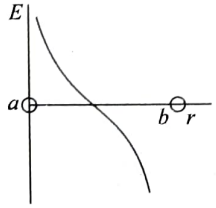
Hence required acceleration $= \frac{q}{m}\left(-b \cdot \frac{2a}{b} + a\right) = \frac{q}{m}(-2a + a) = -\frac{qa}{m}$

CONCEPT APPLICATION EXERCISE 1.3

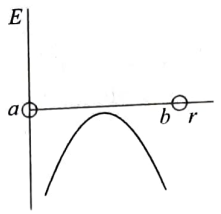
1. Two point electric charges of unknow.n magnitudes and signs are placed a certain distance apart. The electric field intensity is zero at a point not between the charges but on the line joining them. Write two essential conditions for this to happen.
2. Two point charges $+5 \times 10^{-19}$ C and $+20 \times 10^{-19}$ C are separated by a distance of 2 m. The electric field intensity will be zero at a distance $x =$ _____ from the charge of 5×10^{-19} C.
3. Two point-like charges a and b whose strengths are equal in absolute value are positioned at a certain distance from each other. Assuming the field strength is positive in the direction coinciding with the positive direction of the r axis, determine the signs of the charges for each distribution of the field strength between charges shown in Figs. (a)–(d).



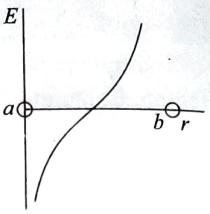
(a)



(b)

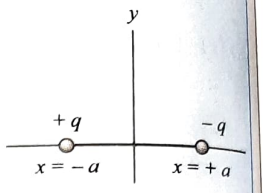


(c)



(d)

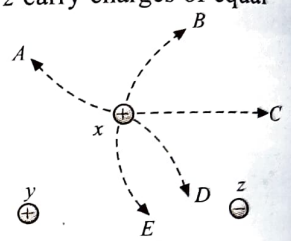
4. Two point charges $\pm q$ are placed on the axis at $x = -a$ and $x = +a$, as shown in figure.



   (a) Plot the variation of E along the x-axis.

   (b) Plot the variation of E along the y-axis.

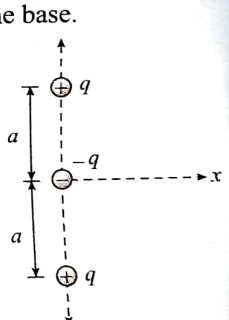
5. Three small spheres x, y, and z carry charges of equal magnitudes and with signs shown in figure. They are placed at the vertices of an isosceles triangle with the distance between x and y equal to the distance between x and z. Spheres y and z are held in place, but sphere x is free to move on a frictionless surface.



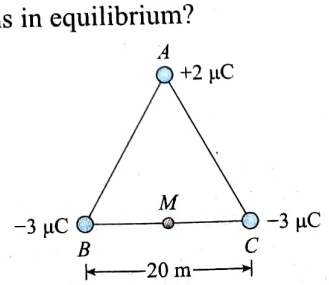
   (a) What is the direction of the electric force on sphere x at the point shown in the figure?

   (b) Which path is sphere x likely to take when released?

6. Three identical positive charges Q are arranged at the vertices of an equilateral triangle. The side of the triangle is a. Find the intensity of the field at the vertex of a regular tetrahedron of which the triangle is the base.
7. Two identical point charges having magnitude q each are placed as shown in figure. If we place a negative charge (of magnitude $-q$ and mass m) at the midpoint of charges and displaced along the x-axis, examine whether it will perform simple harmonic motion. If yes, then find the time period of oscillation of the particle.

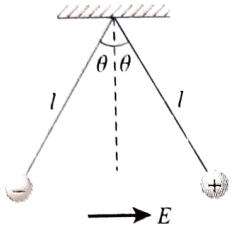


8. Three-point charges of $+2\ \mu C$, $-3\ \mu C$ and $-3\ \mu C$ are kept at the vertices A, B and C respectively of an equilateral triangle of side 20 cm as shown in the figure. What should be the sign and magnitude of the charge (q) to be placed at the mid-point (M) of side BC so that the charge at A remains in equilibrium?



9. Four particles each having a charge q are placed on the four vertices of a regular pentagon. The distance of each corner from the centre is 'a'. Find the electric field at the centre of pentagon.

10. Two small spheres, each of mass m, are suspended by light strings of length l as shown in the figure. A uniform electric field is applied in the x-direction. The spheres have charges equal to $-q$ and $+q$. Determine the electric field that enables the spheres to be in equilibrium at an angle of θ.



ANSWERS

2. $\frac{2}{3}$ m

3. (a) $q_a \to +, q_b \to -$ (b) $q_a \to +, q_b \to +$
(c) $q_a \to -, q_b \to +$ (d) $q_a \to -, q_b \to -$

5. (a) Along C (b) Path D

6. $\frac{\sqrt{3}Q}{2\sqrt{2}\pi\varepsilon_0 a^2}$

7. Yes; $2\pi\sqrt{\frac{2\pi\varepsilon_0 ma^3}{q^2}}$

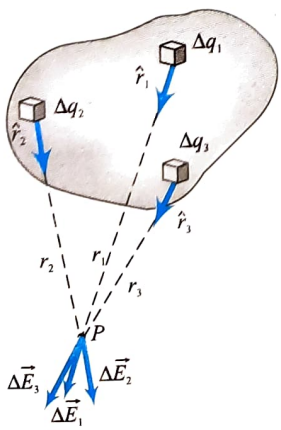
8. $\frac{9\sqrt{3}}{4}$ μC, Positive charge

9. $\frac{kq}{a^2}$

10. $\left[\frac{mg\tan\theta}{q}+\frac{q}{16\pi\varepsilon_0 l^2\sin^2\theta}\right]$

ELECTRIC FIELD DUE TO CONTINUOUS DISTRIBUTION OF CHARGE

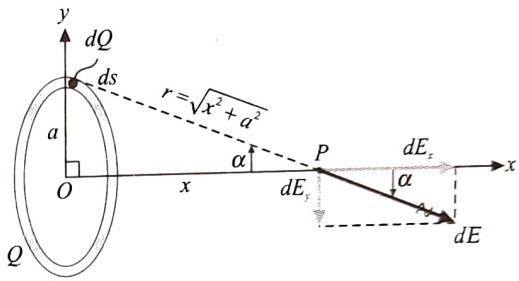
A system of closely spaced electric charges forms a continuous charge distribution. To find the field of a continuous charge distribution, we divide the charge into infinitesimal charge elements. Each infinitesimal charge element is then considered a point charge, and the electric field $d\vec{E}$ is determined due to this charge at a given point. The net field at the given point is the summation of fields of all the elements, i.e., $\vec{E}=\int d\vec{E}$.



Linear charge distribution	Surface charge distribution	Volume charge distribution
In this distribution, charge is distributed on a line, for example charge on a wire, charge on a ring, etc. The linear charge density is given by $\lambda=\frac{\text{Charge}}{\text{Length}}=\frac{Q}{2\pi R}$	In this distribution, charge is distributed on the surface, for example charge on a conducting sphere, charge on a sheet, etc. The surface charge density is given by $\sigma=\frac{\text{Charge}}{\text{Area}}=\frac{Q}{4\pi R^2}$	In this distribution, charge is distributed in the whole volume of the body, for example a solid uniformly charged object, say a solid charged sphere. The volume charge density is given by $\rho=\frac{\text{Charge}}{\text{Volume}}=\frac{Q}{\frac{4}{3}\pi R^3}$

FIELD OF RING CHARGE

A ring-shaped conductor with radius a carries a total charge Q uniformly distributed around it. Let us calculate the electric field at a point P that lies on the axis of the ring at a distance x from its center.



As shown in figure, the ring is divided into infinitesimal segments each of length ds. Each segment has charge dQ and acts as a point charge source of electric field. Let $d\vec{E}$ be the electric field from one such segment. The net electric field at P is then the sum of all contributions $d\vec{E}$ from all the segments that make up the ring. (The same technique works for any situation in which charge is distributed along a line or a curve.) The calculation of $\vec{E}$ is greatly simplified because the field point P is on the symmetry axis of the ring. If we consider two ring segments at the top and bottom of the ring, we see that the contributions $d\vec{E}$ to the field at P from these segments have the same x-component but opposite y-components. Hence, the total y-component of field due to this pair of segments is zero. When we add up the contributions from all such pairs of segments, the total field $\vec{E}$ will have only a component along the ring's symmetry axis (the x-axis), with no component perpendicular to that axis (that is, no y-component or z-component). So the field at P is described completely by its x-component E_x.

To calculate E_x, note that the square of the distance r from a ring segment to the point P is $r^2=x^2+a^2$. Hence, the magnitude of this segment's contribution to the electric field at P is

$$dE=\frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}$$

Using, $\cos\alpha=\frac{x}{r}=\frac{x}{(x^2+a^2)^{1/2}}$

the component dE_x of this field along the x-axis is

$$dE_x=dE\cos\alpha=\frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}}=\frac{1}{4\pi\varepsilon_0}\frac{xdQ}{(x^2+a^2)^{3/2}}$$

To find the total x-component E_x of the field at P, we integrate this expression over all segments of the ring, i.e.,

$$E_x = \int \frac{1}{4\pi\varepsilon_0}\frac{x\,dQ}{(x^2+a^2)^{3/2}}$$

Since x does not vary as we move from point to point around the ring, all the factors on the right side except dQ are constant and can be taken outside the integral. The integral of dQ is just the total charge Q, and we finally get

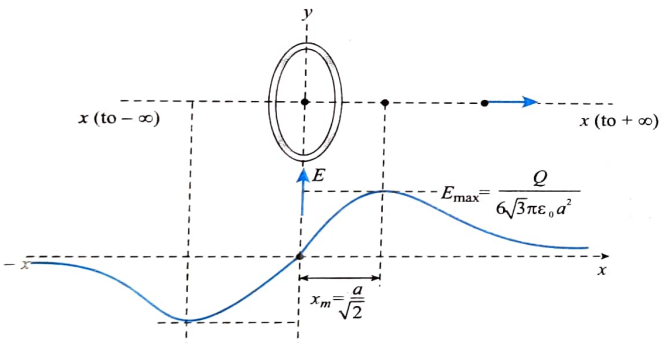
$$\vec{E} = E_x\hat{i} = \frac{1}{4\pi\varepsilon_0}\frac{xQ}{(x^2+a^2)^{3/2}}\hat{i} \qquad ...(i)$$

Electric field is directed away from positively charged ring. For $x = 0$, $E = 0$, this conclusion may be arrived at by the symmetry consideration. At a large distance from the ring, the electric field will be zero. Hence, it should have certain maximum value between $x = 0$ and $x = \infty$ (or $x = -\infty$). If we maximize Eq. (i), we can get the value of x_m as well as E_{max}. For the maximum value of E_x, we get

$$\frac{d}{dx}\left\{\frac{1}{4\pi\varepsilon_0}Q\frac{x}{(x^2+a^2)^{3/2}}\right\} = 0$$

or $$\frac{(x^2+a^2)^{3/2}(1-x)\frac{3}{2}(x^2+a^2)^{1/2}(2x)}{(x^2+a^2)^3}$$

or $$(x^2+a^2)-3x^2 = 0 \text{ or } x = \pm\frac{a}{\sqrt{2}}$$



The maximum value of the electric field is

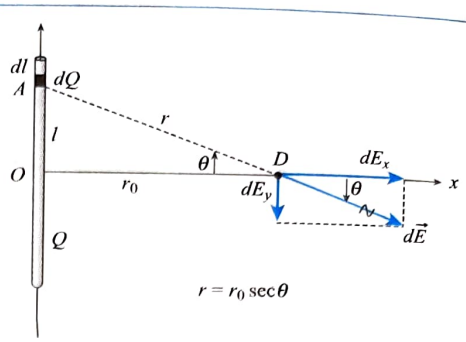
$$E_{x(max)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{3\sqrt{3}\,a^2}\right)$$

ELECTRIC FIELD DUE TO AN INFINITE LINE CHARGE

A positive electric charge Q is distributed uniformly along a line, lying along the y-axis. Let us find the electric field at point D on the x-axis at a distance r_0 from the origin.

We divide the line charge into infinitesimal segments, each of which acts as a point charge; let the length of a typical segment at height l be dl. If the charge is distributed uniformly with the linear charge density λ, then the charge dQ in a segment of length dl is $dQ = \lambda dl$. At point D, the differential electric field dE created by this element is

$$dE = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{\lambda dl}{4\pi\varepsilon_0 r^2} = \frac{\lambda dl}{4\pi\varepsilon_0 r_0^2\sec^2\theta} \qquad ...(i)$$



In triangle AOD, $OA = OD\tan\theta$, i.e., $l = r_0\tan\theta$. Differentiating this equation with respect to θ, we get $dl = r_0\sec^2\theta d\theta$. Substituting the value of dl in Eq. (i), we get

$$dE = \frac{\lambda d\theta}{4\pi\varepsilon_0 r_0}$$

Field dE has components dE_x, dE_y given by

$$dE_x = \frac{\lambda\cos\theta d\theta}{4\pi\varepsilon_0 r_0} \text{ and } dE_y = \frac{\lambda\sin\theta d\theta}{4\pi\varepsilon_0 r_0}$$

On integrating the expression for dE_x and dE_y in limits $\theta = -\pi/2$ to $\theta = +\pi/2$, we obtain E_x and E_y. Note that as the length of the wire increases, the angle θ also increases. For a very long wire (infinitely long wire), θ approaches $\pi/2$.

$$E_x = \int_{-\pi/2}^{+\pi/2}\frac{\lambda\cos\theta\,d\theta}{4\pi\varepsilon_0 r_0} = \frac{\lambda}{2\pi\varepsilon_0 r_0}$$

and $$E_y = \int_{-\pi/2}^{+\pi/2}\frac{\lambda\sin\theta\,d\theta}{4\pi\varepsilon_0 r_0} = 0$$

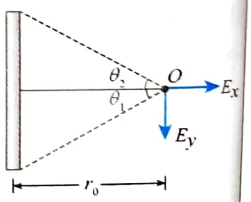
Thus, $$E = E_x = \frac{\lambda}{4\pi\varepsilon_0 r_0}$$

Note: Using a symmetry argument, we could have guessed that E_y would be zero; if we place a positive test charge at D, the upper half of the line of charge pushes downward on it, and the lower half pushes upward with equal magnitude.

If the wire has finite length and the angles subtended by the ends of the wire at a point are θ_1 and θ_2, the limits of integration would change.

$$E_x = \int_{-\theta_1}^{+\theta_2}\frac{\lambda\cos\theta\,d\theta}{4\pi\varepsilon_0 r_0}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r_0}(\sin\theta_1 + \sin\theta_2)$$



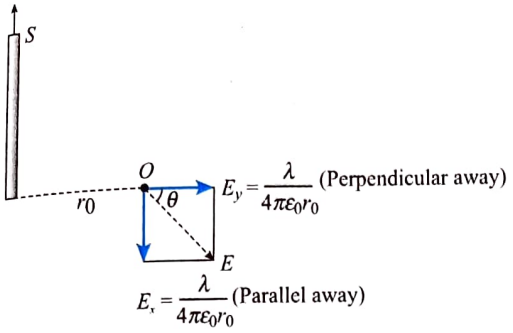
$$E_y = \int_{-\theta_1}^{+\theta_2}\frac{\lambda\sin\theta d\theta}{4\pi\varepsilon_0 r_0}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r_0}(\cos\theta_1 - \cos\theta_2)$$

If we wish to determine field at the end of a long wire, we may substitute $\theta_1 = 0$ and $\theta_2 = \pi/2$ in the expressions for E_x and E_y, i.e.,

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r_0}\left[\sin(0) + \sin\left(\frac{\pi}{2}\right)\right] = \frac{\lambda}{4\pi\varepsilon_0 r_0}$$

and $E_y = \frac{\lambda}{4\pi\varepsilon_0 r_0}\left[\cos(0) - \cos\left(\frac{\pi}{2}\right)\right] = \frac{\lambda}{4\pi\varepsilon_0 r_0}$



Magnitude of the resultant field $\vec{E}$ is

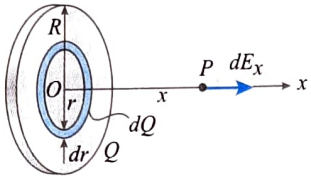
$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r_0}$$

$\vec{E}$ makes an angle θ with the x-axis, where

$$\tan\theta = |E_y|/|E_x| = 1;\ \theta = 45°$$

FIELD OF UNIFORMLY CHARGED DISK

Let us find the electric field caused by a disk of radius R with a uniform positive surface charge density (charge per unit area) σ at a point on the axis of the disk at a distance x from its center.



The situation is shown in figure. We can represent this charge distribution as a collection of concentric rings of charge. We already know how to find the field of a single ring on its axis of symmetry; therefore, we will add the contribution of all the rings. As shown in the figure, a typical ring has charge dQ, inner radius r, and outer radius $r + dr$. Its area dA is approximately equal to its width dr times its circumference $2\pi r$, or $dA = 2\pi r\,dr$. The charge per unit area is $\sigma = dQ/dA$, so the charge of the ring is $dQ = \sigma(2\pi r dr)$, or $dQ = 2\pi\sigma r dr$. The field component dE_x at point P due to charge dQ of a ring of radius r is

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{(2\pi\sigma r dr)x}{(x^2 + r^2)^{3/2}}$$

To find the total field due to all the rings, we integrate dE_x over r. To include the whole disk, we must integrate from 0 to R (not from $-R$ to R), i.e.,

$$E_x = \int dE_x = \int_0^R dE_x = \int_0^R \frac{1}{4\pi\varepsilon_0}\frac{(2\pi\sigma r dr)x}{(x^2 + r^2)^{3/2}}$$

Remember that x is a constant during the integration and that the integration variable is r. The integral can be evaluated by the use of the substitution $z = x^2 + r^2$. After calculation, the result is

$$E_x = \frac{\sigma x}{2\varepsilon_0}\left[-\frac{1}{\sqrt{x^2 + R^2}} + \frac{1}{x}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right] \quad ...(i)$$

In figure, the charge is assumed to be positive. At a point on the symmetry axis of a uniformly charged ring, the electric field due to the ring has no components perpendicular to the axis. Hence, at point P in the figure, $dE_y = dE_z = 0$ for each ring, and thus the total field has $E_y = E_z = 0$.

Again, we can ask what happens if the charge distribution gets very large. Suppose we keep increasing the radius R of the disk, simultaneously adding charge so that the surface charge density σ (charge per unit area) is constant. In the limit that R is much larger than the distance x of the field point from the disk ($R >> x$), i.e., the situation becomes the electric field near infinite plane sheet of charge. From Eq. (i), we get

$$E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{x\sqrt{1 + \frac{R^2}{x^2}}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{1 + \frac{R^2}{x^2}}}\right]$$

As $R \gg x$, $\frac{1}{\sqrt{1 + \frac{R^2}{x^2}}} \to 0$

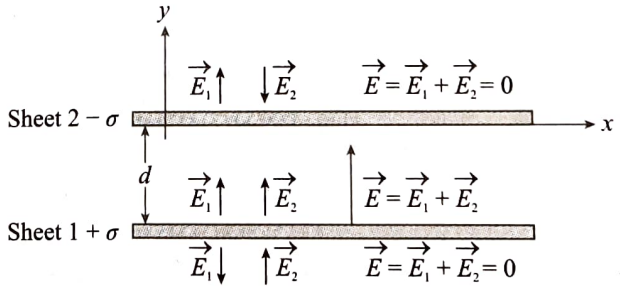
and we get $E_x = \frac{\sigma}{2\varepsilon_0}$

Our final result does not contain the distance x from the plane. This is a correct but rather surprising result. We observe the following:

- The electric field produced by an infinite plane sheet of charge is independent of the distance from the sheet.
- The electric field is uniform; everywhere its direction is perpendicular to the sheet and away from it.
- The infinite plane sheet of charge is a hypothetical case. In real practice, there is no such infinite plane sheet of charge. But if the dimensions of the sheet are much larger than the distance x of the observation point P from the sheet, the field is very nearly the same as for an infinite sheet.

FIELD OF TWO OPPOSITELY CHARGED SHEETS

Two infinite plane sheets are placed parallel to each other, separated by a distance d (as shown in figure). The lower sheet has a uniform positive surface charge density σ, and the upper sheet has a uniform negative surface charge density $-\sigma$ with the same magnitude. Let us find the electric field between the two sheets, above the upper sheet and below the lower sheet.



The situation described in this example in an idealization of two finite, oppositely charged sheets, like the plates shown in figure. If the dimensions of the sheets are large in comparison to the separation d, then to good approximation we can consider the sheets to be infinite in extent. We know the field due to

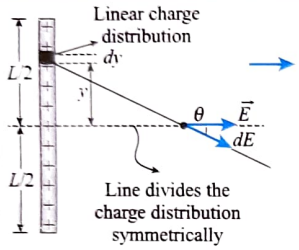
a single infinite plane sheet of charge. We can then find the total field by using the principle of superposition of electric fields. Let sheet 1 be the lower sheet of positive charge, and let sheet 2 be the upper sheet of negative charge; the fields due to each sheet are $\vec{E}_1$ and $\vec{E}_2$, respectively, and both have the same magnitude at all points, no matter how far from either sheet, i.e., $E_1 = E_2 = \sigma/2\varepsilon_0$.

At all points, the direction of $\vec{E}_1$ is away from the positive charge of sheet 1, and the direction of $\vec{E}_2$ is toward the negative charge of sheet 2. These fields, as well as the x- and y-axes, are shown in the figure. At points between the sheets, the fields add each other, and at points above the upper sheet or below the lower sheet, they cancel each other. Thus, the total field is

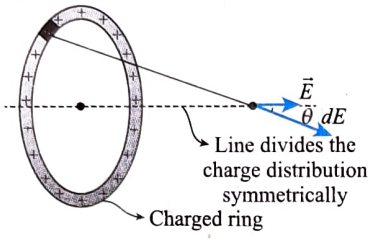
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} 0 & \text{above the upper sheet} \\ \sigma & \text{between the sheet} \\ -\hat{j} & \\ \varepsilon_0 & \text{above the upper sheet} \\ 0 & \end{cases}$$

Because we considered the sheets to be infinite, our result does not depend on the separation d.

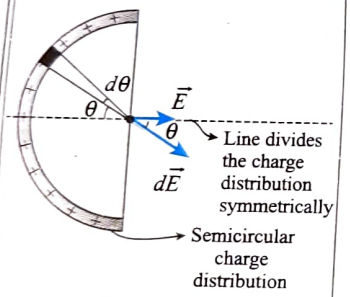
Symmetry plays a very important role in problem solving. Electric field is in the direction along the line that divides the charge distribution symmetrically. Some cases of symmetry are as follows:



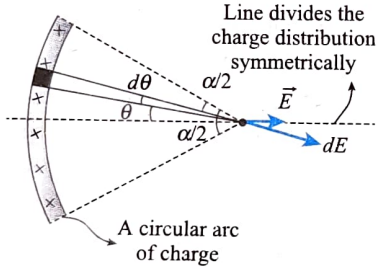
$$E_{net} = \int dE\cos\theta$$



$$E_{net} = \int dE\cos\theta$$

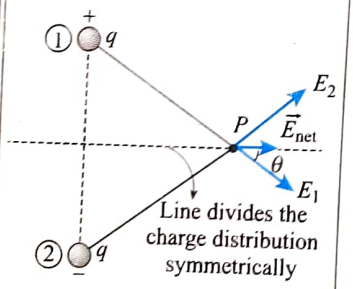


$$E_{net} = \int dE\cos\theta$$



$$E_{net} = \int dE\cos\theta$$

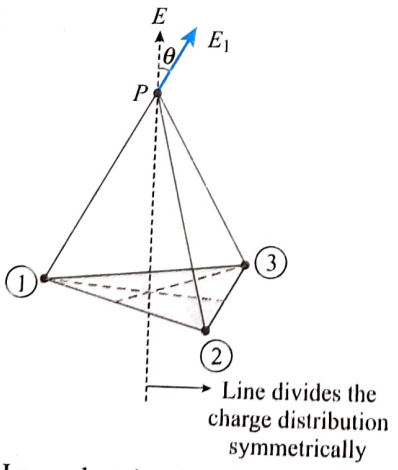
Two point charges



Here, $|\vec{E}_1| = |\vec{E}_2|$

$$E_{net} = 2|E_1|\cos\theta$$

Three point charges, at the corners, of an equilateral triangle

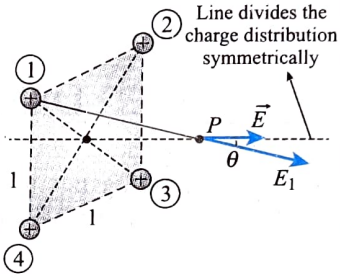


Here, electric fields at P due to charges (1), (2), and (3) are equal, i.e.,

$$|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3|$$

Hence, $E_{net} = 3|\vec{E}_1|\cos\theta$

Four point charges at the corners of a square



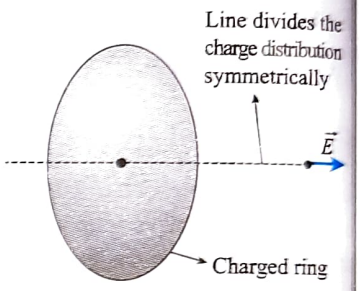
The electric field at point P due to charges (1), (2), (3), and (4) is

$$|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3| = |\vec{E}_4|$$

Hence, the net electric field at P is

$$|\vec{E}_{net}| = 4|\vec{E}_1|\cos\theta$$

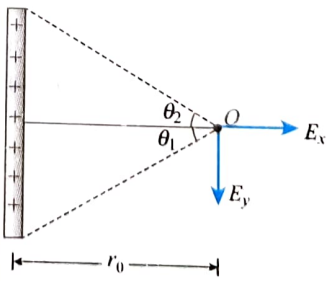
Charged disk



Net field at a point on the axis is along the axis of the disk.

Some Useful Results

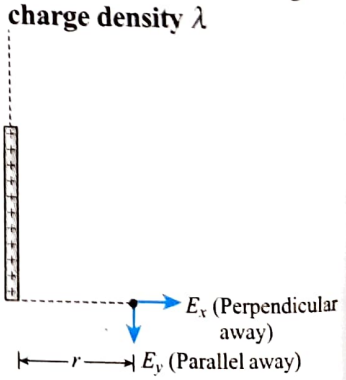
A charged rod of fixed length having charge density λ



$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 r}(\cos\theta_1 - \cos\theta_2)$$

Semi-infinite rod having charge density λ



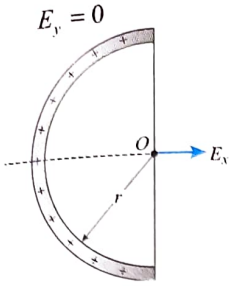
$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r_0}$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 r_0}$$

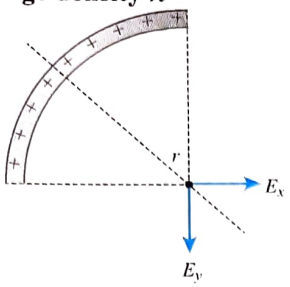
Semicircular ring having charge density λ

$$E_x = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E_y = 0$$

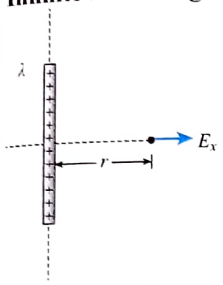


Quarter circular ring having charge density λ



$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r}, \quad E_y = \frac{\lambda}{4\pi\varepsilon_0 r}$$

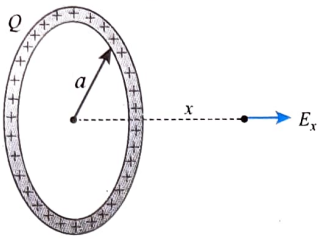
Infinite line charge



$$E_x = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E_y = 0$$

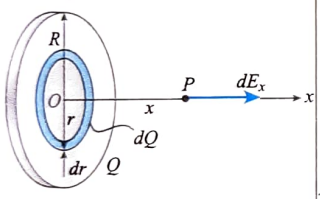
Charged ring



$$E_x = \frac{kQx}{(x^2 + a^2)^{3/2}}$$

$$E_y = 0$$

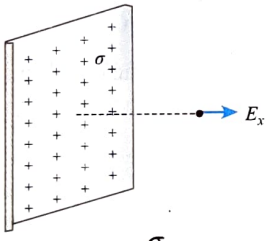
Charged disk



$$E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

$$E_y = 0$$

Infinite sheet of charge

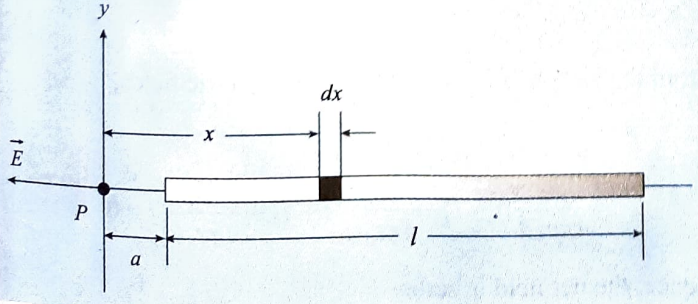


$$E_x = \frac{\sigma}{2\varepsilon_0}$$

$$E_y = 0$$

ILLUSTRATION 1.32

A rod of length l, has a uniform positive charge per unit length and a total charge Q. Calculate the electric field at a point P located along the long axis of the rod and at a distance a from one end (figure).



Sol. The field $d\vec{E}$ at P due to each segment of charge on the rod is in the negative x direction because every segment carries a positive charge.

Because the rod is continuous, we are evaluating the field due to a continuous charge distribution rather than a group of individual charges. Because every segment of the rod produces an electric field in the negative x direction, the sum of their contributions can be handled without the need to add vectors.

Let us assume the rod is lying along the x-axis, dx is the length of one small segment, and dq is the charge on that segment. Because the rod has a charge per unit length λ, the charge dq on the small segment is $dq = \lambda\, dx$. The magnitude of the electric field at P due to one segment of the rod having a charge dq is

$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda\, dx}{x^2}$$

The total field at P is

$$E = \int_a^{l+a} k_e \lambda \frac{dx}{x^2}$$

If k_e and $\lambda = Q/l$ are constants and can be removed from the integral, then

$$E = k_e \lambda \int_a^{l+a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x}\right]_a^{l+a}$$

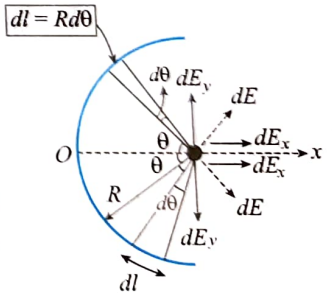
$$= k_e \frac{Q}{l}\left(\frac{1}{a} - \frac{1}{l+a}\right) = \frac{k_e Q}{a(l+a)}$$

If $a \to 0$, which corresponds to sliding the bar to the left until its left end is at the origin, then $E \to \infty$. That represents the condition in which the observation point P is at zero distance from the charge at the end of the rod, so the field becomes infinite.

ILLUSTRATION 1.33

A uniformly charged wire with linear charge density λ is placed in the form of a semicircle of radius R. Find the electric field generated by the semicircle at the center.

Sol. We consider a differential element dl on the ring that subtends an angle $d\theta$ at the center of the ring, i.e., $dl = Rd\theta$. So the charge on this element is $dQ = \lambda\, Rd\theta$. This element creates a field dE, which makes an angle θ at the center as shown in figure.



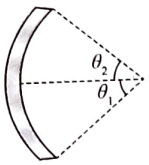
For each differential element in the upper half of the ring, there corresponds a symmetrically placed charge element in the lower half. The y-components of the field due to these symmetric elements cancel out, and the x-components remain. So, we get

$$dE_x = dE\cos\theta = \frac{dQ}{4\pi\varepsilon_0 R^2}\cos\theta = \frac{\lambda(Rd\theta)\cos\theta}{4\pi\varepsilon_0 R^2}$$

On integrating the expression for dE_x with respect to angle θ in limits $\theta = -\pi/2$ to $\theta = +\pi/2$, we obtain

$$E = \int_{-\pi/2}^{+\pi/2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\cos\theta\, d\theta = \frac{\lambda}{2\pi\varepsilon_0 R}$$

In terms of the total charge, say Q, on the ring, $\lambda = Q/\pi R$ and we get $E = Q/2\pi^2\varepsilon_0 R^2$.



If we consider the wire in the form of an arc as shown in figure, the symmetry consideration is not useful in canceling out x- and y-components of the fields, if θ_1 and θ_2 are different. We will integrate dE_x as well as dE_y in limits $\theta = -\theta_1$ to $\theta = +\theta_2$.

$$E_x = \int_{-\theta_1}^{+\theta_2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}(\sin\theta_1 + \sin\theta_2)$$

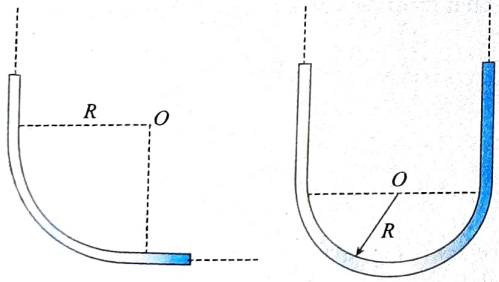
$$E_y = -\int_{-\theta_1}^{+\theta_2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\sin\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}(\cos\theta_2 - \cos\theta_1)$$

For a symmetrical arc, $\theta_1 = \theta_2$. Thus, E_y vanishes and

$$E_x = \frac{\lambda \sin\theta}{2\pi\varepsilon_0 R}$$

ILLUSTRATION 1.34

A long wire with a uniform charge density λ is bent in two configurations shown in Figs. **(a)** and **(b)**. Determine the electric field intensity at point O.



(a) **(b)**

Sol. For Fig. (a), field due to segment 1 is

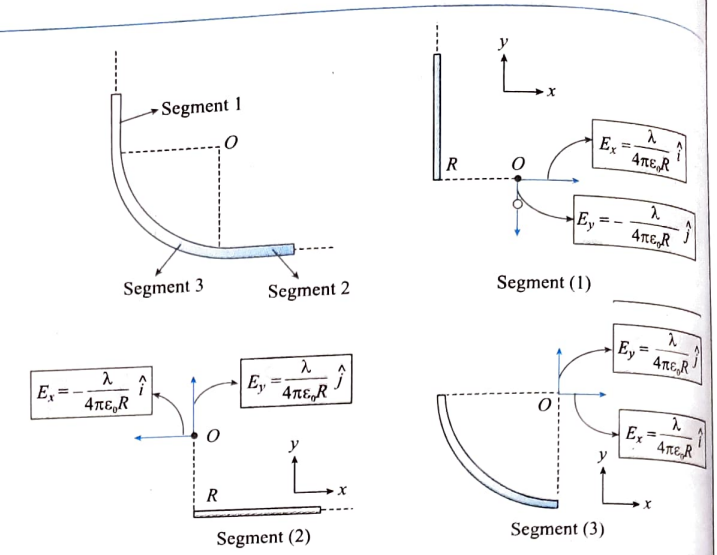
$$\vec{E}_1 = \left[\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{i} + \left[-\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{j}$$

Field due to segment 2 is

$$\vec{E}_2 = \left[-\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{i} + \left[\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{j}$$

Field due to quarter shape wire segment 3 is

$$\vec{E}_3 = \left[\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{i} + \left[\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{j} \quad (\because \theta_1 = 90°, \theta_2 = 0°)$$



The resultant field is the superposition of the fields due to each part, i.e.,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \quad ...(i)$$

Substituting the values of $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$ in Eq. (i), we get

$$\vec{E} = \left[\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{i} + \left[\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{j}$$

$$|\vec{E}| = \left[\left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2 + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2\right]^{1/2} = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R}$$

Here, $E_x = E_y = \dfrac{\lambda}{4\pi\varepsilon_0 R}$

Hence, the resultant field will make an angle of 45° with the axis
For Fig. (b), field due to segment 1 is

$$\vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} - \hat{j}]$$

Field due to segment 2 is

$$\vec{E}_{x_2} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{i}$$

$$\vec{E}_{y_2} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{j}$$

$$\vec{E}_2 = -\frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} + \hat{j}]$$

Field due to segment 3 is

$$\vec{E}_3 = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j}$$

From the principle of superposition of electric fields,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} - \hat{j}] - \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} + \hat{j}] + \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j} = 0$$

Hence, the net field is zero.

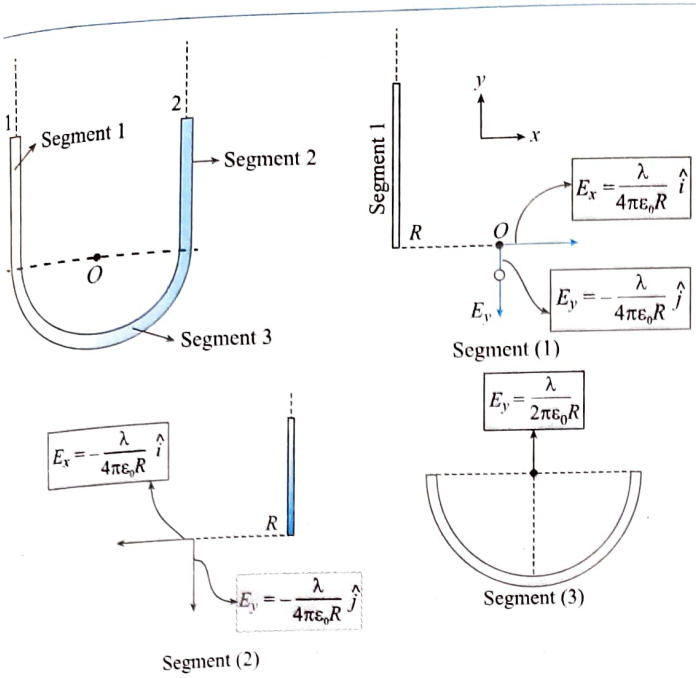
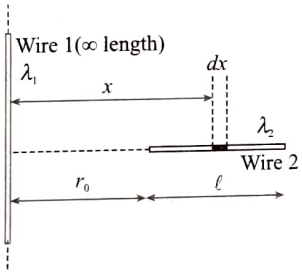


ILLUSTRATION 1.35

A segment of a charged wire of length l, charge density λ_2, and an infinitely long charged wire, charge density λ_1, lie in a plate at right angles to each other. The separation between the wires is r_0. Determine the force of interaction between the wires.

Sol. Electric field near a long wire is given by the expression

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$



The second wire lies in the nonuniform field of the first wire. Each element of the second wire experiences different magnitude of field. Therefore, we consider differential element dx, and charge $dQ = \lambda_2 dx$, at a distance x from the long wire. The force acting on this element dF is

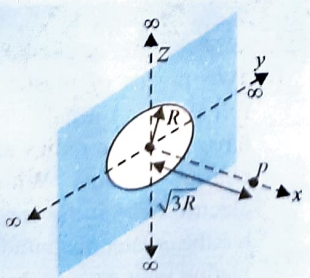
$$dF = EdQ = \left[\frac{\lambda_1}{2\pi\varepsilon_0 x}\right]\lambda_2 dx$$

The force acting on each element depends on x, the separation between wire 1 and 2. Integrating the expression for dF in the limits $x = r_0$ to $x = r_0 + l$, we obtain

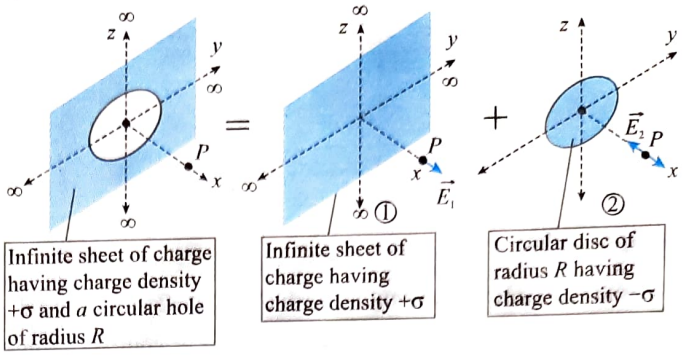
$$F = \int_{r_0}^{r_0+l} \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}\frac{dx}{x} = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}\ln\left[1 + \frac{l}{r_0}\right]$$

ILLUSTRATION 1.36

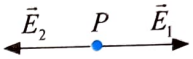
An infinite dielectric sheet having charge density σ has a hole of radius R in it. An electron is released from point P on the axis of the hole at a distance $\sqrt{3}R$ from the center. Find the speed with which it crosses the plane of the sheet.



Sol. The infinite charged sheet with a circular hole can be considered superposition of an infinite sheet of charge density σ and charged disc of charge density $-\sigma$. From the principle of superposition, we have the following:



Net electric field at P is the vector sum of electric fields due to both infinite sheet of charge $(\vec{E}_1)$ and disc $(\vec{E}_2)$.



Electric field due to infinite dielectric sheet is

$$\vec{E}_1 = \frac{\sigma}{2\varepsilon_0}(\hat{i})$$

Electric field at the axis of a disc of radius R is

$$E_2 = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right](-\hat{i})$$

Resultant electric field is

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\frac{x}{\sqrt{x^2 + R^2}}(\hat{i})$$

Force on the electron is

$$\vec{F} = -\frac{\sigma e x}{2\varepsilon_0\sqrt{x^2 + R^2}}(\hat{i})$$

$$mv\frac{dv}{dx} = -\frac{\sigma e x}{2\varepsilon_0\sqrt{x^2 + R^2}}$$

Let the speed of the electron when it crosses the sheet be v. Then,

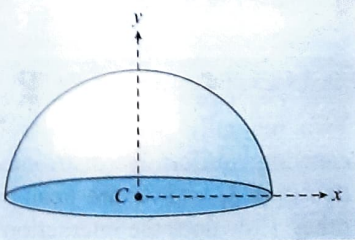
$$m\int_0^v v\,dv = -\frac{\sigma e}{2\varepsilon_0}\int_{\sqrt{3}R}^{0}\frac{x}{\sqrt{x^2 + R^2}}dx$$

or $$m\frac{v^2}{2} = -\frac{\sigma e}{2\varepsilon_0}\left[\sqrt{x^2 + R^2}\right]_{\sqrt{3}R}^{0}$$

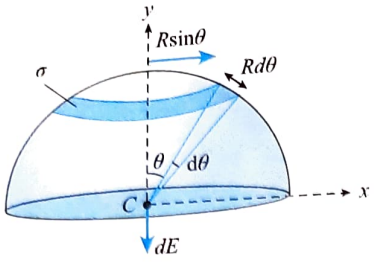
or $$v = \sqrt{\frac{\sigma e R}{m\varepsilon_0}}$$

ILLUSTRATION 1.37

Figure shows a hollow hemisphere, uniformly charged with surface charge density σ C/m², find electric field strength at its centre C.



Sol. Let us consider an elemental ring on its surface of angular width $d\theta$ at an angle θ from its axis as shown.



The surface area of this ring will be $ds = 2\pi R\sin\theta \times Rd\theta$

Charge on this elemental ring is $dq = \sigma ds = \sigma \cdot 2\pi R^2 \sin\theta\, d\theta$

Now due to this ring, electric field strength at centre C can be given as

$$dE = \frac{kdq\,(R\cos\theta)}{(R^2\sin^2\theta + R^2\cos^2\theta)^{3/2}}$$

$$= \frac{k\sigma \cdot 2\pi R^2 \sin d\theta \cdot R\cos\theta}{R^3} = \pi k\sigma \sin 2\theta\, d\theta$$

Net electric field at centre can be obtained by integrating this expression between limits 0 to $\pi/2$ as

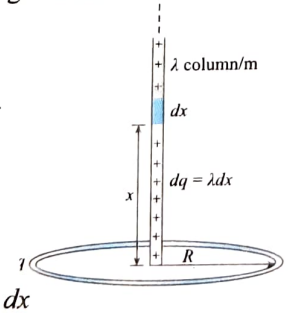
$$E = \int dE = \pi k\sigma \int_0^{\pi/2} \sin 2\theta\, d\theta$$

$$= \frac{\sigma}{4\varepsilon_0}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi/2} = \frac{\sigma}{4\varepsilon_0}\left[\frac{1}{2}+\frac{1}{2}\right] = \frac{\sigma}{4\varepsilon_0}$$

ILLUSTRATION 1.38

A system consists of a thin charged wire ring of radius R and a very long uniformly charged thread oriented along the axis of the ring, with one of its ends coinciding with the centre of the ring. The total charge of the ring is equal to q. The charge of the thread (per unit length) is equal to λ. Find the interaction force between the ring and the thread.

Sol. Let us take an elemental charge on the thread at a distance x from the centre of the ring.



Force dF on the element $dF = (dq)\cdot E$

dq = charge in element $= \lambda \cdot dx$

E = electric field due to ring.

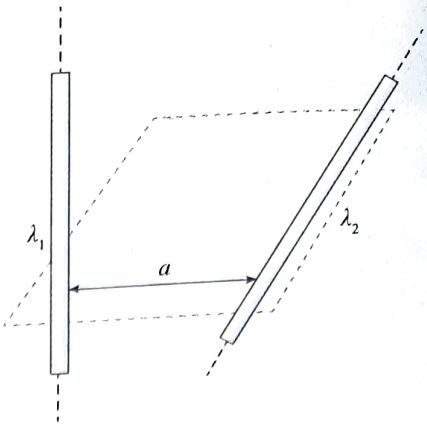
$$\Rightarrow \quad dF = (\lambda dx).\frac{kqx}{(x^2+R^2)^{3/2}}$$

Net force on the wire, $F = kq\lambda \int_0^{\infty} \frac{x\,dx}{(R^2+x^2)^{3/2}}$

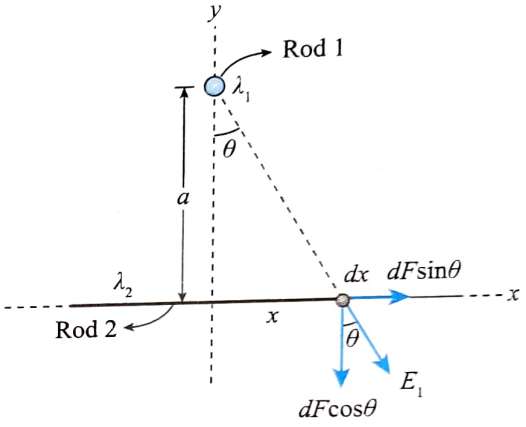
On integration we get, $F = \frac{\lambda q}{4\pi\varepsilon_0 R}$

ILLUSTRATION 1.39

Two mutually perpendicular long straight conducting rods carrying uniformly distributed charges of linear charge densities λ_1 and λ_2 are positioned at a distance a from each other. How does the interaction between the rods depend on a.



Sol. Let us make two dimensional view of the situation because of symmetry we can say the force on rod 2 (λ_2) will be along negative y direction. Let us consider an element of length dx charged, having charge $d\theta$.



Force on the element, $dF = E_1 dq = \left(\frac{\lambda}{2\pi\varepsilon_0\sqrt{a^2+x^2}}\right)(\lambda_2 dx)$

The net force on rod '2', $F_{net} = \int dF\cos\theta$

$$= \int \frac{\lambda_1}{2\pi\varepsilon_0\sqrt{a^2+x^2}}(\lambda_2 dx).\cos\theta$$

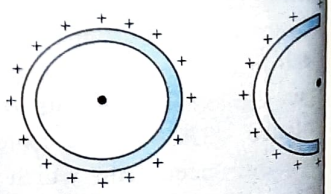
where $\cos\theta = \frac{a}{\sqrt{a^2+x^2}}$

$$\Rightarrow \quad F_{net} = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0} a\int_{-\infty}^{\infty} \frac{dx}{a^2+x^2} = \frac{\lambda_1\lambda_2 a}{2\pi\varepsilon_0} \times \frac{1}{a}\left[\tan^{-1}\frac{x}{a}\right]_{-\infty}^{\infty}$$

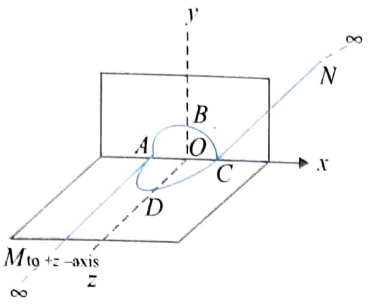
$$\Rightarrow \quad F_{net} = \frac{\lambda_1\lambda_2}{2\varepsilon_0}$$

CONCEPT APPLICATION EXERCISE 1.4

1. Two pieces of plastic, a full ring and a half ring, have the same radius and charge density. Which electric field at the center has the greater magnitude? Define your answer.

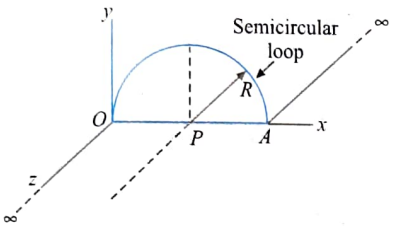


2. Two semicircular wires ABC and ADC, each of radius R, are lying on xy and xz planes, respectively, as shown in figure.



If the linear charge density of the semicircular parts and straight parts is λ, find the electric field intensity $\vec{E}$ at the origin.

3. An infinite wire having linear charge density λ is arranged as shown in figure. A charge particle of mass m and charge q is released from point P. Find the initial acceleration of the particle (at $t=0$) just after the particle is released.



4. A ring of radius R has charge $-Q$ distributed uniformly over it. Calculate the charge that should be placed at the center of the ring such that the electric field becomes zero at a point on the axis of the ring at distant R from the center of the ring.

5. A charged particle q of mass m is in equilibrium at a height h from a horizontal infinite line charge with uniform linear charge density λ. The charge lies in the vertical plane containing the line charge. If the particle is displaced slightly (vertically), prove that the motion of the charged particle will be simple harmonic. Also find its time period.

ANSWERS

1. Due to semicircular ring

2. $\dfrac{-\lambda}{2\pi\varepsilon_0 R}(\hat{j}+\hat{k})$ 3. $\dfrac{q\lambda}{2\pi\varepsilon_0 mR}(-\hat{j})$ 4. $\dfrac{Q}{2\sqrt{2}}$

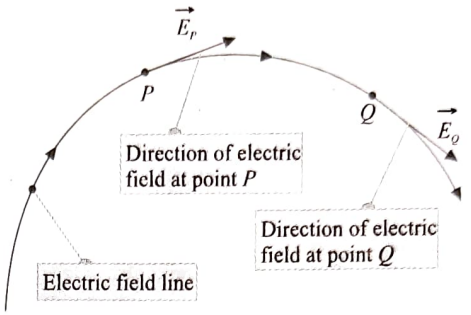
5. $2\pi\sqrt{\dfrac{mh^2}{2kq\lambda}}$

LINES OF FORCE

We have defined the electric field mathematically. Let us now explore a means of visualizing the electric field in a pictorial representation. A convenient way of visualizing electric field patterns is to draw lines, called electric field lines. The lines of force provide a nice idea to visualize the pattern of electric field in a given space. We assume that the space around a charged body is filled with some lines known as electric lines of force. "These lines of force are drawn in space in such a way that the tangent to the line at any point gives the direction of the electric field at that point." It has been found quite convenient to visualize the electric field in terms of lines of force.

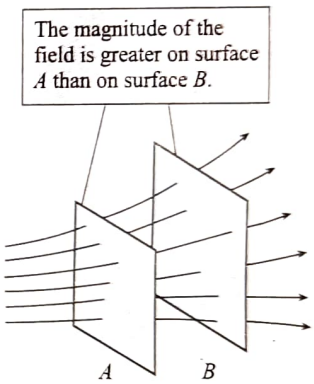
PROPERTIES OF ELECTRIC LINES OF FORCE

The electric field vector $\vec{E}$ is tangent to the electric field line at each point. The line has a direction, indicated by an arrowhead, that is the same as that of the electric field vector. The direction of the line is that of the force on a positive test charge placed in the field.



The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. Therefore, the field lines are close together where the electric field is strong and far apart where the field is weak.

No field line originates or terminates in the space surrounding a charge. Every field line is a continuous and smooth curve originating from a positive charge and ending on a negative charge. If one type of charge is in excess, some lines will begin or end infinitely far away.

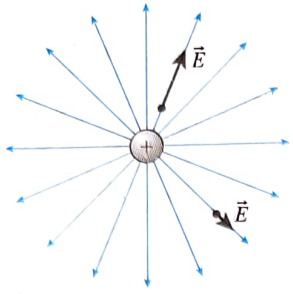


Two electric lines of force never cross each other, because if they do, intensity will have two directions at the point of intersection, which is absurd.

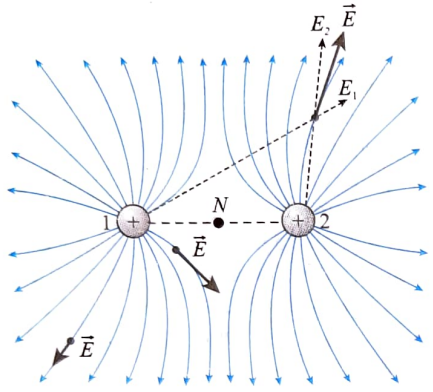
Field lines do not pass through the conductor. This indicates that the **electric field within the conductor is zero**. The field lines are perpendicular to the surface of the conductor. Had it been not so, there would have been a component of the field along the surface of the conductor and a current would flow through it. But no current flows in such an electrostatic situation. "Thus, the electric field just outside the surface of the conductor is perpendicular to its surface."

Note: The tangent to the line of force at a point in an electric field gives the direction of intensity or force or acceleration, which a positive charge will experience there, but not the direction of motion always. So a positive point charge free to move in an electric field may or may not follow the line of force. It will follow the line of force if it is a straight line (as direction of velocity and acceleration will be same) and will not follow the line if it is curved as the direction of motion will be different from that of acceleration and the particle will move neither in the direction of motion nor in the direction of acceleration (line of force). So, in general, the trajectory of a charged particle is not the same as that of a field line.

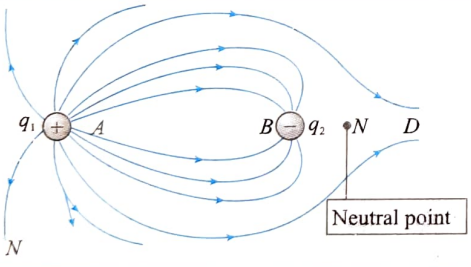
DIFFERENT PATTERNS OF ELECTRIC FIELD LINES



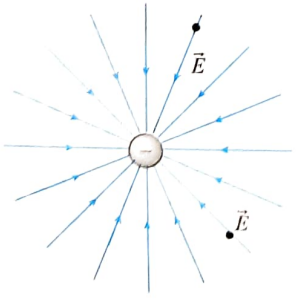
Electric field lines of an isolated positive point charge are directed radially outwards. They extend to infinity. The field is spherically symmetric, i.e., it looks same in all directions, as seen from the point charge.



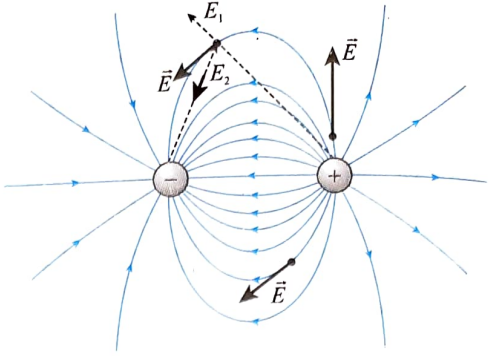
Two equal positive charges. N is the neutral point lying at the middle of the charges. The field is cylindrically symmetric about the line joining the charges, i.e., the field pattern is same in all planes passing through the line joining charges.



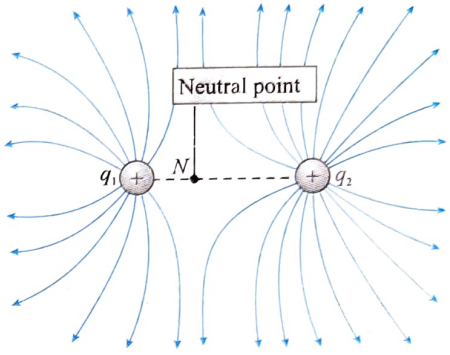
A is the positive and B is the negative charge of different magnitudes ($|q_2|<|q_1|$). The field is cylindrically symmetric about the line joining the charges, i.e., the field pattern is same in all planes passing through the line joining charges.



Electric field lines of an isolated negative point charge are directed radially inward. The field is spherically symmetric, i.e., it looks same in all directions, as seen from the point charge.



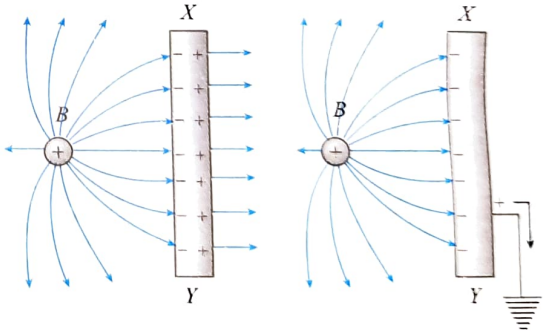
A positive charge and a negative charge of equal magnitude (an electric dipole). The field is cylindrically symmetric about the line joining the charges, i.e., the field pattern is same in all planes passing through the line joining charges.



Two positive charges of different magnitudes ($|q_1|<|q_2|$). The field is cylindrically symmetric about the line joining the charges, i.e., the field pattern is same in all planes passing through the line joining the charges.

ELECTROSTATIC SHIELDING (OR SCREENING)

Electrostatic screening is the process of limiting the electric field to a certain region of space.



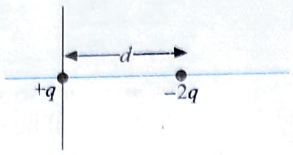
(a) (b)

An earthed conductor acts as a screen (or a shield) against the electric field. If we place a metal screen XY near a charged body B, the resultant electrostatic field due to electrostatic induction is as shown in Fig. (a). But when the conductor XY is earthed the free induced positive charge on it flows to the earth and with it the lines of force emanating from it disappear as shown in Fig. (b). Thus, the region to the right of XY is shielded from the field due to B. It is to be noted that the shielding surface may or may not be continuous. Continuous shielding surfaces, called Faraday cages, are hollow conductors and are based on the fact that electric field inside a hollow conductor is zero. To protect people and dielectric instruments from external electric field

we enclose them in such cages and leave them unearthed. An apparatus generating high voltage is also enclosed in such a cage to prevent its field from spreading out. But such a cage is earthed.

ILLUSTRATION 1.40

Charges $+q$ and $-2q$ are fixed at distance d apart as shown in figure.

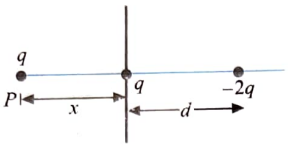


(a) Sketch roughly the pattern of electric field lines, showing the position of neutral point.

(b) Where should a charge particle q be placed so that it experiences no force?

Sol. Let the net force on q at P be zero. Then,

$$\frac{kq^2}{x^2} = \frac{kq\ 2q}{(d+x)^2} \text{ or } x = \frac{d}{\sqrt{2}-1}$$



P is the neutral potential where electric field will be zero.

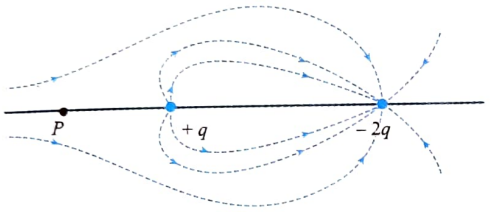
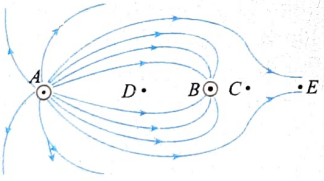


ILLUSTRATION 1.41

The field lines for two point charges are shown in figure.



(a) Is the field uniform?

(b) Determine the ratio q_A/q_B.

(c) What are the sign of q_A and q_B?

(d) Apart from infinity, where is the neutral point?

(e) If q_A and q_B are separated by a distance $10(\sqrt{2}-1)$ cm, find the position of neutral point.

(f) Where will the lines meet which are coming from A and are not meeting at q_B?

(g) Will a positive charge follow the line of force if free to move?

Sol.

(a) No

(b) The number of lines coming from or coming to a charge is proportional to the magnitude of the charge, so

$$\frac{q_A}{q_B} = \frac{12}{6} = 2$$

(c) q_A is positive and q_B is negative.

(d) C is the other neutral point.

(e) For neutral point

$$E_A = E_B \Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q_A}{(l+x)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_B}{x^2}$$

or $$\left(\frac{l+x}{x}\right)^2 = \frac{q_A}{q_B} = 2 \text{ or } x = 10 \text{ cm}$$

(f) At infinity.

(g) No. As lines of force are curved, the direction of velocity and acceleration will be different. Hence, a charge cannot follow strictly the lines of force. Also to move on some curved path, centripetal force is required, whereas the lines of force will provide only tangential force.

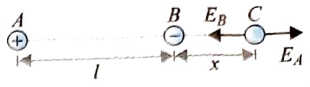
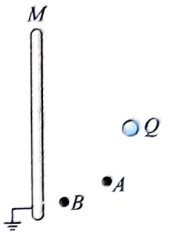


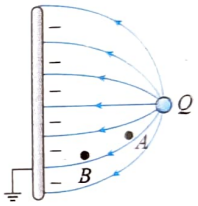
ILLUSTRATION 1.42

A metal plate M is grounded. A point charge $+Q$ is placed in front of it. Consider two points A and B as shown in figure. At which point (A or B) is the electric field stronger? At which point is the potential higher?



Sol. Negative charge gets induced on the metal surface. Field lines are as shown. Density of lines is higher at A

$E_A > E_B$



MOTION OF CHARGED PARTICLE IN UNIFORM ELECTRIC FIELD

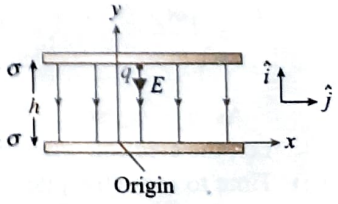
When a charged particle is placed in an electric field, it experiences an electrical force. If this is the only force on the particle, it must be the net force. The net force will cause the particle to accelerate according to Newton's second law. So

$$\vec{F}_e = q\vec{E} = m\vec{a}$$

If $\vec{E}$ is uniform, then $\vec{a}$ is constant and $\vec{a} = q\vec{E}/m$. If the particle has a positive charge, its acceleration is in the direction of the field. If the particle has a negative charge, its acceleration is in the direction opposite to the electric field. Since the acceleration is constant, the kinematic equations can be used.

ILLUSTRATION 1.43

A particle of mass m and charge q is released from rest in a uniform field of magnitude E. The uniform field is created between two parallel plates of charge densities $+\sigma$ and $-\sigma$, respectively. The particle accelerates toward the other plate a distance d apart. Determine the speed at which it strikes the opposite plate.



Sol. The applied electric field is $\vec{E} = -E_0\hat{j}$. The force experienced by the charge q is

$$\vec{F} = q\vec{E} = -qE_0\hat{j}$$

The force is constant, and so the acceleration is constant as well, i.e.,

$$\vec{a} = \frac{\vec{F}}{m} = -\frac{qE_0}{m}\hat{j}$$

Because of constant acceleration, the particle moves in the negative y-direction; the problem is analogous to the motion of a mass released from rest in a gravitational field. From equations of motion, we get

$$v_y = v_{y0} + a_y t = 0 - \frac{qE_0}{m}t \quad \ldots(i)$$

and $$y = y_0 + v_0 t + \frac{1}{2}a_y t^2 \text{ or } 0 = d + 0 - \frac{1}{2}\frac{qE_0}{m}t^2 \quad \ldots(ii)$$

The particle starts at $y_0 = d$, and impact occurs at $y = 0$.

From Eq. (ii), we get $t = \left(\frac{2dm}{qE_0}\right)^{1/2}$

From Eq. (i), we get $v_y = -\frac{qE_0}{m}\left(\frac{2dm}{qE_0}\right)^{1/2} = -\sqrt{\frac{2qE_0 d}{m}}$

ILLUSTRATION 1.44

A uniform electric field E exists between two metal plates, one negative and other positive. The plate length is l and the separation of the plates is d.

(a) An electron and a proton start from the negative plate and positive plate, respectively, and go to opposite plates. Which one of them wins this race?

(b) An electron and a proton start moving parallel to the plates toward the other end from the midpoint of the separation of plates at one end of the plates. Which of the two will have greater deviation when they come out of the plates if they start with the

(i) same initial velocity,

(ii) same initial kinetic energy, and

(iii) same initial momentum.

Sol.

(a) $a_e = \frac{eE}{m_e}, a_p = \frac{eE}{m_p}; d = \frac{1}{2}at^2$

$$t = \sqrt{\frac{2d}{a}} \text{ or } t = \sqrt{\frac{2md}{eE}}$$

Therefore, we have

$$\frac{t_e}{t_p} = \sqrt{\frac{m_e}{m_p}}$$

As $m_e < m_p$, so $t_e < t_p$. Hence, electron will take less time, i.e., the electron wins the race.

(b) Time to cross the plates is $t = 1/u$. Deviation is

$$y = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m}\left(\frac{1}{u}\right)^2$$

or $$\frac{y_e}{y_p} = \frac{m_p}{m_e}\left(\frac{u_p}{u_e}\right)^2 \quad \ldots(i)$$

(i) If $u_p = u_e$, then

$$\frac{y_e}{y_p} = \frac{m_p}{m_e}$$

As $m_p > m_e$, so $y_e > y_p$. Hence, the deviation of the electron will be more.

(ii) From Eq. (i),

$$\frac{y_e}{y_p} = \left(\frac{m_p u_p^2}{m_e u_e^2}\right) = 1 \text{ (as given)}$$

Hence, the deviation of both the electron and the proton will be the same.

(iii) From Eq. (i),

$$\frac{y_e}{y_p} = \left(\frac{m_p u_p}{m_e u_e}\right)^2 \frac{m_e}{m_p} = \frac{m_e}{m_p}$$

As $m_e < m_p$, so $y_e < y_p$. Hence, the deviation of proton will be more.

ILLUSTRATION 1.45

A particle having charge that of an electron and mass 1.6×10^{-30} kg is projected with an initial speed u at an angle 45° to the horizontal from the lower plate of a parallel-plate capacitor as shown in figure. The plates are sufficiently long and have a separation of 2 cm. Find the maximum value of the velocity of the particle so that it does not hit the upper plate. Take the electric field between the plates as 10^3 Vm^{-1} directed upward.

Sol. Resolving the velocity of the particle parallel and perpendicular to the plate, we get

$$u_{\parallel} = u\cos 45° = \frac{u}{\sqrt{2}} \text{ and } u_{\perp} = u\sin 45° = \frac{u}{\sqrt{2}}$$

Force on the charged particle in the downward direction and normal to the plate is eE. Therefore, acceleration $a = eE/m$, where m is the mass of the charged particle. The particle will not hit the upper plate if the velocity component normal to the plate becomes zero before reaching it, i.e., $0 = -u_{\perp}^2 - 2ay$ with $y \le d$, where d is the distance between the plates. Therefore, the maximum velocity for the particle not to hit the upper plate (for this $y = d = 2$ cm) is

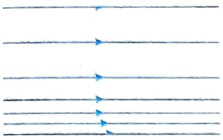
$$u_{\perp} = \sqrt{2ay} = \sqrt{\frac{2\times1.6\times10^{-19}\times10^3\times2\times10^{-2}}{1.6\times10^{-30}}}$$

$$= 2\times10^6 \text{ ms}^{-1}$$

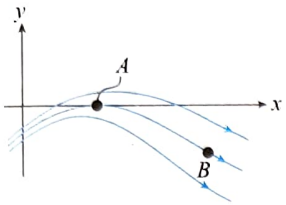
or $$u_{max} = u_{\perp}/\cos 45° = 2\sqrt{2}\times10^6 \text{ ms}^{-1}$$

CONCEPT APPLICATION EXERCISE 1.5

1. Is an electric field of the type shown by the electric lines in figure physically possible?



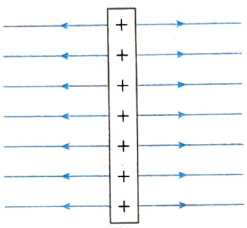
2. Figure shows three electric field lines.



(a) What is the direction of the electrostatic force on a positive test charge placed at points A and B ?

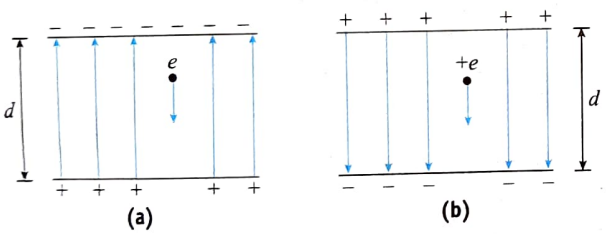
(b) At which point, A or B, will the acceleration of the test charge be greater if the charge is released?

3. Figure shows that E has the same value for all points in front of an infinitely charged sheet. Is this reasonable? One might think that the field should be stronger near the sheet because the charges are so much closer.



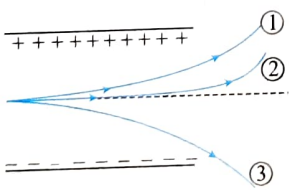
4. Two point charges Q and $4Q$ are fixed at a distance of 12 cm from each other. Sketch lines of force and locate the neutral point, if any.

5. An electron (mass m_e) falls through a distance d in a uniform electric field of magnitude E.



The direction of the field is reversed keeping its magnitude unchanged, and a proton (mass m_p) falls through the same distance. If the times taken by the electron and the proton to fall the distance d is $t_{electron}$ and t_{proton}, respectively, then the ratio $t_{electron}/t_{proton}$ = _____

6. Figure shows the tracks of three charged particles in a uniform electrostatic field projected parallel to a plate with the same velocity. Give the signs of the three charges. Which of the three particles has the highest charge-to-mass ratio?

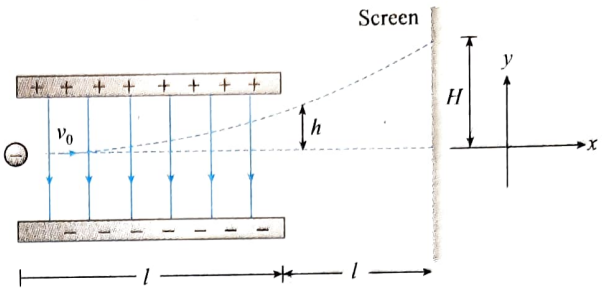


7. Two charged metal plates in vacuum are 10 cm apart. A uniform electric field of intensity $(45/16) \times 10^3$ NC^{-1} is applied between the plates. An electron is released between the plates from rest at a point just outside the negative plate.

(a) Calculate how long (t) will the electron take to reach the other plate.

(b) At what velocity (v) will it be going just before it hits the other plate?

8. An electron moving in a gravitational free space enters a uniform electric field E with an initial velocity v_0 as shown in figure.

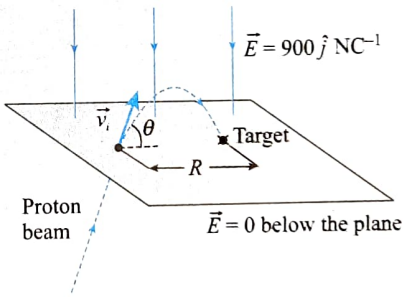


(a) Find the deflection distance h in the field.

(b) Find an expression for the velocity of electron when it just emerges from the field.

(c) Find an expression for the total deflection distance H at a vertical screen placed at a distance l from the region of uniform field. (Assume that the field abruptly ends outside the field.)

9. Protons are projected with an initial speed $v_i = 6$ km s^{-1} from a field-free region $\vec{E} = -900\hat{j}$ NC^{-1} present above the plane as shown in figure. The initial velocity vector of the protons makes an angle u with the plane. The protons are to hit a target that lies at a horizontal distance of $R = 2$ mm from the point where the protons cross the plane and enter the electric field. Find the angle θ at which the protons must pass through the plane to strike the target.



ANSWERS

2. (a) Along tangent at points A and B (b) A

5. $\sqrt{\frac{m_e}{m_p}}$ **6.** Particle 3

7. (a) 2×10^{-8} s (b) 10^7 m/s

8. (a) $\left(\frac{eE}{2mv_0^2}\right)l^2$ (b) $v_0\hat{i} + \left(\frac{eEl}{mv_0}\right)\hat{j}$ (c) $\frac{3eEl^2}{2mv_0^2}$

9. 15°

ELECTRIC DIPOLE

An electric dipole is a system of two equal and opposite point charges separated by a very small and finite distance. Figure shows an electric dipole consisting of two equal and opposite

point charges $-q$ and $+q$ separated by a small distance $2l$. The strength of an electric dipole is measured by a vector quantity known as electric dipole moment. Its magnitude is equal to the product of the magnitude of either charge and the distance between the two charges, i.e., $p = 2ql$. The direction of p is from the negative charge to the positive charge. In the SI system of units, p is measured in coulomb-meter.

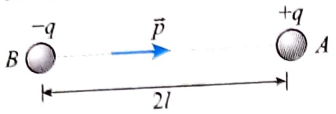
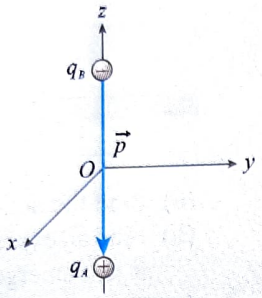


ILLUSTRATION 1.46

A system has two charges $q_A = +2.5 \times 10^{-7}$ C and $q_B = +2.5 \times 10^{-7}$ C located at points A: (0, 0, −15 cm) and B: (0, 0, +15 cm), respectively. What are the total charge and electric dipole moment of the system?



Sol. The two charges q_A and q_B are equal and opposite and situated a fixed distance apart, from an electric dipole. Thus, the total charge of the system is

$$q_A + q_B = 2.5 \times 10^{-7}\,\text{C} - 2.5 \times 10^{-7}\,\text{C} = 0$$

The distance between the two charges is $2a = 30$ cm $= 0.3$ m. So the dipole moment of the system is either charge $\times\, 2a$

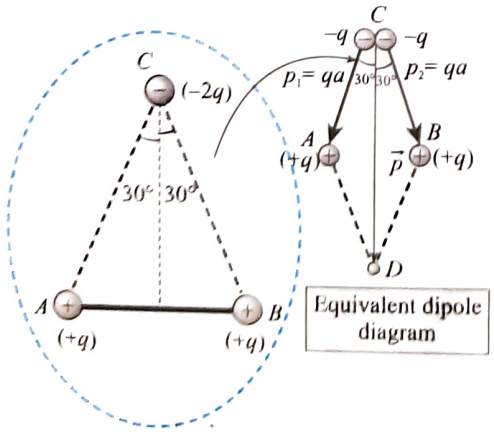
$$= (2.5 \times 10^{-7}\,\text{C})(0.3\,\text{m}) = 7.5 \times 10^{-8}\,\text{Cm}$$

The direction of $\vec{p}$, as shown in figure, is from B to A, i.e., along the negative z-axis.

ILLUSTRATION 1.47

Three charges $+q$, $+q$, and $-2q$ are placed at the vertices of an equilateral triangle. What is the dipole moment of the system?

Sol. Charge $-q$ at C and $+q$ at A form one dipole, and the remaining charge $-q$ at C and $+q$ at B form another dipole (figure).



Dipole moment along CA is qa, and dipole moment along CB is qa. So the net dipole moment is

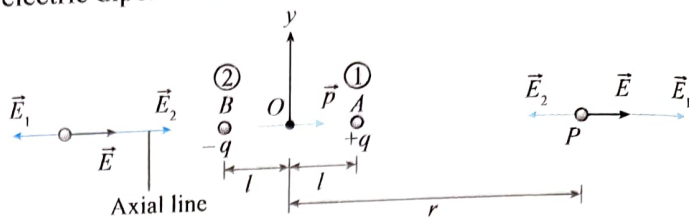
$$p = \sqrt{(qa)^2 + (qa)^2 + 2(qa)(qa)\cos 60^\circ} = \sqrt{3}\,qa$$

$\vec{p}$ acts along CD, which bisects the angle (60°) at C.

ELECTRIC FIELD DUE TO A DIPOLE

ELECTRIC FIELD INTENSITY DUE TO AN ELECTRIC DIPOLE AT A POINT ON THE AXIAL LINE

A line passing through the negative and positive charges of the electric dipole is called the axial line of the electric dipole.



Suppose an electric dipole AB is located in a medium of dielectric constant K (as shown in figure). Let the dipole consists of two point charges $-q$ and $+q$ separated by a short distance $2l$. Let P be an observation point on the axial line such that its distance from the midpoint O of the electric dipole is r. We are interested to calculate the intensity of the electric field at P.

$$E_1 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r-l)^2} \quad \text{due to } q \text{ at } P$$

(along the direction OX)

and $$E_2 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r+l)^2} \quad \text{due to } -q \text{ at } P$$

(along the direction OB)

The intensities E_1 and E_2 are along the same line but in opposite directions. Since $E_1 > E_2$, the resultant intensity E at the point P will be equal to their differences and in the direction $\overrightarrow{AP}$. Thus,

$$E = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r-l)^2} - \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r+l)^2}$$

$$= \frac{q}{4\pi\varepsilon_0 K}\left[\frac{4lr}{(r^2-l^2)^2}\right] = \frac{1}{4\pi\varepsilon_0 K}\left[\frac{2(2ql)r}{(r^2-l^2)^2}\right]$$

But $2ql = p$ = electric dipole moment, so

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{2pr}{(r^2-l^2)^2}$$

If l is very small as compared to r ($l \ll r$), then l^2 is negligible in comparison to r^2. Then the electric field intensity at the point P due to a short dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{2pr}{r^4} = \frac{1}{4\pi\varepsilon_0 K}\frac{2p}{r^3}$$

If the dipole is placed in air or vacuum, then $K = 1$ and

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

Note: The direction of electric field E is in the direction of $\vec{p}$, i.e., parallel to the axis of dipole from the negative charge toward the positive charge.

In vector form, we can write $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$

Electric Field Intensity Due to an Electric Dipole at a Point on the Equatorial Line

An equatorial line of the electric dipole is a line perpendicular to the axial line, and it passes through a point midway between charges. Let us now suppose that the observation point P is situated on the equatorial line of the dipole such that its distance from the midpoint O of the electric dipole is r as shown in figure. Let us assume that the medium between the electric dipole and the observation point has dielectric constant K.

$$E_1 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)} \qquad \text{(along the direction } PD\text{)}$$

and $$E_2 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)} \qquad \text{(along the direction } PC\text{)}$$

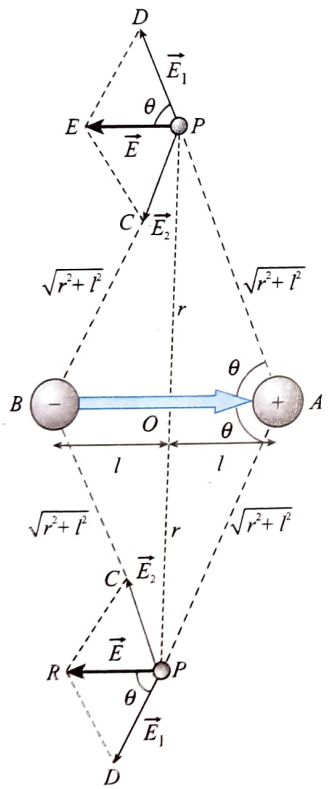
The magnitudes of E_1 and E_2 are equal but directions are different.

Net intensity is

$$E = E_1\cos\theta + E_2\cos\theta \qquad \text{(since components cancel out)}$$

$$= \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}\cos\theta + \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}\times 2\cos\theta \qquad \text{(along the direction } PR\text{)}$$



But from figure,

$$\cos\theta = \frac{OA}{PA} = \frac{OA}{(OP^2+OA^2)^{1/2}} = \frac{l}{(r^2+l^2)^{1/2}}$$

$$\therefore\quad E = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}\times\frac{2l}{(r^2+l^2)^{1/2}} = \frac{1}{4\pi\varepsilon_0 K}\times\frac{2ql}{(r^2+l^2)^{3/2}}$$

But $2ql = p$ = electric dipole moment, so

$$E = \frac{1}{4\pi\varepsilon_0 K}\times\frac{p}{(r^2+l^2)^{3/2}}$$

If l is very small as compared to r ($l \ll r$), then l^2 is negligible in comparison to r^2. Then the electric field intensity at the point P due to a short dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0 K}\times\frac{p}{(r^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0 K}\frac{p}{r^3}$$

If the dipole is placed in air or vacuum, then $K = 1$ and

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

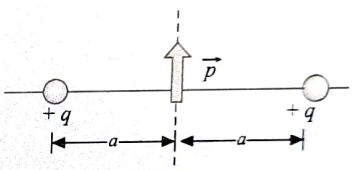
The direction of the resultant electric field is opposite to the direction of the dipole moment, so we can write

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\times\frac{\vec{p}}{r^3}$$

Note: The direction of the electric field E is opposite to the direction of $\vec{p}$, i.e., antiparallel to the axis of dipole from the positive charge toward the negative charge.

Illustration 1.48

What is the force on a dipole of dipole moment p placed as shown in figure.



Sol. Force on any q by dipole is

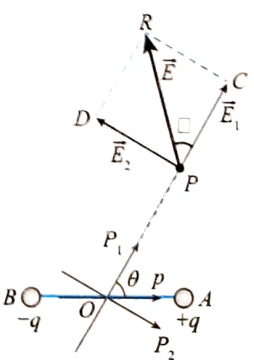
$$F = qE_{\text{dipole}} = \frac{q}{4\pi\varepsilon_0}\frac{p}{a^3} \text{ downward}$$

So from Newton's third law, force on the dipole due to both charges is

$$2F = \frac{qp}{2\pi\varepsilon_0 a^3} \text{ upward}$$

Electric Field Intensity Due to a Short Dipole at Some General Point

Let AB be a short electric dipole of dipole moment $\vec{p}$ (directed from B to A). We are interested to find the electric field at some general point P. The distance of the observation point P with respect to midpoint O of the dipole is r, and the angle made by the line OP with the axis of the dipole is θ.



We know that the dipole moment of a dipole is a vector quantity. It can be resolved into two rectangular components $\vec{p}_1$ and $\vec{p}_2$ as shown in figure, so that $\vec{p} = \vec{p}_1 + \vec{p}_2$. The magnitudes of $\vec{p}_1$ and $\vec{p}_2$ are $p_1 = p\cos\theta$ and $p_2 = p\sin\theta$.

It is clear from the figure that point P lies on the axial line of the dipole with moment $\vec{p}_1$. Hence, the magnitude of the electric field intensity $\vec{E}_1$ at P due to $\vec{p}_1$ is

$$E_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{2p\cos\theta}{r^3} \text{ (along the direction } OC) \quad ...(i)$$

Similarly, P lies on the equatorial line of dipole with moment $\vec{p}_2$. Hence, the magnitude of electric field intensity $\vec{E}_2$ at P due to $\vec{p}_2$ is

$$E_2 = \frac{1}{4\pi\varepsilon_0} \frac{p\sin\theta}{r^3} \text{ (opposite to } p_2) \quad ...(ii)$$

Hence, the resultant intensity at P is $\vec{E} = \vec{E}_1 + \vec{E}_2$.

The magnitude of $\vec{E}$ is $E = \sqrt{E_1^2 + E_2^2}$ (as $\vec{E}_1$ and $\vec{E}_2$ are mutually perpendicular).

$$E = \sqrt{\left(\frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\right)^2 + \left(\frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\right)^2}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1 + 3\cos^2\theta}$$

If the resultant field intensity vector $\vec{E}$ makes an angle ϕ with the direction of $\vec{E}$, then

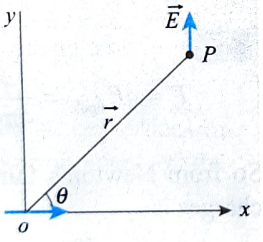
$$\tan\phi = \frac{E_2}{E_1} = \frac{p\sin\theta / 4\pi\varepsilon_0 r^3}{2p\cos\theta / 4\pi\varepsilon_0 r^3} = \frac{1}{2}\tan\theta$$

ILLUSTRATION 1.49

An electric dipole having dipole moment p is placed at origin O, such that its equator is y-axis. At point P, located at a distance r from the origin, the electric field direction is along y-axis. Find the

(a) angular position of point P (value of θ)

(b) magnitude of electric field at P.



Sol.

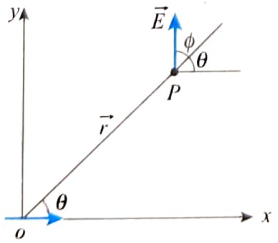
(a) The electric field $\vec{E}$ due to given dipole makes an angle ϕ with line OP such that

$$\tan\phi = \frac{1}{2}\tan\theta \quad ...(i)$$

As electric field is along y-direction. Hence $\phi = 90 - \theta$

From (i) $\tan(90-\theta) = \frac{1}{2}\tan\theta$

$\Rightarrow \tan^2\theta = 2$ or $\tan\theta = \sqrt{2}$



(b) Net electric field at P is given as

$$E = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1 + 3\cos^2\theta} \quad ...(ii)$$

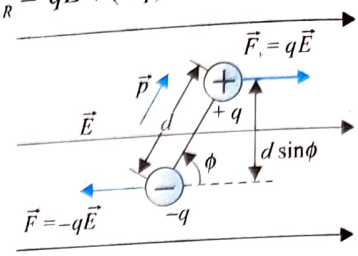
As $\tan\theta = \sqrt{2} \Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$

from (ii), $E = \frac{p}{4\pi\varepsilon r^3}\sqrt{1 + 3\left(\frac{1}{\sqrt{3}}\right)^2} \Rightarrow E = \frac{p}{2\sqrt{2}\pi\varepsilon_0 r^3}$

DIPOLE IN A UNIFORM ELECTRIC FIELD

TORQUE

When a dipole is placed in a uniform field as shown in figure, the net force on it is $F_R = q\vec{E} + (-q)\vec{E} = 0$



Hence, the net force on a dipole is zero in a uniform electric field. While the torque is

$$\tau = qE \times d\sin\phi = pE \times \sin\phi \text{ (as } p = qd)$$

$\therefore$ $\vec{\tau} = \vec{p} \times \vec{E}$ (by electric field)

and $\vec{\tau} = \vec{E} \times \vec{p}$ (if the dipole is in equilibrium)

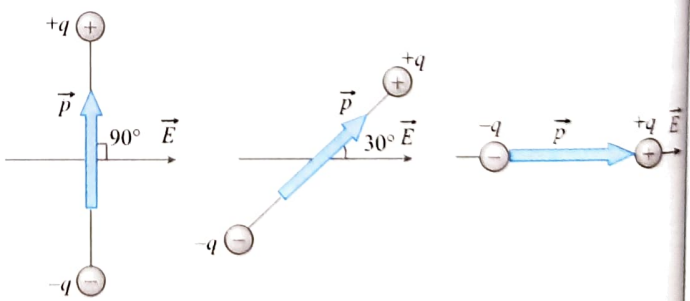
From the expression, it is clear that the couple acting on a dipole is maximum ($= pE$) when the dipole is perpendicular ($\phi = 90°$) to the field and minimum ($= 0$) when the dipole is parallel ($\phi = 0°$) or antiparallel ($\phi = 180°$) to the field.

By applying a torque, the electric field tends to align a dipole in its own direction.

ILLUSTRATION 1.50

An electric dipole of dipole moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$. Write the expression for the torque $\vec{\tau}$ experienced by the dipole. Identify two pairs of perpendicular vectors in the expression. Show diagrammatically the orientation of the dipole in the field for which the torque is **(a)** maximum, **(b)** half the maximum value, and **(c)** zero.

Sol. As $\vec{\tau} = \vec{p} \times \vec{E}$, the pair of perpendicular vectors are (i) $\vec{\tau}$ and $\vec{p}$ and (ii) $\vec{\tau}$ and $\vec{E}$. The required orientations of the dipole in the electric field are shown in figure.



$\tau = pE\sin 90°$ $= pE$ = maximum (τ_{max}) **(a)**

$\tau = pE\sin 30°$ $= (1/2)pE = 1/2\ (\tau_{max})$ **(b)**

$\tau = pE\sin 0°$ $= 0$ **(c)**

ILLUSTRATION 1.51

An electric dipole consists of two charges of 0.1 μC separated by a distance of 2.0 cm. The dipole is placed in an external field of 10^5 NC^{-1}. What maximum torque does the field exert on the dipole?

Sol. $\tau = pE\sin\theta = q\times 2a\times E\sin\theta$. The value of τ will be maximum when $\sin\theta = 1$. So

$$\tau_{max} = 10^{-7}\times 2\times 10^{-2}\times 10^{5}\times 1 = 2\times 10^{-4}\ \text{Nm}$$

ELECTRIC DIPOLE IN A NONUNIFORM ELECTRIC FIELD

If an electric dipole is placed in a nonuniform electric field, there is a net force on the dipole in addition to the torque tending to align the dipole with the field. The net force depends on

- the orientation of the dipole with respect to the electric field,
- the dipole moment of the dipole, and
- how rapidly the field varies in space.

QUALITATIVE DISCUSSION

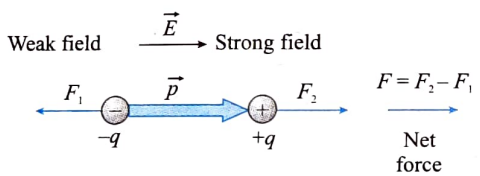
To clarify the basic ideas, let us consider only the simplest situation where

- the electric field has a constant direction and
- its magnitude increases steadily as we move in the direction of the electric field.

Further, consider the following two cases:

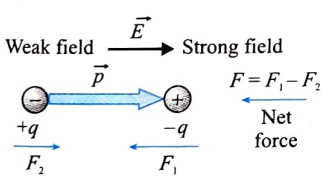
When $\vec{p}$ is parallel to $\vec{E}$, i.e., $\vec{p}\uparrow\uparrow\vec{E}$

Since the force F_2 acting on $+q$ (lying in the strong field) is greater than the force F_1 acting on $-q$ (lying in the weak field), the net force $F = F_2 - F_1$ is in the direction of increasing $\vec{E}$, i.e., toward the region of the strong field as shown in figure.



When $\vec{p}$ is antiparallel to $\vec{E}$, i.e., $\vec{p}\uparrow\downarrow\vec{E}$

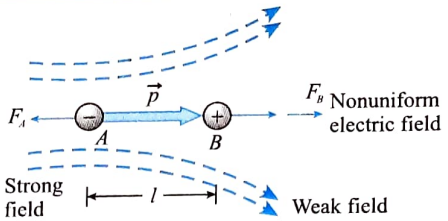
Since the force F_1 acting on $-q$ (placed in the strong field) is greater than the force F_2 acting on $+q$ (placed in the weak field), the net force $F = F_1 - F_2$ is in the direction of decreasing field, i.e., toward the region of the weak field as shown in figure.



Thus, when $\vec{p}\uparrow\uparrow\vec{E}$, the dipole moves toward the region of strong electric field (i.e., in the direction of increasing $\vec{E}$) and when $\vec{p}\uparrow\downarrow\vec{E}$, the dipole moves toward the region of weak field (i.e., in the direction of decreasing $\vec{E}$).

QUANTITATIVE DISCUSSION

An electric dipole in a nonuniform electric field is subject to the action of a resultant force because the negative and the positive charges ($-q$ and $+q$) of the dipole are located at different points in the field, having different field strengths as shown in figure.



The resultant force is

$$F = F_A - F_B = qE_A - qE_B = ql\left[\frac{E_A - E_B}{l}\right] \quad \text{...(i)}$$

But $ql = p$ is the dipole moment (l being the length of the dipole) and $(E_A - E_B)/l = \Delta E/\Delta l$ is field strength gradient (i.e., the quantity showing the change in field strength per unit length).

Thus, from Eq. (i), $F = p\dfrac{\Delta E}{\Delta l}$ ...(ii)

In differential form, $|\vec{F}| = \left|p\dfrac{d\vec{E}}{dx}\right|$

where $d\vec{E}/dx$ is the gradient of the field in the x-direction.

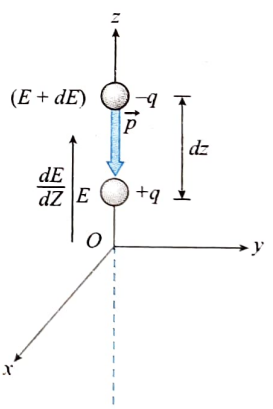
This force pulls the dipole into the region with a stronger field. It is due to this reason that a charged body attracts light objects such as pieces of paper, dust, pieces of foil, etc. By the action of the field, these objects first acquire a dipole moment and then pulled to a region where the field strength is greater, i.e., closer to the electrified body.

ILLUSTRATION 1.52

In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive z-direction, at the rate of 10^5 NC^{-1} m^{-1}. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} Cm in the negative z-direction?

Sol. Here, $p = 10^{-7}$ Cm

$$\frac{dE}{dZ} = 10^5\ \text{NC}^{-1}\ \text{m}^{-1}$$



From figure,

$$\vec{F} = q\vec{E} + (-q)(\vec{E} + d\vec{E}) - qd\vec{E}$$

$$= -q\left[\frac{d\vec{E}}{dZ}\right]dZ$$

$$= -qdZ\left[\frac{d\vec{E}}{dZ}\right] = -p\frac{d\vec{E}}{dZ}$$

(as $q\,dZ = p$ = dipole moment)

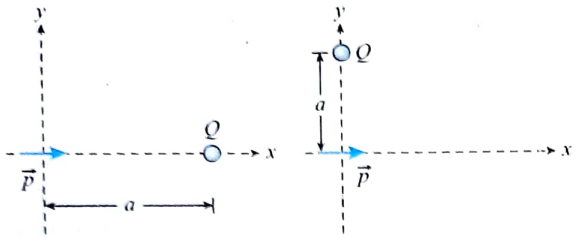
Thus, $\vec{F}$ acts in a direction opposite to $d\vec{E}/dZ$, i.e., along the negative z-direction. Further,

$$F = |\vec{F}| = p\frac{dE}{dZ} = (10^{-7}\ \text{Cm})\left(10^{-5}\ \frac{\text{N}}{\text{Cm}}\right) = 10^{-2}\ \text{N}$$

Since $\vec{p}\downarrow\uparrow\vec{E}$, $\tau = 0$ (or $\tau = pE\sin\theta = pE\sin 180° = 0$).

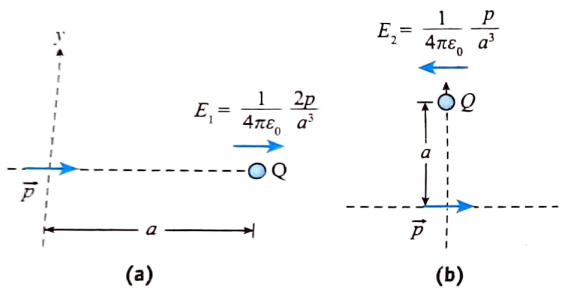
ILLUSTRATION 1.53

A positive point charge Q and a dipole of dipole moment p are arranged as shown in the given Figs. (a) and (b). Find the force acting on the dipole in each case.



Sol. In Fig. (a), the charged particle is placed at the axial position of the dipole and in Fig. (b) the charged particle is placed on the equatorial position of the dipole.

The force on the charged particle because of the dipole should be equal to the force on the dipole because of the charged particle as both the forces form the action reaction pair.



From the above figure it is clear. Force on the charged particles in Figs. (a) and (b) are

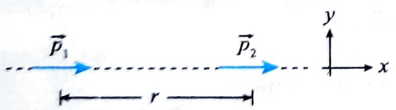
$$\vec{F}_1 = Q\vec{E}_2 = Q\cdot\frac{1}{4\pi\varepsilon_0}\frac{p}{a^3}(-\hat{i})$$

or $$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{pQ}{a^3}(-\hat{i})$$

Hence the force on the dipole $\vec{F}_{2,\text{dipole}} = \frac{1}{4\pi\varepsilon_0}\frac{pQ}{a^3}(\hat{i})$

ILLUSTRATION 1.54

Two dipoles of dipole moments $\vec{P}_1$ and $\vec{P}_2$ are aligned along the x-axis and separated by a distance r as shown in figure. Find the magnitude of force of interaction between the dipoles.



Sol. The electric field due to first dipole on the second is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

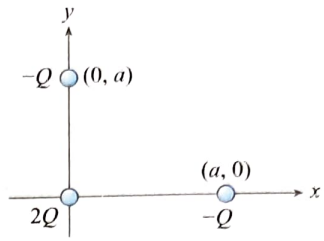
The gradient of electric field on the axis is $\frac{dE}{dr} = -\frac{1}{4\pi\varepsilon_0}\frac{6p_1}{r^4}$

The magnitude of force of interaction between the two dipoles is

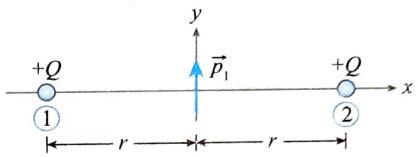
$$|F| = p_2\left|\frac{dE}{dr}\right| - = \frac{1}{4\pi\varepsilon_0}\frac{6p_1p_2}{r^4}$$

CONCEPT APPLICATION EXERCISE 1.6

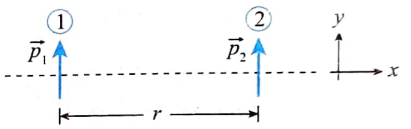
1. Find the dipole moment of the combination of the charges as shown in figure.



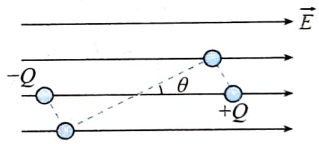
2. For the electrostatic charge system as shown in the figure, find the force acting on dipole.



3. Two dipoles of dipole moments $\vec{p}_1$ and $\vec{p}_2$ are placed as shown in figure. Find the force of interaction between the dipoles.

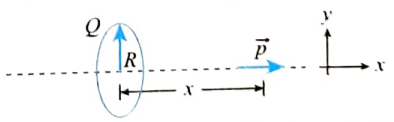


4. An electric dipole formed by two particles fixed at the end of a light rod of length l. The mass of each particle is m and the charges are $-q$ and $+q$.



The system is placed in such a way that the dipole axis is parallel to a uniform electric field E that exists in the region. The dipole is slightly rotated about its centre and released. Show that for small angular displacement, the motion is angular SHM and evaluate its time period.

5. A dipole having dipole moment 'p' is place at the axis of a charged ring having charge Q and radius R, at a distance x from the centre as shown in figure. Find the force experienced by the dipole of dipole moment $\vec{p} = p\hat{i}$ placed along the axes osf the uniformly charged ring.



ANSWERS

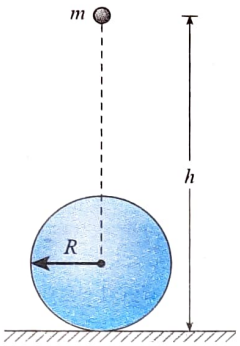
1. $\sqrt{2}Qa$
2. $\frac{1}{2\pi\varepsilon_0}\frac{pQ}{r^3}(\hat{j})$
3. $\frac{1}{4\pi\varepsilon_0}\frac{3p_1p_2}{r^4}$
4. $T = 2\pi\sqrt{\frac{ml}{2qE}}$
5. $\frac{Qp}{4\pi\varepsilon_0}\left(\frac{R^2-2x^2}{(R^2+x^2)^{5/2}}\right)$

Solved Examples

EXAMPLE 1.1

Electrically charged drops of mercury fall from an altitude h into a spherical metal vessel of radius R in the upper part of which there is a small opening. The mass of each drop is m and the charge on the drop is Q. What will be the number n of the last drop that can still enter the sphere?

Sol. Each charged drop that falls into the conducting vessel increases the charge of the vessel by Q. The accumulated charge on the conducting vessel is uniformly distributed on the surface of the vessel. The charged spherical vessel creates its own electric field that can be calculated by assuming the entire charge of the sphere to be concentrated at its centre. Let n drops have accumulated in the vessel and $(n+1)^{th}$ be in the state of equilibrium at a height h. The equilibrium exists under the influence of repulsive Coulombic force (directed upward) and force of gravity (downward). Thus, we have



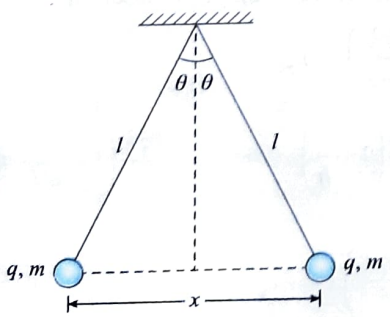
$$F_{electric} = F_{gravity}$$

$$\frac{(nQ)Q}{4\pi\varepsilon_0(h-R)^2} = mg$$

Hence $n = \dfrac{4\pi\varepsilon_0 mg(h-R)^2}{Q^2}$

EXAMPLE 1.2

Two similar balls each of mass m and charge q are hung from a common point by two silk threads each of length l. Prove that separation between the balls is $x = \left[\dfrac{q^2 l}{2\pi\varepsilon_0 mg}\right]^{1/3}$, if θ is small.



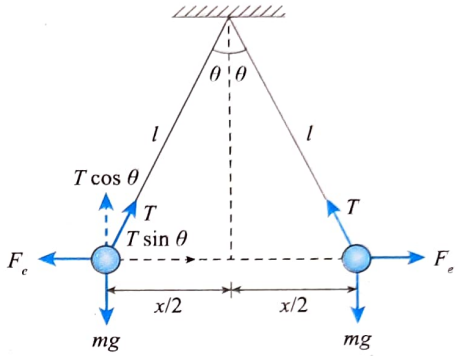
Find the rate $\dfrac{dq}{dt}$ with which the charge should leak off each sphere if their velocity of approach varies as $v = a/\sqrt{x}$, where a is a constant.

Sol. Considering the equilibrium of the charges.

$$T\sin\theta = F_e = \frac{kq^2}{x^2} \quad \ldots(i)$$

and $T\sin\theta = mg$ $\ldots(ii)$

Dividing (i) with (ii), we get $\tan\theta = \dfrac{F}{mg} = \dfrac{kq^2}{x^2 mg}$



$$\frac{x}{2l} = \frac{q^2}{4\pi\varepsilon_0 x^2 mg} \Rightarrow x = \left[\frac{q^2 l}{2\pi\varepsilon_0 mg}\right]^{1/3}$$

Now $q^2 = \dfrac{2\pi\varepsilon_0 mg\, x^3}{l}$

or $q = \sqrt{\dfrac{2\pi\varepsilon_0 mg}{l}}\, x^{3/2}$

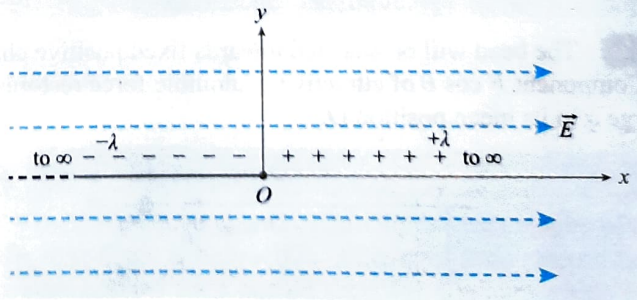
or $\dfrac{dq}{dt} = \sqrt{\dfrac{2\pi\varepsilon_0 mg}{l}}\, \dfrac{3}{2}\sqrt{x}\dfrac{dx}{dt}$

$\because$ $\left(\sqrt{x}\dfrac{dx}{dt} = \sqrt{x}(-v) = -\dfrac{\sqrt{x}a}{\sqrt{x}} = -a\right)$

or $\dfrac{dq}{dt} = -\dfrac{3}{2}a\sqrt{\dfrac{2\pi\varepsilon_0 mg}{l}}$

EXAMPLE 1.3

A uniform non-conducting rod of mass m and length l, with the charge density λ as shown in the adjacent figure, is hinged at mid point at origin so that it can rotate in a horizontal plane without any friction. A uniform electric field E exists parallel to x-axis in the entire region. Calculate the period of small oscillations of the rod.



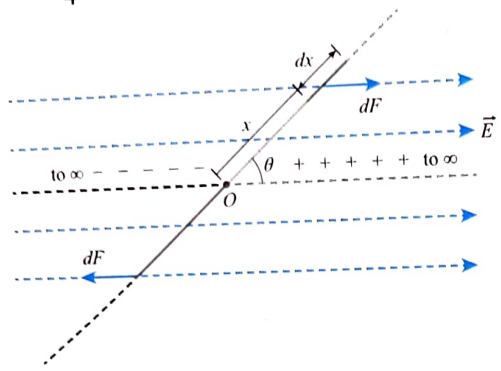
Sol. Let the rod is displaced a small angle θ with horizontal. The torque due to elemental charge

$$d\tau = dF \cdot 2x\sin\theta$$

Hence, $\tau = \int d\tau = 2\int_0^{l/2} (\lambda \cdot dx) E \cdot (x \sin\theta)$

$$= 2\lambda E \sin\theta \int_0^{l/2} x\,dx = 2\lambda E \sin\theta \cdot \frac{1}{2} \cdot \frac{l^2}{4}$$

$$\Rightarrow \quad \tau = \frac{\lambda E l^2}{4} \sin\theta$$



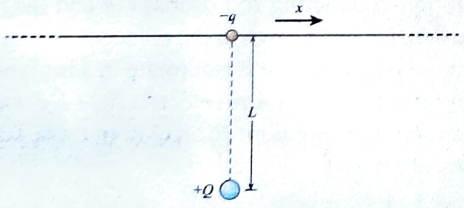
As θ is small, $\tau \approx \frac{E\lambda l^2}{4}\theta$

$$\frac{ml^2}{12}\alpha = -\frac{E\lambda l^2 \theta}{4} \Rightarrow \omega = \sqrt{\frac{3E\lambda}{m}}$$

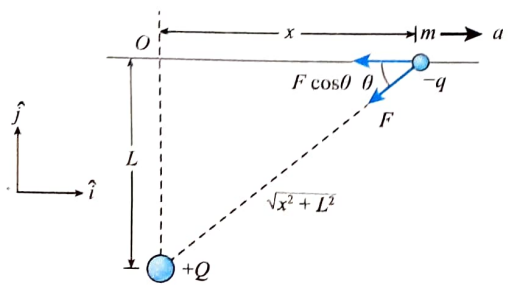
$$\therefore \quad T = 2\pi\sqrt{\frac{m}{3E\lambda}}$$

EXAMPLE 1.4

A small bead of mass m having charge $-q$ is constrained to move along a frictionless wire. A positive charge Q lies at a distance L from the wire. Initially bead is just above charge $+Q$. Show that if the bead is displaced a distance x, where $x \ll L$, and released, it will exhibit simple harmonic motion. Obtain an expression for the time period of simple harmonic motion.



Sol. The bead will be attached towards fixed positive charge Q. Component $F\cos\theta$ of attractive coulombic force restores the charge q to its mean position O.



From Newton's second law, $F\cos\theta = ma$

$$\frac{Qq}{4\pi\varepsilon_0(x^2+L^2)} \frac{x}{\sqrt{x^2+L^2}} = ma$$

$$a = \frac{Qq}{4\pi\varepsilon_0 m} \frac{x}{(x^2+L^2)^{3/2}}$$

For a small linear displacement, $x \ll L$, we can ignore x^2 in comparison to L^2.

$$a \simeq -\frac{Qq}{4\pi\varepsilon_0 mL^3} x$$

Negative sign is introduced because the direction of 'x' and the acceleration 'a' are opposite to each other,

which is equation of SHM with $\omega = \sqrt{\frac{Qq}{4\pi\varepsilon_0 mL^3}}$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mL^3}{Qq}}$$

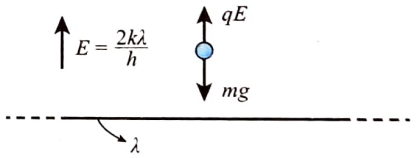
EXAMPLE 1.5

A charged particle q of mass m is in equilibrium at a height h from a horizontal infinite line charge with uniform linear charge density λ. The charge lies in the vertical plane containing the line charge. If the particle is displaced slightly (vertically). Prove that the motion of the charged particle will be simple harmonic and find its time period. (write time period of oscillation.)

Sol. From FBD of the charged particle, at equilibrium $mg = qE$

But $E = \frac{2k\lambda}{h}$

Hence $mg = q \cdot \frac{2k\lambda}{h}$ ...(i)



When particle is displaced by a small distance x in upward direction then net force in upward direction

$$F = q\left(\frac{2k\lambda}{h+x}\right) - mg$$

$$= \frac{2kq\lambda}{h}\left(1+\frac{x}{h}\right)^{-1} - mg = \frac{2kq\lambda}{h}\left(1-\frac{x}{h}\right) - mg$$

[As x is small

$$F = -\left(\frac{2kq\lambda}{h^2}\right)x = -c_0 x$$

[c is constant

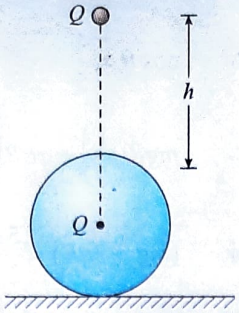
$\therefore$ $F \propto -x$ i.e., Motion is SHM

Time period of motion,

$$T = 2\pi\sqrt{\frac{m}{c}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{mh^2}{2kq\lambda}}$$

EXAMPLE 1.6

A point charge Q is fixed at the center of an insulated disc of mass M. The disc rests on a rough horizontal plane. Another charge having same value as point charge is fixed vertically above the centre of the disc at a height of h. After the disc is displaced slightly in the horizontal direction (friction is sufficient to prevent slipping). Find the period of oscillation of disc.



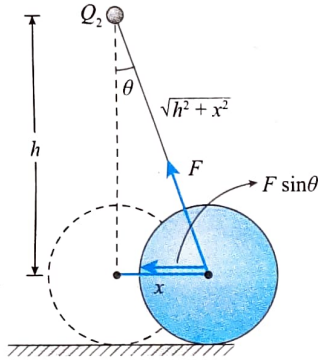
Sol. Let the radius of the disc be R. If the disc is displaced x, the corresponding angular displacement $\theta = x/R$.

The restoring torque τ about the point of contact of disc with ground,

$$\tau_p = (F \sin\theta)R$$

$$(F \sin\theta)R = I\alpha = \left[\frac{MR^2}{2} + MR^2\right]\alpha \quad \text{...(i)}$$

$$\text{where } F = \frac{Q^2}{4\pi\varepsilon_0(h^2 + x^2)} \quad \text{...(ii)}$$



$$\text{and} \qquad \sin\theta = \frac{x}{\sqrt{h^2 + x^2}} \quad \text{...(iii)}$$

From (i), (ii) and (iii), we get $\dfrac{Q^2 xR}{4\pi\varepsilon_0(h^2 + x^2)^{3/2}} = \left[\dfrac{MR^2}{2} + MR^2\right]\alpha$

$$\text{or} \qquad \alpha = \frac{Q^2 x}{6\pi\varepsilon_0 MR(h^2 + x^2)^{3/2}}$$

For $x \ll h$, $\alpha \simeq -\dfrac{Q^2 x}{6\pi\varepsilon_0 MRh^3}$ and $\alpha = \dfrac{a}{R}$

Negative sign is being introduced because angular acceleration and angular displacement are opposite to each other.

Thus, $a = -\dfrac{Q^2 x}{6\pi\varepsilon_0 Mh^3}$

Hence, $\omega = \sqrt{\dfrac{Q^2}{6\pi\varepsilon_0 Mh^3}}$

or $T = 2\pi\sqrt{\dfrac{6\pi\varepsilon_0 Mh^3}{Q^2}} = 2\pi\dfrac{h}{Q}\sqrt{6\pi\varepsilon_0 Mh}$

EXAMPLE 1.7

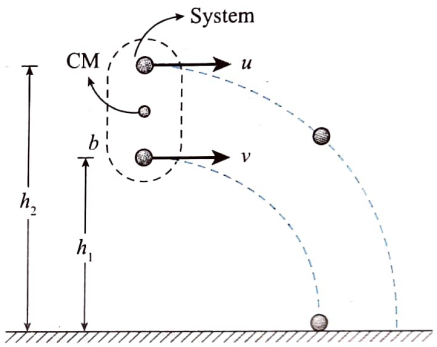
Two small balls having the same mass and charge and located on the same vertical at heights h_1 and h_2 are thrown in the same direction along the horizontal at the same velocity v. The first ball touches the ground at a horizontal distance R from the initial vertical position. At what height h_2 will the second ball be at this instant? Neglect any frictional resistance of air and the effect of any induced charges on the ground.

Sol. Let us take two balls as a system. The electrical force between the two balls is an internal force for the system. As internal forces do not affect the motion of the center of mass, the motion of center of mass takes place only under the influence of gravity. The centre of mass moves along a parabolic trajectory. Since the initial velocity of the two balls is horizontal, the time taken to travel distance x is x/v and the vertical height fallen by the centre of mass in this time is

$$y = \frac{1}{2}g\left(\frac{x}{v}\right)^2$$

Position of centre of mass at this moment from the ground is

$$h = \frac{h_1 + h_2}{2} - \frac{1}{2}g\left(\frac{x}{v}\right)^2$$



When the first ball touches the ground at a distance $x = R$, the height of the centre of mass from the ground is

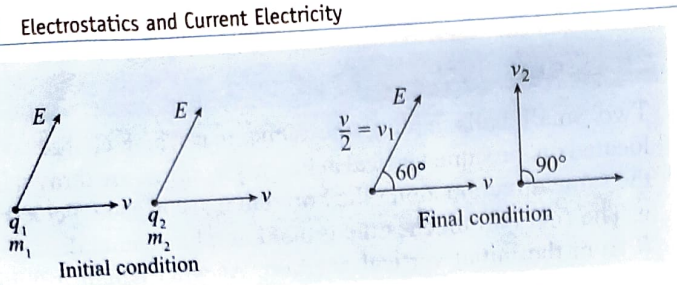
$$h = \frac{h_1 + h_2}{2} - \frac{1}{2}g\left(\frac{R}{v}\right)^2$$

As the masses of the balls are equal, the second ball will be at a height $h_2' = 2h$ at this instant. Therefore,

$$h_2' = h_1 + h_2 - g\left(\frac{R}{v}\right)^2$$

EXAMPLE 1.8

Two balls of charges q_1 and q_2 initially have a velocity of the same magnitude and direction. After a uniform electric field has been applied for a certain time, the direction of the first ball changes by 60° and the velocity magnitude is reduced by half. The direction of velocity of the second ball changes by 90°. In what ratio will the velocity of the second ball change? Determine the magnitude of the charge-to-mass ratio of the second ball if it is equal to α_1 for the first ball. Ignore the electrostatic interaction between the balls.



Sol. Let the electric field on each ball be given by

$$E = E_x\hat{i} + E_y\hat{j}$$

From the impulse–momentum equation, we have

Impulse = Change in momentum

Let the final velocities of the balls be v_1 and v_2. Noting that $v_1 = v/2$, we have

$$q_1(E_x\hat{i} + E_y\hat{j})\Delta t = m_1\left(\frac{v}{2}\cos 60°\hat{i} + \frac{v}{2}\sin 60°\hat{j}\right) - m_1 v\hat{i} \quad ...(i)$$

$$q_2(E_x\hat{i} + E_y\hat{j})\Delta t = m_2\left(v_2\cos 90°\hat{i} + v_2\sin 90°\hat{j}\right) - m_2 v\hat{i} \quad ...(ii)$$

On comparing the x- and y-components on both sides of Eq. (i), we get

$$\frac{q_1}{m_1}E_x\Delta t = -\frac{3}{4}v \text{ and } \frac{q_1}{m_1}E_y\Delta t = \frac{\sqrt{3}}{4}v \quad ...(iii)$$

Similarly, for Eq. (ii), we get

$$\frac{q_2}{m_2}E_x\Delta t = -v \text{ and } \frac{q_2}{m_2}E_y\Delta t = v_2 \quad ...(iv)$$

From Eqs. (iii) and (iv), by dividing the equations for x-components, we get

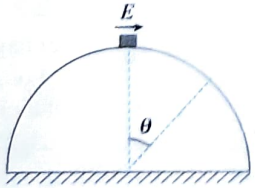
$$\frac{q_1/m_1}{q_2/m_2} = \frac{3}{4} \quad ...(v)$$

or $$\frac{q_2}{m_2} = \frac{4}{3}\frac{q_1}{m_1} = \frac{4}{3}\alpha_1$$

Also $$\frac{q_1/m_1}{q_2/m_2} = \frac{\sqrt{3}v}{4v_2} \text{ or } \frac{\sqrt{3}v}{4v_2} = \frac{3}{4} \text{ or } v_2 = \frac{v}{\sqrt{3}}$$

EXAMPLE 1.9

In a horizontal uniform electric field, a small charged disk is gently released on the top of a fixed spherical dome. The disk slides down the dome without friction and breaks away from the surface of the dome at the angular position $\theta = \sin^{-1}(3/5)$ from the vertical. Determine the ratio of the force of gravity acting on the disk to the force of its interaction with the field.



Sol. Motion from A to B,

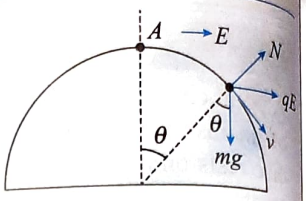
$$mgR(1 - \cos\theta) + qER\sin\theta = \frac{mv^2}{2}$$

Given $\sin\theta = \frac{3}{5}$

$$\therefore \quad \cos\theta = \frac{4}{5}$$

$$\therefore \quad mgR\left(\frac{1}{5}\right) + qER\left(\frac{3}{5}\right) = \frac{mv^2}{2}$$

or $$(mg + 3qE) = \frac{5}{2}\frac{mv^2}{R} \quad ...(i)$$



Also the equation of circular motion is

$$mg\cos\theta - N - qE\sin\theta = \frac{mv^2}{R}$$

When the particle loses contact, $N = 0$, $\sin\theta = \frac{3}{5}$, and $\cos\theta = \frac{4}{5}$. Therefore,

$$mg\left(\frac{4}{5}\right) - qE\left(\frac{3}{5}\right) = \frac{mv^2}{R}$$

or $$4mg - 3qE = 5\frac{mv^2}{R} \quad ...(ii)$$

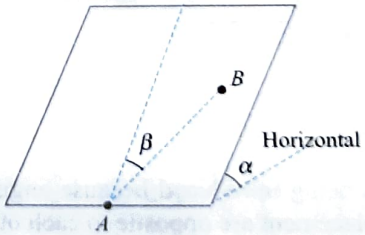
From Eqs. (i) and (ii), $$\frac{mg + 3qE}{4mg - 3qE} = \frac{1}{2}$$

or $$2mg + 6qE = 4mg - 3qE$$

or $$2mg = 9qE \text{ or } \frac{mg}{qE} = \frac{9}{2}$$

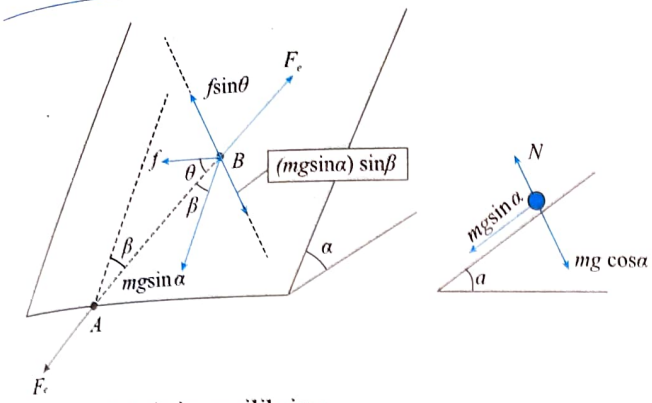
EXAMPLE 1.10

A charge particle A is fixed at the base of a uniform slope of inclination α. Another charge particle B is placed on the slope at an angular position β measured from the line of greatest slope passing through the position of the first particle. The coefficient of friction between the particle B and the slope is μ ($\mu < \tan\alpha$). For the particle at B to stay in equilibrium, what would be the maximum value of the angle β?



Sol. The electrostatic repulsive force will act along the li joining the two charges along AB. The driving forces on partic B are F_e and $mg\sin\alpha$. The friction will act in the direction opp site to the resultant of these forces. Let friction act at an angle to the line AB.

Let us analyze the forces parallel and perpendicular to AB. T forces acting on B perpendicular to AB are the components friction force $f\sin\theta$ and $(mg\sin\alpha)\sin\beta$.



If the particle is in equilibrium,

$$f\sin\theta = (mg\sin\alpha)\sin\beta$$

or $$f = \frac{mg\sin\alpha\sin\beta}{\sin\theta} \quad \text{or} \quad f \le \mu mg\cos\alpha$$

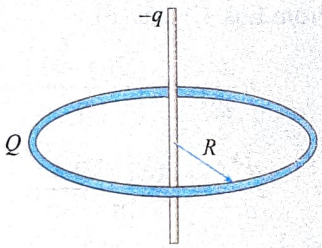
or $$\frac{mg\sin\alpha\sin\beta}{\sin\theta} \le \mu mg\cos\alpha$$

or $$\sin\beta \le \mu\cot\alpha \times \sin\theta$$

β is maximum when $\sin\theta = 1$. So $\beta \le \sin^{-1}(\mu\cot\alpha)$

EXAMPLE 1.11

In a free space, a thin rod carrying uniformly distributed negative charge $-q$ is placed symmetrically along the axis of a thin ring of radius R carrying uniformly distributed charge Q. The mass of the rod is m and length is $l = 2R$. The ring is fixed and the rod is free to move. The rod is displaced slightly along the axis of the ring and then released. Find the period T of the small amplitude oscillations of the rod.

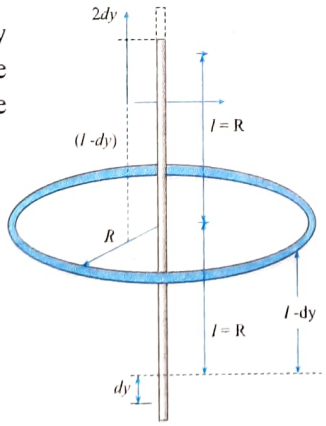


Sol. If we displace the rod by length upward, the unbalanced force will be due to length $2dy$. Hence, the unbalanced force on the element is

$$dF = (dq)E$$

$$= (\lambda\, 2dy)\frac{1}{4\pi\varepsilon_0}\frac{QR}{(R^2+R^2)^{3/2}}$$

$$a = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{2\sqrt{2}R^3 m}dy$$



Hence,

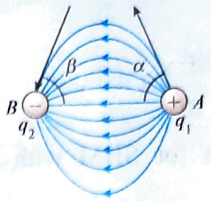
$$\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{2\sqrt{2}R^3 m}$$

or $$\frac{2\pi}{T} = \left(\frac{1}{4\pi\varepsilon_0}\frac{Qq}{2\sqrt{2}R^3 m}\right)^{1/2}$$

or $$T = 2\pi\left(\frac{4\pi\varepsilon_0 2\sqrt{2}R^3 m}{Qq}\right)^{1/2} = 4\pi R\left(\frac{2\sqrt{2}\pi\varepsilon_0 Rm}{Qq}\right)^{1/2}$$

EXAMPLE 1.12

Two charges $+q_1$ and $-q_2$ are placed at A and B, respectively. A line of force emanates from q_1 at an angle α with the line AB. At what angle will it terminate at $-q_2$?



Sol. It is the property of lines of force that their number within a tube remains unchanged, and the number of lines of force is equal to the charge. The lines of force emanating from q_1 spread out equally in all directions. Hence, lines of force per unit solid angle are $q_1/4\pi$, and the number of lines through the cone of half-angle a is

$$N_1 = \frac{q_1}{4\pi}2\pi(1-\cos\alpha)$$

because the solid angle of a cone is $2\pi(1-\cos\alpha)$. Similarly, the number of lines of force terminating on $-q_2$ at β is

$$N_2 = \frac{q_2}{4\pi}2\pi(1-\cos\beta)$$

By the property of lines of force, $N_1 = N_2$

$$\frac{q_1}{4\pi}2\pi(1-\cos\alpha) = \frac{q_2}{4\pi}2\pi(1-\cos\beta)$$

or $$\frac{q_1}{2}2\sin^2\frac{\alpha}{2} = \frac{q^2}{2}2\sin^2\frac{\beta}{2}$$

or $$\sin\frac{\beta}{2} = \sin\frac{\alpha}{2}\sqrt{\frac{q_1}{q_2}}$$

or $$\beta = 2\sin^{-1}\left[\sin\frac{\alpha}{2}\sqrt{\frac{q_1}{q_2}}\right]$$

EXAMPLE 1.13

A small point mass m has a charge q, which is constrained to move inside a narrow frictionless cylinder. At the base of the cylinder is a point mass of charge Q having the same sign as q. If the mass m is displaced by a small amount from its equilibrium position and released, it will exhibit simple harmonic motion. Find the angular frequency of the mass.

Sol. In the equilibrium position, gravitational force is balanced by Coulomb repulsive force $mg = \frac{Qq}{4\pi\varepsilon_0 y_0^2}$

If charge q is displaced in positive y-direction, such that $y \ll y_0$, from Newton's second law,

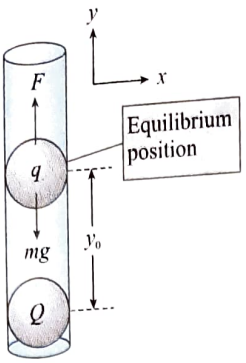
$$\frac{Qq}{4\pi\varepsilon_0(y_0+y)^2} - mg = ma$$

or $$\frac{Qq}{4\pi\varepsilon_0 y_0^2}\left[\frac{1}{(1+y/y_0)^2}\right]-mg=ma$$

or $$mg\left[1-\frac{2y}{y_0}\right]-mg=ma$$

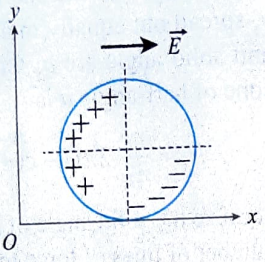
or $$a=-\frac{2g}{y_0}\cdot y \quad \text{or} \quad \frac{d^2y}{dt^2}+\frac{2g}{y_0}y=0$$

which is the equation for SHM with $\omega=\sqrt{2g/y_0}$.

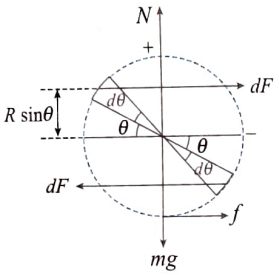


EXAMPLE 1.14

A nonconducting ring of mass m and radius R, with charge per unit length λ, is shown in figure. It is then placed on a rough nonconducting horizontal plane. At time $t = 0$, a uniform electric field $\vec{E}=E_0\hat{i}$ is switched on and the ring starts rolling without sliding. Determine the friction force (magnitude and direction) acting on the ring.



Sol. Consider a differential element subtending an angle $d\theta$ at the center and at an angle θ as shown in figure.



$$dF=\lambda R d\theta E_0$$

Net torque due to electric field is $\tau_{el}=\int d\tau$

Torque due to electrical charges is $d\tau = dF(2R\sin\theta)$

Torque equation is

$$\tau_{el}-fR=I\alpha$$

or $$\int_0^{\pi/2}\lambda R\,d\theta E_0(2R\sin\theta)-fR=mR^2\alpha$$

$$2\lambda R^2E_0-fR=mR^2\alpha \quad ...(i)$$

Force equation $F=f=ma$ ...(ii)

For rolling $a=R\alpha$ ...(iii)

Solving Eqs. (i), (ii), and (iii), we get $f=\lambda RE_0$ along the positive x-axis.

EXAMPLE 1.15

Two small identical balls lying on a horizontal plane are connected by a weightless spring. One ball (ball 2) is fixed at O, and the other (ball 1) is free. The balls are charged identically, as a result of which the spring length increases $\eta = 2$ times. Determine the change in frequency.

Sol. When the balls are uncharged

$$f_0=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where k is the force constant of the spring and m is the mass of the oscillating ball (ball 1). When the balls are charged, ball 1 will oscillate about the new equilibrium position. At the equilibrium position of ball 1,

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\eta l)^2}=k(\eta l-l)=kl(\eta-1)$$

or $$l^3=\frac{q^2}{4\pi\varepsilon_0\eta^2(\eta-1)k} \quad ...(i)$$

When ball 1 is displaced by a small distance from the equilibrium position to the right, the unbalanced force to the right is given by

$$F=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\eta l+x)^2}-k(\eta l+x-l)$$

From Newton's law, we have

$$m\frac{d^2x}{dt^2}=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\eta^2l^2}\left[1+\frac{x}{\eta l}\right]^{-2}-kl(\eta-1)-kx$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\eta^2l^2}\left[1-\frac{2x}{\eta l}\right]-kl(\eta-1)-kx$$

$$=-\left[\frac{1}{4\pi\varepsilon_0}\frac{2q^2}{\eta^3l^3}+k\right]x \quad ...(ii)$$

From Eqs. (i) and (ii),

$$m\frac{d^2x}{dt^2}=-\left[\frac{2(\eta-1)}{\eta}k+k\right]x=-\frac{3\eta-2}{\eta}kx$$

or $$\frac{d^2x}{dt^2}=-\frac{(3\eta-2)}{\eta}\frac{k}{m}x \quad \text{or} \quad a=-\left[\frac{(3\eta-2)k}{m\eta}\right]x$$

Comparing with $a=-\omega^2x$,

$$\omega^2=\frac{(3\eta-2)}{\eta}\frac{k}{m} \quad \text{or} \quad f=\frac{1}{2\pi}\sqrt{\left(\frac{3\eta-2}{\eta}\right)\frac{k}{m}}$$

or $$\frac{f}{f_0}=\sqrt{\frac{3\eta-2}{\eta}}$$

Thus, the frequency is increased $\sqrt{(3\eta-2)/\eta}$ times. Here $\eta=2$ and so frequency increases $\sqrt{2}$ times.

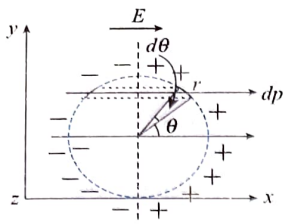
EXAMPLE 1.16

A nonconducting ring of mass m and radius r is lying at rest in the vertical xy plane on a smooth nonconducting horizontal xz plane. Charges $+q$ and $-q$ are distributed uniformly on the ring. On the two sides of the vertical diameter of the ring, a constant and uniform electric field $\vec{E}$ is set up along the x-direction. The ring is given a small rotation about the vertical diameter of the ring and released. Find the period of oscillation of the ring.

Sol. Dipole moment of the ring

$$p=\left[\int_{-\pi/2}^{\pi/2}(\lambda r d\theta 2r\cos\theta)\right], \text{ where } \lambda=\frac{q}{\pi r}$$

$$= 4r^2\lambda = \frac{4qr}{\pi}$$



From the configuration shown in figure,

$$\vec{\tau} = \vec{p} \times \vec{E} = pE \sin\theta$$

or $$mr^2 \frac{d^2\theta}{dt^2} = \frac{4qr}{\pi} E\theta$$ as θ is very small

or $$\frac{d^2\theta}{dt^2} = \frac{4q}{\pi mr} E\theta$$

Angular frequency of oscillation, $\omega = \sqrt{\frac{4qE}{\pi mr}}$

or $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\pi mr}{4qE}}$$

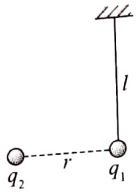
Exercises

Single Correct Answer Type

1. Five balls numbered 1, 2, 3, 4, and 5 are suspended using separate threads. The balls (1, 2), (2, 4) and (4, 1) show electrostatic attraction, while balls (2, 3) and (4, 5) show repulsion. Therefore, ball 1 must be

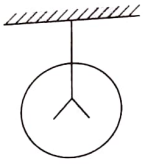
(1) negatively charged (2) positively charge
(3) neutral (4) made of metal

2. An isolated charge q_1 of mass m is suspended freely by a thread of length l. Another charge q_2 is brought near it ($r >> l$). When q_1 is in equilibrium, tension in thread will be



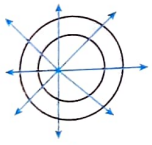
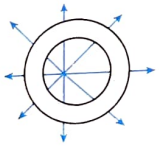
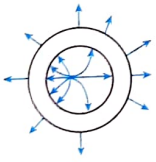
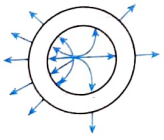
(1) mg (2) $>mg$
(3) $<mg$ (4) none of these

3. An electroscope is given a positive charge, causing its foil leaves to separate. When an object is brought near the top plate of the electroscope, the foils separate even further. We conclude

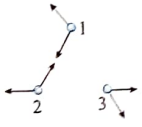


(1) that the object is positively charged
(2) that the object is electrically neutral
(3) that the object is negatively charged
(4) none of these

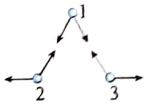
4. A metallic shell has a point charge q kept inside its cavity. Which of the following diagrams correctly represents the electric lines of forces?

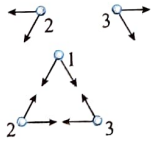
(1) 
(2) 
(3) 
(4) 

5. Which of the following four figures correctly show the forces that three charged particles exert on each other?

(I) 

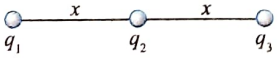
(II)

(III) 

(IV) 

(1) All of the above (2) None of the above
(3) II, III (4) II, III and IV

6. Three charged particles are placed on a straight line as shown in figure. q_1 and q_2 are fixed, but q_3 can be moved. Under the action of the forces from q_1 and q_2, q_3 is in equilibrium. What is the relation between q_1 and q_2?



(1) $q_1 = 4q_2$ (2) $q_1 = -q_2$
(3) $q_1 = -4q_2$ (4) $q_1 = q_2$

7. Two particles A and B (B is right of A) having charges 8×10^{-6} C and -2×10^{-6} C, respectively, are held fixed with separation of 20 cm. Where should a third charged particle be placed so that it does not experience a net electric force?

(1) 5 cm right of B (2) 5 cm left of A
(3) 20 cm left of A (4) 20 cm right of B

8. Two point charges repel each other with a force of 100 N. One of the charges is increased by 10% and the other is reduced by 10%. The new force of repulsion at the same distance would be

(1) 100 N (2) 121 N
(3) 99 N (4) none of these

9. Three equal charges, each $+q$, are placed on the corners of an equilateral triangle. The electric field intensity at the centroid of the triangle is

(1) kq/r^2 (2) $3kq/r^2$ (3) $\sqrt{3}kq/r^2$ (4) zero

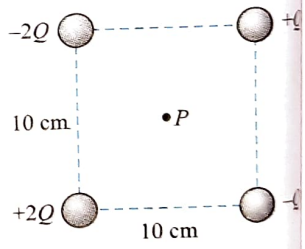
10. Two charges $Q_1 = 18\ \mu C$ and $Q_2 = -2\ \mu C$ are separated by a distance R, and Q_1 is on the left of Q_2. The distance of the point where the net electric field is zero is

(1) between Q_1 and Q_2 (2) left of Q_1 at $R/2$
(3) right of Q_2 at R (4) right of Q_2 at $R/2$

11. An electric charge q exerts a force F on a similar electric charge q separated by a distance r. A third charge $q/4$ is placed midway between the two charges. Now, the force will

(1) become $F/3$ (2) become $F/9$
(3) become $F/27$ (4) remain F

12. Four electrical charges are arranged on the corners of a 10 cm square as shown.



What would be the direction of the resulting electric field at the center point P?

(1) $\rightarrow$ (2) $\uparrow$
(3) $\leftarrow$ (4) $\downarrow$

13. The maximum electric field at a point on the axis of a uniformly charged ring is E_0. At how many points on the axis will the magnitude of the electric field be $E_0/2$.

(1) 1 (2) 2 (3) 3 (4) 4

14. Three charges $+Q_1$, $+Q_2$, and q are placed on a straight line such that q is somewhere in between $+Q_1$ and $+Q_2$. If this system of charges is in equilibrium, what should be the magnitude and sign of charge q?

(1) $\dfrac{Q_1Q_2}{\left(\sqrt{Q_1}+\sqrt{Q_2}\right)^2}$, positive (2) $\dfrac{Q_1+Q_2}{2}$, positive

(3) $\dfrac{Q_1Q_2}{\left(\sqrt{Q_1}+\sqrt{Q_2}\right)^2}$, negative (4) $\dfrac{Q_1+Q_2}{2}$, negative

15. A point charge of 100 μC is placed at $3\hat{i}+4\hat{j}$ m. Find the electric field intensity due to this charge at a point located at $9\hat{i}+12\hat{j}$ m.

(1) 8000 Vm^{-1} (2) 9000 Vm^{-1}

(3) 2250 Vm^{-1} (4) 4500 V m^{-1}

16. An oil drop, carrying six electronic charges and having a mass of 1.6×10^{-12} g, falls with some terminal velocity in a medium. What magnitude of vertical electric field is required to make the drop move upward with the same speed as it was formerly moving downward with? Ignore buoyancy.

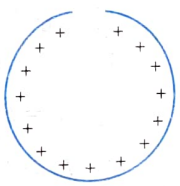
(1) 10^5 NC^{-1} (2) 10^4 NC^{-1}

(3) 3.3×10^4 NC^{-1} (4) 3.3×10^5 NC^{-1}

17. Five point charges, $+q$ each, are placed at the five vertices of a regular hexagon. The distance of the center of the hexagon from any of the vertices is a. The electric field at the center of the hexagon is

(1) $\frac{q}{4\pi\varepsilon_0 a^2}$ (2) $\frac{q}{8\pi\varepsilon_0 a^2}$ (3) $\frac{q}{16\pi\varepsilon_0 a^2}$ (4) zero

18. A ring of charge with radius 0.5 m has 0.002 π m gap. If the ring carries a charge of +1 C, the electric field at the center is



(1) 7.5×10^7 NC^{-1} (2) 7.2×10^7 NC^{-1}

(3) 6.2×10^7 NC^{-1} (4) 6.5×10^7 NC^{-1}

19. Three positive charges of equal magnitude q are placed at the vertices of an equilateral triangle of side l. How can the system of charges be placed in equilibrium?

(1) by placing a charge $Q=-q/\sqrt{3}$ at the centroid of the triangle

(2) by placing a charge $Q=q/\sqrt{3}$ at the centroid of the triangle

(3) by placing a charge $Q=q$ at a distance l from all the three charges

(4) by placing a charge $Q=-q$ above the plane of the triangle at a distance l from all the three charges

20. Three identical spheres, each having a charge q and radius R, are kept in such a way that each touches the other two. The magnitude of the electric force on any sphere due to the other two is

(1) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$ (2) $\frac{\sqrt{3}}{4\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$

(3) $\frac{\sqrt{3}}{16\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$ (4) $\frac{\sqrt{5}}{16\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$

21. Five point charges, each of value $+q$, are placed on five vertices of a regular hexagon of side L. The magnitude of the force on a point charge of value $-q$ coulomb placed at the center of the hexagon is

(1) $\frac{1}{\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$ (2) $\frac{2}{\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$

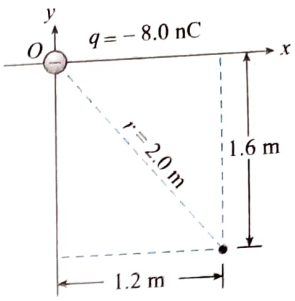
(3) $\frac{1}{2\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$ (4) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$

22. It is required to hold equal charges q in equilibrium at the corners of a square. What charge when placed at the center of the square will do this?

(1) $-\frac{q}{2}(1+2\sqrt{2})$ (2) $\frac{q}{2}(1+2\sqrt{2})$

(3) $\frac{q}{4}(1+2\sqrt{2})$ (4) $-\frac{q}{4}(1+2\sqrt{2})$

23. A point charge $q=-8.0$ nC is located at the origin. The electric field (in NC^{-1}) vector at the point $x=1.2$ m, $y=-1.6$ m, as shown in figure, is



(1) $-14.4\hat{i}+10.8\hat{j}$

(2) $-14.4\hat{i}-10.8\hat{j}$

(3) $-10.8\hat{i}+14.4\hat{j}$

(4) $-10.8\hat{i}-14.4\hat{j}$

24. A positive point charge 50 μC is located in the plane xy at a point with radius vector $\vec{r}_0=2\hat{i}+3\hat{j}$. The electric field vector $\vec{E}$ at a point with radius vector $\vec{r}=8\hat{i}-5\hat{j}$, where r_0 and r are expressed in meter, is

(1) $(1.4\hat{i}-2.6\hat{j})$ kNC^{-1} (2) $(1.4\hat{i}+2.6\hat{j})$ kNC^{-1}

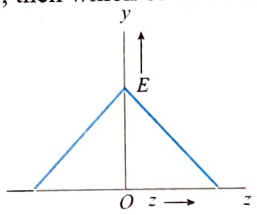
(3) $(2.7\hat{i}-3.6\hat{j})$ kNC^{-1} (4) $(2.7\hat{i}+3.6\hat{j})$ kNC^{-1}

25. Four identical charges Q are fixed at the four corners of a square of side a. The electric field at a point P located symmetrically at a distance $a/\sqrt{2}$ from the center of the square is

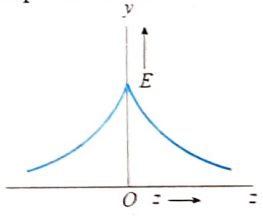
(1) $\frac{Q}{2\sqrt{2}\pi\varepsilon_0 a^2}$ (2) $\frac{Q}{\sqrt{2}\pi\varepsilon_0 a^2}$ (3) $\frac{2\sqrt{2}Q}{\pi\varepsilon_0 a^2}$ (4) $\frac{\sqrt{2}Q}{\pi\varepsilon_0 a^2}$

26. A circular ring carries a uniformly distributed positive charge and lies in the xy plane with center at the origin of the coordinate system. If at a point $(0, 0, z)$, the electric field is E, then which of the following graphs is correct?

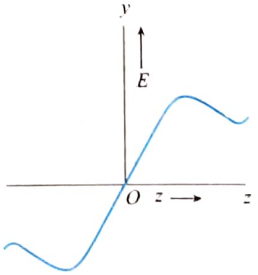
(1)



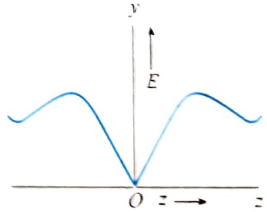
(2)



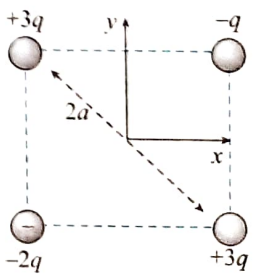
(3)



(4)



27. Four point charges are placed at the corners of a square with diagonal $2a$ as shown. What is the total electric field at the center of the square?



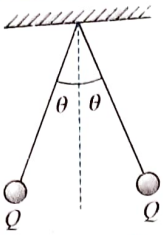
(1) kq/a^2 at an angle 45° above the $+x$-axis

(2) kq/a^2 at an angle 45° below the $-x$-axis

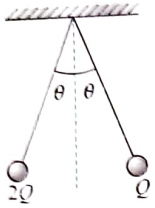
(3) $3kq/a^2$ at an angle 45° above the $-x$-axis

(4) $3kq/a^2$ at an angle 45° below the $+x$-axis

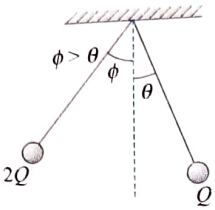
28. Two pith balls each with mass m are suspended from insulating threads. When the pith balls are given equal positive charge Q, they hang in equilibrium as shown. We now increase the charge on the left pith ball from Q to $2Q$ while leaving its mass essentially unchanged. Which of the following diagrams best represents the new equilibrium configuration?



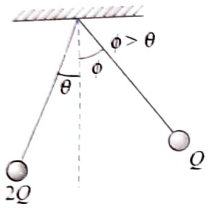
(1)



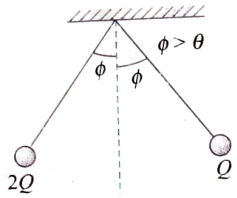
(2)



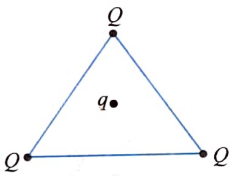
(3)



(4)



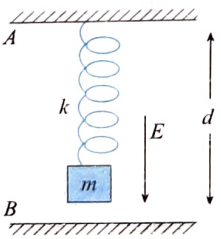
29. Three charges (each Q) are placed at the three corners of an equilateral triangle. A fourth charge q is placed at the center of the triangle. The ratio $|q/Q|$ so as to make the system in equilibrium is



(1) 1:3 (2) $1:\sqrt{3}$

(3) $\sqrt{3}:1$ (4) $2:\sqrt{3}$

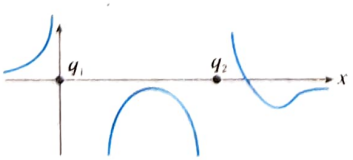
30. A block of mass m is suspended vertically with a spring of spring constant k. The block is made to oscillate in a gravitation field. Its time period is found to be T. Now the space between the plates is made gravity free, and an electric field E is produced in the downward direction. Now the block is given a charge q. The new time period of oscillation is



(1) T (2) $T+2\pi\sqrt{\dfrac{qE}{md}}$

(3) $2\pi\sqrt{\dfrac{qE}{md}}$ (4) none of the above

31. Two charges q_1 and q_2 are kept on the x-axis, and the electric field at different points on the x-axis is plotted against x. Choose the correct statement about the nature and magnitude of q_1 and q_2.



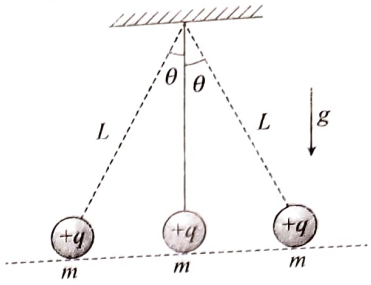
(1) q_1 is positive, q_2 is negative; $|q_1| > |q_2|$

(2) q_1 is positive, q_2 is negative; $|q_1| < |q_2|$

(3) q_1 is negative, q_2 is positive; $|q_1| > |q_2|$

(4) q_1 is negative, q_2 is positive; $|q_1| < |q_2|$

32. Three identical point charges, each of mass m and charge q, hang from three strings as shown in figure. The value of q in terms of m, L, and q is



(1) $q=\sqrt{(16/5)\pi\varepsilon_0 mg L^2\sin^2\theta\tan\theta}$

(2) $q=\sqrt{(16/15)\pi\varepsilon_0 mg L^2\sin^2\theta\tan\theta}$

(3) $q=\sqrt{(15/16)\pi\varepsilon_0 mg L^2\sin^2\theta\tan\theta}$

(4) none of these

33. A and B are two points on the axis and the perpendicular bisector, respectively, of an electric dipole. A and B are far away from the dipole and at equal distances from it. The fields at A and B are $\vec{E}_A$ and $\vec{E}_B$. Then

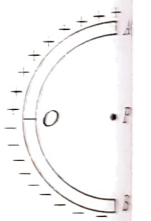
(1) $\vec{E}_A=\vec{E}_B$

(2) $\vec{E}_A=2\vec{E}_B$

(3) $\vec{E}_A=-2\vec{E}_B$

(4) $|\vec{E}_B|=\dfrac{1}{2}|\vec{E}_A|$, and $\vec{E}_A$ is perpendicular to $\vec{E}_B$.

34. A thin glass rod is bent into a semicircle of radius r. A charge $+Q$ is uniformly distributed along the upper half, and a charge $-Q$ is uniformly distributed along the lower half, as shown in figure. The electric field E at P, the center of the semicircle, is



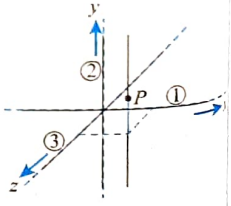
(1) $\dfrac{Q}{\pi^2\varepsilon_0 r^2}$ (2) $\dfrac{2Q}{\pi^2\varepsilon_0 r^2}$

(3) $\dfrac{4Q}{\pi^2\varepsilon_0 r^2}$ (4) $\dfrac{Q}{4\pi^2\varepsilon_0 r^2}$

35. A system consists of a thin charged wire ring of radius r and a very long uniformly charged wire oriented along the axis of the ring, with one of its ends coinciding with the center of the ring. The total charge on the ring is q, and the linear charge density on the straight wire is λ. The interaction force between the ring and the wire is

(1) $\dfrac{\lambda q}{4\pi\varepsilon_0 r}$ (2) $\dfrac{\lambda q}{2\sqrt{2}\pi\varepsilon_0 r}$ (3) $\dfrac{2\sqrt{2}\lambda q}{\pi\varepsilon_0 r}$ (4) $\dfrac{4\lambda q}{\pi\varepsilon_0 r}$

36. Find the electric field vector at P (a, a, a) due to three infinitely long lines of charges along the x-, y- and z-axes, respectively. The charge density, i.e., charge per unit length of each wire is λ.



(1) $\frac{\lambda}{3\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$ (2) $\frac{\lambda}{2\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

(3) $\frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$ (4) $\frac{\sqrt{2}\lambda}{\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

37. A particle of mass m carrying a positive charge q moves simple harmonically along the x-axis under the action of a varying electric field E directed along the x-axis. The motion of the particle is confined between $x = 0$ and $x = 2l$. The angular frequency of the motion is ω. Then which of the following is correct?

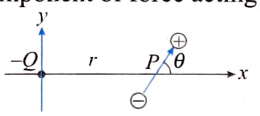
(1) Electric field right of $x = l$ is towards left and to the left of $x = l$ is towards right.

(2) Electric field right of $x = l$ is towards right and to the left of $x = l$ is towards left.

(3) Electric field to the right of origin is directed along the positive x-axis for all values of x.

(4) Electric field to the right of origin is directed along the negative x-axis for all values of x.

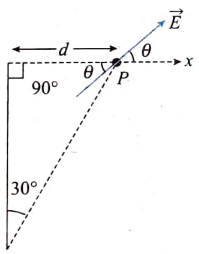
38. A point negative charge $-Q$ is placed at a distance r from a dipole with dipole moment P in the xy plane as shown in figure. The x-component of force acting on the charge $-Q$ is



(1) $-\frac{PkQ}{r}\cos\theta\hat{i}$ (2) $\frac{PkQ}{r}\cos\theta\hat{i}$

(3) $-\frac{2PkQ}{r^3}\cos\theta\hat{i}$ (4) $\frac{2PkQ}{r^3}\cos\theta\hat{i}$

39. The direction (θ) of $\vec{E}$ at point P due to uniformly charged finite rod will be



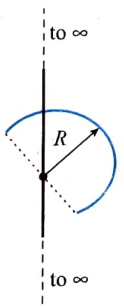
(1) 30° from the x-axis

(2) 45° from x-axis

(3) 60° from x-axis

(4) none of these

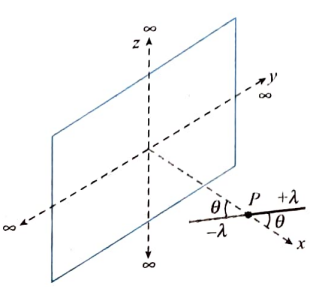
40. Find the force experienced by a semicircular rod having a charge q as shown in figure. Radius of the wire is R, and the line of charge with linear charge density λ passes through its center and is perpendicular to the plane of wire.



(1) $\frac{\lambda q}{2\pi^2\varepsilon_0 R}$ (2) $\frac{\lambda q}{\pi^2\varepsilon_0 R}$

(3) $\frac{\lambda q}{4\pi^2\varepsilon_0 R}$ (4) $\frac{\lambda q}{4\pi\varepsilon_0 R}$

41. A large sheet carries uniform surface charge density σ. A rod of length $2l$ has a linear charge density λ on one half and $-\lambda$ on the second half. The rod is hinged at the midpoint O and makes an angle θ with the normal to the sheet. The torque experienced by the rod is



(1) 0 (2) $\frac{\sigma\lambda l^2}{2\varepsilon_0}\sin\theta$

(3) $\frac{\sigma\lambda l^2}{\varepsilon_0}\sin\theta$ (4) $\frac{\sigma\lambda l}{2\varepsilon_0}$

42. A point charge $+Q$ is placed at the centroid of an equilateral triangle. When a second charge $+Q$ is placed at a vertex of the triangle, the magnitude of the electrostatic force on the central charge is 8 N. The magnitude of the net force on the central charge when a third charge $+Q$ is placed at another vertex of the triangle is

(1) zero (2) 4 N (3) $4\sqrt{2}$ N (4) 8 N

43. Five Styrofoam balls are suspended from insulating threads. Several experiments are performed on the balls and the following observations are made:

(i) Ball A repels C and attracts B.

(ii) Ball D attracts B and has no effect on E.

(iii) A negatively charged rod attracts both A and E.

An electrically neutral Styrofoam ball gets attracted if placed nearby a charged body due to induced charge. What are the charges, if any, on each ball A, B, C, D, and E?

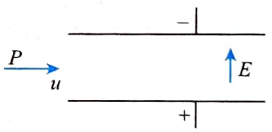
(1) $++-+O+$ (2) $+-++O$

(3) $+-+OO$ (4) $-+-OO$

44. A charged particle of mass $m = 2$ kg and charge 1 μC is projected from a horizontal ground at an angle $\theta = 45°$ with speed 10 ms^{-1}. In space, a horizontal electric field toward the direction of projection $E = 2 \times 10^7$ NC^{-1} exists. The range of the projectile is

(1) 20 m (2) 60 m (3) 200 m (4) 180 m

45. A positively charged particle P enters the region between two parallel plates with a velocity u, in a direction parallel to the plates. There is a uniform electric field in this region. P emerges from this region with a velocity v.

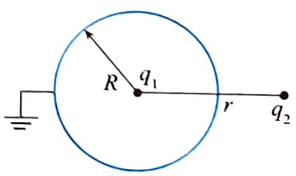


Taking C as a constant, v will depend on u as

(1) $v = Cu$ (2) $v = \sqrt{u^2 + Cu}$

(3) $v = \sqrt{u^2 + \frac{C}{u}}$ (4) $v = \sqrt{u^2 + \frac{C}{u^2}}$

46. A conducting spherical shell is earthed. A positive charge $+q_1$ is placed at the center and another small positive charge $+q_2$ is placed at a distance r from q_1 (see figure). Ignore the effect of induced charge due to q_2 on the sphere. Then the coulomb force on q_2 is



(1) zero (2) $\frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

(3) $\frac{q_1 q_2}{4\pi\varepsilon_0 (r-R)^2}$ (4) $\frac{q_1 q_2}{4\pi\varepsilon_0 (r^2-R^2)}$

47. In question 46, if the conducting spherical shell is not earthed but is neutral, then the force on q_2 is

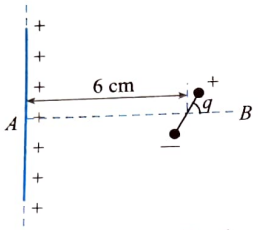
(1) zero (2) $\frac{q_1q_2}{4\pi\varepsilon_0 r^2}$

(3) $\frac{q_1q_2}{4\pi\varepsilon_0(r-R)^2}$ (4) none of these

48. A charge particle moves in a circle around an infinite line charge with the center of circle at the line and line being perpendicular to the plane of circle. Let r be the radius of circle. The velocity of the particle depends upon which power of r.

(1) 1 (2) 2 (3) –1 (4) none of these

49. An electric dipole consists of charges $\pm 2.0 \times 10^{-8}$ C separated by distance of $\pm 2.0 \times 10^{-3}$ m. It is placed at a distance of 6 cm from a line charge of linear charge density 4.0×10^{-4} Cm^{-1} as shown in figure such that the dipole makes an angle of $\theta = 60°$ with line AB. Find the force acting on the dipole.



(1) 0.012 N (2) 1.8 N

(3) 0.04 N (4) none of these

50. An electric dipole is placed at origin in the xy plane with its orientation along the positive x-axis. The direction of electric field

(1) at $(-x_0, 0, 0)$ is along the positive x-axis

(2) at $(0, y_0, 0)$ is along the negative x-axis

(3) at $(0, 0, z_0)$ is along the negative x-axis

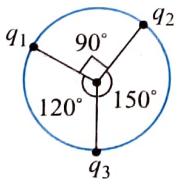
(4) all the above

51. Three charges q, q, and $-2q$ are fixed on the vertices of an equilateral triangular plate of edge length a. This plate is in equilibrium between two very large plates having surface charge density σ_1 and σ_2, respectively. Find the time period of small angular oscillation about an axis passing through its centroid and perpendicular to the plane. Moment of inertia of the system about this axis is I.

(1) $2\pi\sqrt{\frac{\varepsilon_0 I}{qa|\sigma_1-\sigma_2|}}$ (2) $2\pi\sqrt{\frac{\varepsilon_0 I}{2qa|\sigma_1-\sigma_2|}}$

(3) $2\pi\sqrt{\frac{2\varepsilon_0 I}{\sqrt{3}qa|\sigma_1-\sigma_2|}}$ (4) $2\pi\sqrt{\frac{2\varepsilon_0 I}{qa|\sigma_1-\sigma_2|}}$

52. Three positive point charges q_1, q_2, and q_3 form an isolated system. Suppose the charges have generated a property due to which like charges also attract. The charges are moving along a circle with same speed maintaining angles as shown in the figure.



The charge q_1 experiences a force f_1 due to other two charges. Similarly, q_2 experiences a force f_2 and q_3, a force f_3. The ratio $f_1 : f_2 : f_3$ is

(1) 1 : 1 : 1 (2) $q_1 : q_2 : q_3$

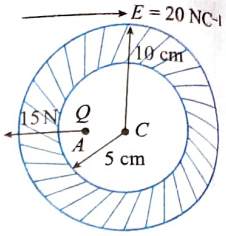
(3) 1: $\sqrt{3}$: 2 (4) This ratio cannot be calculated.

53. In the above question, the ratio of masses of the charge particles q_1, q_2, and q_3 is

(1) 1 : 1 : 1 (2) $q_1 : q_2 : q_3$ (3) 1 : $\sqrt{3}$: 2

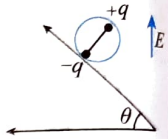
(4) This ratio cannot be calculated

54. In a conducting hollow sphere of inner and outer radii 5 cm and 10 cm, respectively, a point charge 1 C is placed at point A, that is 3 cm from the center C of the hollow sphere. An external uniform electric field of magnitude 20 NC^{-1} is also applied. Net electric force on this charge is 15 N, away from the center of the sphere as shown. Find the magnitude of net force exerted on the charge placed at point A by the induced charges on the sphere.



(1) 15 N (2) 20 N (3) 5 N (4) 35 N

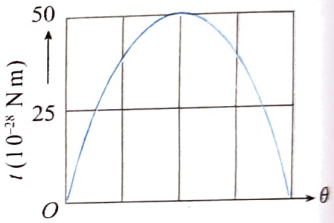
55. A wheel having mass μ has charges $+q$ and $-q$ on diametrically opposite points. It remains in equilibrium on a rough inclined plane in the presence of uniform vertical electric field E =



(1) $\mu g/q$ (2) $\mu g/2q$

(3) $\mu g \tan\theta/2q$ (4) none of these

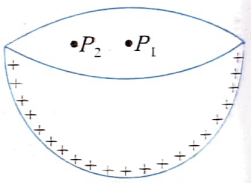
56. An electric dipole is placed in a uniform electric field $\vec{E}$ of magnitude 40 NC^{-1}. Graph shows the magnitude of the torque on the dipole versus the angle θ between the field $\vec{E}$ and the dipole moment $\vec{p}$. The magnitude of dipole moment $\vec{p}$ is equal to



(1) 2.5×10^{-28} Cm (2) 2.0×10^{-25} Cm

(3) 1.25×10^{-28} Cm (4) 5.0×10^{-28} Cm

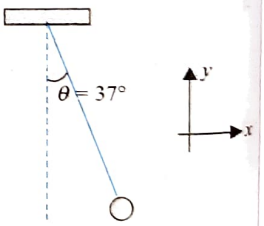
57. Consider a uniformly charged hemispherical shell shown in figure. Indicate the directions (not magnitude) of the electric field at the central point P_1 and an off-center point P_2 on the drum-head of the shell.



(1) ↑; ↖ (2) ↑; ↗

(3) ↑ ; ↑ (4) ↑;→

58. A charged ball of mass 9 kg is suspended from a string in a uniform electric field $\vec{E} = (3\hat{i} + 5\hat{j}) \times 10^5$ N/C. The ball is in equilibrium with $\theta = 37°$. If direction of electric field is reversed, find the new equilibrium position of the ball. Give your answer in terms of angle made by string with vertical. Take $g = 10$ ms^{-2}.



(1) $\tan^{-1}\left(\frac{3}{4}\right)$ (2) $\cot^{-1}\left(\frac{3}{14}\right)$

(3) $\cot^{-1}\left(\frac{3}{4}\right)$ (4) $\tan^{-1}\left(\frac{3}{14}\right)$

59. Given are four arrangements of three fixed electric charges. In each arrangement, a point labeled P is also identified. A test charge $+q$ is placed at P. All of the charges are of the same magnitude Q, but they can be either positive or negative as indicated. The charges and point P all lie on a

straight line. The distances between adjacent items, either between two charges or between a charge and point P, are all the same.

I. $\oplus\oplus\oplus\ P$ II. $\oplus\oplus\ P\ \ominus$

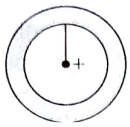
III. $\oplus\oplus\ominus\ P$ IV. $\oplus\ominus\oplus\ P$

Correct order of choices in a decreasing order of magnitude of force on P is

(1) II > I > III > IV (2) I > II > III > IV

(3) II > I > IV > III (4) III > IV > I > II

60. An electrically isolated hollow (initially uncharged) conducting sphere has a small positively charged ball suspended by an insulating rod from its inside surface (see figure).



This causes the inner surface of the sphere to become negatively charged. When the ball is centered in the sphere the electric field outside the conducting sphere is

(1) zero

(2) the same as if the sphere was not there

(3) twice what it would be if the sphere was not there

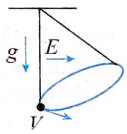
(4) equal in magnitude but opposite in direction to what it would be if the sphere was not there

61. Electric dipole of moment $\vec{p} = p\hat{i}$ is kept at a point (x, y) in an electric field $\vec{E} = 4xy^2\hat{i} + 4x^2y\hat{j}$. Find the force on the dipole.

(1) $4py^2$ (2) $4py(y^2 + 2x^2)^{1/2}$

(3) $4py^2 + 8pxy$ (4) $4py(y^2 + 4x^2)^{1/2}$

62. A positively charged particle of charge q and mass m is suspended from a point by a string of length l. In the space a uniform horizontal electric field E exists. The particle is drawn aside so that the string becomes vertical and then it is projected horizontally with velocity v such that the particle starts to move along a circle with same constant speed v. Find the speed v.



(1) $\frac{2qE}{m}\sqrt{\frac{l}{g}}$ (2) $\frac{qE}{m}\sqrt{\frac{l}{g}}$ (3) $\frac{qE}{m}\sqrt{\frac{2l}{g}}$ (4) $\frac{qE}{2m}\sqrt{\frac{l}{g}}$

63. The corners A, B, C, and D of a square are occupied by charges q, $-q$, $2Q$, and Q, respectively. The side of square is $2b$. The field at the midpoint of side CD is zero. What is the value of q/Q ?

(1) $5\sqrt{5}/2$ (2) $2\sqrt{2}/5$ (3) 2/5 (4) 5/2

64. In a region where intensity of electric field is 5 NC^{-1}, 40 lines of electric force are crossing per square meter. The number of lines crossing per square meter where intensity of electric field is 10 NC^{-1} will be

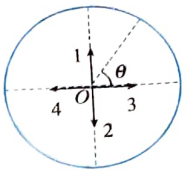
(1) 20 (2) 80 (3) 100 (4) 200

65. A tiny electric dipole of dipole moment $\vec{P} = P_0\hat{j}$ is placed at point $(l, 0)$. There exists an electric field $\vec{E} = 2ax^2\hat{i} + (2by^2 + 2cy)\hat{j}$.

(1) Force on dipole is $2P_0a\hat{i}$ (2) Force on dipole is $2P_0b\hat{j}$

(3) Force on dipole is $2P_0c\hat{j}$ (4) Force on dipole is $-2P_0c\hat{j}$

66. Charge over a nonconducting ring is distributed so that the linear charge density varies as $\lambda = \lambda_0 \sin\theta$. What is the direction of force on a charge q_0 placed at the center?



(1) along 1 if q_0 is positive and 2 if q_0 is negative

(2) along 2 if q_0 is positive and 1 if q_0 is negative

(3) along 3 if q_0 is positive and 4 if q_0 is negative

(4) along 4 if q_0 is positive and 3 if q_0 is negative

Multiple Correct Answers Type

1. A conducting ball is positively charged and another positive point charge is brought closer to the ball.

(1) The ball may attract the point charge

(2) The ball may repel the point charge

(3) There may be no force between them

(4) The ball will only repel the point charge and in no condition it can attract the point charge

2. When an electron moves in a circular path around a stationary nucleus charge at the center,

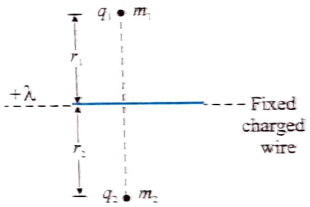
(1) the acceleration of the electron changes

(2) the velocity of the electron changes

(3) electric field due to the nucleus at the electron changes

(4) none of these

3. In the arrangement shown in figure, the two point charges are in equilibrium. The infinite wire is fixed in the horizontal plane, and the two point charges are placed one above and the other below the wire.



Considering the gravitational effect of the earth, the nature of q_1 and q_2 can be

(1) $q_1 \rightarrow$ positive, $q_2 \rightarrow$ positive

(2) $q_1 \rightarrow$ positive, $q_2 \rightarrow$ negative

(3) $q_1 \rightarrow$ negative, $q_2 \rightarrow$ negative

(4) $q_1 \rightarrow$ negative, $q_2 \rightarrow$ positive

4. Two point charges (Q each) are placed at $(0, y)$ and $(0, -y)$. A point charge q of the same polarity can move along the x-axis. Then

(1) the force on q is maximum at $x = \pm y/\sqrt{2}$

(2) the charge q is in equilibrium at the origin

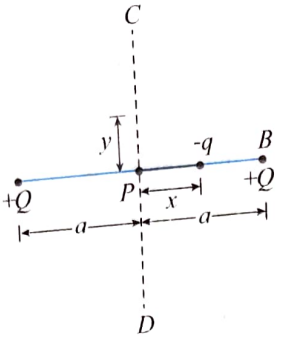
(3) the charge q performs an oscillatory motion about the origin

(4) for any position of q other than origin, the force is directed away from origin

5. Imagine a short dipole at the center of a spherical surface. If the magnitude of the electric field at a certain point on the surface of the sphere is 10 NC^{-1}, then which of the following cannot be the magnitude of the electric field anywhere on the surface of the sphere?

(1) 4 NC^{-1} (2) 8 NC^{-1} (3) 16 NC^{-1} (4) 32 NC^{-1}

6. In the arrangement shown in figure, two positive charges, $+Q$ each, are fixed. Mark the correct statement(s) regarding a third charged particle q placed at the midpoint P that can be displaced along or perpendicular to the line connecting the charges.



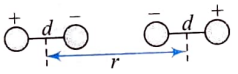
(1) The particle will perform SHM for $x << a$.

(2) The particle will oscillate about P but not harmonically for any x.

(3) The particle will perform SHM for $y << a$.

(4) The particle will oscillate about P but not harmonically for y comparable to a.

7. We have two electric dipoles. Each dipole consists of two equal and opposite point charges at the ends of an insulating rod of length d. The dipoles sit along the x-axis a distance r apart, oriented as shown in figure.
Their separation $r \gg d$. The dipole on the left:



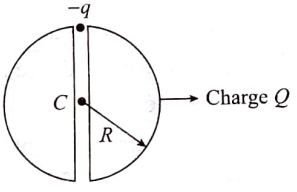
(1) will feel a force to the left

(2) will feel a force to the right

(3) will feel a torque trying to make it rotate counterclockwise

(4) will feel no torque

8. In a uniformly charged dielectric sphere, a very thin tunnel has been made along the diameter as shown in figure. A charge particle $-q$ having mass m is released from rest at one end of the tunnel. For the situation described, mark the correct statement(s). (Neglect gravity.)



(1) Charge particle will perform SHM about center of the sphere as mean position.

(2) Time period of the particle is $2\pi\sqrt{2\pi\varepsilon_0 mR^3/qQ}$.

(3) Particle will perform oscillation but not SHM.

(4) Speed of the particle while crossing the mean position is $\sqrt{qQ/4\pi\varepsilon_0 mR}$.

9. A particle of mass 2 kg and charge 1 mC is projected vertically with a velocity 10 ms^{-1}. There is a uniform horizontal electric field of 10^4 NC^{-1}. Then

(1) the horizontal range of the particle is 10 m

(2) the time of flight of the particle is 2 s

(3) the maximum height reached is 5 m

(4) the horizontal range of the particle is 0

10. When a positively charged sphere is brought near a metallic sphere, it is observed that a force of attraction exists between the two. It means

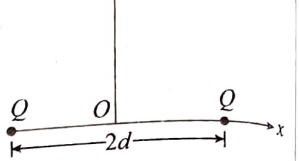
(1) the metallic sphere may be electrically neutral

(2) the metallic sphere is necessarily negatively charged

(3) nothing can be said about the charge of the metallic sphere

(4) the metallic sphere may be negatively charged

11. Consider two identical charges placed distance $2d$ apart, along the x-axis (see figure). The equilibrium of a positive test charge placed at point O midway between them is



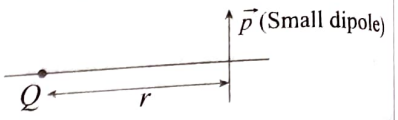
(1) stable for displacements along the x-axis

(2) neutral

(3) unstable for displacement along the y-axis

(4) stable for displacements along the y-axis

12. For the situation shown in figure (assume $r >>$ length of dipole) mark out the correct statement(s).



(1) Force acting on the dipole is zero.

(2) Force acting on the dipole is approximately $pQ/4\pi\varepsilon_0 r^3$ and is acting upward.

(3) Torque acting on the dipole is $pQ/4\pi\varepsilon_0 r^2$ in clockwise direction.

(4) Torque acting on the dipole is $pQ/4\pi\varepsilon_0 r^2$ in anti-clockwise direction.

13. A dipole is placed in xy plane parallel to the line $y = 2x$. There exists a uniform electric field along z-axis. Net force acting on the dipole will be zero. But it can experience some torque. We can show that the direction of this torque will be parallel to the line

(1) $y = 2x + 1$ (2) $y = -2x$

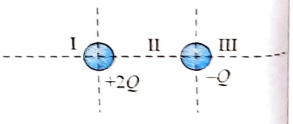
(3) $y = -\frac{1}{2}x$ (4) $y = -\frac{1}{2}x + 2$

14. A point charge μ is placed at origin. Let $\vec{E}_A, \vec{E}_B$, and $\vec{E}_C$ be the electric field at three points A (1, 2, 3), B (1, 1, −1), and C (2, 2, 2) due to charge μ. Then

(1) $\vec{E}_A \perp \vec{E}_B$ (2) $\vec{E}_A \| \vec{E}_B$

(3) $|\vec{E}_B| = 4|\vec{E}_C|$ (4) $\vec{E}_B = 16|\vec{E}_C|$

15. The figure shows two point charges $2Q$ (> 0) and $-Q$. The charges divide the line joining them in three parts I, II, and III.



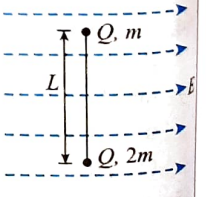
(1) Region III has a local maxima of electric field.

(2) Region I has a local minima of electric field.

(3) Equilibrium position for a test charge lies in region II.

(4) The equilibrium (if there is any) is stable for a negative test charge.

16. Two small balls A and B of positive charge Q each and masses m and 2m, respectively, are connected by a nonconducting light rod of length L. This system is released in a uniform electric field of strength E as shown.



Just after the release (assume no other force acts on the system)

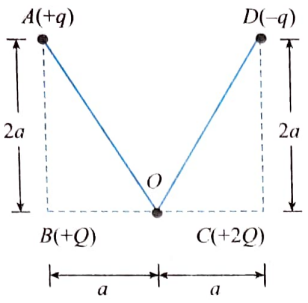
(1) rod has angular acceleration $QE/2mL$ in clockwise direction

(2) rod has angular acceleration $QE/2mL$ in anticlockwise direction

(3) acceleration of point A is $2QE/3m$ toward right

(4) acceleration of point A is QE/m toward right

17. A ball at O is in equilibrium as it is attached with two strings AO and DO, which are tied at A and D. $AO = DO = a\sqrt{5}$. The charges at A, B, C, and D are $+q$, $+Q$, $+2Q$, and $-q$, respectively. Find the correct options. The ball at O is positively charged.



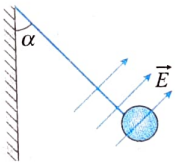
(1) The ball O cannot remain in equilibrium.

(2) If the charge at C is $+Q$, the ball will remain in equilibrium.

(3) The ball O will remain in equilibrium

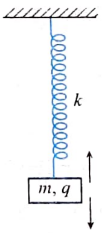
(4) If the charge at A and D and charge at B and C are interchanged, the ball will remain in equilibrium.

18. A charged cork of mass m suspended by a light string is placed in uniform electric field of strength $E = (\hat{i} + \hat{j}) \times 10^5$ NC^{-1} as shown in the figure. If in equilibrium position tension in the string is $2mg/(1+\sqrt{3})$ then angle α with the vertical can be



(1) 60° (2) 30° (2) 45° (4) 18°

19. The following figure shows a block of mass m suspended from a fixed point by means of a vertical spring. The block is oscillating simple harmonically and carries a charge q. There also exists a uniform electric field in the region. Consider four different cases. The electric field is zero in case 1, mg/q downward in case 2, mg/q upward in case 3, and $2mg/q$ downward in case 4. The speed at mean position is same in all cases. Select the correct alternative(s).



(1) Time periods of oscillation are equal in case 1 and case 3.

(2) Amplitudes of displacement are same in case 2 and case 3.

(3) The maximum elongation (increment in length from natural length) is maximum in case 4.

(4) Time periods of oscillation are equal in case 2 and case 4.

20. Three charged particles are in equilibrium under their electrostatic forces only.

(1) The particles must be collinear.

(2) All the charges cannot have the same magnitude.

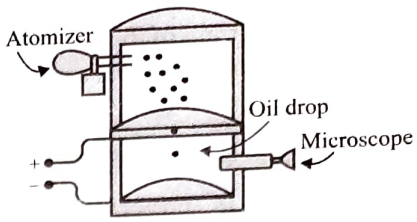
(3) All the charges cannot have the same sign.

(4) The equilibrium is unstable.

Linked Comprehension Type

For Problems 1–2

In 1909, Robert Millikan was the first to find the charge of an electron in his now-famous oil-drop experiment. In that experiment, tiny oil drops were sprayed into a uniform electric field between a horizontal pair of oppositely charged plates. The drops were observed with a magnifying eyepiece, and the electric field was adjusted so that the upward force on some negatively charged oil drops was just sufficient to balance the downward force of gravity. That is, when suspended, upward force qE just equaled mg. Millikan accurately measured the charges on many oil drops and found the values to be whole number multiples of 1.6×10^{-19} C— the charge of the electron. For this, he won the Nobel prize.



1. If a drop of mass 1.08×10^{-14} kg remains stationary in an electric field of 1.68×10^5 NC^{-1}, then the charge of this drop is

(1) 6.40×10^{-19} C (2) 3.2×10^{-19} C

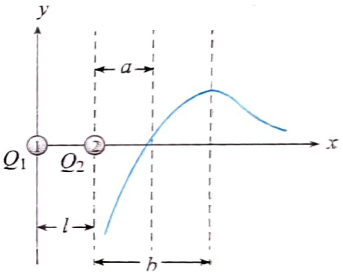
(3) 1.6×10^{-19} C (4) 4.8×10^{-19} C

2. Extra electrons on this particular oil drop (given the presently known charge of the electron) are

(1) 4 (2) 3 (3) 5 (4) 8

For Problems 3–5

Two point like charges Q_1 and Q_2 are positioned at points 1 and 2. The field intensity to the right of the charge Q_2 on the line that passes through the two charges varies according to a law that is represented schematically in figure. The field intensity is assumed to be positive if its direction coincides with the positive direction on the x-axis. The distance between the charges is l.



3. The sign of each charge Q_1 and Q_2 is

(1) +, − (2) −, + (3) +, + (4) −, −

4. The ratio of the absolute values of the charges $|Q_1/Q_2|$ is

(1) $\left(\frac{a+l}{a}\right)^2$ (2) $\left(\frac{l}{a}\right)^2$ (3) $\left(\frac{a}{a+l}\right)^2$ (4) $\left(\frac{a}{l}\right)^2$

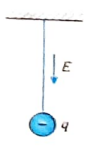
5. The value of b, where the field intensity is maximum, is

(1) $\frac{l}{(Q_1/Q_2)^{1/3}+1}$ (2) $\frac{l}{(Q_1/Q_2)^{1/3}-1}$

(3) $\frac{l}{(Q_2/Q_1)^{1/3}+1}$ (4) $\frac{l}{(Q_2/Q_1)^{1/3}-1}$

For Problems 6–7

6. A simple pendulum of mass m charged negatively to q coulomb oscillates with a time period T in a downward electric field E such that $mg > qE$. If the electric field is withdrawn, the new time period



(1) $= T$

(2) $> T$

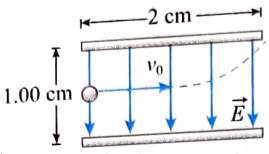
(3) $< T$

(4) any of the above three is possible

7. At equilibrium of the bob, the change in tension in the string will be (assuming rest condition)
(1) mg (2) qE (3) $2qE$ (4) $qE/2$

For Problems 8–11

An electron is projected with an initial speed $v_0 = 1.60 \times 10^6$ ms^{-1} into the uniform field between the parallel plates as shown in figure. Assume that the field between the plates is uniform and directed vertically downward, and that the field outside the plates is zero. The electron enters the field at a point midway between the plates. Mass of electron is 9.1×10^{-31} kg.



8. If the electron just misses the upper plate, the time of flight of the electron up to this instant is
(1) 1.25×10^{-9} s (2) 32.5×10^{-6} s
(3) 1.25×10^{-8} s (4) 32.5×10^{-8} s

9. For the condition of the previous situation, the magnitude of electric field is
(1) 124 NC^{-1} (2) 364 NC^{-1}
(3) 224 NC^{-1} (4) 520 NC^{-1}

10. If instead of an electron, a proton were projected with the same speed, then compare the paths travelled by the electron and the proton.
(1) The proton will hit the upper plate.
(2) The proton will hit the lower plate.
(3) The proton will not hit either plate.
(4) None of these.

11. The vertical displacement traveled by the proton as it exits the region between the plates is (mass of proton is 1.67×10^{-27} kg.)
(1) 1.6×10^{-8} m (2) 3.25×10^{-8} m
(3) 5.25×10^{-6} m (4) 2.73×10^{-6} m

For Problems 12–13

Four equal positive charges, each of value Q, are arranged at the four corners of a square of diagonal $2a$. A small body of mass m carrying a unit positive charge is placed at a height h above the center of the square.

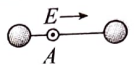
12. What should be the value of Q in order that this body may be in equilibrium?
(1) $\pi\varepsilon_0 \frac{mg}{2h}(h^2+2a^2)^{3/2}$ (2) $\pi\varepsilon_0 \frac{mg}{h}(h^2+a^2)^{3/2}$
(3) $\pi\varepsilon_0 \frac{2mg}{h}(h^2+2a^2)^{3/2}$ (4) $\pi\varepsilon_0 \frac{mg}{2h}(h^2-a^2)^{3/2}$

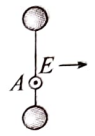
13. The type of equilibrium of the point mass is (consider only vertical displacement)
(1) stable equilibrium (2) unstable equilibrium
(3) neutral equilibrium (4) cannot be determined

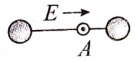
For Problems 14–16

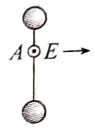
There is an insulator rod of length L and of negligible mass with two small balls of mass m and electric charge Q attached to its ends. The rod can rotate in the horizontal plane around a vertical axis crossing it at a distance $L/4$ from one of its ends.

14. At first the rod is in unstable equilibrium in a horizontal uniform electric field of field strength E. Then we gently displace it from this position. Determine the maximum velocity attained by the ball that is closer to the axis in the subsequent motion.
(1) $\sqrt{\frac{2QEL}{m}}$ (2) $\sqrt{\frac{2QEL}{5m}}$
(3) $\sqrt{\frac{QEL}{5m}}$ (4) $\sqrt{\frac{4QEL}{5m}}$

15. In what position is the rod to be set so that if displaced a little from that position, it begins a harmonic oscillation about the axis A?
(1) 

(2) 

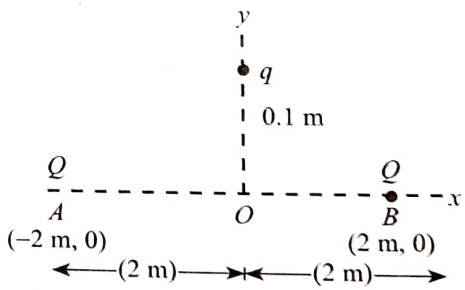
(3) 

(4) 

16. What is the time period of the SHM as mentioned in the above question?
(1) $2\pi\sqrt{\frac{mL}{QE}}$ (2) $2\pi\sqrt{\frac{2mL}{3QE}}$
(3) $2\pi\sqrt{\frac{5mL}{8QE}}$ (4) $2\pi\sqrt{\frac{5mL}{4QE}}$

For Problems 17–20

Consider a system of two equal point charges, each $Q = 8$ μC, which are fixed at points (2 m, 0) and (–2 m, 0). Another charge q is held at a point (0, 0.1 m) on the y-axis. Mass of the charge q is 91 mg. At $t = 0$, μ is released from rest and it is observed to oscillate along y-axis in a simple harmonic manner. It is also observed that at $t = 0$, the force experienced by it is 9×10^{-3} N.



17. Charge q is
(1) −8 μC (2) −6.5 μC (3) −5 μC (4) +6.5 μC

18. Amplitude of motion is
(1) 10 cm (2) 20 cm (3) 30 cm (4) 40 cm

19. Frequency of oscillation is
(1) 8 (2) 10 (3) 5 (4) 2

20. Equation of SHM (displacement from mean position) can be expressed as
(1) $y = 0.1\sin(10\pi t)$ (2) $y = 0.1\sin(10\pi t + \pi/2)$
(3) $y = 0.1\sin(5\pi t + \pi/2)$ (4) $y = 0.2\sin(5\pi t)$

For Problems 21–24

In a certain experiment to measure the ratio of charge to mass of elementary particles, a surprising result was obtained in which two particles moved in such a way that the distance between them always remained constant. It was also noticed that this two-particle system was isolated from all other particles and no force was acting on this system except the force between these two masses. After careful observation followed by intensive calculation, it was deduced that velocity of these two particles was always opposite in direction and magnitude of velocity was $10^3\ ms^{-1}$ and $2\times10^3\ ms^{-1}$ for first and second particle, respectively, and masses of these particles were 2×10^{-30} kg and 10^{-30} kg, respectively. Distance between them were 12Å ($1Å=10^{-10}$m).

21. Acceleration of the first particle was

(1) zero (2) $4\times10^{16}\ ms^{-2}$

(3) $2\times10^{16}\ ms^{-2}$ (4) $2.5\times10^{15}\ ms^{-2}$

22. Acceleration of the second particle was

(1) $5\times10^{15}\ ms^{-2}$ (2) $4\times10^{16}\ ms^{-2}$

(3) $2\times10^{16}\ ms^{-2}$ (4) zero

23. If the first particle is stopped for a moment and then released, the velocity of center of mass of the system just after the release will be

(1) $\frac{1}{3}\times10^{-30}\ ms^{-1}$ (2) $\frac{1}{3}\times10^{3}\ ms^{-1}$

(3) $\frac{2}{3}\times10^{3}\ ms^{-1}$ (4) none of these

24. Paths of the two particles were

(1) intersecting straight lines

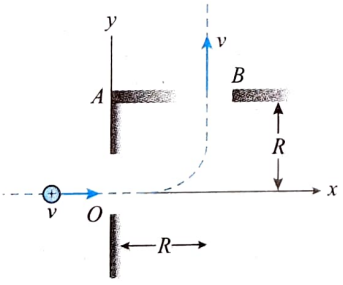
(2) parabolic

(3) circular

(4) straight line with respect to each other

For Problems 25–26

A researcher studying the properties of ions in the upper atmosphere wishes to construct an apparatus with the following characteristics: Using an electric field, a beam of ions, each having charge q, mass m, and initial velocity $v\hat{i}$, is turned through an angle of 90° as each ion undergoes displacement $R\hat{i}+R\hat{j}$. The ions enter a chamber as shown in figure and leave through the exit port with the same speed they had when they entered the chamber. The electric field acting on the ions is to have constant magnitude.



25. Suppose the electric field is produced by two concentric cylindrical electrodes not shown in the diagram, and hence is radial. What magnitude should the field have?

(1) $\frac{mv^2}{2qR}$ centered at A (2) $\frac{mv^2}{2qR}$ centered at O

(3) $\frac{mv^2}{qR}$ centered at A (4) $\frac{mv^2}{qR}$ centered at O

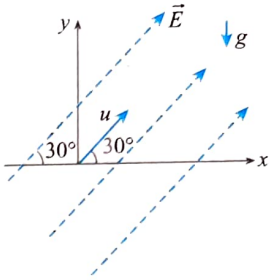
26. If the field is produced by two flat plates and is uniform in direction, what value should the field have in this case?

(1) $\frac{mv^2}{2qR}(\hat{i}+\hat{j})$ (2) $\frac{mv^2}{2qR}(-\hat{i}+\hat{j})$

(3) $\frac{2mv^2}{qR}(\hat{i}-\hat{j})$ (4) $\frac{2mv^2}{qR}(-\hat{i}+\hat{j})$

For Problems 27–28

In a region, an electric field $E = 10\ NC^{-1}$ making an angle of $\theta = 30°$ with the horizontal plane is present. Coefficient of restitution between ball and surface is $e = 0.2$. Ball has charge $q = -2C$ over it and mass $m = 2$ kg. Ball is projected at an angle of $\theta = 30°$ with the horizontal with speed $u = 10\ ms^{-1}$.



27. Horizontal distance traveled up to first rebound is

(1) $20\sqrt{3}$ m (2) $\frac{20}{\sqrt{3}}$ m (3) $\frac{20}{3\sqrt{3}}$ m (4) 2 m

28. After first impact with the ground,

(1) the ball rebounds at an angle 30° with the horizontal

(2) the ball rebounds at an angle 60° with the horizontal.

(3) the ball rebounds in the vertical direction with velocity $1\ ms^{-1}$

(4) the ball moves in the horizontal direction with velocity $1\ ms^{-1}$

For problems 29–30

We have two electric dipoles. Each dipole consists of two equal and opposite point charges at the end of an insulating rod of length d. The dipoles are placed along the x-axis at a large distance r apart oriented as shown in figure.

29. The dipole on the left

(1) will feel a force upward and a torque trying to make it rotate clockwise

(2) will feel a force upward and a torque trying to make it rotate counterclockwise

(3) will feel a force upward and no torque about its center

(4) will feel a force downward and a torque trying to make it rotate clockwise

30. The dipole on the right

(1) will feel a force downward and a torque trying to make it rotate clockwise

(2) will feel a force downward and a torque trying to make it rotate counterclockwise

(3) will feel a force upward and no torque about its center

(4) will feel no force and a torque trying to make it rotate counterclockwise

Matrix Match Type

1. Two objects A and B are charged. Let q_A = charge on A and q_B = charge on B, then match column I and column II.

Column I	Column II
i. $q_A = q_B$	a. charge transfer by conduction
ii. $q_A \geq q_B$	b. charging by friction
iii. $q_B = \frac{q_A}{2}$	c. charging by induction

Codes:

	i.	ii.	iii.
(1)	a	c	b
(2)	c	a	b
(3)	b	c	a
(4)	b	a	c

2. Let A and B are two identical objects of same mass initially are charged electrostatically by rubbing against each other. Let A has got a positive charge and B has got a negative charge, then match column I and column II.

Column I	Column II
i. Mass of A	a. increases
ii. Mass of B	b. decreases
iii. Work function of A	c. is less
iv. Work function of B	d. is more

Codes:

	i.	ii.	iii.	iv.
(1)	d	c	b	a
(2)	b	d	c	a
(3)	b	a	c	d
(4)	c	d	a	b

3. Figure (a) shows a uniformly charged ring of radius r. Its axis is along the x-axis and the ring is in the yz plane. Point P can be anywhere on the x-axis and p' in the xy plane. Figure (b) shows a uniformly charged disk of radius r. Its axis is along the x-axis and the disk is in the yz plane. Point P can be anywhere on the x-axis and p' in the xy plane.

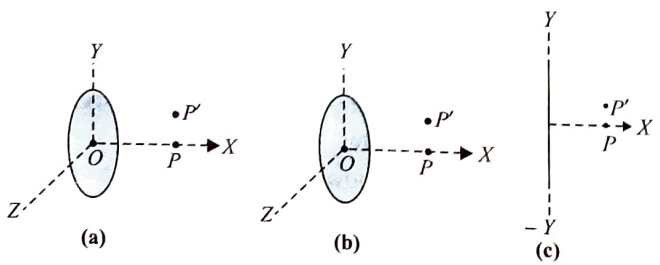
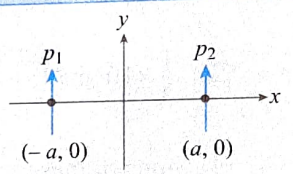
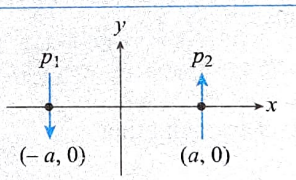
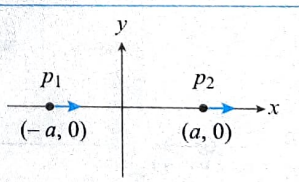
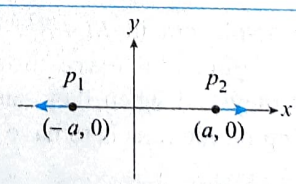


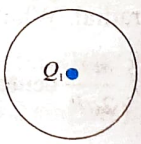
Figure (c) shows an infinite line charged uniformly placed along y-axis. P is a point on x-axis and P' in the xy plane. E_x, E_y and E_z are the components of electric field along x-, y-, and z-axis and E_p and $E_{p'}$ are net electric fields at P and P', respectively. Now match the following columns.

Column I	Column II
i. $E_x = 0,\ E_y = 0,\ E_z = 0$	a. Point P' in part (c)
ii. $E_x \neq 0, E_y = 0,\ E_z = 0$	b. Point P' in part (b)
iii. $E_x \neq 0, E_y \neq 0,\ E_z = 0$	c. Point P in part (c)
iv. $E_P = E_{P'}$	d. Point P in part (a) and (b)

4. In each situation of Column I, two electric dipoles having dipole moments $\vec{p}_1$ and $\vec{p}_2$ of same magnitude (that is $p_1 = p_2$) are placed on the x-axis symmetrically about origin in different orientations as shown. In Column II, certain inferences are drawn for these two dipoles. Then, match the different orientations of dipoles in Column I with the corresponding results in Column II.

Column I	Column II
 i. $\vec{p}_1$ and $\vec{p}_2$ are perpendicular to the x-axis as shown	a. The torque on one dipole due to other is zero.
 ii. $\vec{p}_1$ and $\vec{p}_2$ are perpendicular to the x-axis as shown	b. The potential energy of one dipole in electric field of other dipole is negative.
 iii. $\vec{p}_1$ and $\vec{p}_2$ are parallel to the x-axis as shown	c. There is one straight line in xy plane (not at infinity), which is equipotential.
 iv. $\vec{p}_1$ and $\vec{p}_2$ are parallel to the x-axis as shown	d. Electric field at origin is zero.

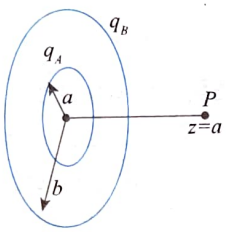
5. Inside a neutral metallic spherical shell, a charge Q_1 is placed, and outside the shell, a charge Q_2 is placed.



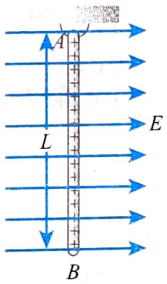
Column I	Column II
i. If Q_1 is at the center of shell,	a. electric field inside the shell remains zero
ii. If Q_1 is not at the center of shell,	b. electric field inside the shell remains non-zero
iii. If position of Q_1 is changed within the shell keeping Q_2 fixed,	c. electric field inside changes
iv. If position of Q_2 is changed outside keeping Q_1 fixed inside at any point,	d. electric field outside changes

Numerical Value Type

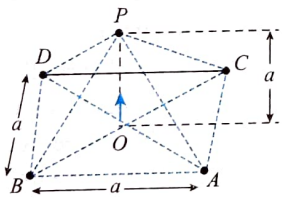
1. Two concentric rings, one of radius a and the other of radius b, have the charges $+q$, and $-(2/5)^{-3/2}q$, respectively, as shown in figure. Find the ratio b/a if a charge particle placed on the axis at $z = a$ is in equilibrium.



2. A rod AB of length L and mass m is uniformly charged with a charge Q, and it is suspended from end A as shown in figure. The rod can freely rotate about A in the plane of the figure. An electric field E is suddenly switched on in the horizontal direction due to which the rod gets turned by a maximum angle of 90°. The magnitude of E is equal to nMg/Q. Find the value of n.

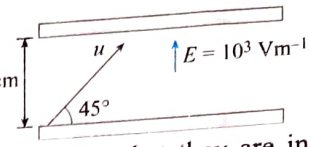


3. Four charge particles each having charge $Q = 1$ C are fixed at the corners of the base (A, B, C, and D) of a square pyramid with slant length a ($AP = BP = PD = PC = a = \sqrt{2}m$), a charge $-Q$ is fixed at point P. A dipole with dipole moment $p = 1$ Cm is placed at the center of the base and perpendicular to its plane as shown in figure. Force on the dipole due to the charge particles is $\frac{x}{4\pi\varepsilon_0}$ N. Find the value of x ?



4. A ring of radius R has charge $-Q$ distributed uniformly over it. Calculate the charge (q) that should be placed at the center of the ring such that the electric field becomes zero at a point on the axis of the ring distant R from the center of the ring. The value of q is $(Q/4)\sqrt{N}$. Find the value of N?

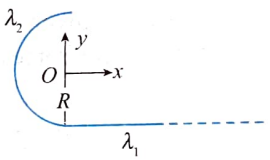
5. Two identical small equally charged conducting balls are suspended from long threads secured at one point. The charges and masses of the balls are such that they are in equilibrium when the distance between them is a (the length of the thread $L >> a$). One of the balls is then discharged. Again for the certain value of distance b ($b << l$) between the balls, the equilibrium is restored. Find the value of (a^3/b^3).



6. A particle having charge that of an electron and mass 1.6×10^{-30} kg is projected with an initial speed u at an angle 45° to the horizontal from the lower plate of a parallel plate capacitor as shown in the figure. The plates are sufficiently long and have separation 2 cm. The maximum value of velocity of particle not to hit the upper plate is $\sqrt{W} \times 10^6$ ms^{-1}. Take electric field between the plates as 10^3 Vm^{-1} directed upward. Find W.

7. Excess electrons are placed on a small lead sphere of mass 6.90 g so that its net charge is -3.20×10^{-9} C. Find the number of excess electrons (in terms of $\times 10^{10}$) on the sphere.

8. In the figure shown, find the ratio of the linear charge densities λ_1 (on semi-infinite straight wire) and λ_2 (on semi-circular part) that is, λ_1/λ_2 so that the field at O is along y direction.



9. Two small metal spheres having equal charge and mass are suspended from some point on the ceiling of a damp room with silk threads of equal length. Let center to center distance between sphere be x, $x << l$, l is length of silk thread. Due to ionization of medium, charge leaks off from each sphere and they keep on coming closer to each other at a constant rate. Let their approach velocity v varies as $v \propto x^{-1/2}$. If mass of each sphere is m then the rate at which charge varies with respect to time is $\frac{dq}{dt} \propto \frac{N}{2}\sqrt{\frac{2\pi\varepsilon_0 mg}{l}}$. Find the value of N.

10. A charged dust particle of radius 5×10^{-7} m is moving in a horizontal electric field of intensity 6.28×10^5 V/m. The surrounding medium is air with coefficient of viscosity $\eta = 1.6 \times 10^{-5}$ N-s/m^2. If this particle is moving with a uniform horizontal speed of 0.02 m/s, the number of excess electrons on the drop are $6k$. Find the value of k.

Archives

JEE Main

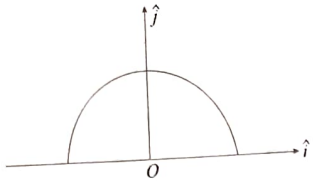
Single Correct Answer Type

1. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then the Q/q equals

(1) $-2\sqrt{2}$ (2) -1

(3) 1 (4) $-\frac{1}{\sqrt{2}}$ **(AIEEE 2009)**

2. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field E at the centre O is



(1) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (2) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

(3) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (4) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ **(AIEEE 2010)**

3. Two identically charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g/cm^3, the angle remains the same. If the density of the material of the sphere is 16 g/cm^3, the dielectric constant of the liquid is

(1) 4 (2) 3 (3) 2 (4) 1 **(AIEEE 2010)**

4. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d << l)$ apart becuase of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them

(1) $v \propto x^{-1/2}$ (2) $v \propto x^{-1}$

(3) $v \propto x^{1/2}$ (4) $v \propto x$ **(AIEEE 2011)**

5. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = q/2$ is placed at the origin. If charge q_0 is given a small displacement $(y << a)$ along the y-axis, the net force acting on the particle is proportional to

(1) $-y$ (2) $\frac{1}{y}$

(3) $-\frac{1}{y}$ (4) y **(JEE Main 2013)**

6. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then:

(1) $[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$

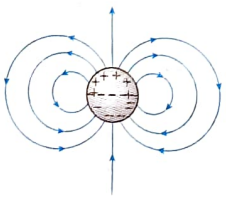
(2) $[\varepsilon_0] = [M^{-1}L^2T^1A^{-2}]$

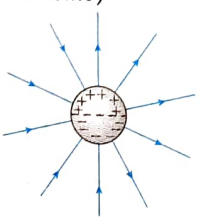
(3) $[\varepsilon_0] = [M^{-1}L^2T^1A]$

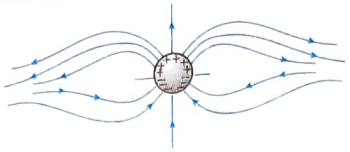
(4) $[\varepsilon_0] = [M^{-1}L^{-3}T^2A]$ **(JEE Main 2013)**

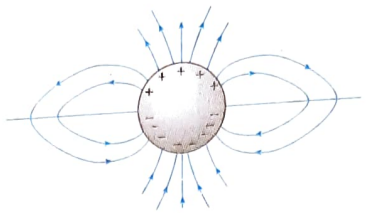
7. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in:

(figures are schematic and not drawn to scale)

(1) 

(2) 

(3) 

(4) 

(JEE Main 2015)

8. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is

(1) 60° (3) 90°

(3) 30° (4) 45° **(JEE Main 2017)**

JEE ADVANCED

Single Correct Answer Type

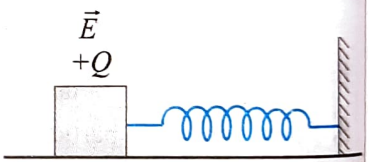
1. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $(81\pi/7) \times 10^5$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} ms^{-1}. Given $g = 9.8$ ms^{-2}, viscosity of the air is 1.8×10^{-5} Ns m^{-2}, and the density of oil is 900 kg m^{-3}, the magnitude of q is

(1) 1.6×10^{-19} C (2) 8.0×10^{-19} C

(3) 4.8×10^{-19} C (4) 8.0×10^{-10} C

(IIT-JEE, 2010)

2. A wooden block performs SHM on a frictionless surface with frequency ν_0. The block carries a charge $+Q$ on its surface. If a uniform electric field E is switched on as shown, then the SHM of the block will be



(1) of the same frequency and with shifted mean position

(2) of the same frequency and with the same mean position

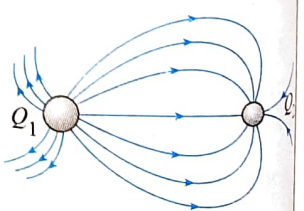
(3) of changed frequency and with shifted mean position

(4) of changed frequency and with the same mean position

(IIT-JEE, 2011)

Multiple Correct Answers Type

1. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that



(1) $|Q_1| > |Q_2|$

(2) $|Q_1| < |Q_2|$

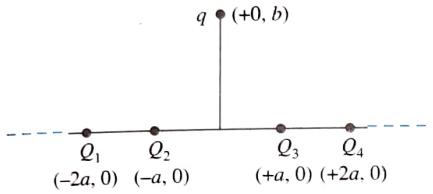
(3) at a finite distance to the left of Q_1, the electric field is zero

(4) at a finite distance to the right of Q_2, the electric field is zero

(IIT-JEE, 2010)

Matrix Match Type

1. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at $x = -2a$, $-a$, $+a$ and $+2a$, respectively. A positive charge q is placed on the positive y axis at a distance $b > 0$. Four options of the signs of these charges are given in List I. The direction of the forces on the charge q is given in List II. Match List I with List II and select the correct answer using the code given below the lists.



List I	List II
P. Q_1, Q_2, Q_3, Q_4 all positive	1. $+x$
Q. Q_1, Q_2 positive; Q_3, Q_4 negative	2. $-x$
R. Q_1, Q_4 positive ; Q_2, Q_3 negative	3. $+y$
S. Q_1, Q_3 positive; Q_2, Q_4 negative	4. $-y$

Code:

(1) P-3, Q-1, R-4, S-2

(2) P-4, Q-2, R-3, S-1

(3) P-3, Q-1, R-2, S-4

(4) P-4, Q-2, R-1, S-3

(JEE Advanced 2014)

Numerical Value Type

1. A particle, of mass 10^{-3} kg and charge 1.0 C, is initially at rest. At time $t = 0$, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin \omega t \hat{i}$ where $E_0 = 1.0\,\text{NC}^{-1}$ and $\omega = 10^3\,\text{rad s}^{-1}$. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms^{-1}, attained by the particle at subsequent times is..... .

(JEE Advanced 2018)

Answers Key

EXERCISES

Single Correct Answer Type

1. (3)	**2.** (2)	**3.** (1)	**4.** (3)	**5.** (3)
6. (3)	**7.** (4)	**8.** (3)	**9.** (4)	**10.** (4)
11. (4)	**12.** (2)	**13.** (4)	**14.** (3)	**15.** (2)
16. (3)	**17.** (1)	**18.** (2)	**19.** (1)	**20.** (3)
21. (4)	**22.** (3)	**23.** (3)	**24.** (3)	**25.** (2)
26. (3)	**27.** (2)	**28.** (4)	**29.** (2)	**30.** (1)
31. (3)	**32.** (1)	**33.** (3)	**34.** (1)	**35.** (1)
36. (2)	**37.** (1)	**38.** (3)	**39.** (1)	**40.** (2)
41. (2)	**42.** (4)	**43.** (3)	**44.** (1)	**45.** (4)
46. (1)	**47.** (2)	**48.** (4)	**49.** (3)	**50.** (4)
51. (3)	**52.** (3)	**53.** (3)	**54.** (4)	**55.** (2)
56. (3)	**57.** (3)	**58.** (4)	**59.** (3)	**60.** (2)
61. (4)	**62.** (2)	**63.** (1)	**64.** (2)	**65.** (3)
66. (2)				

Multiple Correct Answers Type

1. (1),(2),(3)	**2.** (1),(2),(3)	**3.** (2),(3)
4. (1),(2),(4)	**5.** (1),(4)	**6.** (3),(4)
7. (1),(4)	**8.** (1),(2),(4)	**9.** (1),(2),(3)
10. (1),(4)	**11.** (1),(3)	**12.** (2),(3)
13. (3),(4)	**14.** (1),(3)	**15.** (1),(4)
16. (1),(4)	**17.** (3),(4)	**18.** (1),(2)
19. (1),(2),(3),(4)	**20.** (1),(2),(3),(4)	

Linked Comprehension Type

1. (1)	**2.** (1)	**3.** (1)	**4.** (1)	**5.** (2)
6. (3)	**7.** (2)	**8.** (3)	**9.** (2)	**10.** (3)
11. (4)	**12.** (2)	**13.** (1)	**14.** (3)	**15.** (1)
16. (4)	**17.** (3)	**18.** (1)	**19.** (3)	**20.** (2)
21. (4)	**22.** (1)	**23.** (3)	**24.** (3)	**25.** (3)
26. (2)	**27.** (3)	**28.** (3)	**29.** (2)	**30.** (2)

Matrix Match Type

1. (4) **2.** (3)

3. i. → d., ii. → a., c., d.; iii. → b.; iv. → a., c.

4. i. → a., c.; ii. → a., b., c., d.; iii. → a., b., c.; iv. → a., d.

5. i. → b.; ii. → b.; iii. → b., c.; iv. → b., d.

Numerical Value Type

1. (2)	**2.** (1)	**3.** (6)	**4.** (2)	**5.** (4)
6. (8)	**7.** (2)	**8.** (2)	**9.** (3)	**10.** (5)

ARCHIVES

JEE Main

Single Correct Answer Type

1. (1)	**2.** (3)	**3.** (3)	**4.** (1)	**5.** (4)
6. (1)	**7.** (1)	**8.** (1)		

JEE Advanced

Single Correct Answer Type

1. (1) **2.** (2)

Multiple Correct Answers Type

1. (1), (4)

Matrix Match Type

1. (1)

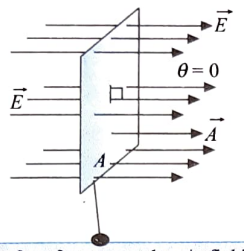
Numerical Value Type

1. (2)

2 Electric Flux and Gauss's Law

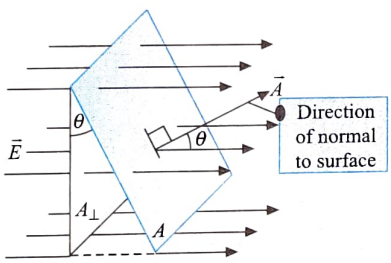
ELECTRIC FLUX

The concept of electric field lines was described qualitatively in the previous chapter. We now treat electric field lines in a more quantitative way. Consider an electric field that is uniform in both magnitude and direction as shown in figure.



Surface face on to electric field $\vec{E}$ and $\vec{A}$. Parallel angle between $\vec{E}$ and $\vec{A}$ is $\theta = 0$; flux $\Phi_E = \vec{E} \cdot \vec{A} = EA$

The field lines penetrate a rectangular surface of area A, whose plane is oriented perpendicular to the field. As we know that the number of lines per unit area (in other words, the **line density**) is proportional to the magnitude of the electric field, the total number of lines penetrating the surface is proportional to the product EA. This product of the magnitude of the electric field E and surface area A perpendicular to the field is called the **electric flux** Φ_E (uppercase Greek letter phi) given by



The number of field lines that go through area A is the same as the number of lines that go through area $A_\perp$.

$$\Phi_E = EA \qquad \text{...(i)}$$

From the SI units of E and A, we see that Φ_E has the units of Nm^2C^{-1}. Electric flux is proportional to the number of electric field lines penetrating some surface.

If the surface under consideration is not perpendicular to the field, the flux through it must be less than that given by Eq. (i). Consider figure, where the normal to the surface of area A is at an angle θ to the uniform electric field. Notice that the number of lines that cross area A is equal to the number of lines that cross area $A_\perp$, which is a projection of area A onto a plane oriented perpendicular to the field. Figure shows that the two areas are related by $A_\perp = A \cos\theta$. Because the flux through A equals the flux through $A_\perp$, the flux through A is

$$\Phi_E = EA_\perp = EA\cos\theta \qquad \text{...(ii)}$$

We assumed a uniform electric field in the preceding discussion. In more general situations, the electric field may vary over a large surface. Therefore, the definition of flux given by Eq. (ii) has meaning only for a small element of the area over which the field is approximately constant. Consider a general surface divided into a large number of small elements, each of area ΔA. It is convenient to define a vector $\Delta\vec{A}_i$, whose magnitude represents the area of the ith element of the large surface and whose direction is defined to be perpendicular to the surface element as shown in figure. The electric field $\vec{E}_i$ at the location of this element makes an angle θ_i with the vector $\Delta\vec{A}_i$. The electric flux $\Phi_{E,i}$ through this element is

$$\Phi_{E,i} = E_i \Delta A_i \cos\theta_i = \vec{E}_i \cdot \Delta\vec{A}_i \qquad \text{...(iii)}$$

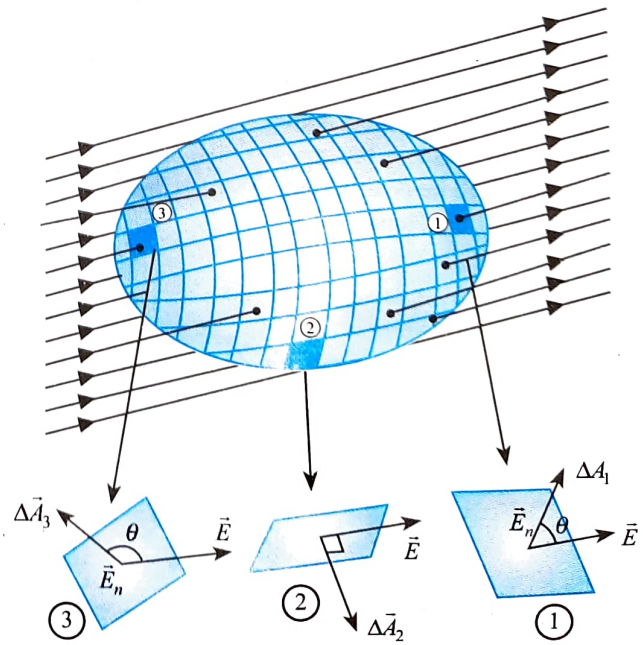
Summing the contributions of all elements gives an approximation to the total flux through the surface:

$$\Phi_E \approx \Sigma \vec{E}_i \cdot \Delta\vec{A}_i$$

If the area of each element approaches zero, the number of elements approaches infinity and the sum is replaced by an integral. Therefore, the general definition of electric flux is

$$\Phi_E \equiv \int_{\text{Surface}} \vec{E} \cdot d\vec{A} \qquad \text{...(iv)}$$

Equation (iv) is a surface integral, which means it must be evaluated over the surface in question. In general, the value of Φ_E depends both on the field pattern and on the surface.



Element 1	Element 2	Element 3
In figure, the field lines are crossing the surface from inside to outside and $\theta < 90°$; hence, the flux $\Phi_{E,1} = \vec{E} \cdot \Delta\vec{A}_1$ through this element is positive.	In figure, the field lines graze the surface (perpendicular to $\Delta\vec{A}_2$); therefore, $\theta = 90°$ and the flux is zero.	In figure where the field lines are crossing the surface from outside to inside, $180° > \theta > 90°$, and the flux is negative because cos θ is negative.

We are often interested in evaluating the flux through a closed surface, defined as a surface that divides space into an inside and an outside region so that one cannot move from one region to the other without crossing the surface. The surface of a sphere, for example, is a closed surface.

ILLUSTRATION 2.1

A point charge Q is placed at the corner of a square of side a. Find the flux through the square.

Sol. The electric field due to Q at any point on the square will be along the plane of the square and perpendicular to the area vector ($\vec{A}$). As $\phi = \vec{E} \cdot \vec{A}$ and $\vec{E} \perp \vec{A}$, hence $\phi = 0$. In other words, we can say that no line is crossing the square and so flux is 0.

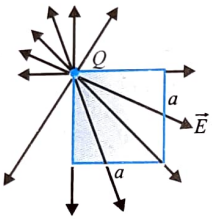
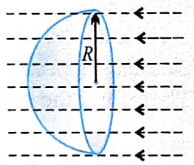


ILLUSTRATION 2.2

Find out the flux through the curved surface of a hemisphere of radius R if it is placed in a uniform electric field E as shown in figure.



Sol. The number of electric lines passing through the base of the hemisphere is the same as that of the lines passing through the hemisphere.

The flux associated with the base of the hemisphere is $\phi = -E\pi R^2$ (negative as it is the incoming flux). Hence, the same amount of flux will be associated with the curved surface, but the sign of the flux will be positive as it is as outgoing flux. Hence,

$$\phi_{curve} = E\pi R^2$$

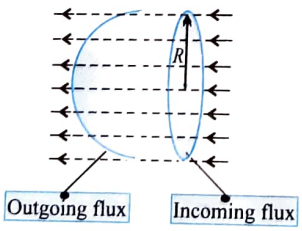
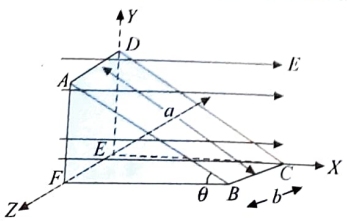


ILLUSTRATION 2.3

Find the flux of the electric field through each of the five surfaces of the inclined plane as shown in figure. What is the total flux through the entire closed surface?

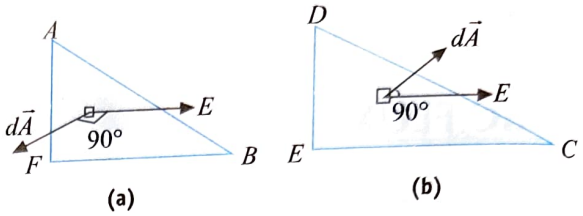


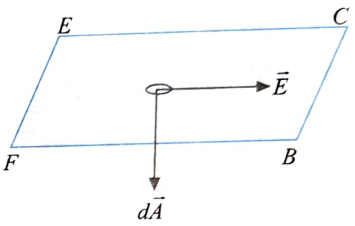
Sol. Note that flux through the faces ABF, CDE, and $BCEF$ is zero [refer Fig. (a)]. Area vector of face ABF points in the positive z-direction, area vector of CDF points in the negative z-direction, and area vector of $BCEF$ points in the negative y-direction. In all the three cases, field E is normal to area vector.

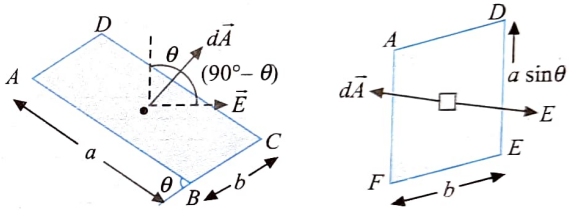
Flux through face $ABCD$ [Fig. (b)]:

Magnitude of area vector of face $ABCD = ab$

$$\phi_E = \vec{E} \cdot \vec{A} = E(ab)\cos(90° - \theta) = Eab \sin\theta$$







Flux through face $ADEF$ [Fig. (e)]:

Magnitude of area vector of face $ADEF = (a \sin\theta)b = ab \sin\theta$

$$\phi_E = E\cos 180° (ab \sin\theta) = -Eab \sin\theta$$

Thus, we obtain

$$(\phi_E)_{ABF} = 0, \quad (\phi_E)_{CDE} = 0, \quad (\phi_E)_{BCEF} = 0,$$

$$(\phi_E)_{ABCD} = +Eab\sin\theta \text{ and } (\phi_E)_{ADEF} = -Eab\sin\theta$$

Flux is a scalar quantity, therefore the total flux is algebraic sum of flux through each surface.

$$\phi_{total} = (\phi_E)_{ABF} + (\phi_E)_{CDE} + (\phi_E)_{BCEF} + (\phi_E)_{ABCD} + (\phi_E)_{ADEF}$$

$$= 0 + 0 + 0 + Eab \sin\theta - Eab \sin\theta = 0$$

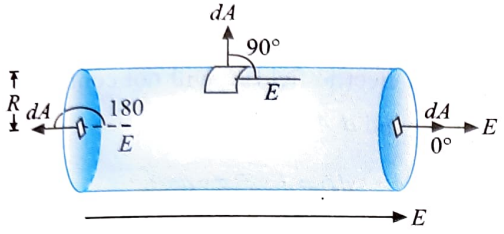
- Note that the contribution to the flux for a closed surface is positive for the surface where the field is directed out ($ABCD$) and negative for the surface where the field is directed into the surface ($ADEF$).
- The net flux for this closed surface can also be seen to be zero from the examination of the field lines. If the field is uniform, the number of lines that enter the closed surface equals the number of lines that come out.
- The flux of a constant vector through any closed surface is zero.

ILLUSTRATION 2.4

Consider a cylindrical surface of radius R and length l in a uniform electric field E. Compute the electric flux if the axis of the cylinder is parallel to the field direction.

Sol. We can divide the entire surface into three parts, right and left plane faces and curved portion of its surface. Hence, the surface integral consists of the sum of the three terms:

$$\phi_E = \oint E \cdot dA = \oint_{\text{left end}} E \cdot dA + \oint_{\text{right end}} E \cdot dA + \oint_{\text{curved}} E \cdot dA$$



All the area elements on the left end and electric field E are at an angle of 180°.

$$(\phi_E)_{\text{left end}} = \oint_{\text{left end}} \vec{E} \cdot \vec{dA} = \oint_{\text{left end}} E\, dA \cos 180°$$

$$= -E \oint_{\text{left end}} dA = -E\pi R^2$$

Note that E is constant over the entire plane surface of left end; therefore, we take it out from the integral.

Similarly, all the area elements on the right end are parallel to electric field E, i.e., angle is 0°.

$$(\phi_E)_{\text{right end}} = \oint_{\text{right end}} \vec{E} \cdot \vec{dA} = \oint_{\text{right end}} E\, dA \cos 0°$$

$$= +E \oint_{\text{right end}} dA = E\pi R^2$$

Finally, at every point on the curved surface the area vectors are perpendicular to the direction of the electric field. Thus,

$$(\phi_E)_{\text{curved}} = \oint_{\text{curved surface}} \vec{E} \cdot \vec{dA} = \oint_{\text{curved surface}} E\, dA(\cos 90°) = 0$$

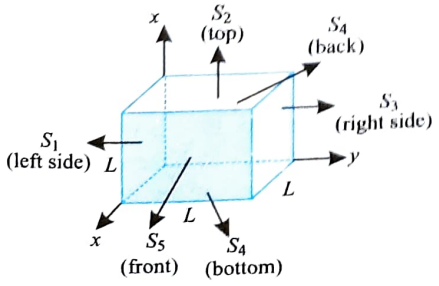
$$\text{Total flux} = (\phi_E)_{\text{right end}} + (\phi_E)_{\text{left end}} + (\phi_E)_{\text{curved surface}}$$

$$= (+E\pi R^2) + (-E\pi R^2) + 0 = 0$$

Hence, we see that in a uniform field the flux through a closed surface is zero. This is true for any shape of closed surface.

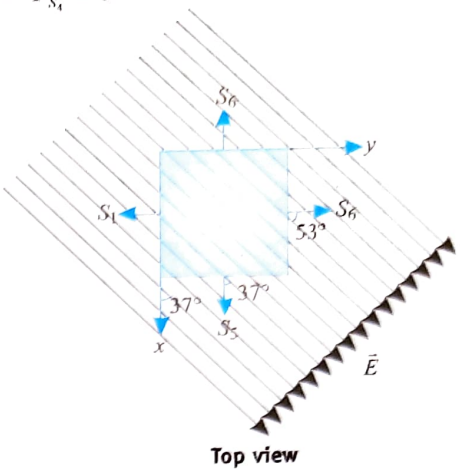
ILLUSTRATION 2.5

The cube as shown in figure has sides of length L = 10 cm. The electric field is uniform, has a magnitude $E = 4 \times 10^3$ N/C, and is parallel to the xy-plane at an angle of 37° measured from the $+x$-axis toward the $+y$-axis. What is the electric flux through each of the six cube faces S_1, S_2, S_3, S_4, S_5 and S_6?



Sol. The electric field is parallel to the xy-plane. For surfaces S_2 and S_4 the electric field and area vectors are perpendicular to each other hence flux through S_2 and S_4 should be zero.

Hence, $\Phi_{S_2} = \Phi_{S_4} = 0$



Top view

The flux through S_1: $\Phi_{S_1} = \vec{E} \cdot \vec{A} = EA\cos\theta$

$$= (4\times10^3)(0.1)\cos(180° - 53°)$$

$$= -(4\times10^3)(0.1)\cos 53° = -24 \text{Nm}^2/\text{C}$$

The flux through S_2: $\Phi_{S_2} = \vec{E} \cdot \vec{A} = EA\cos\theta$

$$= (4\times10^3)(0.1)\cos 53°$$

$$= 24 \text{Nm}^2/\text{C}$$

The flux through S_5: $\Phi_{S_5} = \vec{E} \cdot \vec{A} = EA\cos\theta$

$$= (4\times10^3)(0.1)\cos 37°$$

$$= 32 \text{Nm}^2/\text{C}$$

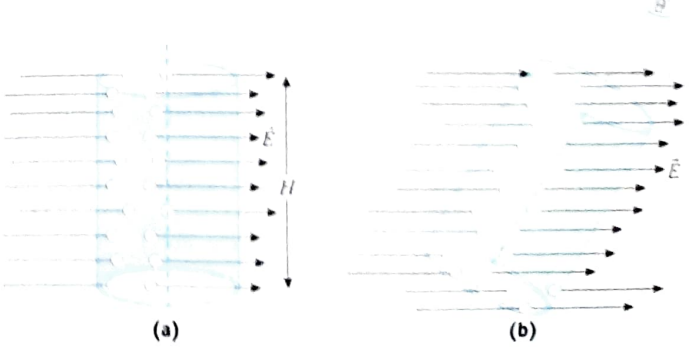
The flux through S_6: $\Phi_{S_6} = \vec{E} \cdot \vec{A} = EA\cos\theta$

$$= (4\times10^3)(0.1)\cos(180° - 37°)$$

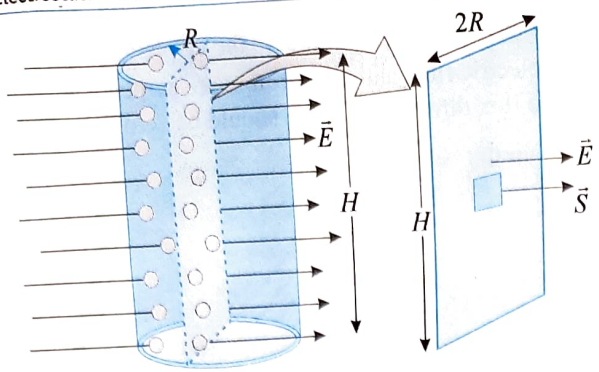
$$= -(4\times10^3)(0.1)\cos 37° = -32 \text{Nm}^2\text{ C}$$

ILLUSTRATION 2.6

A cylinder of radius R and height h is placed in uniform electric field as shown in Fig. (a), if the axis of the cylinder made inclined θ with vertical as shown in Fig. (b). Calculate the flux of electric field entering the cylinder.



Sol. The flux of electric field entering the cylinder in position shown in Fig. (a).



$\Phi = \vec{E} \cdot \vec{S} = E(2R.H) = 2ERH$

Now the axis of the cylinder is made inclined θ with vertical. We know the number of the electric field lines passing through any surface is proportional to the flux passing through the surface. The electric field lines will cross the cylinder in new orientation as shown in figure.

The flux passing through axial plane, $\phi_1 = E(2RH) \cdot \cos\theta$

The flux passing through upper top surface,

$$\phi_2 = E\left(\frac{\pi R^2}{2}\right) \cdot \cos(90° - \theta) = E\frac{\pi R^2}{2}\sin\theta$$

The flux passing through lower bottom surface,

$$\phi_3 = E\left(\frac{\pi R^2}{2}\right) \cdot \cos(90° - \theta) = E\frac{\pi R^2}{2}\sin\theta$$

The flux of electric field entering the cylinder in this position.

$$\phi_{total} = \phi_1 + \phi_2 + \phi_3 = 2ERH + E\frac{\pi R^2}{2}\sin\theta + E\frac{\pi R^2}{2}\sin\theta$$

$$= 2ERH + E\pi R^2 \sin\theta = ER(2H + \pi R\sin\theta)$$

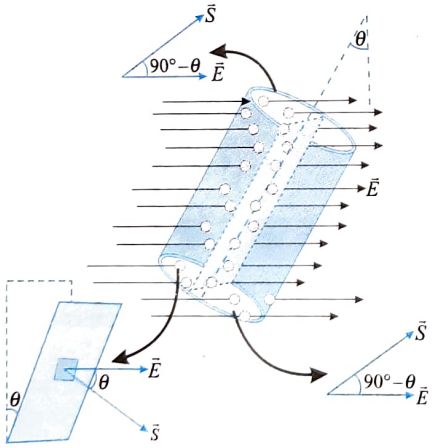
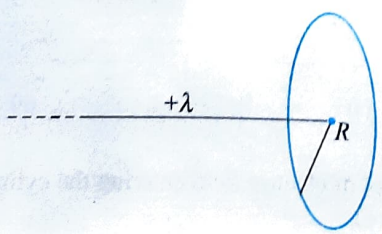
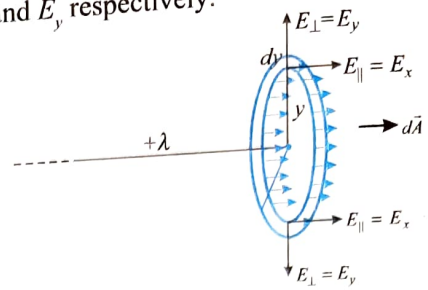


ILLUSTRATION 2.7

A very long uniformly charged wire oriented along the axis of a circle of radius R rests on its centre with one of the ends. The linear charge density on the wire is λ. Evaluate the flux of vector $\vec{E}$ across the circle area.



Sol. Let us consider a circular ring of radius y and thickness dy as shown in figure. The electric field due to wire at the position of the ring will be constant. We can make two mutual perpendicular components of this electric field, parallel and perpendicular to the wire, E_x and E_y respectively.



The component of electric field E_y will not contribute to flux as angle between $\vec{E}_y$ and $d\vec{A}$ is 90°.

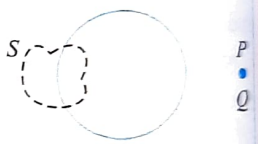
Hence flux due to $\vec{E}_x$, $d\phi = E_x \cdot 2\pi y\, dy$

Total flux, through circular loop, $\phi = \int d\phi = \int_0^R \frac{\lambda}{4\pi\varepsilon_0 \cdot y} \cdot 2\pi y dy$

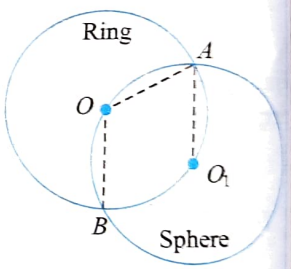
$$\Rightarrow \quad \phi = \frac{\lambda}{2\varepsilon_0}\int_0^R dy = \frac{\lambda R}{2\varepsilon_0}$$

CONCEPT APPLICATION EXERCISE 2.1

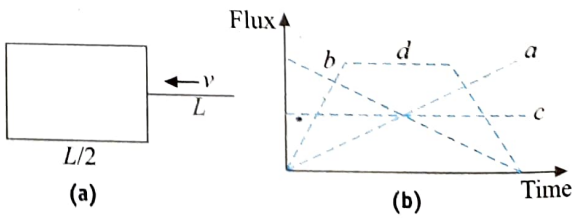
1. Figure shows a closed surface which intersects a conducting sphere. If a positive charge is placed at point P, find the sign of flux passing through the curved surface S.



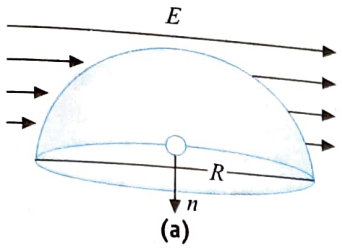
2. A charge Q is distributed uniformly on a ring of radius r. A sphere of equal radius r is constructed with its center at the periphery of the ring. Find the flux of the electric field through the surface of the sphere.



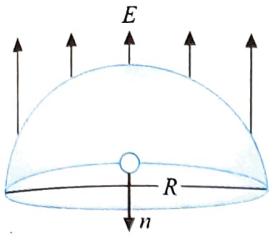
3. Figure (a) shows an imaginary cube of edge $L/2$. A uniformly charged rod of lenght L moves toward left at a small but constant speed v. At $t = 0$, the left end of the rod just touches the right face of the cube. Which of the graphs in Fig. (b) represents the flux of the electric field through the cube as the rod goes through it?



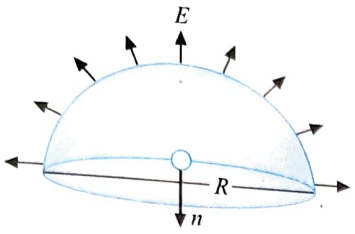
4. A hemispherical body is placed in a uniform electric field E. What is the flux linked with the curved surface if the field is (i) parallel to base of the body [Fig. (a)]; (ii) perpendicular to base of the body [Fig. (b)] and (iii) perpendicular to the curved surface at every points as in Fig. (c).



(a)

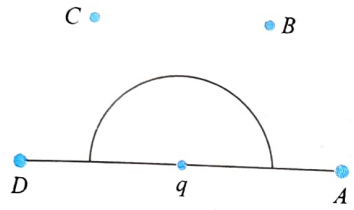


(b)

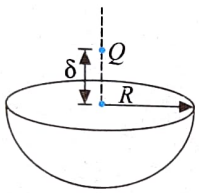


(c)

5. In which position (A, B, C, or D) of second charge, the flux of the electric field through the hemisphere remains unchanged?



6. A point charge Q is located just above the center of the flat face of a hemisphere of radius R as in figure. What is the flux:

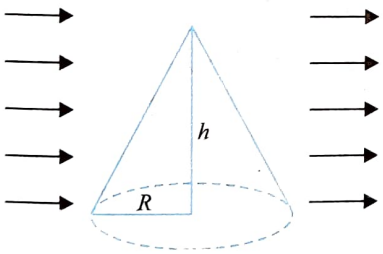


(a) through the curved surface, and

(b) through the flat face?

(c) Repeat parts (a) and (b) if the charge is exactly at the center.

7. In figure, a cone lies in a uniform electric field E. Determine the electric flux entering the cone.

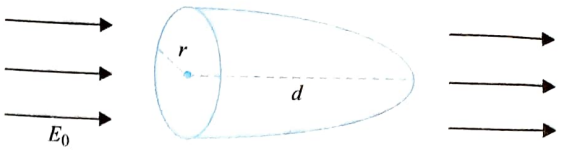


8. A uniform electric field $a\hat{i}+b\hat{j}$ intersects a surface of area A. What is the flux through this area if the surface lies **(a)** in the yz plane, **(b)** in the xz plane, **(c)** in the xy plane?

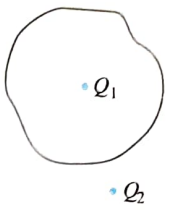
9. **(a)** A point charge q is located at distance d from an infinite plane. Determine the electric flux through the plane due to the point charge.

(b) A point charge q is located at a very small distance from the center of a very large square on the line perpendicular to the square passing through its center. Determine the approximate electric flux through the square due to the point charge.

10. Calculate the total electric flux through the paraboloidal surface due to a uniform electric field of magnitude E_0 in the direction shown in figure.



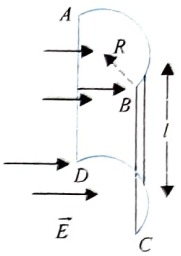
11. Consider a closed surface of arbitatry shape as shown in figure. Suppose a single charge Q_1 is located at some point within the surface and second charge Q_2 is located outside the surface.



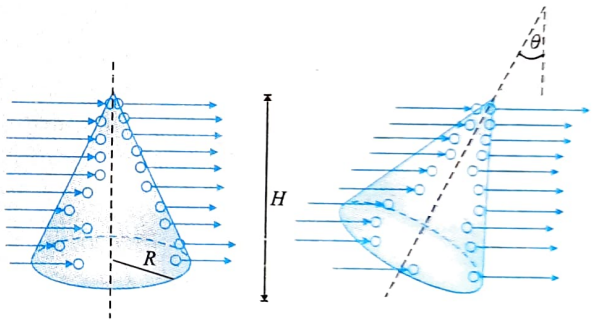
(a) What is the total flux passing through the surface due to charge Q_1?

(b) What is the total flux passing through the surface due to charge Q_2?

12. A hollow half cylinder surface of radius R and length l is placed in a uniform electric field $\vec{E}$. Electric field is acting perpendicular on the plane $ABCD$. Find the flux through the curved surface of the hollow cylindrical surface.

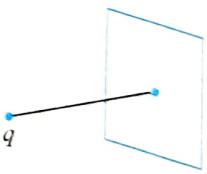


13. A cone of radius R and height h is placed in uniform electric field as shown in Fig. (a), if the axis of the cone made inclined θ with vertical as shown in Fig. (b). Calculate the flux of electric field entering the cone.



(a) (b)

14. Point charge q is placed at a point on the axis of a square non-conducting surface. The axis is perpendicular to the square surface and is passing through its centre. Flux of Electric field through the square caused due to charge q is ϕ. If the square is given a surface charge of uniform density σ, find the magnitude of force on the square surface due to point charge q.



ANSWERS

1. positive **2.** $Q/3\varepsilon_0$

3. Graph (d) **4.** (a) zero (b) $E\pi R^2$ (c) $E2\pi R^2$ **5.** A or D

6. (a) $Q/2\varepsilon_0$ (b) $-Q/2\varepsilon_0$ (c) Zero, $Q/2\varepsilon_0$

7. ERh **8.** (a) Aa (b) Ab (c) 0 (Zero)

9. (a) $q/2\varepsilon_0$ (b) $q/2\varepsilon_0$ **10.** $E_0\pi r^2$

11. (a) Q_1/ε_0 (b) zero **12.** $E2Rl$

13. $ELR\cos\theta+\dfrac{E\pi R^2}{2}\sin\theta$ **14.** $\sigma\phi$

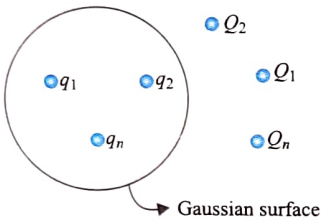
GAUSS'S LAW

Gauss's law is an alternative to Coulomb's law. Although completely equivalent to Coulomb's law, Gauss' law provides a different way to express the relationship between electric charge and electric field.

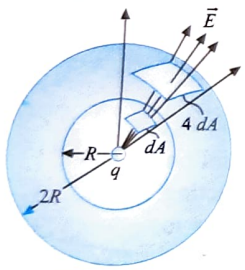
Gauss's law states that the total electric flux through a closed surface is proportional to the total electric charge enclosed within the surface. This law is useful in calculating field caused by charge distributions that have various symmetry properties. Mathematically, Gauss's law can be written as

$$\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

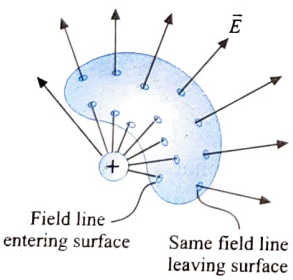
One thing is to note that the electric field appearing in Gauss's law is the resultant electric field due to all the charges present inside as well as outside the given closed surface. On the other hand, the charges q_{in} appearing in the law are only the charges contained within the closed surface. The contribution of the charges outside the closed surface in producing the flux is zero. A surface on which Gauss's law is applied is sometimes called the Gaussian surface (figure).



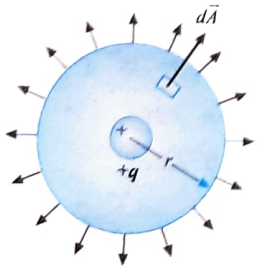
Caution: Remember that the closed surface in Gauss's law is imaginary; there need not be any material object at the position of the surface.



Projection of an element of area dA of a sphere of radius R onto a concentric sphere of radius $2R$. The projection multiplies each linear dimension by 2, so the area element on the larger sphere is 4 dA. The same number of field lines and the same flux pass through these two area elements.

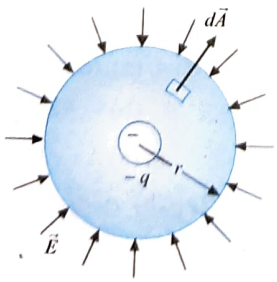


A point charge outside a closed surface that encloses no charge. If an electric field line from the external charge enters the surface at one point, it must leave at another. Figure illustrates this point. Electric field lines can begin or end inside a region of space only when there is charge in that region.



(a)

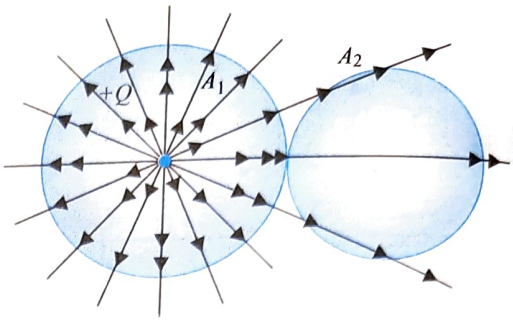
Spherical Gaussian surface around positive charge; positive (outward) flux.



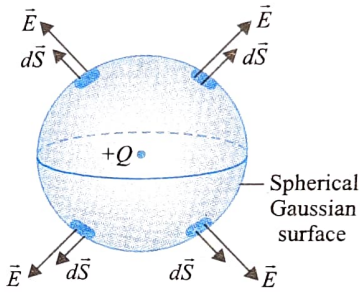
(b)

Spherical Gaussian surface around negative charge; negative (inward) flux.

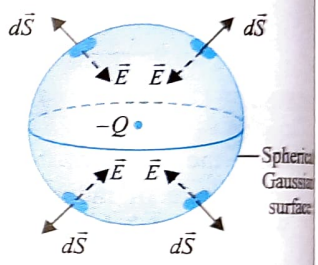
Note that the electric field in the expression $\oint \vec{E} \cdot d\vec{A} = q_{in}/\varepsilon_0$ the resultant field at any point on the Gaussian surface, where q_{in} is the charge enclosed by the Gaussian surface. Consider two Gaussian surfaces A_1 and A_2 as shown in figure.



Charge Q lies at the center of the Gaussian surface A_1. For surface A_1, the net flux through A_1 is Q/ε_0. For surface A_2, charge Q is outside A_2 so that the net flux through A_2 is zero. Note the field lines that enter the Gaussian surface A_2 (net flux in) also leave it (net flux out).



(a)



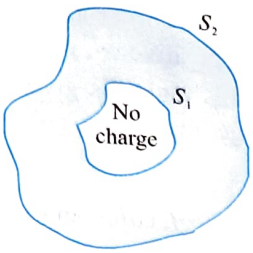
(b)

Some important points on Gauss's law are as follows:

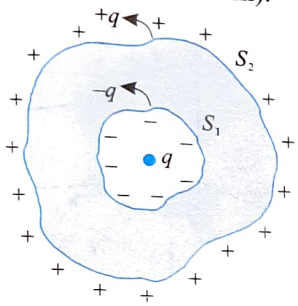
- The net flux through the surface is proportional to the net number of lines leaving the surface, where the net number means the number of lines leaving the surface minus the number of lines entering the surface. If more lines are leaving than entering, the net flux is positive. If more lines are entering than leaving, the net flux is negative.
- If no net flux passes through any closed surface, E may not be zero, but the flux of $\vec{E}$ must be zero and the net charge enclosed must be zero.
- In Gauss's law, $\vec{E}$ is the net field due to all charges present inside and outside the Gaussian surface.
- No net flux is contributed by the charges present outside the Gaussian surface because the number of E-lines (flux) coming into is equal to the number of E-lines going out of the surface.
- The net flux is contributed only by the charges inside (or enclosed by) the Gaussian surface.
- If we change the configuration by displacing the charges inside the Gaussian surface, the electric field at any point may change leaving the net flux passing through the Gaussian surface unchanged.
- However, any change in configuration of the charges outside a Gaussian surface may change E at a point, but the net flux remains the same as external charges contribute nothing to the total flux.

Application of Gauss's theorem for finding induced charge in a conductor:

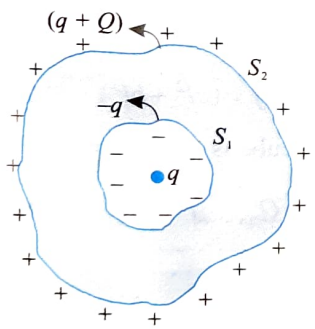
(i) Net electric field in a charge free cavity in a closed conductor is zero. There can be charges outside the conductor and on the surface also. Then also this result is true. No charge will be induced on the inner most surface of the conductor.



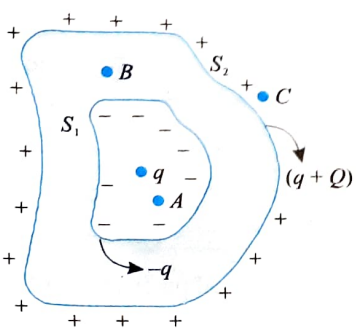
(ii) If we place a charge q inside the cavity of a conductor then $-q$ will be induced on the inner surface (S_1) and $+q$ will be induced on the outer surface of the conductor (S_2) (We can prove it by using Gauss's theorem).



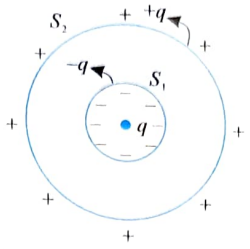
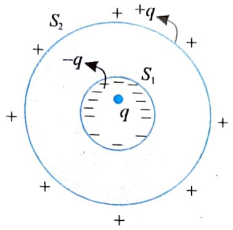
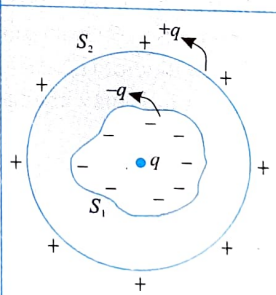
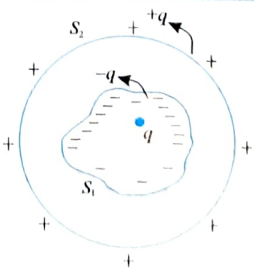
(iii) If we place a charge q inside the cavity of a conductor and conductor is given a charge Q then $-q$ charge will be induced on inner surface (S_1) and total charge on the outer surface (S_2) will be $q + Q$. (We can prove it by using Gauss's theorem).

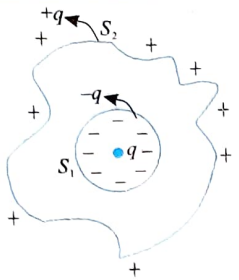
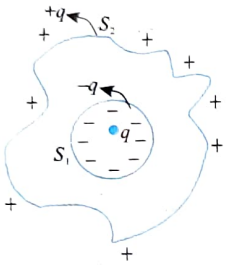


(iv) Net electric field, due to q placed inside the cavity and induced charge on S_1, at any point outside S_1 (like B, C) is zero. Net electric field due to $q + Q$ on S_2 and any other charge outside S_2, at any point inside of surface S_2 (like A, B) is zero.



Charge distribution for different types of cavities in conductors

	Situation	Pattern of the induced charge
1.	 The surfaces S_1 and S_2 both are spherical and the charge is placed at center of cavity.	The induced charge on the surfaces S_1 and S_2 will be uniformly distributed.
2.	 The surfaces S_1 and S_2 both are spherical and the charge is not placed at the center of cavity.	The induced charge on the surface S_1 will be non-uniform and on S_2, it will be uniformly distributed.
3.	 The surfaces S_1 is not spherical and S_2 is spherical and the charge is placed at the center of S_2.	The induced charge on the surface S_1 will be non-uniform and on S_2, it will be uniformly distributed.
4.	 The surfaces S_1 is not spherical and S_2 is spherical and the charge is not placed at the center of S_2.	The induced charge on the surface S_1 will be non-uniform and on S_2, it will be uniformly distributed.

5.	 The surfaces S_1 is spherical and S_2 is not spherical and the charge is placed at the center of S_1.	The induced charge on the surface S_1 will be uniform and on S_2 ,it will be non-uniformly distributed.
6.	 The surfaces S_1 is spherical and S_2 is not spherical and the charge is not placed at the center of S_1.	The induced charge on the surfaces S_1 and S_2 will be non-uniformly distributed.

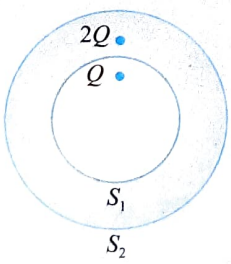
Note: In all cases charge on inner surface $S_1 = -q$ and on outer surface $S_2 = q$. The distribution of charge on S_1 will not change even if some charges are kept outside the conductor (i.e. outside the surface S_2). But the charge distribution on S_2 may change if some charge(s) is/are kept outside the conductor.

ILLUSTRATION 2.8

S_1 and S_2 are two hollow concentric spheres enclosing charges Q and $2Q$, respectively, as shown in figure.

(a) What is the ratio of the electric flux through S_1 and S_2?

(b) How will the electric flux through sphere S_1 change if a medium of dielectric constant 5 is introduced in the space inside S_1 in place of air.



Sol. **(a)** Flux through S_1 is $\phi_1 = \dfrac{Q}{\varepsilon_0}$

Flux through S_2 is $\phi_2 = \dfrac{Q+2Q}{\varepsilon_0} = \dfrac{3Q}{\varepsilon_0}$

Thus, $\dfrac{\phi_1}{\phi_2} = \dfrac{Q/\varepsilon_0}{3Q/\varepsilon_0} = \dfrac{1}{3} = 1:3$

(b) As $E_m = \dfrac{E}{\varepsilon_r}$ and $\varepsilon_r = 5$, so $E_m = \dfrac{E}{5}$

ϕ_1' (changed flux through S_1) $= \oint \vec{E}_m \cdot \vec{dS} = \oint \vec{E} \cdot \vec{dS}$

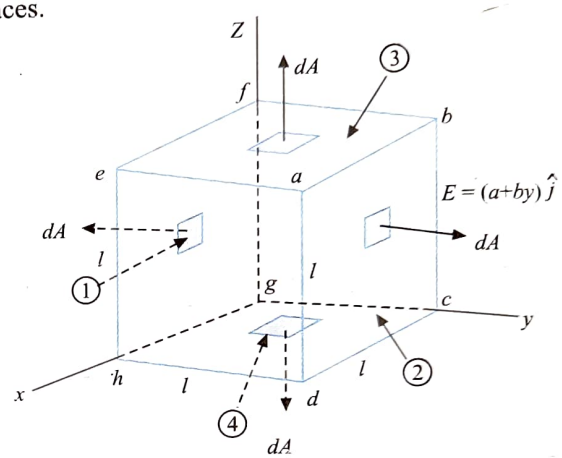
$= \dfrac{1}{5}\phi_1$ (as $\oint \vec{E} \cdot \vec{dS} = \phi_1$)

$= \dfrac{Q}{5\varepsilon_0}$ $\left(\text{as } \phi_1 = \dfrac{Q}{\varepsilon_0}\right)$

ILLUSTRATION 2.9

A cube of side l has one corner at the origin of coordinates and extends along the positive x-, y- and z-axes. Suppose that the electric field in this region is given by $\vec{E} = (a + by)\,\hat{j}$. Determine the charge inside the cube (a and b are some constants).

Sol. The faces *adhe*, *bcgf*, *cdhg*, and *abfe* will contribute zero flux because the area vector is normal to the electric field for these faces.



Flux through face *efgh* is

$$\phi_1 = \int \vec{E} \cdot d\vec{A} \quad = a(\hat{j}) \cdot l^2(-\hat{j}) = -al^2$$

The field at the face *efgh* (that lies in the yz-plane, $y = 0$) is $\vec{E} = a\hat{j}$ and the area vector is $l^2(-\hat{j})$ (direction outward normal).

Flux through face *abcd* is

$$\phi_2 = (a+bl)\hat{j} \cdot l^2\hat{j} = (al^2 + bl^3) \qquad \text{(for this } y = l\text{)}$$

Net flux through the cube is $\phi_1 + \phi_2 = bl^3$. From Gauss's law,

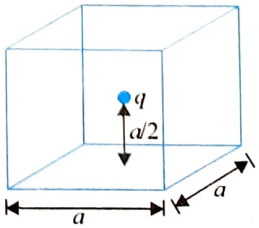
$$\phi = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \text{ or } Q_{\text{enclosed}} = \varepsilon_0\phi_E = \varepsilon_0 bl^3$$

ILLUSTRATION 2.10

A point charge q is placed at a distance $a/2$ from the center of a square of side a as shown in figure. Calculate the electric flux passing through the square.

Sol.

This problem can be solved by symmetry consideration and Gauss's law.

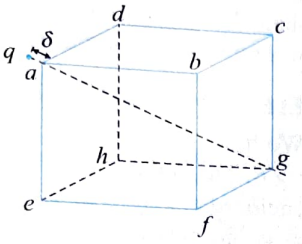


We can enclose the charged particle by a cube of side 'a' and keep the particle at the center of the cube.

The total flux passing through the close cube is $\phi = q/\varepsilon_0$. All the six surfaces are symmetrical with respect to charge, hence they will have equal contribution of the flux. So, flux through any one face is $\phi' = \phi/6 = q/6\varepsilon_0$.

ILLUSTRATION 2.11

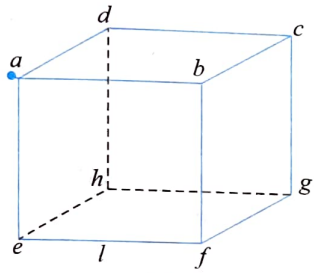
In figure, a charge q is placed at a distance $\delta \to 0$ near one of the corners of a cube of edge l on a line of symmetry along diagonal.



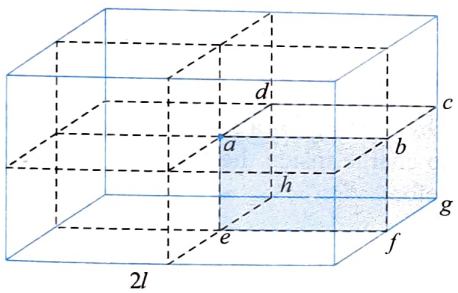
(a) What is the flux through each of the faces containing the point a?

(b) What is the flux through the other three faces?

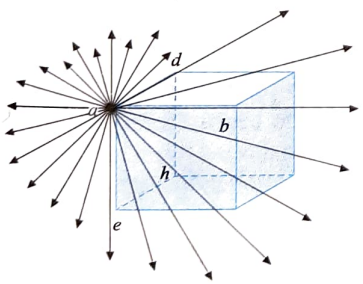
Sol. Use of symmetry consideration may be useful in problems of flux calculation.



(a)



(b)



(c)

We can imagine a charged particle placed at the center of a cube of side $2l$. The flux enclosed with the cube is $\phi = q/\varepsilon_0$. The flux passing through one of the faces of the cube is $\phi' = \phi/6 = q/6\varepsilon_0$. Hence, the flux passing through the face $bcgf$ is $\phi/4 = q/24\varepsilon_0$.

Each of the faces $(efgh)$, $(bcgf)$, and $(dcgh)$ is symmetrical with respect to charge. Hence, the flux passing through each of the face is $q/24\varepsilon_0$.

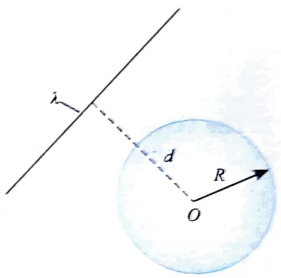
The electric field lines for the faces $(efgh)$, $(bcgf)$ and $(dcgh)$ are away from the faces. Hence, the flux associated with each of the faces will be positive $(\text{i.e.}, +q/24\varepsilon_0)$. Hence, the total flux through these sides is $\frac{3\times q}{24\varepsilon_0} = \frac{q}{8\varepsilon_0}$. As $\delta \to 0$, we can say the faces $(abcd)$, $(abfe)$ and $(adhe)$ are also symmetrical about charge. Charge is slightly outside the cube.

The number of electric field lines passing through the faces, which do not contain the point a, is same as the number of electric field lines passing through the faces containing the point a. Hence, the same amount of flux will pass through the faces containing the point a, i.e., $q/8\varepsilon_0$.

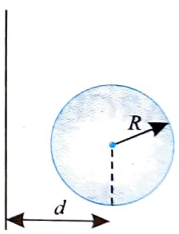
The electric field lines are toward the faces containing the point a. Hence, the flux will be negative, i.e., $\phi'' = -q/8\varepsilon_0$. Hence, the flux through each of the faces containing the point 'a' will be $\phi''/3 = q/24\varepsilon_0$.

ILLUSTRATION 2.12

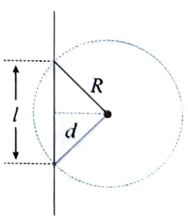
An infinitely long line charge having a uniform charge per unit length λ lies a distance d from point O as shown in figure. Determine the total electric flux through the surface of a sphere of radius R centered at O resulting from this line charge. Consider both cases where $R < d$ and $R > d$.



Sol. For $R < d$, flux will be zero. It is because no charge lies inside the sphere.



For $R > d$, length inside sphere $l = 2(\sqrt{R^2 - d^2})$



From Gauss's theorem, the flux through closed surface, $\phi = \frac{q_{in}}{\varepsilon_0}$

So charge inside the sphere $q_{in} = \lambda \times l = \lambda 2\sqrt{R^2 - d^2}$

So, $\phi = \dfrac{2\lambda\sqrt{R^2 - d^2}}{\varepsilon_0}$

Problem-Solving Strategy

Identify the relevant concepts: Gauss's law is most useful in situations where the charge distribution has spherical or cylindrical symmetry or is distributed uniformly over a plane. In these situation, we determine the direction of $\vec{E}$ from the symmetry of the charge distribution. If we are given the charge distribution, we can use Gauss's law to find the

magnitude of $\vec{E}$. Alternatively, if we are given the field, we can use Gauss's law to determine the details of the charge distribution. In either case, begin your analysis by asking the question, 'What is the symmetry?'

The problem uses the following steps:

1. Select the surface that you will use with Gauss's law. We often call it a Gaussian surface. If you are trying to find the field at a particular point, then that point must lie on the Gaussian surface.

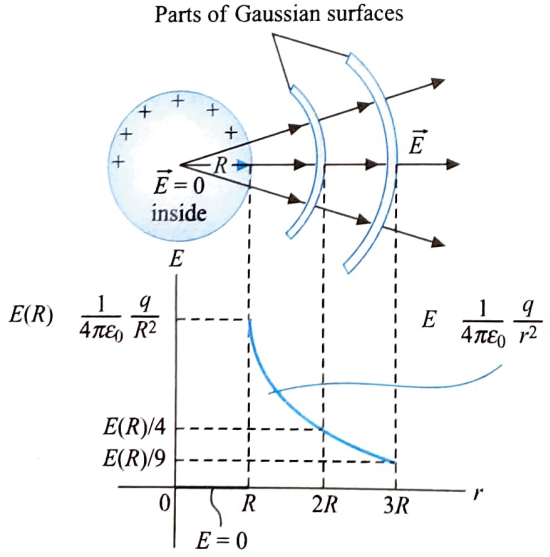
Charge distribution	Gaussian surface	Electric field
Point charge	Spherical	Radial
Spherical charge distribution	Spherical	Radial
Line of charge	Cylindrical	Radial
Planer charge	Plane parallel to charge distribution	Normal to surface

2. The Gaussian surface does not have to be a real physical surface, such as a surface of a solid body. Often the appropriate surface is an imaginary geometric surface; it may be in empty space, embedded in a solid body or both.

3. Usually, you can evaluate the integral in Gauss's law (without using a computer) only if the Gaussian surface and the charge distribution have some symmetry property. If the charge distribution has cylindrical or spherical symmetry, choose the Gaussian surface to be a coaxial cylinder or a concentric sphere, respectively.

FIELD OF A CHARGED CONDUCTING SPHERE

We place positive charge q on a solid conducting sphere with radius R (as shown in figure). All the charges must be on the surface of the sphere.



Under electrostatic conditions, the electric field inside a solid conducting sphere is zero. Outside the sphere, the electric field drops off as $1/r^2$ as though all the excess charge on the sphere is concentrated at its centre.

SELECTION OF GAUSSIAN SURFACE

The system has spherical symmetry. To take advantage of the symmetry, we take an imaginary sphere as our Gaussian surface of radius r centered on the conductor. To calculate the field outside the conductor, we take r to be greater than the radius of the conductor R; to calculate the field inside, we take r to be less than R. In either case, the point where we want to calculate $\vec{E}$ lies on Gaussian surface.

ELECTRIC FIELD OUTSIDE THE SPHERE

We first consider the field outside the conductor, so we choose $r > R$. The entire conductor is within the Gaussian surface, so the enclosed charge is q. The area of the Gaussian surface is $4\pi r^2$, $\vec{E}$ is uniform over the surface and perpendicular to it at each point. The flux integral $\oint E_\perp dA$ in Gauss's law is therefore $E(4\pi r^2)$ which gives

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$ (outside a charged conducting sphere)

This expression for the field at any point outside the sphere ($r > R$) is the same as for a point charge; the field due to the charged sphere is the same as if the entire charge were concentrated at its centre. Just outside the surface of the sphere, where $r = R$, we get

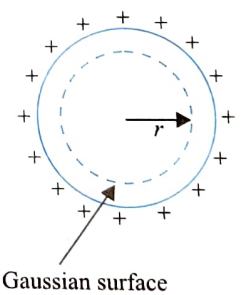
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

(at the surface of a charged conducting sphere)

ELECTRIC FIELD INSIDE THE SPHERE

We know that extra charge on a conductor lies on its outer surface. So there is no charge inside the Gaussian surface, i.e., $q_{in} = 0$ (figure). Therefore,

$$\therefore \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = 0 \Rightarrow E4\pi r^2 = 0 \Rightarrow E = 0$$



Gaussian surface

Hence, at a point inside the sphere, electric field is zero.

ILLUSTRATION 2.13

A point charge $+Q$ is placed at the center of an uncharged spherical conducting shell of inner radius a and outer radius b.

(a) Find the electric field for $r < a$.

(b) What is the magnitude and sign of the induced charge q' on the inner shell surface?

(c) What is the electric field at points $r > b$?

(d) What is the surface charge on the outer surface of the conductor?

Sol.

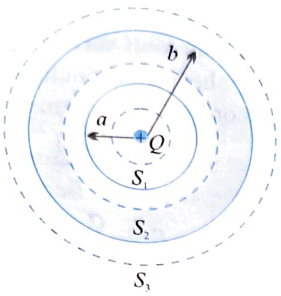
(a) Consider a Gaussian surface S_1 of radius $r < R$ inside the cavity, centered on charge Q. From Gauss's law,

$$\phi_E = \oint E \cdot dA = E(4\pi r^2) = \frac{+Q}{\varepsilon_0}$$

From this we find the electric field to be

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

(b) Consider a Gaussian surface S_2 inside the conducting material. We do not know if there is a charge on the inside surface of the conductor or not. We assume that the charge is q'; if q' is zero, the result of Gauss's law will show it. Because the Gaussian surface is inside the conductor, the electric field is zero. From Gauss's law,



$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = 0$$

or $$\frac{Q_{\text{enclosed}}}{\varepsilon_0} = \frac{Q+q'}{\varepsilon_0} = 0 \quad \text{or } q' = -Q$$

There is a charge on the inside surface of the conductor. The total charge induced on the inside surface of the cavity is the negative of the charge placed at its center.

(c) For $E\,(r > b)$, consider a Gaussian surface S_3. From Gauss's law,

$$\phi_E = \oint E \cdot dA = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

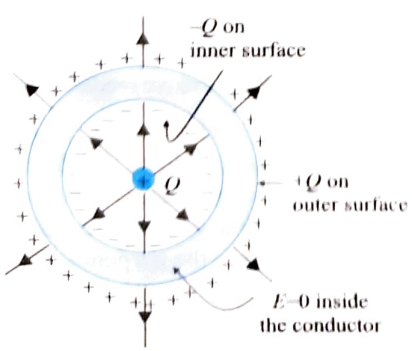
or $$E(4\pi r^2) = \frac{+Q}{\varepsilon_0}$$

or $$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

It was stated in the problem that the conducting sphere has no net charge. Consequently, the total charge inside our Gaussian surface S_3 is the sum of charge $+Q$ and induced charges $-Q$ on the inner surface of the conductor and $+Q$ on the surface. Once more we can see that the field outside the sphere is the same as for a point charge. The conducting sphere has no shielding effect at all.

However, such a conducting shield prevents electrostatic fields from charges outside the shell from entering it.

(d) The conducting shell has no net charge, yet there is a surface charge $-Q$ on its surface. Because the net charge on the shell is zero and no charge can reside inside a conductor, there must be $+Q$ on the outer surface of the conductor.

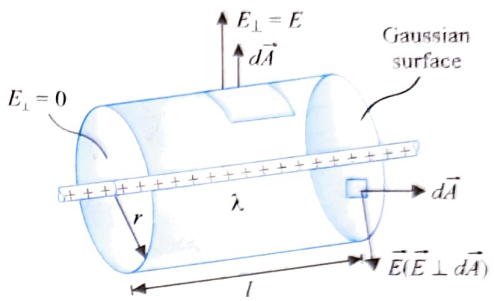


FIELD OF A LINE CHARGE

Electric charge is distributed uniformly along an infinitely long, thin wire. The charge per unit length is λ (assumed positive) known as linear charge density.

SELECTION OF GAUSSIAN SURFACE

The system has cylindrical symmetry. This property suggests that we use a cylinder as a Gaussian surface with arbitrary radius r and arbitrary length l, with its ends perpendicular to the wire (see figure). We break the surface integral for the flux Φ_E into an integral over each flat end and one over the curved side wall.



There is no flux through the ends because $\vec{E}$ lies in the plane of the surface. To find the flux through the side walls, note that $\vec{E}$ is perpendicular to the surface at each point; by symmetry, E has the same value everywhere on the walls (curved surface). The area of the side wall is $2\pi rl$. Hence, the total flux Φ_E through the entire cylinder is the sum of the flux through the side wall, which is $(E)(2\pi rl)$, and the zero flux through the two ends. Finally, we need the total enclosed charge, which is the charge per unit length multiplied by the length of wire inside the Gaussian surface, or $Q_{\text{encl}} = \lambda l$. From Gauss's law, we get

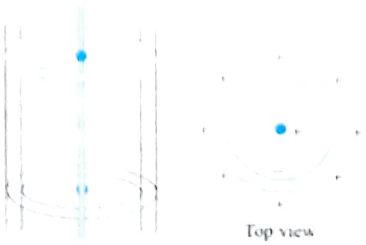
$$\Phi_E = (E)(2\pi rl) = \frac{\lambda l}{\varepsilon_0} \text{ and } E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

(field of an infinite line of charge)

We have assumed that λ is positive. If it is negative, $\vec{E}$ is directed radially inward toward the line of charge, and in the above expression for the field magnitude E, we must interpret λ as the magnitude (absolute value) of the charge per unit length.

ILLUSTRATION 2.14

A long, straight wire is surrounded by a hollow metal cylinder whose axis coincides with that of the wire. The wire has a charge per unit length of λ, and the cylinder has a net charge per unit length of 2λ. From this information, use Gauss's law to find **(a)** the charge per unit length on the inner and outer surfaces of the cylinder and **(b)** the electric field outside the cylinder, a distance r from the axis.

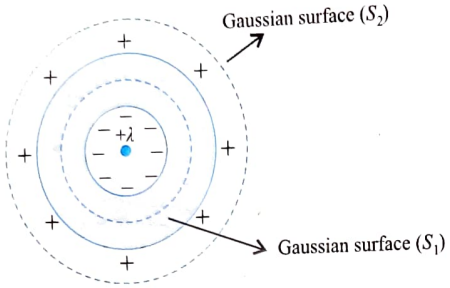


Sol.

(a) Inside surface: Consider a cylindrical Gaussian surface (S_1) of length l, inside the conducting material. Let us assume the charge induced per unit length on the inner surface of the cylinder be λ'. As the Gaussian surface is inside the conductor, the electric field is zero. From Gauss's law

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0} \Rightarrow 0 = \frac{(\lambda l + \lambda' l)}{\varepsilon_0}$$

which gives $\lambda' = -\lambda$, hence the charge per unit length on inner surface of the cylinder is $-\lambda$.



Outside surface: The cylinder has net charge per unit length 3λ. Now considering the cylinder of length l.
Total charge on metal cylinder,

$$2\lambda l = q_{in} + q_{out} = (-\lambda l) + q_{out} \Rightarrow q_{out} = 3\lambda l$$

Hence charge per unit length on outer cylinder,

$$\lambda_{out} = \frac{q_{out}}{l} = 3\lambda$$

(b) Now consider a cylindrical Gaussian surface length 'l' passing through a point which is located outside the cylinder at a distance r from the axis.

Applying Gauss's law $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$

$$E\cdot 2\pi r l = \frac{(\lambda l - \lambda l + 3\times l)}{\varepsilon_0}$$

$$\Rightarrow E = \frac{3\lambda}{2\pi\varepsilon_0 r} \quad \text{(radial outward)}$$

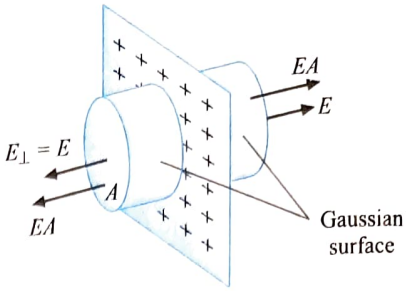
FIELD OF AN INFINITE PLANE SHEET OF CHARGE

Let us consider a thin, flat, infinite sheet on which there is a uniform positive charge per unit area σ.

SELECTION OF GAUSSIAN SURFACE

To take advantage of these symmetry properties, we use a cylinder as our Gaussian surface with its axis perpendicular to the sheet of charge, with ends of area A (refer figure).

The charged sheet passes through the middle of the cylinder's length, so the cylinder is perpendicular to the surface; hence, the flux through each end is EA. Because $\vec{E}$ is perpendicular to the charged sheet, it is parallel to the curved side wall of the cylinder, and there is no flux through these wall.



The total flux integral in Gauss's law is then $2EA$ (EA from each end and zero from the side wall). The net charge within the Gaussian surface is the charge per unit area multiplied by the sheet area enclosed by the surface, or $Q_{encl} = \sigma A$. Hence, according to the Gauss's law

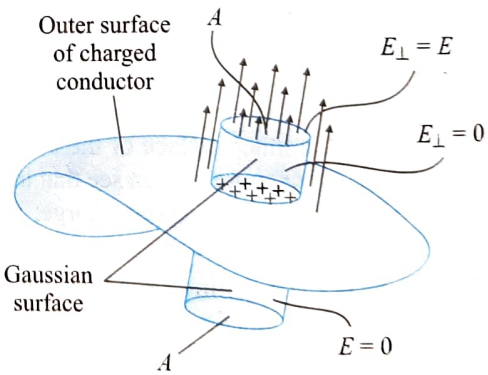
$$2EA = \frac{\sigma A}{\varepsilon_0}$$

or $$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(field of an infinite sheet of charge)}$$

If the charge density is negative, $\vec{E}$ is directed toward the sheet the flux through the Gaussian surface in figure is negative, and in the expression $E = \sigma/2\varepsilon_0$, σ denotes the magnitude (absolute value) of the charge density.
The assumption that the sheet is infinitely large is an idealization nothing in nature is really infinitely large. But the result $E = \sigma/2\varepsilon_0$ is a good approximation for points that are close to the sheet (compared to the sheet's dimensions) and not too near its edges. At such points, field is very nearly uniform and perpendicular to plane.

FIELD AT THE SURFACE OF A CONDUCTOR

To find a relation between σ at any point on the surface and the perpendicular component of the electric field at that point, we construct a Gaussian surface in the form of a small cylinder (as shown in figure).



The field just outside a charged conductor is perpendicular to the surface and its perpendicular component $E_\perp$ is equal to σ/ε_0.

One end face, with area A, lies within the conductor and the other lies just outside. The electric field is zero at all points within the conductor. Outside the conductor, the component of $\vec{E}$ perpendicular to the side walls of the cylinder is zero, and over the end face the perpendicular component is equal to $E_\perp$. (If σ is positive, the electric field points out of the conductor and $E_\perp$ is positive; if σ is negative, the field points inward and $E_\perp$ is negative.) Hence, the total flux through the surface

$E_\perp A$. The charge enclosed within the Gaussian surface is σ_A. So from Gauss's law, we get

$$E_\perp A = \frac{\sigma A}{\varepsilon_0}$$

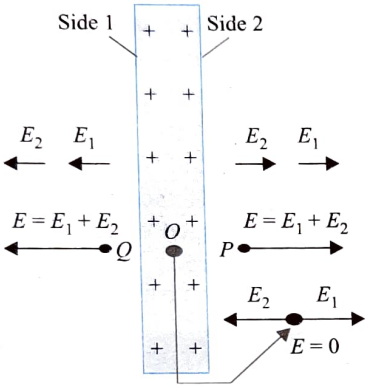
or $$E_\perp = \frac{\sigma}{\varepsilon_0}$$ (field at the surface of a conductor)

We can check this with the results we have obtained for spherical, cylindrical, and plane surfaces.

Electric Field Due to a Charged Isolated Conducting Plate

The electric field due to a charged isolated conducting plate is twice the field due to a plane sheet of charge. This is due to the reason that in the case of the sheet, the same charge is present on both its sides.

The above result can be obtained very easily from the principle of superposition of fields. The electric field at P is due to two sheets of charge, one on each surface of the plate.



Further, the fields due to both the sheets are perpendicular to the plate and are in the same direction.

Thus, E is the electric field (E_1) at P due to the sheet of charge on side 1 plus the electric field (E_2) at P due to the sheet of charge on side 2 of the plate, i.e.,

$$E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

Same is true for a point Q on the left side of the plate.

For a point inside the plate,

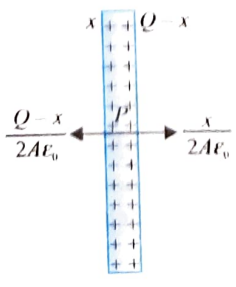
$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

or $E = E_1 - E_2 = 0$ (as $\vec{E}_1$ and $\vec{E}_2$ are equal and opposite)

Illustration 2.15

Prove that if an isolated (isolated means no charges are near the sheet) large conducting sheet is given a charge, then the charge distributes equally on its two surfaces.

Sol. Let there be x charge on the left side of the sheet and $Q-x$ charge on the right side of the sheet. Let us take a point P inside the plate.



Since P lies inside the conductor,

$$E_P = 0$$

$$\Rightarrow \frac{x}{2A\varepsilon_0} - \frac{Q-x}{2A\varepsilon_0} = 0 \quad \text{or} \quad \frac{2x}{2A\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

or $$x = \frac{Q}{2}$$ and $$Q - x = \frac{Q}{2}$$

So charge in equally distributed on both sides.

Illustration 2.16

If an isolated infinite plate contains a charge Q_1 on one of its surfaces and a charge Q_2 on its other surface, then prove that electric field intensity at a point in front of the plate will be $Q/2A\varepsilon_0$, where $Q = Q_1 + Q_2$.

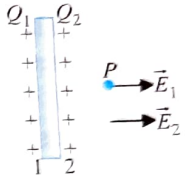
Sol. Let us take a point P right of the plate.

The electric field at P is the vector sum of the field due to both the surfaces. Hence,

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{Q_1}{2A\varepsilon_0}\hat{n} + \frac{Q_2}{2A\varepsilon_0}\hat{n}$$

$$= \frac{Q_1 + Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q}{2A\varepsilon_0}\hat{n}$$



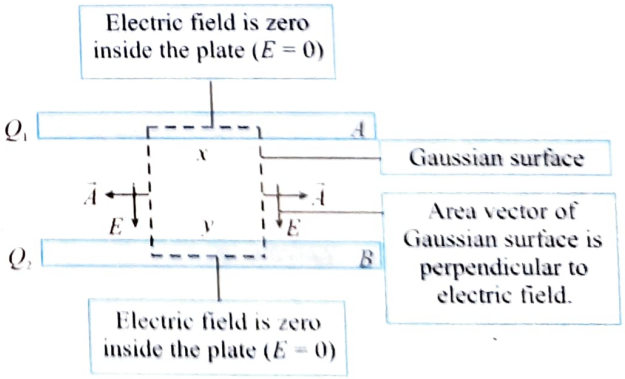
Important Point:

The illustration shows that the resultant field due to a sheet depends only on the total charge of the sheet and not on the distribution of charge on individual surfaces.

Illustration 2.17

Two conducting plates A and B are placed parallel to each other. A is given a charge Q_1 and B a charge Q_2. Prove that the charges on the inner surfaces are of equal magnitude and opposite sign.

Sol. Consider a Gaussian surface as shown in figure. Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The flux through these faces is, therefore, zero. The other parts of the closed surface, which are outside the conductor, are parallel to the electric field, and hence the flux on these parts is also zero. The total flux of the electric field through the closed surface is, therefore, zero.



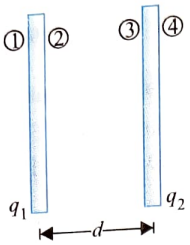
From Gauss's law, the total charge inside this closed surface should be zero. Thus, $x + y = 0$ or $x = -y$.

Hence, the charge on the inner surface of A should be equal and opposite to that on the inner surface of B.

ILLUSTRATION 2.18

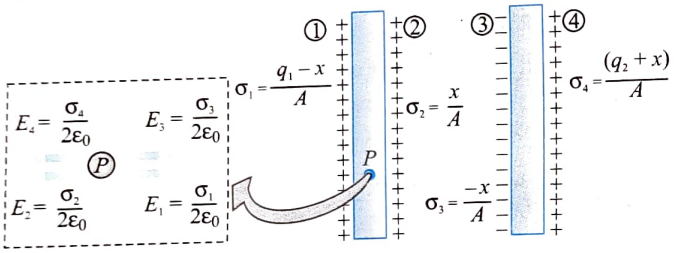
Two identical metal plates each having surface area A, having charges q_1 and q_2, are placed facing each other at a separation d. Find the charge appearing on surface (1), (2), (3), and (4). Assume the size of the plate is much larger than the separation between the plates.

Sol. Facing surfaces have equal and opposite charge (by Gauss's theorem). Let the facing surfaces have charges x and $-x$ [surfaces (2) and (3), respectively].



Then the charge on surfaces (1) and (2) should be $(q_1 - x)$ and x, respectively (by conservation of charge). Facing metallic surfaces always have equal and opposite charge. Hence, charge appearing on surfaces (3) and (4) will be $-x$ and $(q_2 + x)$, respectively.

Let us consider a point P inside the left plate. The net electric field at P should be zero.



The net electric field at P will be due to the resultant of electric fields due to the charges appearing on all four surfaces. Thus,

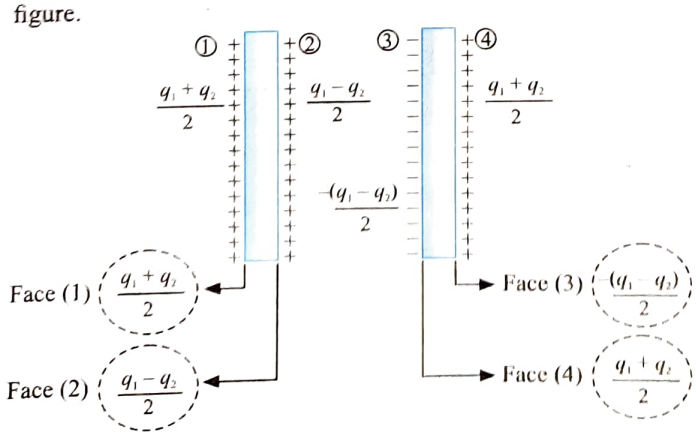
$$E_1 + E_3 = E_2 + E_4$$

$$\Rightarrow \quad \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} = \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_4}{2\varepsilon_0}$$

$$\Rightarrow \quad q_1 - x + x = q_2 + x + x$$

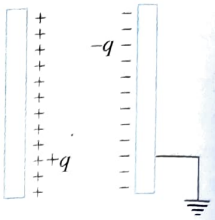
$$\text{or} \quad x = \frac{q_1 - q_2}{2}$$

Hence, charge appearing on different surfaces are as shown in figure.



Important Points:

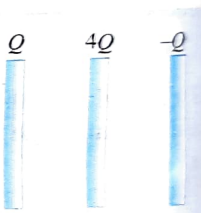
- Facing surfaces have equal but opposite nature of charges with magnitude half the difference of the charges on different plates, i.e., $(q_1 - q_2)/2$ in surface (2) and $(q_2 - q_1)/2$ or $-(q_1 - q_2)/2$ in surface (3).
- Outer surfaces always have equal charges of magnitude half the summation of charges, i.e., $(q_1 + q_2)/2$ in each surfaces (1) and (4).
- If we have this type of charge distribution, then the electric field inside any metal plate will be zero.
- The charge appearing on surfaces (2) and (3) is called bounded charge, and the charge appearing on surfaces (1) and (4) is called free charge.



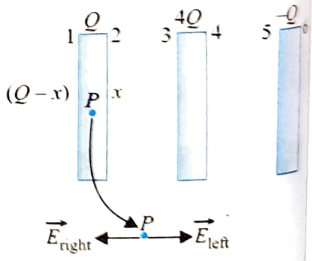
- If we join the second plate (right plate) with ground, the charge appearing on surface (4) will go to the earth.
- Any metal plate or object connected to the earth need not have zero charge. If the conducting body is isolated and connected to earth, then it will have no charge. If the conducting body connected to earth has any charged object near it, then the body will not have zero charge.

ILLUSTRATION 2.19

Figure shows three large metallic plates with charges Q, $4Q$, and $-Q$, respectively. Determine the final charges on all the surfaces.



Sol. **Method 1:** We assume that the charge on surface 2 is x. Following conservation of charge, we see that surface 1 has charge $(Q-x)$. The electric field inside the metal plate is zero, so fields at P is zero.



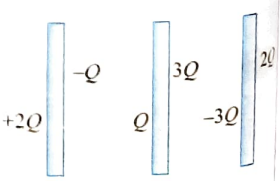
So the resultant field at P is $E_P = 0$

$$\Rightarrow \quad \frac{(Q-x)}{2A\varepsilon_0} = \frac{(x + 4Q - Q)}{2A\varepsilon_0}$$

$$\text{or} \quad Q - x = x + 3Q$$

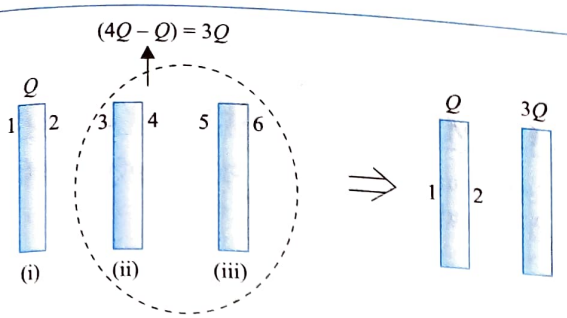
$$\text{or} \quad x = -Q$$

The charges on the facing surfaces of the plates are of equal magnitude and opposite sign. This can be proved by Gauss's theorem also. It is an important result.



Thus, the final charge distribution on all the surfaces is shown in figure.

Method 2: The electric field due to an infinite sheet of charge is independent of distance. We can take plates (ii) and (iii) as one system or one plate having charge $4Q - Q = 3Q$.

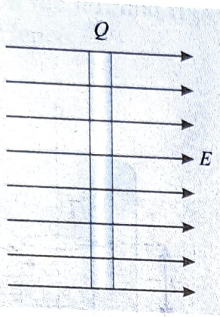


The charge appearing on surface (2) is $q_2 = \frac{Q - 3Q}{2} = -Q$

Hence, final charge distribution is same as we calculated in method 1.

ILLUSTRATION 2.20

An isolated conducting sheet of area A and carrying a charge Q is placed in a uniform electric field E, such that the electric field is perpendicular to the sheet and covers all the sheet. Find out the charges appearing on its two surfaces.



Sol. Let there be x charge on the left side of the plate and $Q - x$ charge on the right side of the plate. Then

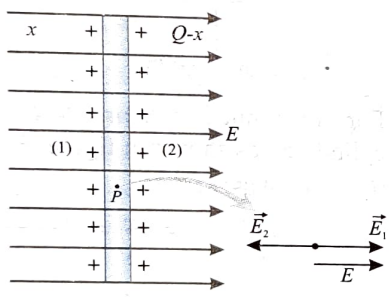
$$E_P = \vec{E}_1 + \vec{E}_2 + \vec{E} = 0$$

$$\Rightarrow \quad \frac{x}{2A\varepsilon_0} - \frac{Q-x}{2A\varepsilon_0} + E = 0$$

$$\text{or} \quad \frac{x}{A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} - E$$

$$\text{or} \quad x = \frac{Q}{2} - EA\varepsilon_0$$

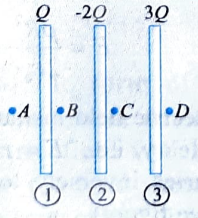
$$\text{and} \quad Q - x = \frac{Q}{2} + EA\varepsilon_0$$



So the charge on one side is $\left(\frac{Q}{2} - EA\varepsilon_0\right)$ and the other side is $\left(\frac{Q}{2} + EA\varepsilon_0\right)$.

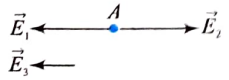
ILLUSTRATION 2.21

Three large conducting sheets placed parallel to each other at a finite distance contain charges Q, $-2Q$, and $3Q$, respectively. Find the electric fields at points A, B, C, and D.



Sol.

(a) Electric field at A is $\vec{E}_A = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

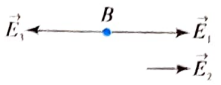


$$\because \quad \vec{E}_1 = \frac{Q}{2\varepsilon_0 A}(-\hat{i}),\ \vec{E}_2 = \frac{2Q}{2\varepsilon_0 A}(\hat{i}),\ \text{and}\ \vec{E}_3 = \frac{3Q}{2\varepsilon_0 A}(-\hat{i})$$

$$\therefore \quad \vec{E}_A = \frac{-Q + 2Q - 3Q}{2\varepsilon_0 A}(\hat{i}) = \frac{Q}{A\varepsilon_0}(-\hat{i})$$

$$\text{or} \quad E_A = \frac{Q}{A\varepsilon_0} \quad \text{(toward left)}$$

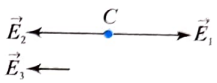
(b) Electric field at B is $\vec{E}_B = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$



$$\because \quad \vec{E}_1 = \frac{Q}{2\varepsilon_0 A}(\hat{i}),\ \vec{E}_2 = \frac{2Q}{2\varepsilon_0 A}(\hat{i}),\ \text{and}\ \vec{E}_3 = \frac{3Q}{2\varepsilon_0 A}(-\hat{i})$$

$$\therefore \quad \vec{E}_B = \frac{Q + 2Q - 3Q}{2\varepsilon_0 A}(\hat{i}) = 0$$

(c) Electric field at C is $\vec{E}_C = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

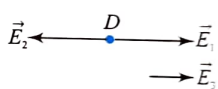


$$\because \quad \vec{E}_1 = \frac{Q}{2\varepsilon_0 A}(\hat{i}),\ \vec{E}_2 = \frac{2Q}{2\varepsilon_0 A}(-\hat{i}),\ \text{and}\ \vec{E}_3 = \frac{3Q}{2\varepsilon_0 A}(-\hat{i})$$

$$\therefore \quad \vec{E}_C = \frac{Q - 2Q - 3Q}{2\varepsilon_0 A}(\hat{i}) = \frac{2Q}{A\varepsilon_0}(-\hat{i})$$

$$\text{or} \quad E_C = \frac{2Q}{A\varepsilon_0} \quad \text{(toward left)}$$

(d) Electric field at D is $\vec{E}_D = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$



$$\because \quad \vec{E}_1 = \frac{Q}{2\varepsilon_0 A}(\hat{i}),\ \vec{E}_2 = \frac{2Q}{2\varepsilon_0 A}(-\hat{i}),\ \text{and}\ \vec{E}_3 = \frac{3Q}{2\varepsilon_0 A}(\hat{i})$$

$$\therefore \quad \vec{E}_D = \frac{(Q - 2Q + 3Q)}{2\varepsilon_0 A}(\hat{i}) = \frac{Q}{\varepsilon_0 A}(\hat{i})$$

$$\text{or} \quad E_D = \frac{Q}{A\varepsilon_0} \quad \text{(toward right)}$$

FIELD OF A UNIFORMLY CHARGED SPHERE

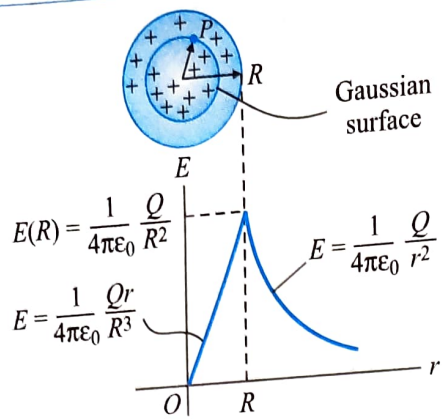
Positive electric charge Q is distributed uniformly throughout the volume of an insulating sphere with radius R.

SELECTION OF GAUSSIAN SURFACE

The system is spherically symmetric. To make use of this symmetry, we choose a sphere as our Gaussian surface with radius r, concentric with the charge distribution.

ELECTRIC FIELD INSIDE THE SPHERE

From symmetry, the magnitude E of the electric field has the same value at every point on the Gaussian surface, and the direction of $\vec{E}$ is radial at every point on the surface. Hence, the total electric flux through the Gaussian surface is the product of E and the total area of the surface $A = 4\pi r^2$, that is, $\Phi_E = 4\pi r^2 E$.



The amount of charge enclosed within the Gaussian surface depends on the radius r. Let us first find the field magnitude inside the charged sphere of radius R; so we choose $r < R$.

The volume charge density ρ is the charge Q divided by volume of the entire charged sphere of radius R, i.e.,

$$\rho = \frac{Q}{4\pi R^3/3}$$

$$Q_{\text{encl}} = \rho V_{\text{encl}} = \left(\frac{Q}{4\pi R^3/3}\right)\left(\frac{4}{3}\pi r^3\right) = Q\frac{r^3}{R^3}$$

Then using Gauss's law, we get $\phi = \oint E dA = \frac{Q_{\text{encl}}}{\varepsilon_0}$

$$E\oint dA = E4\pi r^2 = \frac{Q}{\varepsilon_0}\frac{r^3}{R^3}$$

or $E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}$ (field inside a uniformly charged sphere)

The field magnitude is proportional to the distance r of the field point from the center of the sphere (refer figure).

At the center ($r = 0$), $E = 0$.

Electric field in terms of charge density (at inside point) is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\left(\rho\frac{4}{3}\pi R^3\right)r}{R^3} = \frac{\rho r}{3\varepsilon_0}$$

$\Rightarrow$ $E = \frac{\rho r}{3\varepsilon_0}$ (field inside a uniformly charged sphere)

Electric Field Outside the Charged Sphere

We use a spherical Gaussian surface of radius $r > R$. This surface encloses the entire charged sphere, so $Q_{\text{encl}} = Q$ and Gauss's law gives

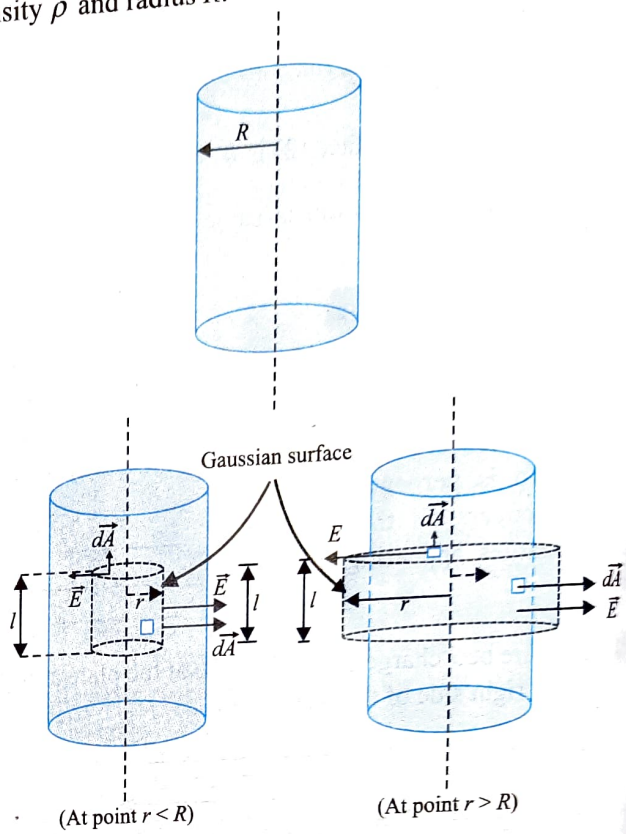
$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

or $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ (field outside a uniformly charged sphere)

For any spherically symmetric charged body, the electric field outside the body is the same; however, the entire charge is concentrated at the center.

Electric Field Due to a Long Uniformly Charged Cylinder

Consider a long uniformly charged cylinder of volumetric charge density ρ and radius R.



For any point $r < R$ or $r > R$, the Gaussian surface will be cylindrical as shown in figure. For any point inside the cylinder ($r < R$), we get

$$E(2\pi rl) = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{(\rho\pi r^2 l)}{\varepsilon_0}$$

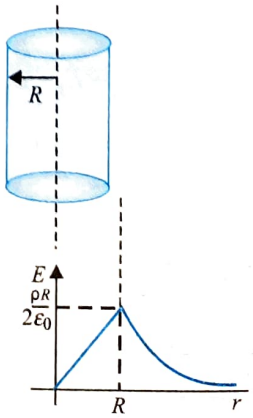
$$E = \frac{\rho r}{2\varepsilon_0} \Rightarrow E \propto r$$

For any point outside the cylinder ($r > R$), we have

$$E(2\pi rl) = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{(\rho\pi R^2 l)}{\varepsilon_0}$$

$$E = \frac{\rho R^2}{2\varepsilon_0 r} \Rightarrow E \propto \frac{1}{r}$$

Electric field inside the long uniformly charged cylinder varies lineary, i.e., $E \propto r$ and outside the cylinder the electric field varies inversely to the distance from the axis, i.e., $E \propto \frac{1}{r}$ (see figure).



ELECTRIC FIELD NEAR UNIFORMLY VOLUME CHARGED PLANE

Let there be charge distributed uniformly in an infinite plane of thickness d with the volume charge density ρ. Due to symmetry, the electric field will be normally away and same in magnitude at same distances from the plane of symmetry.

FIELD INSIDE THE PLANE

Consider the Gaussian surface of the form of a cylinder of area S and thickness $2r(<d)$ placed symmetrically (figure). On the curved surface, flux of electric field will be zero as the area vector is perpendicular to the field vector. On the left and right surfaces, flux is positive (outcoming flux). Hence,

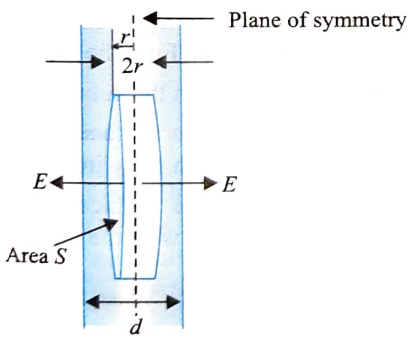
$$\oint_S \vec{E}d\vec{S} = \frac{q_{\text{enclosed}}}{\varepsilon_0},$$

where $\oint_S \vec{E}d\vec{S} = \oint_{\text{Left}} \vec{E}d\vec{S} + \oint_{\text{Right}} \vec{E}d\vec{S} + 0$, $q_{\text{enclosed}} = V\rho = S2r\rho$

$$\Rightarrow \quad \oint_S \vec{E}d\vec{S} = 2ES = \frac{S(2r)\rho}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho r}{\varepsilon_0} \quad \text{(field inside a uniformly charged plane)}$$

Hence, the electric field inside the plane sheet ($r < d/2$) is directly proportional to the distance of point r from the central plane.

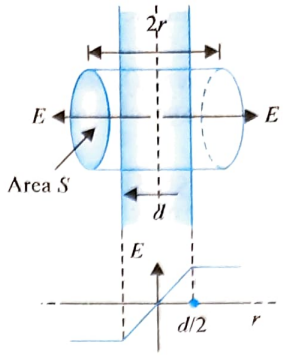


Now, consider the Gaussian surface of the form of a cylinder of area S and thickness $2r$ ($> d$) placed symmetrically. On the curved surface, the flux of electric field will be zero as the area vector is perpendicular to the field vector. On left and right surfaces, flux is positive (outcoming flux). Hence,

$$\oint_S \vec{E}d\vec{S} = \frac{q_{\text{inclosed}}}{\varepsilon_0}$$

$$\Rightarrow \quad 2ES = \frac{Sd\rho}{\varepsilon_0}$$

$$\Rightarrow \quad E = \left(\frac{\rho}{\varepsilon_0}\right)\frac{d}{2} \quad \text{(field outside uniformly charged plane)}$$



Hence, the electric field outside the plane sheet ($r < d/2$) is constant and does not depend on the distance of point r from the central plane (see figure).

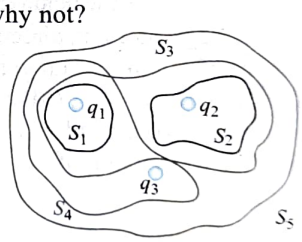
CONCEPT APPLICATION EXERCISE 2.2

1. The three small spheres as shown in figure carry charges $q_1 = 4$ nC, $q_2 = -7.8$ nC and $q_3 = 2.4$ nC. Find the net electric flux through each of the following closed surfaces shown in cross section in the figure.

(a) S_1 **(b)** S_2 **(c)** S_3

(d) S_4 **(e)** S_5

Do your answers to parts from (a) to (e) depend on how the charge is distributed over each small sphere? Why or why not?



Surface	What it encloses
S_1	q_1
S_2	q_2
S_3	q_1 and q_2
S_4	q_1 and q_3
S_5	q_1, q_2 and q_3

2. A conducting sphere carrying charge Q is surrounded by a spherical conducting shell.

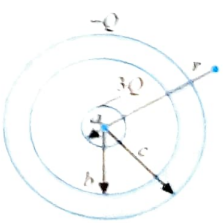
(a) What is the net charge on the inner surface of the shell?

(b) Another charge q is placed outside the shell. Now, what is the net charge on the inner surface of the shell?

(c) If q is moved to a position between the shell and the sphere, what is the net charge on the inner surface of the shell?

(d) Are your answers valid if the sphere and shell are not concentric?

3. A solid insulating sphere of radius a carries a net positive charge $3Q$, uniformly distributed throughout its volume. Concentric with this sphere is a conducting spherical shell with inner radius b and outer radius c and having a net charge $-Q$, as shown in figure.



(a) Consider a spherical Gaussian surface of radius $r > c$, the net charge enclosed by this surface is _____.

(b) The direction of the electric field $r > c$ is _____.

(c) The electric field at $r > c$ is _____.

(d) The electric field in the region with radius r, where $c > r > b$, is _____.

(e) Consider a spherical Gaussian surface of radius r, where $c > r > b$, the net charge enclosed by this surface is _____.

(f) Consider a spherical Gaussian surface of radius r, where $b > r > a$, the net charge enclosed by this surface is _____.

(g) The electric field in the region $b > r > a$ is _____.

(h) Consider a spherical Gaussian surface of radius $r < a$. Find an expression for the net charge $Q(r)$ enclosed by this surface as a function of r. Note that the charge inside this surface is less than $3Q$.

(i) The electric field in the region $r < a$ is _____.

(j) The charge on the inner surface of the conducting shell is _____.

(k) The charge on the outer surface of the conducting shell is _____.

(l) Make a plot of the magnitude of the electric field versus r.

4. A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting sphereical shell with inner radius c and outer radius d. The inner shell has total charge $+2q$ and the outer shell has charge $+4q$.

(a) Make a plot of the magnitude of the electric field versus r.

(b) Calculate the electric field (magnitude and direction in terms of q) and the distance r from the common centre of the two shell for **(i)** $r < a$; **(ii)** $a < r < b$; **(iii)** $b < r < c$; **(iv)** $c < r < d$; **(v)** $r > d$. Show your result in a graph with radial component of $\vec{E}$ as function of r.

(c) What is the total charge on the

(i) inner surface of the small shell;

(ii) outer surface of the small shell;

(iii) inner surface of the large shell;

(iv) outer surface of the large shell?

5. Which of the following statements is/are correct?

(a) Electric field calculated by Gauss law is the field due to only those charges which are enclosed inside the Gaussian surface.

(b) Gauss law is applicable only when there is a symmetrical distribution of charge.

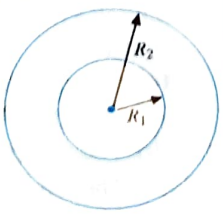
(c) Electric flux through a closed surface is equal to total flux due to all the charges enclosed within that surface only.

6. Which of the following statements is correct? If $E = 0$, at all points of a closed surface

(a) the electric flux through the surface is zero.

(b) the total charge enclosed by the surface is zero.

7. A hollow dielectric sphere, as shown in figure, has inner and outer radii of R_1 and R_2, respectively. The total charge carried by the sphere is $+Q$, this charge is uniformly distributed R_1 and R_2.



Then,

(a) the electric field for $r < R_1$ is zero. (Yes/No)

(b) the electric field for $R_1 < r < R_2$ is given by _____.

(c) the electric field for $r > R_2$ is given by _____.

8. A ring of diameter d is rotated in a uniform electric field until the position of maximum electric flux is found. The flux is found to be ϕ. What is the electric field strength?

9. Consider two concentric conducting spheres. The outer sphere is hollow and initially has a charge $-7Q$ on it. The inner sphere is solid and has a charge $+2Q$ on it.

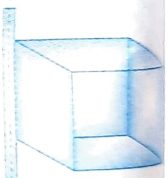
(a) How much charge is on the outer surface and inner surface of the outer sphere.

(b) If a wire is connected between the inner and outer sphere, after electrostatic equilibrium is established how much total charge is on the outer sphere? How much charge is on the outer surface and inner surface of the outer sphere? Does the electric field at the surface of the inside sphere change when the wire is connected?

(c) We return to original condition in (a). We now connect the outer sphere to ground with a wire and then disconnect it. How much total charge will be on the outer sphere? How much charge will be on the inner surface and outer surface of the outer sphere?

10. In an insulating medium (dielectric constant = 1) the charge density varies with y co-ordinate as $\rho = by$, where b is a positive constant. The electric field is zero at $y = 0$ and everywhere else it is along y direction. Calculate the electric field as a function of y.

11. An infinite wire having charge density λ passes through one of the edges of a cube having edge length 'l'. Find the



(a) total flux passing through the cube,

(b) flux passing through the surfaces which are in contact with the wire,

(c) flux passing through the surfaces which are not in contact with the wire.

ANSWERS

1. (a) 452 Nm²/C (b) −881 Nm²/C (c) −429 Nm²/C (d) 723 Nm²/C (e) −158 Nm²/C

2. (a) $-Q$ (b) $-Q$ (c) $-(Q + q)$ (d) Yes

3. (a) $2Q$ (b) radially outward (c) $1/4\pi\varepsilon_0\, 2Q/r^2$ (d) zero (e) zero (f) 3Q (g) $1/4\pi\varepsilon_0\, 3Q/r^2$ (h) $Q(r) = 3Qr^3/a^3$ (i) $1/4\pi\varepsilon_0\, 3Qr/a^3$ (j) $-3Q$ (k) $2Q$

5. (a) False (b) False (c) True **6.** (a) True (b) True

7. (a) Yes (b) $\dfrac{Q}{4\pi\varepsilon_0 r^2}\left(\dfrac{r^3 - R_1^3}{R_2^3 - R_1^3}\right)$ (c) $1/4\pi\varepsilon_0\, Q/r^2$

8. $4\phi/\pi d^2$ **9.** (a) $-5Q$; $-2Q$ (b) $-5Q$; $-5Q$, 0; Yes

10. $6y^2/2\varepsilon_0$

11. (a) $\lambda l/4\varepsilon_0$ (b) zero (c) $\lambda l/8\varepsilon_0$

FORCE ACTING ON THE SURFACE OF A CHARGED CONDUCTOR

To find the electrostatic force on a charged conductor, let us take an elementary patch of area dA, which encloses a charge $dq = \sigma dA$, where σ is the surface charge density.

The elementary charge dq will experience a force dF due to the electric field E_2 of other charges (except dq) because a charge cannot be interacted by its own field. So

$$dF = (dq)\,E_2 \qquad ...(i)$$

Let E_1 be the electric field of dq and E_2 be the electric field of others (except dq). Since $\vec{E}_1$ and $\vec{E}_2$ cancel to give zero net electric field,

$$E_{inside} = E_1 - E_2 = 0$$

or $$E_1 = E_2 \qquad ...(ii)$$

Since $\vec{E}_1$ and $\vec{E}_2$ are parallel and combine to give a net field $E = \sigma/\varepsilon_0$ outside the conductor,

$$E_{outside} = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} \qquad ...(iii)$$

Using Eqs. (ii) and (iii),

$$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$$

Then from Eq. (i), the force $dF = E_2\,dq$, where

$$E_2 = \frac{\sigma}{2\varepsilon_0} \text{ and } dq = \sigma\,dA.$$

$$\therefore \quad \frac{dF}{dA} = \frac{\sigma^2}{2\varepsilon_0}$$

$$\because \quad E_{net}(=E) = \frac{\sigma}{\varepsilon_0} \text{ (just outside the conductor)}$$

Putting $\sigma = \varepsilon_0 E$ in the above expression, we have

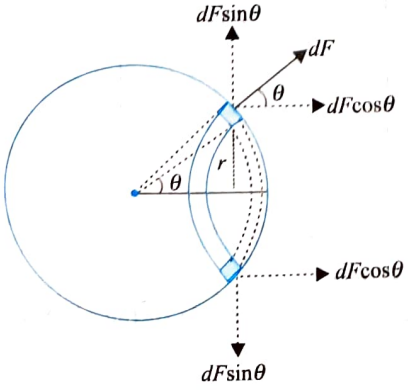
$$\frac{dF}{dA} = p = \frac{\varepsilon_0 E^2}{2}$$

The above expression tells us that the electrostatic stress $p \propto \sigma^2$ and E^2, which makes the charge fly apart but held by the mechanical force of the conductor.

ILLUSTRATION 2.22

Find the electrostatic force of interaction between two halves of a spherical conductor of radius R carrying a charge Q.

Sol. Take an elementary area dA containing a charge dq on the elementary ring of radius r.



The force acting on dq is $dF = \frac{\sigma^2}{2\varepsilon_0} dA$ (as derived earlier)

The net force on the ring is

$$F_{ring} = \int dF \sin\theta = \int \frac{\sigma^2}{2\varepsilon_0} dA \sin\theta$$

$$= \frac{\sigma^2}{2\varepsilon_0} \sin\theta \int dA = \frac{\sigma^2}{2\varepsilon_0} \sin\theta\,(A_{ring})$$

where, $A_{ring} = 2\pi r R d\theta = 2\pi(R\cos\theta)R\,d\theta = 2\pi R^2 \cos\theta\,d\theta$

or $$F_{ring} = \frac{\sigma^2 \pi R^2}{\varepsilon_0} \sin\theta\cos\theta\,d\theta \qquad ...(i)$$

$$F_{hemisphere} = \int F_{ring} \qquad ...(ii)$$

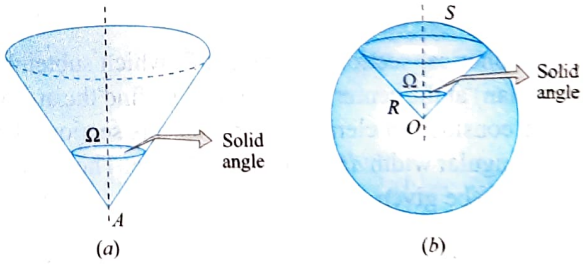
Substituting F_{ring}, from Eq. (i) in Eq. (ii), we get

$$F_{hemisphere} = \frac{\sigma^2 \pi R^2}{\varepsilon_0} \int_0^{+\pi/2} \sin\theta\cos\theta, \text{ where } \sigma = \frac{Q}{4\pi R^2}$$

$$F_{hemisphere} = \frac{Q^2}{32\pi\varepsilon_0 R}$$

CONCEPT OF SOLID ANGLE

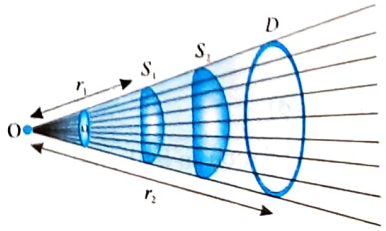
Solid angle is the three-dimensional angle enclosed by the lateral surface of a cone at its vertex as shown in Fig. (a). Solid angle can also be defined as the three-dimensional angle subtended by a spherical section at its center of curvature as shown in Fig. (b). As in the figure shown point O is the center of curvature of a spherical section S of the radius R which subtend a solid angle Ω at point O.



CALCULATION OF SOLID ANGLE OF A RANDOM SURFACE AT A GIVEN POINT

In figure we have a disc 'D'. To find the solid angle subtended by this disc at point O, we join all the points of the periphery of the surface D to the point O by straight lines. This gives a cone with a vertex at O.

Now by taking center at O, we draw several spherical sections on this cone of different radii as shown. Let the area of spherical section which is of radius r_1 is S_1 and the area of section of radius r_2 is S_2.



If is found that the ratio of area of any sphere interrupted by cone to the square of radius of that sphere is a constant and this constant is called solid angle. It is denoted by Greek letter Ω.
In the figure shown the solid angle Ω can be given as

$$\Omega = \frac{S_1}{r_1^2} = \frac{S_2}{r_2^2}$$

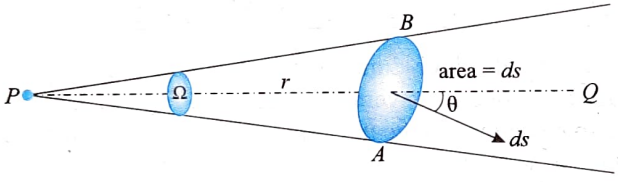
Solid angle is a dimensionless physical quantity and its SI units is streradian.
One steradian is the solid angle subtended at the center of sphere by the surface of the sphere having area equal to square of the radius of sphere.

Solid Angle of a Surface Not Normal to Axis of Cone

Consider a small surface AB of area 'ds' as shown in figure. Let PQ is the axis of cone formed by this surface at point P. Here PQ is not normal to the surface AB. Here the solid angle Ω subtended at point P can be given as.

$$\Omega = \frac{dS\cos\theta}{r^2} \qquad ...(i)$$

Here θ is the angle between surface area vector $\vec{S}$ and the axis of cone PQ as shown in figure. For small surfaces, solid angle can be obtained by Eq. (i) for any point in the surrounding.



Relation in Half Angle of Cone and Solid Angle at Vertex

Approach 1:

Consider a spherical section S of radius R, which subtend a half angle ϕ (radian) at the center of curvature. To find the area of this section, we consider an elemental strip on this section of radius $R\sin\theta$ and angular width $d\theta$ as shown in figure. The surface area of this strip can be given as

$$dS = 2\pi R\sin\theta \times R d\theta$$

The total area of spherical section can be given by integrating the area of this elemental strip within limits from 0 to ϕ.
Total area of spherical section is

$$S = \int dS = \int_0^{\phi} 2\pi R^2 \sin\theta d\theta = 2\pi R^2[-\cos\theta]_0^{\phi} = 2\pi R^2(1-\cos\phi)$$

If solid angle subtended by this spherical section at this center O is Ω then

$$\Omega = \frac{S}{R^2} = \frac{2\pi R^2(1-\cos\phi)}{R^2} \qquad ...(i)$$

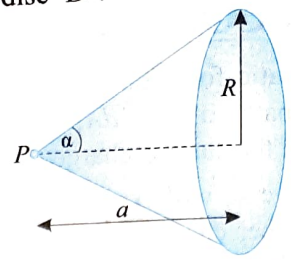
From Eq. (i) we have

$$\Omega = 2\pi(1-\cos\phi) \qquad ...(ii)$$

Equation (i) gives the relation in half angle of a cone ϕ and the solid angle enclosed by the lateral surface of cone at its vertex.

Approach 2:

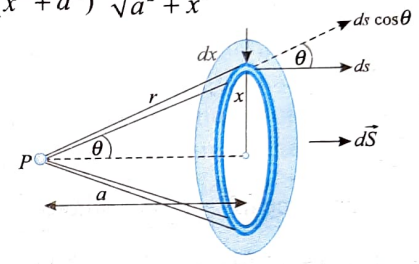
In figure we have a disc 'D'.



To find the solid angle subtended by this disc at point P, we join all the points of the periphery of the surface D to the point P by straight lines. This gives a cone with a vertex at P.
Consider an elemental ring of radius x and width dx.
Area of this ring (strip) is $dS = 2\pi x\,dx$
Solid angle subtended by this element at point P is

$$d\Omega = \frac{dS\cos\theta}{r^2} = \frac{dS\cos\theta}{(x^2+a^2)}$$

$$\Rightarrow \quad d\Omega = \frac{(2\pi x dx)}{(x^2+a^2)}\cdot\frac{a}{\sqrt{a^2+x^2}}$$



Hence total solid angle subtended by the disk is

$$\Omega = \int d\Omega = \int_0^R \frac{2\pi x a}{(x^2+a^2)^{3/2}}dx = 2\pi a\int_0^R \frac{x}{(x^2+a^2)^{3/2}}dx$$

Let $(a^2+x^2) = t$

$$d(a^2+x^2) = dt \Rightarrow 2x\,dx = dt \Rightarrow x\,dx = \frac{1}{2}dt$$

When $x = 0$, $t = a^2$. When $x = R$, $t = (a^2+R^2)$

$$\phi = 2\pi a\int_{a^2}^{(a^2+R^2)} \frac{(dt/2)}{(t)^{3/2}} = \pi a\int_{a^2}^{(a^2+R^2)} t^{-3/2}dt$$

$$= \pi a\left[\frac{t^{\left(-\frac{3}{2}+1\right)}}{\left(-\frac{3}{2}+1\right)}\right]_{a^2}^{(a^2+R^2)} = \pi a(-2)\left[\frac{1}{t^{\frac{1}{2}}}\right]_{a^2}^{(a^2+R^2)}$$

$$\Rightarrow \quad \Omega = -2\pi a\left[\frac{1}{t^{\left(\frac{1}{2}\right)}}\right]_{a^2}^{(a^2+R^2)}$$

$$= -2\pi a\left[\frac{1}{(a^2+R^2)^{\left(\frac{1}{2}\right)}} - \frac{1}{(a^2)^{\left(\frac{1}{2}\right)}}\right]$$

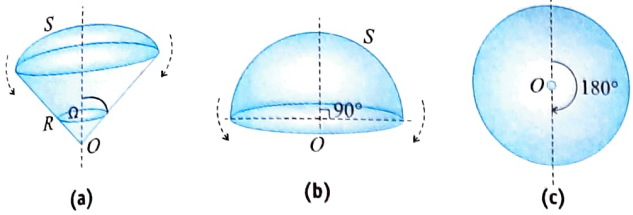
$$= 2\pi\left[1 - \frac{a}{\sqrt{a^2+R^2}}\right]$$

$$\Rightarrow \Omega = 2\pi[1-\cos\alpha]$$

where α is the semi vertical angle of the cone subtended by the disk at P.

SOLID ANGLE ENCLOSED BY A CLOSED SURFACE

Consider Fig. (a) that shows S is a spherical section of radius R which subtend a solid angle Ω at its center of curvature O. The half angle of cone formed for this solid angle is ϕ.



In this situation if we use Eq. (ii) to find the solid angle subtended by a hemispherical surface at its center then it can be given as

$$\Omega = 2\pi(1-\cos 90°)$$

$$\Omega = 2\pi \text{ steradians}$$

If we further increase the angle ϕ, curved surface increases. At $\phi = 180°$, surface S [in Fig. (c)] becomes a complete sphere (closed). At this situation, solid angle at center is

$$\Omega = 2\pi(1-\cos 180°)$$

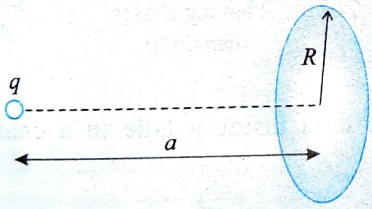
$$= 2\pi(1+1) = 4\pi$$

Thus we can say that every closed surface subtend a solid angle 4π at every interior point.

We also say that 4π is the solid angle of three-dimensional space at every point in the space.

ILLUSTRATION 2.23

A point charge q placed at a distance a from a disc of radius R. Find the electric flux due to the point charge crossing the disc.

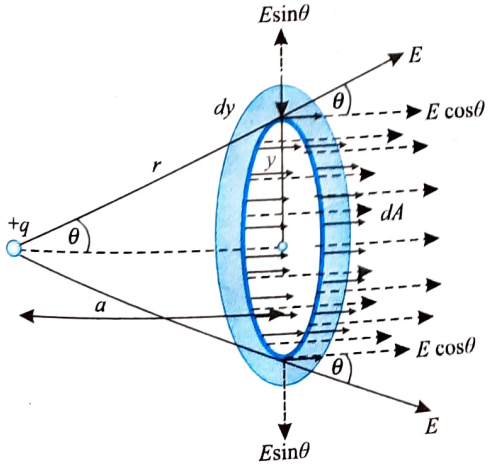


Sol We know a point charge q originates electric field lines in radially outward direction.

Approach 1:

To calculate the flux passing through the disc, we consider an elemental ring of radius y and width dy.

Area of this ring (strip) is $dA = 2\pi y\, dy$



Electric field due to q at this elemental ring,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{(a^2+y^2)}$$

If $d\phi$ is the flux passing through this elemental ring, we have

$$d\phi = EdA\cos\theta$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(a^2+y^2)}\right)(2\pi y dy)\left(\frac{a}{(a^2+y^2)^{1/2}}\right)$$

$$= \frac{qa}{2\varepsilon_0}\cdot\left(\frac{ydy}{(a^2+y^2)^{3/2}}\right)$$

Total flux through the disc surface can be given by integrating this expression over the whole area of ring thus total flux can be given as

$$\phi = \int d\phi = \frac{qa}{2\varepsilon_0}\int_0^R \frac{ydy}{(a^2+y^2)^{3/2}}$$

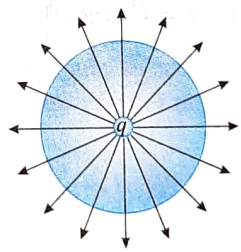
On integration we get,

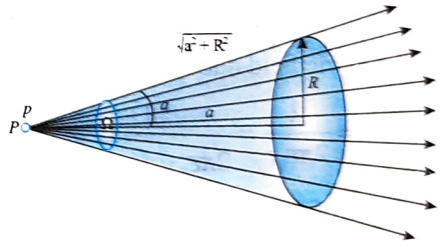
$$\phi = \frac{q}{2\varepsilon_0}\left(1-\frac{a}{\sqrt{a^2+R^2}}\right)$$

Approach 2: Using the concept of solid angle

For a point charge q, total flux originated is $\frac{q}{\varepsilon_0}$ in all directions.

We can say that from a point charge q, $\frac{q}{\varepsilon_0}$ flux is distributed in 4π solid angle.





$\frac{q}{\varepsilon_0}$ flux is distributed in 4π solid angle.

$$\Omega = 2\pi\left(1-\frac{a}{\sqrt{a^2+R^2}}\right)$$

The solid angle enclosed by cone subtended by disc at the point charge can be given as

$$\Omega = 2\pi[1-\cos\alpha] = 2\pi\left(1-\frac{a}{\sqrt{a^2+R^2}}\right)$$

Now we can easily calculate the flux of q which is passing through the disc surface as

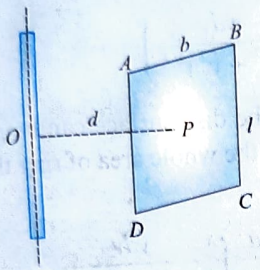
$$\phi_{\text{disc}} = \frac{q/\varepsilon_0}{4\pi}\times\Omega = \frac{q/\varepsilon_0}{4\pi}\times 2\pi\left(1-\frac{a}{\sqrt{a^2+R^2}}\right)$$

$$\phi_{\text{disc}} = \frac{q}{2\varepsilon_0}\left(1-\frac{a}{\sqrt{a^2+R^2}}\right)$$

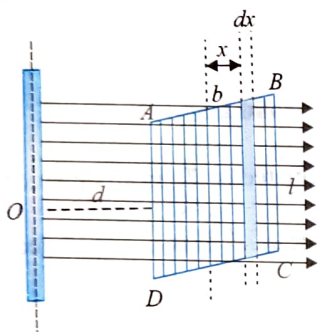
Solved Examples

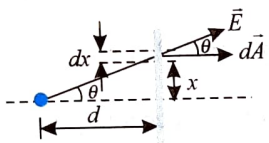
EXAMPLE 2.1

Find the electric flux crossing the wire frame $ABCD$ of length l width b and whose centre is at a distance $OP = d$ from an infinite line of charge with linear charge density λ. Consider that the plane of frame is perpendicular to the line OP.



Sol. Let us consider the wire frame is made of a number of strips as shown in figure. The strips are parallel with wire hence the electric field due to wire is constant on the strip.





The flux passing through a strip of area dA, $d\phi = EdA\cos\theta$

The electric field due to wire on the strip,

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\lambda}{2\pi\varepsilon_0\sqrt{d^2+x^2}}$$

The area of the strip, $dA = (l\,dx)$

and $\cos\theta = \dfrac{d}{\sqrt{d^2+x^2}}$

Hence, $d\phi = \dfrac{\lambda}{2\pi\varepsilon_0\sqrt{d^2+x^2}}.(l\,dx).\dfrac{d}{\sqrt{d^2+x^2}}$

$$\Rightarrow \quad d\phi = \frac{\lambda.d.l}{2\pi\varepsilon_0}\frac{dx}{(d^2+x^2)}$$

$$\phi = \int d\phi = \frac{\lambda.d.l}{2\pi\varepsilon_0}\int_{-b/2}^{b/2}\frac{dx}{d^2+x^2} = \frac{\lambda.d.l}{2\pi\varepsilon_0}\frac{1}{d}\left[\tan^{-1}\frac{x}{d}\right]_{-b/2}^{b/2}$$

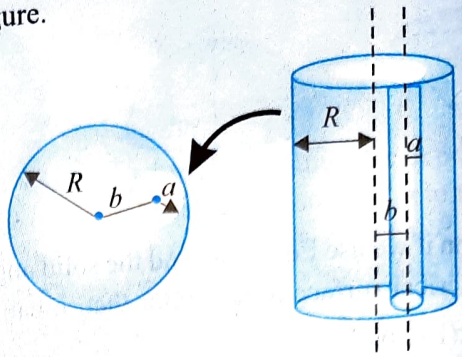
$$= \frac{\lambda.d.l}{2\pi\varepsilon_0}\frac{1}{d}\left[\tan^{-1}\left(\frac{b}{2d}\right) - \tan^{-1}\left(-\frac{b}{2d}\right)\right]$$

$$= \frac{\lambda l}{2\pi\varepsilon_0}.2.\tan^{-1}\left(\frac{b}{2d}\right)$$

$$\Rightarrow \quad \phi = \frac{\lambda l}{\pi\varepsilon_0}.\tan^{-1}\left(\frac{b}{2d}\right)$$

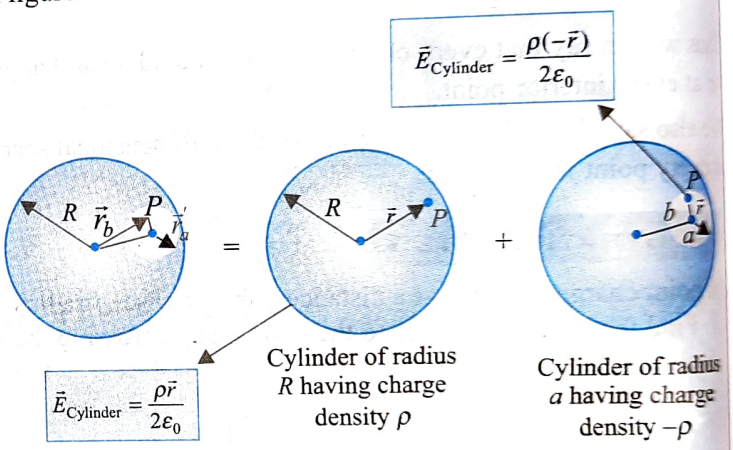
EXAMPLE 2.2

In a long insulating cylinder of radius R a cylindrical tunnel of radius a located at a distance b from the axis of the cylinder as shown in figure.



The solid part of the cylinder has a uniform volume charge density ρ. Find the magnitude and direction of the electric field inside the tunnel.

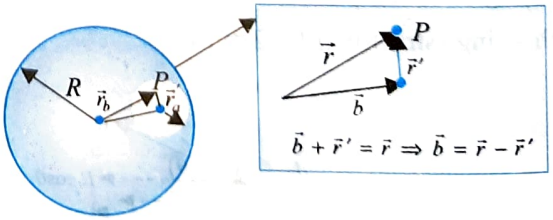
Sol. Here we can use the principle of superposition as shown in figure.

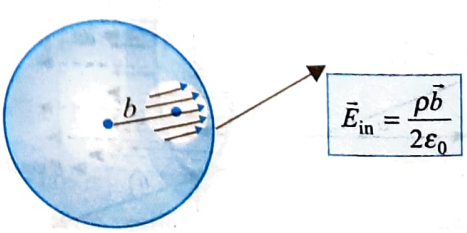


The net electric field inside a hole in a charged insulating cylinder is

$$\vec{E}_P = \vec{E}_{cylinder} + \vec{E}_{cavity} = \frac{\rho\vec{r}}{2\varepsilon_0} + \frac{\rho(-\vec{r}')}{2\varepsilon_0} = \frac{\rho}{2\varepsilon_0}(\vec{r} - \vec{r}')$$

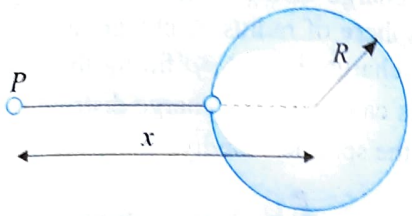
$$\Rightarrow \vec{E}_P = \frac{\rho\vec{b}}{2\varepsilon_0}, \quad \vec{E}_P \text{ is uniform.}$$





EXAMPLE 2.3

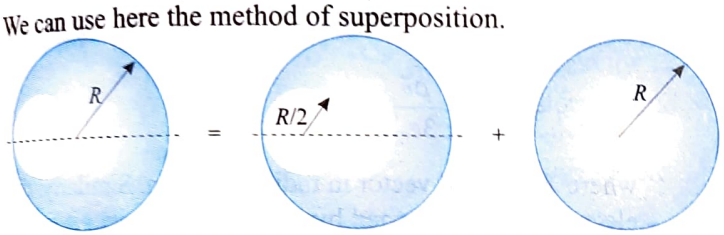
A solid spherical region having a spherical cavity whose diameter 'R' is equal to the radius of the spherical region, has a total charge 'Q'. Find the electric field at a point P as shown.



Sol. Volumetric charge density of structure,

$$\rho = \frac{Q}{\left[\frac{4}{3}\pi R^3 - \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right]} = \frac{8}{7}\cdot\frac{Q}{\frac{4}{3}\pi R^3}$$

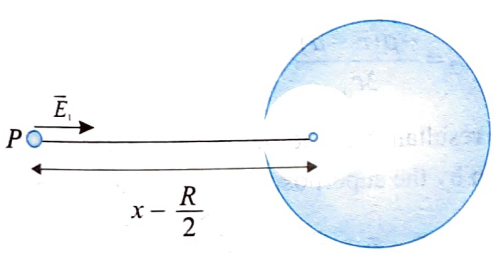
We can use here the method of superposition.



(1) Sphere of radius $R/2$ having charge density $-\rho$

(2) Sphere of radius R having charge density $-\rho$

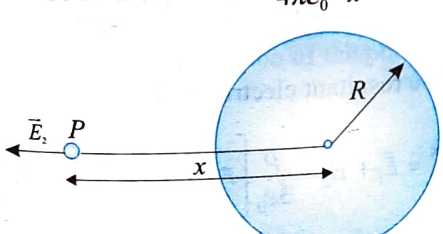
Electric field due to sphere (1) at P, $\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{\left(x-\frac{R}{2}\right)^2}\hat{i}$



$$|Q_1| = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\rho = \frac{1}{7}Q$$

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{1}{7}Q\right)}{\left(x-\frac{R}{2}\right)^2}\hat{i} \qquad \text{...(i)}$$

Electric field due to sphere (2), $\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{x^2}(-\hat{i})$



$$Q_2 = \frac{4}{3}\pi R^3\rho = \frac{8}{7}Q$$

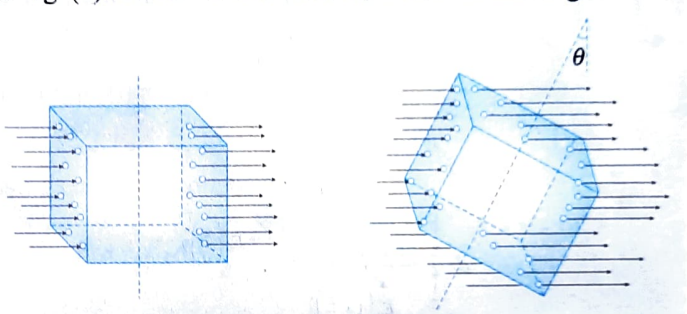
$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{8}{7}Q\right)}{x^2}(-\hat{i}) \qquad \text{...(ii)}$$

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

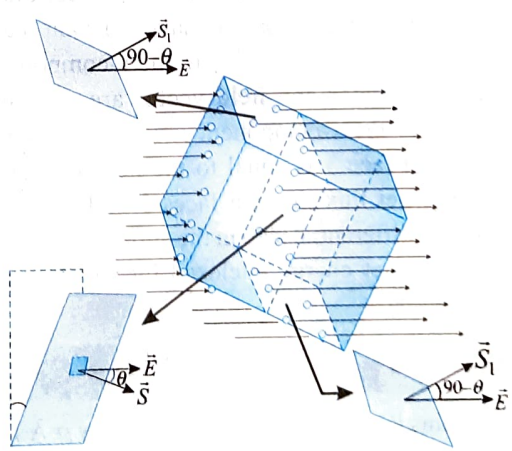
$$\vec{E}_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{7}Q\right)\left[\frac{1}{\left(x-\frac{R}{2}\right)^2} - \frac{8}{x^2}\right]\hat{i}$$

EXAMPLE 2.4

A cube of edge l is placed in uniform electric field as shown in Fig. (a), if the cube is rotated by angle θ with vertical as shown in Fig. (b). Calculate the flux of electric field entering the cube



Sol. The amount of flux passing through any surface is proportional to the number of electric field lines passing through that surface. In new situation the field lines will interact with the different parts of the cube as shown in figure.



The flux passing through the cross section of the cube,

$$\phi_1 = EL^2\cdot\cos\theta$$

The flux passing through the upper face of the cube,

$$\phi_2 = E\left(L\times\frac{L}{2}\right)\cos(90°-\theta) = E\frac{L^2}{2}\sin\theta$$

The flux passing through the lower face of the cube,

$$\phi_3 = E\left(L\times\frac{L}{2}\right)\cos(90°-\theta) = E\frac{L^2}{2}\sin\theta$$

he total flux of electric field entering the cube in this position.

$$\phi_{total} = \phi_1 + \phi_2 + \phi_3 = EL^2 \cos\theta + \frac{EL^2}{2}\sin\theta + \frac{EL^2}{2}\sin\theta$$

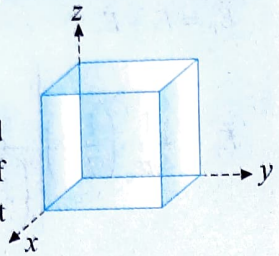
$$= EL^2(\cos\theta + \sin\theta)$$

EXAMPLE 2.5

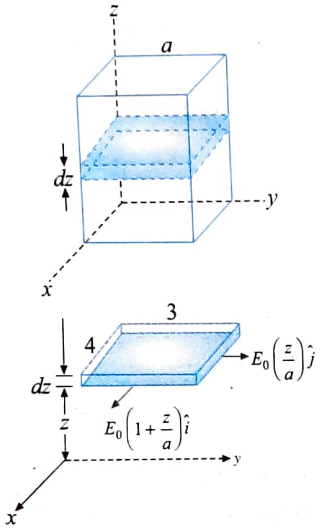
The electric field in a cubical volume is

$$\vec{E} = E_0\left(1+\frac{z}{a}\right)i + E_0\left(\frac{z}{a}\right)j$$

Each edge of the cube measures d, and one of the corners lies at the origin of the coordinates. Determine the net charge within the cube.



Sol.



We choose a differential slab of thickness dz at a distance z from the y-axis. The electric field varies with the z-coordinate only. The field components at this position have constant magnitude. Consider faces 1 and 3. Net flux due to the y-component of the field is zero (area vector and field vector are perpendicular) and net flux due to the x-component is also zero because the net flux in through face 3 is equal to the net flux out through face 1. Similarly, net flux through faces 2 and 4 is also zero. Flux through each differential slab in the cube is zero. Therefore, from Gauss's law, net charge enclosed by cubical volume is zero.

EXAMPLE 2.6

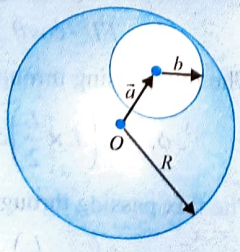
A sphere of radius R has a uniform volume density ρ. A spherical cavity of radius b, whose center lies at $\vec{a}$, is removed from the sphere.

(a) Find the electric field at any point inside the spherical cavity.

(b) Find the electric field outside the cavity

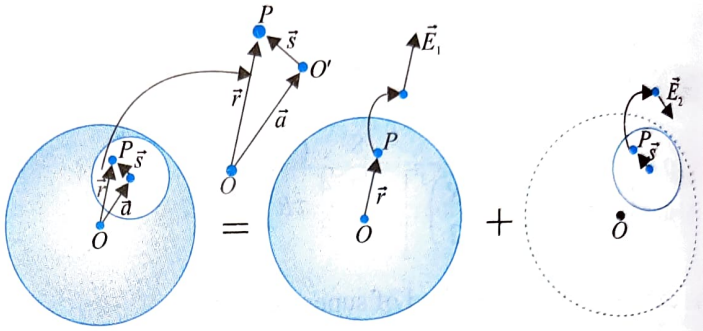
(i) at points inside the large sphere but outside the cavity and

(ii) at points outside the large sphere.



Sol.

(a) The electric field within the cavity or outside is the superposition of the electric field due to the original uncut sphere, plus the electric field due to a sphere of the size of the cavity but with a uniform negative charge density. The effective charge distribution is composed of a uniformly charged sphere of radius R, charge density ρ, superposed on it is a charge density $-\rho$ filling the cavity. An electric field $\vec{E}_1$ is caused by the charge distribution $+\rho$ at a point $\vec{r}$ inside the spherical cavity.



$$\vec{E}_1 = \frac{\rho r}{3\varepsilon_0}\hat{r} = \frac{\rho\vec{r}}{3\varepsilon_0}$$

where $\hat{r}$ is a unit vector in radial direction. Similarly, the electric field $\vec{E}_2$ formed by the charge density $-\rho$ inside the cavity is

$$\vec{E}_2 = \frac{\rho(-\vec{s})}{3\varepsilon_0}$$

Here, $\vec{s}$ is the radius vector from the cavity center to the point P. From vector triangle

$$\vec{r} = \vec{a} + \vec{s} \text{ or } \vec{s} = \vec{r} - \vec{a}$$

$$\therefore \quad \vec{E}_2 = \frac{-\rho(\vec{r}-\vec{a})}{3\varepsilon_0}$$

The resultant electric field inside the cavity is, therefore given by the superposition of $\vec{E}_1$ and $\vec{E}_2$. So

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho\vec{r}}{3\varepsilon_0} + \left[\frac{-\rho(\vec{r}-\vec{a})}{3\varepsilon_0}\right] = + \frac{\rho\vec{a}}{3\varepsilon_0} = \text{constant}$$

$$\therefore \quad \vec{E} = \frac{\rho\vec{a}}{3\varepsilon_0}$$

(b) (i) Electric field at points inside the large sphere but outside the cavity:

$$\vec{E}_1 = \frac{\rho\vec{r}}{3\varepsilon_0}$$

and $$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q(-\vec{s})}{s^3} = -\frac{\left(\frac{4}{3}\pi\rho b^3\right)(\vec{r}-\vec{a})}{4\pi\varepsilon_0|(\vec{r}-\vec{a})|^3}$$

The resultant electric field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}\left[\vec{r} - \left(\frac{b}{\vec{r}-\vec{a}}\right)^3(\vec{r}-\vec{a})\right]$$

(ii) Electric field at points outside the large sphere,

$$\vec{E}_1 = \frac{Q_{\text{total}}}{4\pi\varepsilon_0 r^3}\vec{r} = \frac{\left(\frac{4}{3}\pi R^3\rho\right)}{4\pi\varepsilon_0 r^3}\vec{r} = \frac{R^3\rho}{3\varepsilon_0 r^3}\vec{r}$$

$$\vec{E}_2 = \frac{Q_{\text{total}}}{4\pi\varepsilon_0 s^3}\vec{s} = \frac{-\left(\frac{4}{3}\pi b^3\rho\right)}{4\pi\varepsilon_0(|\vec{r}-\vec{a}|)^3}(\vec{r}-\vec{a})$$

$$= \frac{-\rho b^3}{3\varepsilon_0(|\vec{r}-\vec{a}|)^3}(\vec{r}-\vec{a})$$

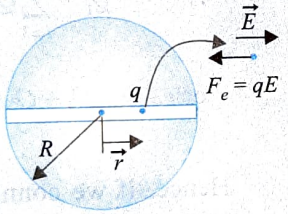
The resultant electric field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}\left[\left(\frac{R}{r}\right)^3\vec{r} - \left(\frac{b}{|\vec{r}-\vec{a}|}\right)^3(\vec{r}-\vec{a})\right]$$

$$\therefore E(\vec{r}) = \begin{cases} \frac{\rho\vec{a}}{3\varepsilon_0} & \text{Electric field inside the cavity} \\ \frac{\rho}{3\varepsilon_0}\left[\vec{r} - \left(\frac{b}{|\vec{r}-\vec{a}|}\right)^3(\vec{r}-\vec{a})\right] & \text{Electric field outside the cavity but inside the large cavity} \\ \frac{\rho}{3\varepsilon_0}\left[\left(\frac{R}{r}\right)^3\vec{r} - \left(\frac{b}{|\vec{r}-\vec{a}|}\right)^3(\vec{r}-\vec{a})\right] & \text{Electric field outside the large sphere} \end{cases}$$

EXAMPLE 2.7

A smooth chute is made in a dielectric sphere of radius R and uniform volume charge density ρ. A charge particle of mass m and charge $-q$ is placed at the center of the sphere. Find the time period of motion of the particle?



Sol. Let the particle displace slightly toward right at a distance r from the center. The force acting on the particle is

$$\vec{F} = -q\vec{E}, \text{ where } E = \frac{\rho}{3\varepsilon_0}\vec{r}$$

$$= -\frac{\rho q}{3\varepsilon_0}\vec{r}$$

or $$m\vec{a} = -\frac{\rho q}{3\varepsilon_0}\vec{r} \quad \text{or} \quad \vec{a} = -\frac{\rho q}{3m\varepsilon_0}\vec{r}$$

Hence, the force acting on the particle is making the particle execute SHM. Comparing with $\vec{a} = -\omega^2\vec{r}$, we get

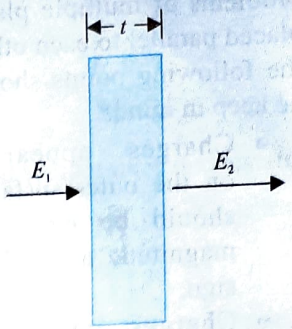
$$\omega^2 = \frac{\rho q}{3m\varepsilon_0}$$

Hence, the time period of oscillation of the particle is

$$T = 2\pi\sqrt{\frac{3\varepsilon_0 m}{\rho q}}$$

EXAMPLE 2.8

An infinitely large layer of charge of uniform thickness t is placed normal to an existing uniform electric field. The charge on the sheet alters the electric field so that it still remains uniform on both the sides of the sheet and assumes values E_1 and E_2 as shown in figure. The charge distribution in the layer is not uniform and depends only on the distance from its faces. Find the expression for the force F per unit area experienced by the charge layer.



Sol. Let E be the external electric field and E_{layer} by the electric field due to the layer. On the left side,

$$E - E_{\text{layer}} = E_1 \qquad \ldots(\text{i})$$

On the right side,

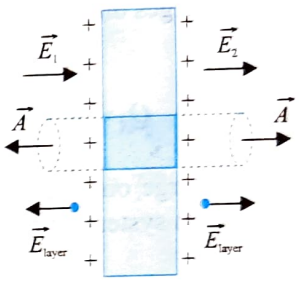
$$E + E_{\text{layer}} = E_2 \qquad \ldots(\text{ii})$$

From Eqs. (i) and (ii),

$$2E = E_1 + E_2 \qquad \ldots(\text{iii})$$

Using Gauss's theorem,

$$-E_1 A + E_2 A = \frac{Q}{\varepsilon_0}$$



Q is the charge inside the Gaussian surface

or $$E_2 - E_1 = \frac{Q}{A\varepsilon_0} \qquad \ldots(\text{iv})$$

So the force experienced by the charge layer per unit area is

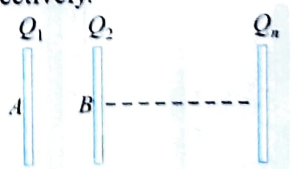
$$\frac{F}{A} = \left(\frac{Q}{A}\right)E \qquad \ldots(\text{v})$$

From Eqs. (iii) and (iv), substituting the values of E and Q/A, we get

$$\frac{F}{A} = \varepsilon_0(E_2 - E_1)\frac{(E_2 + E_1)}{2} = \frac{\varepsilon_0}{2}(E_2^2 - E_1^2)$$

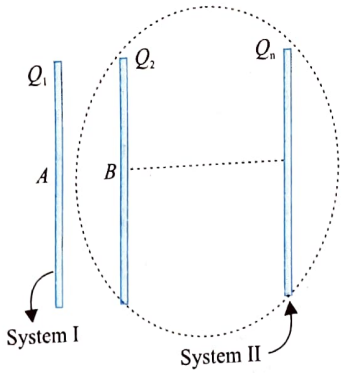
EXAMPLE 2.9

There are n large parallel plate conductors carrying charges $Q_1, Q_2, \ldots, Q_n$ respectively.



(a) Find the charge induced at surface A.
(b) Find the charge induced at surface B.
(c) If the left conductor is earthed, find the magnitude of charge flowing from plate to earth.
(d) If any conductor is earthed, find the magnitude of charge flowing from plate to earth.

Sol. While solving the problems of multiple plates placed parallel to each other, the following points should be keep in mind.



- Charges appearing on the outer surfaces should be equal in magnitude as well as sign.
- Charges appearing on the facing surfaces should be equal in magnitude but opposite in sign.
- If we connect any plate (innermost or outermost) with earth, the charge appearing on the outer surfaces will be zero.
- The charges of the plates other than those connected with earth remain conserved.

(a) Charge on outermost surface A is half the summation of charges on both systems. So

$$Q_A = \frac{Q_1 + \sum_{i=2}^{i=n} Q_i}{2} = \frac{1}{2}\sum_{i=1}^{i=n} Q_i$$

(b) Charge on surface B is half the difference of charges on both systems. So

$$Q_B = -\frac{Q_1 - \sum_{i=2}^{i=n} Q_i}{2} = -\frac{1}{2}\left[Q_1 - \sum_{i=2}^{i=n} Q_i\right]$$

(c) If the outer plate is connected with earth, the charge on the outermost surfaces will be zero.

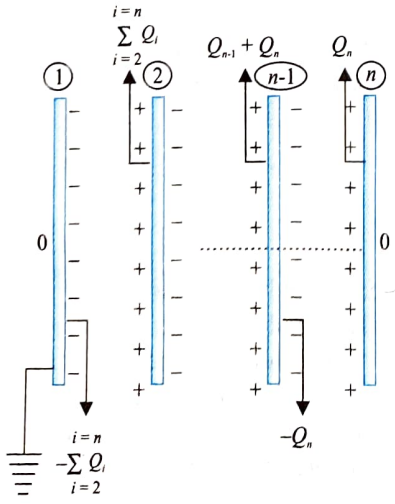
Initial charge in plate 1 = Q_1

Final charge in plate 1 = $-\sum_{i=2}^{i=n} Q_i$

Charge x flowing from plate 1 to earth is

$$-\sum_{i=2}^{i=n} Q_i + x = Q_1$$

or $$x = Q_1 + \sum_{i=2}^{i=n} Q_i = \sum_{i=1}^{i=n} Q_i$$

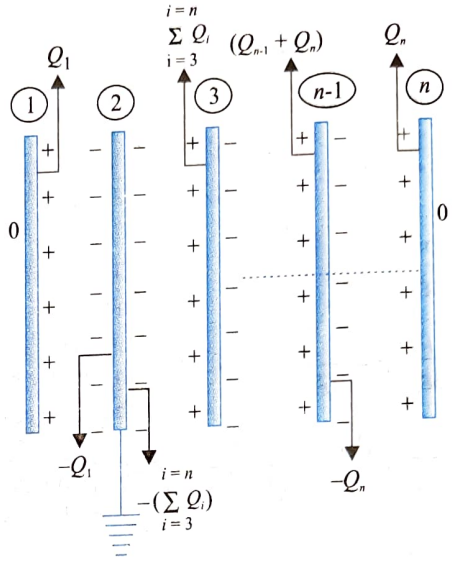


(d) Let the second plate from the left be earthed. The charge on the outermost surface of the plate should be zero.

Initial charge in plate 2 = Q_2

Final charge in plate 2 = $-Q_1 - \sum_{i=3}^{i=n} Q_i$

Let the charge flowing from plate to earth be y.



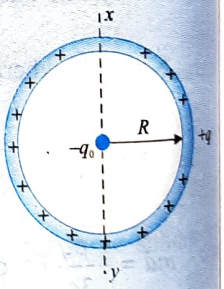
$$\therefore \quad -Q_1 - \sum_{i=3}^{i=n} Q_i + y = Q_2$$

or $$y = Q_1 + Q_2 + \sum_{i=3}^{i=n} Q_i = \sum_{i=1}^{i=n} Q_i$$

Hence, if we connect any plate with earth, the charge flowing from the plate to earth will be $\sum_{i=1}^{i=n} Q_i$.

EXAMPLE 2.10

A thin spherical shell of radius R having uniformly distributed charge q. At the center of shell a negative point charge is placed as shown in figure. If the shell is cut in two identical hemispherical portions by a diametrical section xy as shown, due to mutual repulsion the two hemispherical parts tend to move away from each other but due to the attraction of the charge at centre, they remain in equilibrium. Find the magnitude of the charge to be place at the centre of the shell.



Sol. Let $-q_0$ be the charge to be placed at centre of the hemispherical shells.

Here the outward electric pressure at every point of the spherical shell due to its own charge can be given as

$$P_1 = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2\varepsilon_0}\left(\frac{q}{4\pi R^2}\right)^2 = \frac{q^2}{32\pi^2\varepsilon_0 R^4}$$

Due to charge $-q_0$, the electric field on the surface of shell is

$$E = k\frac{q_0}{R^2}$$

This electric field pulls every point of the shell in inward direction thus inward pressure on the surface of shell due to this negative point charge at center is

$$P_2 = \sigma E = \left(\frac{q}{4\pi R^2}\right)\left(\frac{kq_0}{R^2}\right) = \frac{qq_0}{16\pi^2\varepsilon_0 R^4}$$

Now for equilibrium of hemispherical shell or for the shell do not separate the condition is $P_2 \geq P_1$.

$$\frac{qq_0}{16\pi^2\varepsilon_0 R^4} \geq \frac{q^2}{32\pi^2\varepsilon_0 R^4} \quad \Rightarrow q_0 \geq \frac{q}{2}$$

EXAMPLE 2.11

Surface tension of a soap solution is T. There is a soap bubble of radius r. Calculate the amount of charge that must be spread uniformly on its surface so that its radius becomes $2r$. Atmospheric pressure is P_0. Assume that air temperature inside the bubble remains constant.

Sol. Due to electrostatic repulsion the radius of the bubble will increase. At equilibrium the pressure inside and outside the bubble should be equal.

Pressure inside a soap bubble of radius r is, $P_1 = P_0 + \frac{4T}{r}$.

After expansion, the pressure becomes, $P_2 = \frac{P_0}{8} + \frac{T}{2r}$;

[$\therefore$ The volume becomes 8 times and temperature is constant]

Now, Excess pressure is $\Delta P = P_2 - P_1 = \frac{T}{2r} - \frac{7P_0}{8}$

The electrostatic stress (pressure) due to the charge on bubble,

$$P_{\text{electric}} = \frac{\sigma^2}{2\varepsilon_0}$$

Considering the equilibrium of the bubble, $\frac{\sigma^2}{2\varepsilon_0} + \Delta P = \frac{4T}{2r}$

$$\frac{\sigma^2}{2\varepsilon_0} + \frac{T}{2r} - \frac{7P_0}{8} = \frac{4T}{2r}$$

$$\frac{\sigma^2}{\varepsilon_0} = \frac{3T}{r} + \frac{7P_0}{4} \quad \Rightarrow \sigma = \left[\varepsilon_0\left(\frac{3T}{r} + \frac{7P_0}{4}\right)\right]^{1/2}$$

$$Q = \sigma.4\pi(2r)^2 = 16\pi r^2 \cdot \sigma = 8\pi r^2\left[\varepsilon_0\left(\frac{12T}{r} + 7P_0\right)\right]^{1/2}$$

Exercises

Single Correct Answer Type

1. Under what conditions can the electric flux ϕ_E be found through a closed surface?

(1) If the magnitude of the electric field is known everywhere on the surface.

(2) If the total charge inside the surface is specified.

(3) If the total charge outside the surface is specified.

(4) Only if the location of each point charge inside the surface is specified.

2. If the flux of the electric field through a closed surface is zero, then

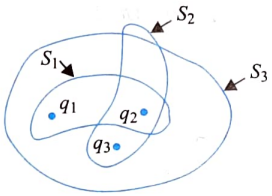
(1) the electric field must be zero everywhere on the surface

(2) the total charge inside the surface must be zero

(3) the electric field must be uniform throughout the closed surface

(4) the charge outside the surface must be zero

3. Three charges $q_1 = 1\times10^{-6}$ C, $q_2 = 2\times10^{-6}$ C, and $q_3 = -3\times10^{-6}$ C have been placed as shown in figure. Then the net electric flux will be maximum for the surface



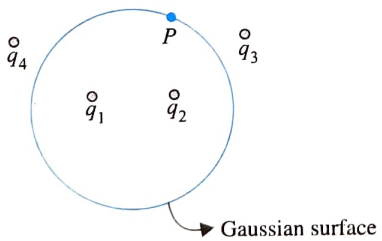
(1) S_1

(2) S_2

(3) S_3

(4) same for all three

4. Consider the Gaussian surface that surrounds part of the charge distribution shown in figure. Then the contribution to the electric field at point P arises from charges



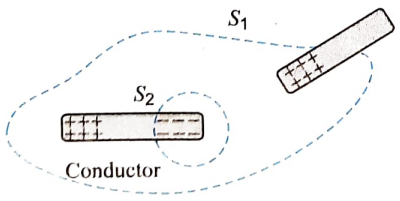
(1) q_1 and q_2 only

(2) q_3 and q_4 only

(3) q_1, q_2, q_3, and q_4

(4) none of the above

5. Charge on an originally uncharged conductor is separated by holding a positively charged rod very closely nearby, as shown in figure. Assume that the induced negative charge on the conductor is equal to the positive charge q on the rod. Then the flux through surface S_1 is



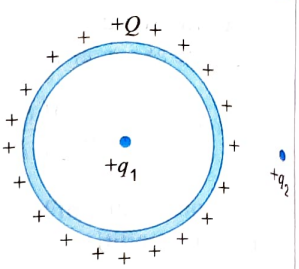
(1) zero

(2) q/ε_0

(3) $-q/\varepsilon_0$

(4) none of these

6. A thin metallic spherical shell contains a charge Q on its surface. A point charge q_1 is placed at the center of the shell, and another charge q_2 is placed outside the shell. All the three charges are positive. Then the force on charge q_1 is



(1) toward right

(2) toward left

(3) zero

(4) none of these

7. The electric flux from a cube of edge l is ϕ. If an edge of the cube is made $2l$ and the charge enclosed is halved, its value will be

(1) 4ϕ

(2) 2ϕ

(3) $\phi/2$

(4) ϕ

8. Consider two concentric spherical surfaces S_1 with radius a and S_2 with radius $2a$, both centered at the origin. There is a charge $+q$ at the origin and there are no other charges. Compare the flux ϕ_1 through S_1 with the flux ϕ_2 through S_2

(1) $\phi_1 = 4\phi_2$

(2) $\phi_1 = 2\phi_2$

(3) $\phi_1 = \phi_2$

(4) $\phi_1 = \phi_2/2$

9. A cylinder of length L and radius b has its axis coincident with the x-axis. The electric field in this region is $\vec{E} = 200\hat{i}$. Find the flux through the left end of the cylinder.

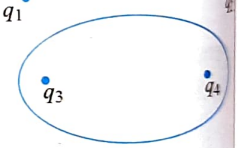
(1) 0

(2) $200\ \pi b^2$

(3) $100\ \pi b^2$

(4) $-200\ \pi b^2$

10. Figure shows four charges q_1, q_2, q_3, and q_4 fixed in space. Then the total flux of the electric field through a closed surface S, due to all the charges, is



(1) not equal to the total flux through S due to q_3 and q_4

(2) equal to the total flux through S due to q_3 and q_4

(3) zero if $q_1 + q_2 = q_3 + q_4$

(4) twice the total flux through S due to q_3 and q_4 if $q_1 + q_2 = q_3 + q_4$

11. Eight charges, 1 μC, -7 μC, -4 μC, 10 μC, 2 μC, -5 μC, -3 μC, and 6 μC, are situated at the eight corners of a cube of side 20 cm. A spherical surface of radius 80 cm encloses this cube. The center of the sphere coincides with the center of the cube. Then, the total outgoing flux from the spherical surface (in units of Vm) is

(1) $36\pi\times10^3$

(2) $684\pi\times10^3$

(3) zero

(4) none of these

12. In a region of space, the electric field is given by $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}$. The electric flux through a surface of area 100 units in the xy plane is

(1) 800 units

(2) 300 units

(3) 400 units

(4) 1500 units

13. A flat, square surface with sides of length L is described by the equations

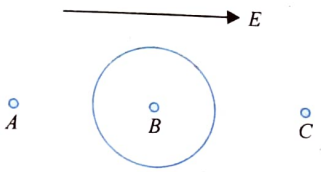
$$x = L,\ 0 \le y \le L,\ 0 \le z \le L$$

The electric flux through the square due to a positive point charge q located at the origin ($x = 0, y = 0, z = 0$) is

(1) $\frac{q}{4\varepsilon_0}$ (2) $\frac{q}{6\varepsilon_0}$

(3) $\frac{q}{24\varepsilon_0}$ (4) $\frac{q}{48\varepsilon_0}$

14. A dielectric in the form of a sphere is introduced into a homogeneous electric field. A, B, and C are three points as shown in figure. Then



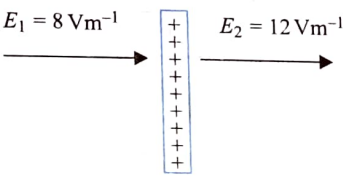
(1) intensity at A increases while that at B and C decreases

(2) intensity at A and B decreases, whereas intensity at C increases

(3) intensity at A and C increases and that at B decreases

(4) intensity at A, B, and C decreases

15. The electric field on two sides of a large charged plate is shown in figure. The charge density on the plate in SI units is given by (ε_0 is the permittivity of free space in SI units)



(1) $2\varepsilon_0$ (2) $4\varepsilon_0$

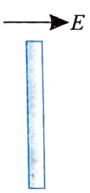
(3) $10\varepsilon_0$ (4) zero

16. A sphere of radius R carries charge such that its volume charge density is proportional to the square of the distance from the center. What is the ratio of the magnitude of the electric field at a distance $2R$ from the center to the magnitude of the electric field at a distance of $R/2$ from the center?

(1) 1 (2) 2

(3) 4 (4) 8

17. An uncharged conducting large plate is placed as shown. Now an electric field E toward right is applied. Find the induced charge density on the right surface of the plate.



(1) $-\varepsilon_0 E$ (2) $\varepsilon_0 E$

(3) $-2\varepsilon_0 E$ (4) $2\varepsilon_0 E$

18. An uncharged aluminum block has a cavity within it. The block is placed in a region where a uniform electric field is directed upward. Which of the following is a correct statement describing conditions in the interior of the block's cavity?

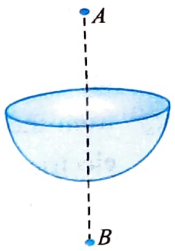
(1) The electric field in the cavity is directed upward.

(2) The electric field in the cavity is directed downward.

(3) There is no electric field in the cavity.

(4) The electric field in the cavity is of varying magnitude and is zero at the exact center.

19. Figure shows a uniformly charged hemisphere of radius R. It has a volume charge density ρ. If the electric field at a point $2R$, above its center is E, then what is the electric field at the point $2R$ below its center?



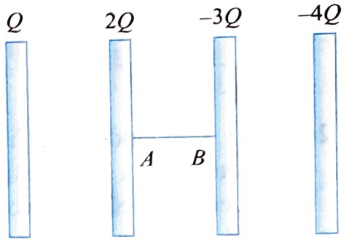
(1) $\rho R/6\varepsilon_0 + E$

(2) $\rho R/12\varepsilon_0 - E$

(3) $-\rho R/6\varepsilon_0 + E$

(4) $\rho R/12\varepsilon_0 + E$

20. Four very large metal plates are given charges as shown in figure. The middle two are then connected through a wire. Find the charge that will flow through the wire.



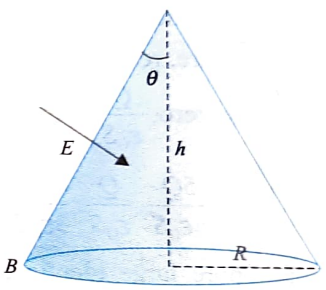
(1) $5Q$ from A to B

(2) $5Q/2$ from A to B

(3) $5Q$ from B to A

(4) no charge will flow

21. A conic surface is placed in a uniform electric field E as shown in figure such that the field is perpendicular to the surface on the side AB. The base of the cone is of radius R, and the height of the cone is h. The angle of the cone is θ.



Find the magnitude of the flux that enters the cone's curved surface from the left side. Do not count the outgoing flux ($\theta < 45°$).

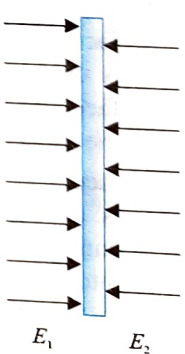
(1) $ER[h\cos\theta + \pi(R/2)\sin\theta]$

(2) $ER[h\sin\theta + \pi R/2\cos\theta]$

(3) $ER[h\cos\theta + \pi R\sin\theta]$

(4) none of these

22. A large charged metal sheet is placed in a uniform electric field, perpendicularly to the electric field lines. After placing the sheet into the field, the electric field on the left side of the sheet is $E_1 = 5 \times 10^5\ \text{Vm}^{-1}$ and on the right it is $E_2 = 3 \times 10^5\ \text{Vm}^{-1}$. The sheet experiences a net electric force of 0.08 N. Find the area of one face of the sheet. Assume the external field to remain constant after introducing the large sheet. Use $\left(\frac{1}{4\pi\varepsilon_0}\right) = 9 \times 10^9\ \text{Nm}^2\text{C}^{-2}$



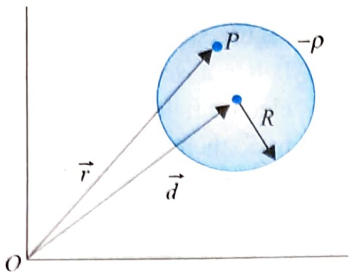
(1) $3.6\pi \times 10^{-2}\ \text{m}^2$

(2) $0.9\pi \times 10^{-2}\ \text{m}^2$

(3) $1.8\pi \times 10^{-2}\ \text{m}^2$

(4) none

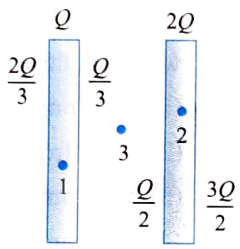
23. A nonconducting sphere of radius R is filled with uniform volume charge density $-\rho$. The center of this sphere is displaced from the origin by $\vec{d}$. The electric field $\vec{E}$ at any point P having position vector inside the sphere is



(1) $\frac{\rho}{3\varepsilon_0}\vec{d}$ (2) $\frac{\rho}{3\varepsilon_0}(\vec{r}-\vec{d})$

(3) $\frac{\rho}{3\varepsilon_0}(\vec{d}-\vec{r})$ (4) $\frac{\rho}{3\varepsilon_0}(\vec{r})$

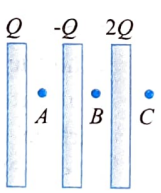
24. Two nonconducting infinite plane sheets having charges Q and $2Q$ are placed parallel to each other as shown in figure. The charge distribution on the four faces of the two plates is also shown. The electric field intensities at three points 1, 2, and 3 are $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, respectively. Then the magnitudes of $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$ are, respectively (surface area of plates)



(1) zero, $\frac{Q}{\varepsilon_0 S}$, zero (2) $\frac{5Q}{6\varepsilon_0 S}, \frac{Q}{2\varepsilon_0 S}$, zero

(3) $\frac{5Q}{6\varepsilon_0 S}, \frac{Q}{\varepsilon_0 S}, \frac{Q}{3\varepsilon_0 S}$ (4) zero, $\frac{Q}{2\varepsilon_0 S}$, zero

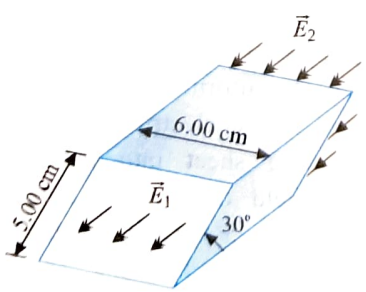
25. Three large identical conducting parallel plates carrying charge $+Q$, $-Q$, and $+2Q$, respectively, are placed as shown in figure. If E_A, E_B, and E_C refer to the magnitudes of the electric fields at points A, B, and C, respectively, then



(1) $E_A > E_B > E_C$ (2) $E_A = E_B > E_C$

(3) $E_A = 0$ and $E_B > E_C$ (4) $E_A = 0$ and $E_B = E_C$

26. The electric field $\vec{E}_1$ at one face of a parallelopiped is uniform over the entire face and is directed out of the face. At the opposite face, the electric field $\vec{E}_2$ is also uniform over the entire face and is directed into that face (as shown in figure). The two faces in question are inclined at 30° from the horizontal, $\vec{E}_1$ and $\vec{E}_2$ (both horizontal) have magnitudes of 2.50×10^4 NC^{-1} and 7.00×10^4 NC^{-1}, respectively. Assuming that no other electric field lines cross the surfaces of the parallelopiped, the net charge contained within is



(1) $-67.5\varepsilon_0$ C (2) $37.5\,\varepsilon_0$ C

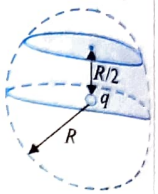
(3) $105\,\varepsilon_0$ C (4) $-105\,\varepsilon_0$ C

27. The number of electric field lines crossing an area ΔS is n_1 when $\Delta\vec{S} \| \vec{E}$, while the number of field lines crossing the same area is n_2 when $\Delta\vec{E}$ makes an angle of 30° with $\vec{E}$. Then

(1) $n_1 = n_2$ (2) $n_1 > n_2$

(3) $n_1 < n_2$ (4) cannot say anything

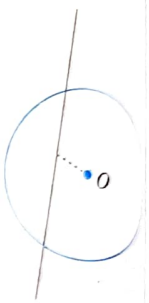
28. Flux passing through the shaded surface of a sphere when a point charge q is placed at the center is (radius of the sphere is R)



(1) q/ε_0 (2) $q/2\varepsilon_0$

(3) $q/4\varepsilon_0$ (4) zero

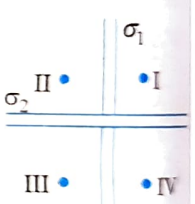
29. A uniformly charged and infinitely long line having a liner charge density λ is placed at a normal distance y from a point O. Consider a sphere of radius R with O as the center and $R > y$. Electric flux through the surface of the sphere is



(1) zero (2) $\frac{2\lambda R}{\varepsilon_0}$

(3) $\frac{2\lambda\sqrt{R^2-y^2}}{\varepsilon_0}$ (4) $\frac{\lambda\sqrt{R^2+y^2}}{\varepsilon_0}$

30. Two infinite sheets having charge densities σ_1 and σ_2 are placed in two perpendicular planes whose two-dimensional view is shown in figure. The charges are distributed uniformly on the sheets in electrostatic equilibrium condition. Four points are marked I, II, III, and IV. The electric field intensities at these points are $\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3$, and $\vec{E}_4$, respectively. The correct expression for the electric field intensities is



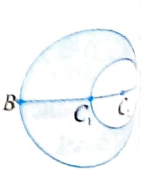
(1) $|\vec{E}_1| = |\vec{E}_2| = \frac{\sqrt{\sigma_1^2+\sigma_2^2}}{2\varepsilon_0} \neq |\vec{E}_4|$

(2) $|\vec{E}_2| = |\vec{E}_4| = \frac{\sqrt{\sigma_1^2+\sigma_2^2}}{2\varepsilon_0}$

(3) $|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3| = |\vec{E}_4| = \frac{\sqrt{\sigma_1^2+\sigma_2^2}}{2\varepsilon_0}$

(4) none of these

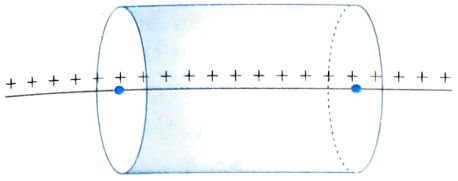
31. A positively charged sphere of radius r_0 carries a volume charge density ρ (figure). A spherical cavity of radius $r_0/2$ is then scooped out and left empty. C_1 is the center of the sphere and C_2 that of the cavity. What is the direction and magnitude of the electric field at point B?



(1) $\frac{17\rho r_0}{54\varepsilon_0}$ left (2) $\frac{\rho r_0}{6\varepsilon_0}$ left

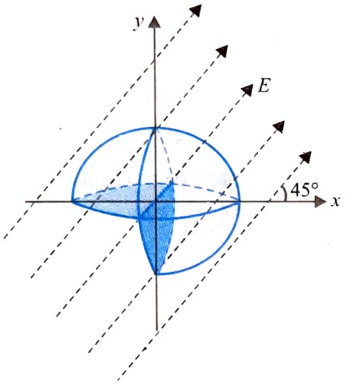
(3) $\frac{17\rho r_0}{54\varepsilon_0}$ right (4) $\frac{\rho r_0}{6\varepsilon_0}$ right

32. Consider an infinite line charge having uniform linear charge density and passing through the axis of a cylinder. What will be the effect on the flux passing through the curved surface if the portions of the line charge outside the cylinder is removed.



(1) decreases (2) increases
(3) remains same (4) cannot say

33. One-fourth of a sphere of radius R is removed as shown in figure. An electric field E exists parallel to the xy plane. Find the flux through the remaining curved part.

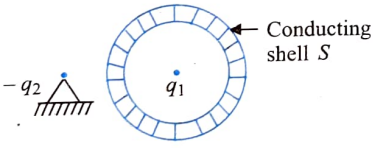


(1) $\pi R^2 E$
(2) $\sqrt{2}\pi R^2 E$
(3) $\pi R^2 E/\sqrt{2}$
(4) none of these

34. A spherical shell of radius $R = 1.5$ cm has a charge $q = 20\ \mu C$ uniformly distributed over it. The force exerted by one half over the other half is

(1) zero (2) 10^{-2} N
(3) 500 N (4) 2000 N

35. The negative charge $-q_2$ is fixed, while positive charge q_1 as well as the conducting sphere S is free to move (refer figure). If the system is released from rest,

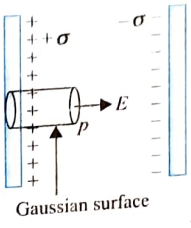


(1) both S and q_1 move toward left
(2) q_1 moves toward right while S moves toward left
(3) q_1 remains at rest, S moves toward left
(4) both q_1 and S remain at rest

36. A wire of linear charge density λ passes through a cuboid of length l, breadth b and height h ($l > b > h$) in such a manner that the flux through the cuboid is maximum. The position of the wire is now changed, so that the flux through the cuboid is minimum. The ratio of maximum flux to minimum flux will be

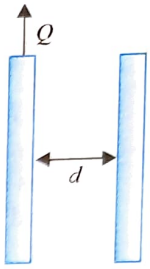
(1) $\dfrac{\sqrt{l^2+b^2}}{h}$ (2) $\dfrac{\sqrt{l^2+b^2+h^2}}{h}$
(3) $\dfrac{h}{\sqrt{l^2+b^2}}$ (4) $\dfrac{h}{\sqrt{l^2+b^2+h^2}}$

37. Consider the situation shown in figure. We find electric field E at point P using Gauss's law and it comes out to be $E = \sigma/\varepsilon_0$. This electric field is due to



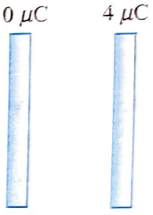
(1) all the amount of charges present on both the plates
(2) all the charges present on positive plate only
(3) positive charge present only inside the Gaussian surface
(4) positive charge present inside the Gaussian surface plus equal magnitude of negative charge present on negative plate in front of Gaussian surface

38. Two parallel conducting plates, each of area A, are separated by a distance d. Now, the left plate is given a positive charge Q. A positive charge q of mass m is released from a point near the left plate. Find the time taken by the charge to reach the right plate.



(1) $\sqrt{\dfrac{3dm\varepsilon_0 A}{qQ}}$ (2) $\sqrt{\dfrac{4dm\varepsilon_0 A}{qQ}}$
(3) $\sqrt{\dfrac{2dm\varepsilon_0 A}{qQ}}$ (4) None of these

39. Two large plates are given the charges as shown in figure. Now, the left plate is earthed. Find the amount of charge that will flow from the earth to the plate.



(1) 14 μC (2) −14 μC
(3) 7 μC (4) −7 μC

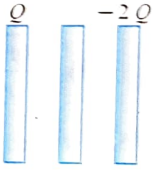
40. The electric field in a region is given by

$$\vec{E} = \frac{E_0 x}{l}\hat{i}$$

Find the charge contained inside a cubical volume bounded by the surfaces $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$, and $z = a$. Take $E_0 = 5 \times 10^3$ NC^{-1}, $l = 0.02$ m, and $a = 0.01$ m.

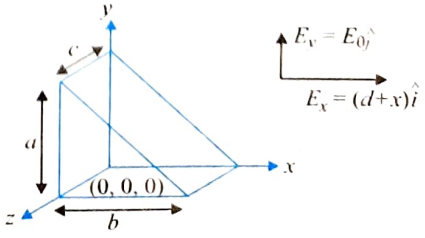
(1) 1.1×10^{-12} C (2) 2.2×10^{-12} C
(3) 4.4×10^{-12} C (4) 5.5×10^{-12} C

41. Three identical metal plates with large surface areas are kept parallel to each other as shown in figure. The leftmost plate is given a charge Q, the rightmost a charge $-2Q$ and the middle one remains neutral. Find the charge appearing on the outer surface of the rightmost plate.



(1) $-Q/4$ (2) $-2Q$
(3) $-Q$ (4) $-Q/2$

42. In the given electric field $\vec{E} = \left[\alpha(d+x)\hat{i} + E_0\hat{j}\right]$NC^{-1}; where $\alpha = 1$ NC^{-1} hypothetical closed surface is taken as shown in figure.

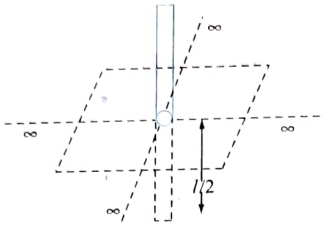


The total charge enclosed within the close surface is

(1) $\frac{abc\varepsilon_0}{2}$ (2) $\frac{acd\varepsilon_0}{2}$

(2) $\frac{abd\varepsilon_0}{2}$ (4) none of these

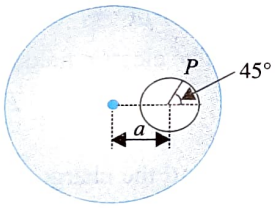
43. An infinite nonconducting sheet has surface charge density σ. There is a small hole in the sheet as shown in the figure. A uniform rod of length l having linear charge density λ is hinged in the hole as shown. If the mass of the rod is m, then the time period of oscillation for small angular displacement is



(1) $\pi\sqrt{\frac{m\varepsilon_0}{3\sigma\lambda}}$ (2) $2\pi\sqrt{\frac{2m\varepsilon_0}{3\sigma\lambda}}$

(3) $\frac{\pi}{2}\sqrt{\frac{m\varepsilon_0}{3\sigma\lambda}}$ (4) $4\pi\sqrt{\frac{m\varepsilon_0}{3\sigma\lambda}}$

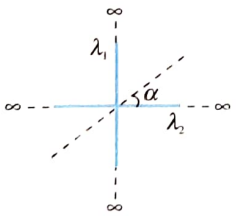
44. A cavity of radius r is made inside a solid sphere. The volume charge density of the remaining sphere is ρ. An electron (charge e, mass m) is released inside the cavity from point P as shown in figure. The center of sphere and center of cavity are separated by a distance a. The time after which the electron again touches the sphere is



(1) $\sqrt{\frac{6\sqrt{2}r\varepsilon_0 m}{epa}}$ (2) $\sqrt{\frac{\sqrt{2}r\varepsilon_0 m}{epa}}$

(3) $\sqrt{\frac{6r\varepsilon_0 m}{epa}}$ (4) $\sqrt{\frac{r\varepsilon_0 m}{epa}}$

45. Two mutually perpendicular wire carry charge densities λ_1 and λ_2. The electric lines of force makes angle α with second wire, then λ_1/λ_2 is



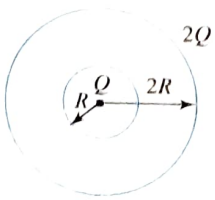
(1) $\tan^2\alpha$

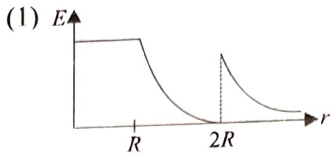
(2) $\cot^2\alpha$

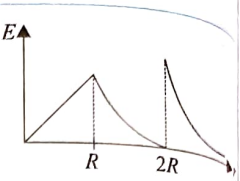
(3) $\sin^2\alpha$

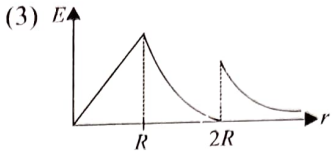
(4) $\cos^2\alpha$

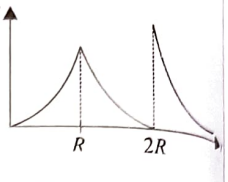
46. A charge Q is uniformly distributed in a dielectric sphere of radius R (having dielectric constant unity). This dielectric sphere is enclosed by a concentric spherical shell of radius $2R$ and having uniformly distributed charge $2Q$. Which of the following graph correctly represents variation of electric field with distance r from the common center?



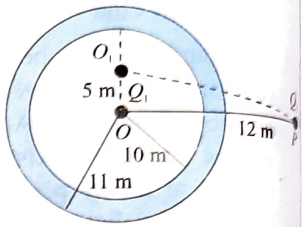
(1) 

(2) 

(3) 

(4) 

47. A charge Q_1 is placed at O inside a hollow conducting sphere having inner and outer radii as 10 m and 11 m as shown. The force experienced by Q_2 at P is F_1 and force experienced by Q_2 when Q_1 is placed at O_1 is F_2. Then F_1/F_2 is equal to



(1) 1 (2) $\left(\frac{12}{13}\right)^2$

(3) $\left(\frac{13}{12}\right)^2$ (4) none of these

Multiple Correct Answers Type

1. For Gauss's law, mark the correct statement(s).

(1) If we displace the enclosed charges (within a Gaussian surface) without crossing the boundary, then both $\vec{E}$ and ϕ remain same.

(2) If we displace the enclosed charges without crossing the boundary, then $\vec{E}$ changes but ϕ remains the same.

(3) If the charge crosses the boundary, then both $\vec{E}$ and would change.

(4) If the charge crosses the boundary, then ϕ changes but $\vec{E}$ remains the same.

2. Consider Gauss's law:

$$\oint \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}$$

Then, for the situation shown in figure at the Gaussian surface

(1) $\vec{E}$ due to q_2 would be zero

(2) $\vec{E}$ due to both q_1 and q_2 would not be zero

(3) ϕ due to both q_1 and q_2 would not be zero

(4) ϕ due to q_2 would be zero

3. Consider a Gaussian spherical surface covering a dipole of charge q and $-q$, then

(1) $q_{in} = 0$ (net charge enclosed by the spherical surface)

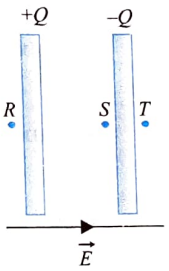
(2) $\phi_{net} = 0$ (net flux coming out the spherical surface)

(3) $E = 0$ at all points on the spherical surface

(4) $\oint \vec{E}\cdot\vec{ds} = 0$ (surface integral of over the spherical surface)

4. A 10 C charge is given to a conducting spherical shell, and a −3 C point charge is placed inside the shell. For this arrangement, find the correct statement(s).
   (1) The charge on the inner surface of the shell will be +3 C, and it can be distributed uniformly or nonuniformly.
   (2) The charge on the inner surface of the shell will be +3 C, and its distribution would be uniform.
   (3) The net charge on the outer surface of the shell will be +7 C, and its distribution can be uniform or nonuniform.
   (4) The net charge on the outer surface of the shell will be +7 C, and its distribution would be uniform.

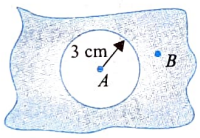
5. Two large thin conducting plates with a small gap in between are placed in a uniform electric field E (perpendicular to the plates). The area of each plate is A, and charges $+Q$ and $-Q$ are given to these plates as shown in figure. If R, S, and T are three points in space, then the



   (1) field at point R is E
   (2) field at point S is E
   (3) field at point T is $\left(E+\frac{Q}{\varepsilon_0 A}\right)$
   (4) field at point S is $\left(E+\frac{Q}{A\varepsilon_0}\right)$

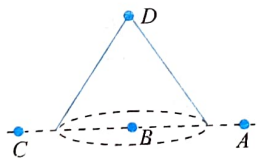
6. Charges Q_1 and Q_2 lie inside and outside, respectively, of a closed surface S. Let E be the field at any point on S and ϕ be the flux of E over S.
   (1) If Q_1 changes, both E and ϕ will change.
   (2) If Q_2 changes, E will change but ϕ will not change.
   (3) If $Q_1 = 0$ and $Q_2 \neq 0$, then $E \neq 0$ but $\phi = 0$.
   (4) If $Q_1 \neq 0$ and $Q_2 = 0$, then $E = 0$ but $\phi \neq 0$.

7. Figure shows a point charge of 0.5×10^{-6} C at the center of the spherical cavity of radius 3 cm of a piece of metal. The electric field at



   (1) A (2 cm from the charge) is 0
   (2) A (2 cm from the charge) is 1.125×10^7 NC^{-1}
   (3) B (5 cm from the charge) is 0
   (4) B (5 cm from the charge) is 1.8×10^6 NC^{-1}

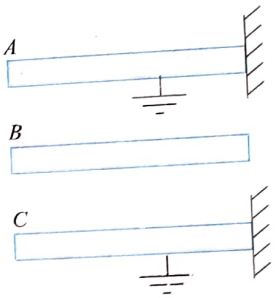
8. A right circular imaginary cone is shown in figure. A, B, and C are the points in the plane containing the base of the cone, while D is the point at the vertex of the cone. If ϕ_A, ϕ_B, ϕ_C, and ϕ_D represent the flux through the curved surface of the cone when a point charge Q is at points A, B, C, and D, respectively, then



   (1) $\phi_A = \phi_C = 0$
   (2) $\phi_D = 0$
   (3) $\phi_B = \frac{Q}{2\varepsilon_0}$
   (4) $\phi_A = \phi_C = \phi_D = 0$

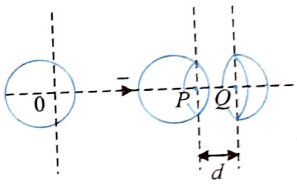
9. A thin-walled spherical conducting shell S of radius R is given charge Q. The same amount of charge is also placed at its center C. Which of the following statements are correct?
   (1) On the outer surface of S, the charge density is $Q/2\pi R^2$.
   (2) The electric field is zero at all points inside S.
   (3) At a point just outside S, the electric field is double the field at a point just inside S in the cavity.
   (4) At any point inside S (i.e., within its cavity), the electric field is inversely proportional to the square of its distance from C.

10. A, B, and C are three large, parallel conducting plates, placed horizontally. A and C are rightly fixed and earthed (figure). B is given some charge. Under electrostatic and gravitational forces, B may be



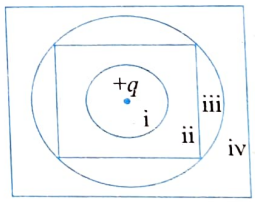
   (1) in equilibrium if it is closer to A than to C
   (2) in equilibrium midway between A and C
   (3) B can never be in stable equilibrium
   (4) in equilibrium if it is closer to C than to A

11. An insulating spherical shell of uniform surface charge density is cut into two parts and placed at a distance d apart as shown in figure. $\vec{E}_P$ and $\vec{E}_Q$ denote the electric fields at P and Q, respectively. As d (i.e. PQ) $\to \infty$



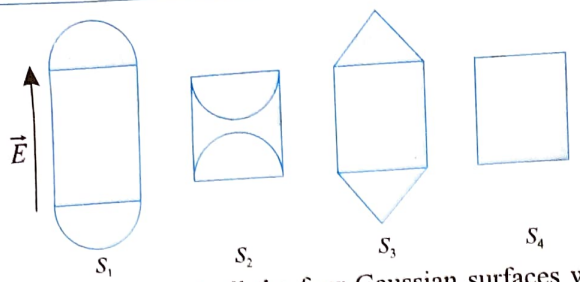
   (1) $|\vec{E}_P| > |\vec{E}_Q|$
   (2) $|\vec{E}_P| = |\vec{E}_Q|$
   (3) $|\vec{E}_P| < |\vec{E}_Q|$
   (4) $\vec{E}_P + \vec{E}_Q = 0$

12. Figure shows, in cross section, two Gaussian spheres and two Gaussian cubes that are centered on a positively charged particle. Rank greatest first, and indicate whether the magnitudes are uniform or variable along each surface.



   (1) net flux through all the four Gaussian surfaces will be equal
   (2) the magnitudes of the electric fields on the surfaces (i) and (iii) will be constant
   (3) the magnitudes of the electric fields on the surfaces (ii) and (iv) will be variable
   (4) the magnitudes of the electric fields on all the surfaces will be constant

13. Figure shows four Gaussian surfaces consisting of identical cylindrical midsections but different end caps. The surfaces are in a uniform electric field that is directed parallel to the central axis of each cylindrical midsection. The end caps have these shapes: S_1, convex hemispheres; S_2, concave hemispheres; S_3, cones; S_4, flat disks. Rank the surfaces according to (a) the net electric flux through them and (b) the electric flux through the top end caps, greatest first.



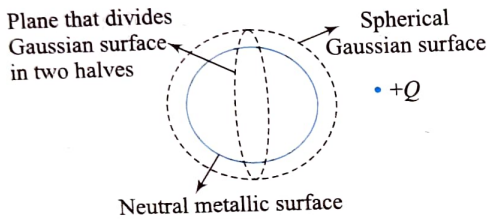
(1) net flux through all the four Gaussian surfaces will be equal

(2) the electric flux through the top end caps will be equal

(3) the electric flux through the top end cap S_1 is greatest

(4) the electric flux through the top end cap S_3 is greatest

14. Figure shows a neutral metallic sphere with a point charge $+Q$ placed near its surface. Electrostatic equilibrium conditions exist on metallic sphere. Mark the correct statements:



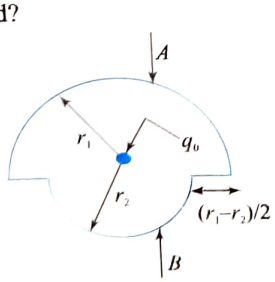
(1) Net flux through Gaussian surface due to charge Q is zero

(2) Net flux through Gaussian surface due to charges appearing on the outer surface of metallic sphere must be zero

(3) If point charge Q is displaced a little towards metallic sphere, magnitude of net flux through right hemispherical closed Gaussian surface increases.

(4) If point charge Q is displaced towards metallic sphere, charge distribution on outer surface of sphere will change

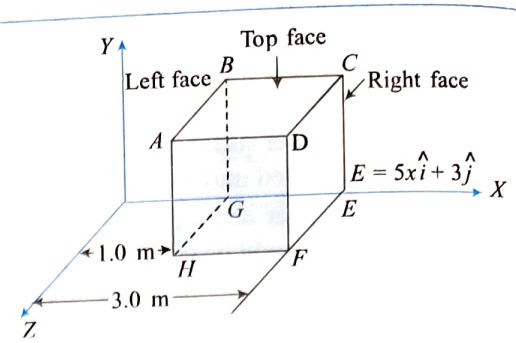
15. A and B are semi-spherical surfaces of radius r_1 and r_2 ($r_1 > r_2$) with E_1 and E_2 as the electric fields at their surfaces. Charge q_0 is placed as shown. What is the condition which may be satisfied?



(1) $\frac{\phi_1}{\phi_2} = 1$ (2) $r_1^2 r_2^2 = E_1 E_2$

(3) $\frac{r_1}{\sqrt{E_2}} = \frac{r_2}{\sqrt{E_1}}$ (4) $r_1\sqrt{E_2} = r_2\sqrt{E_1}$

16. A non-uniform electric field $E = (5x\hat{i} + 3\hat{j})$ N/C goes through a cube of side length 2.0 m. oriented as shown. Then,



(1) flux through face $CDEF = 60\frac{\text{Nm}^2}{\text{C}}$

(2) flux through face $ABCD = 60\frac{\text{Nm}^2}{\text{C}}$

(3) flux through face $ABGH = 20\frac{\text{Nm}^2}{\text{C}}$

(4) flux through face $EFGH = 12\frac{\text{Nm}^2}{\text{C}}$

Linked Comprehension Type

For Problems 1–4

Gauss's law and Coulomb's law, although expressed in differe forms, are equivalent ways of describing the relation betwe charge and electric field in static conditions. Gauss's law $\varepsilon_0\phi = q_{\text{encl}}$, when q_{encl} is the net charge inside an imagina closed surface called Gaussian surface. $\phi = \oint \vec{E}\cdot d\vec{A}$ gives the electric flux through the Gaussian surface. The two equations hold only when the net charge is in vacuum or air.

1. A Gaussian surface encloses two of the four positive charged particles. The particles that contribute to the electr field at a point p on the surface are

(1) q_1 and q_2 (2) q_2 and q_3

(3) q_4 and q_3 (4) q_1, q_2, q_3, and q_4

2. The net flux of the electric field through the surface is

(1) due to q_1 and q_2 only

(2) due to q_3 and q_4 only

(3) equal due to all the four charges

(4) cannot say

3. The net flux of the electric field through the surface due to and q_4 is

(1) zero (2) positive

(3) negative (4) cannot say

4. If the charge q_3 and q_4 are displaced (always remaini outside the Gaussian surface), then consider the followi two statements:

A: Electric field at each point on the Gaussian surface w remain same.

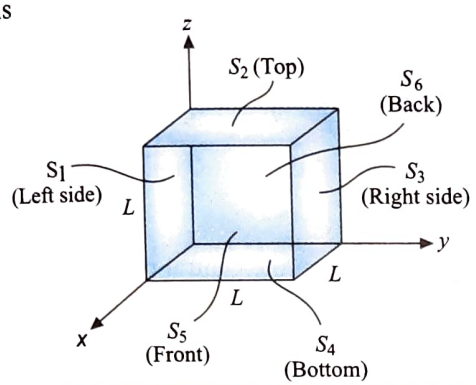
B: The value of $\oint \vec{E}\cdot d\vec{A}$ for the Gaussian surface w remain same.

(1) Both A and B are true.
(2) Both A and B are false.
(3) A is true, but B is false.
(4) B is true, but A is false.

For Problems 5–6

A cube has sides of length L. It is placed with one corner at the origin (refer to figure). The electric field is uniform and given by $\vec{E} = -Bi + Cj - Dk$, where B, C, and D are positive constants.

5. The flux passing through the different surfaces (match the table) is



Surface	Flux
(i) S_1	(m) BL^2
(ii) S_2	(n) $-BL^2$
(iii) S_3	(o) CL^2
(iv) S_4	(p) $-CL^2$
(v) S_5	(q) DL^2
(vi) S_6	(r) $-DL^2$

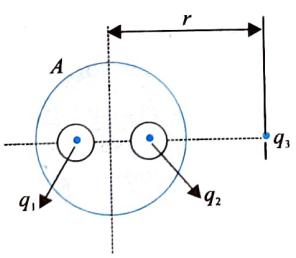
(1) (i, p), (ii, r), (iii, o), (iv, q), (v, n), (vi, m)
(2) (i, r), (ii, p), (iii, q), (iv, n), (v, m), (vi, o)
(3) (i, m), (ii, n), (iii, o), (iv, p), (v, q), (vi, r)
(4) (i, r), (ii, q), (iii, p), (iv, o), (v, n), (vi, m)

6. The total flux passing through the cube is

(1) $(B + C + D)\,L^2$ (2) $2(B + C + D)\,L^2$
(3) $6(B + C + D)\,L^2$ (4) zero

For Problems 7–9

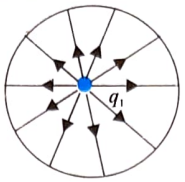
A spherical conductor A contains two spherical cavities as shown in figure. The total charge on conductor itself is zero. However, there is a point charge q_1 at the center of one cavity and q_2 at the center of the other cavity. Another charge q_3 is placed at a large distance r from the center of the spherical conductor.

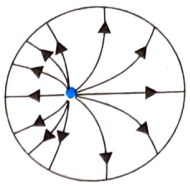


7. Which of the following statements are true?

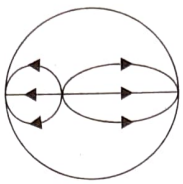
(1) Charge q_3 applies a larger force on charge q_2 than on charge q_1.
(2) Charge q_3 applies a smaller force on charge q_2 than on charge q_1.
(3) Charge q_3 applies equal force on both the charges.
(4) Charge q_3 applies no force on any of the charges.

8. If q_1 is displaced from its center slightly (being always inside the cavity), then the correct representation of field lines inside the same cavity is

(1) 

(2) 

(3) There will be no field lines inside the cavity.

(4) 

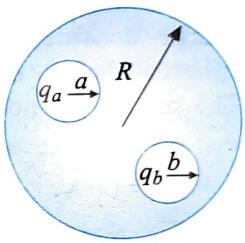
9. The force acting on conductor A will be

(1) zero (2) $\dfrac{q_3(q_1+q_2)}{4\pi\varepsilon_0 r^2}$

(2) $\dfrac{-q_3(q_1+q_2)}{4\pi\varepsilon_0 r^2}$ (4) $\dfrac{q_3q_1+q_2q_3+q_1q_2}{4\pi\varepsilon_0 r^2}$

For Problems 10–12

Two spherical cavities of radii a and b are hollowed out from the interior of a neutral conducting sphere of radius R. At the center of each cavity, a point charge is placed. Call these charges q_a and q_b.



10. Match the table.

Column I	Column II
(i) σ_a	(m) $\dfrac{q_a+q_b}{4\pi R^2}$
(ii) σ_b	(n) $\dfrac{-q_a}{4\pi a^2}$
(iii) σ_R	(o) $\dfrac{-q_b}{4\pi b^2}$

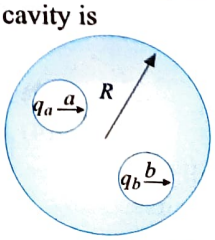
(1) (i, o), (ii, n), (iii, m) (2) (i, n), (ii, o), (iii, m)
(3) (i, m), (ii, o), (iii, n) (4) (i, n), (ii, m), (iii, o)

11. The electric field at a distance r outside the conductor is

(1) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_b}{r^2}$ (2) zero

(3) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_a+q_b}{r^2}$ (4) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_a}{r^2}$

12. The electric field inside the cavity of radius a at a distance r from the center of cavity is

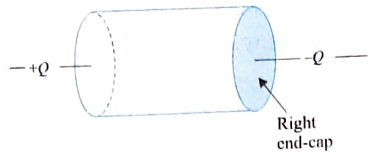


(1) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_a}{r^2}$ (2) $-\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_a}{r^2}$

(3) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_a+q_b}{r^2}$ (4) zero

For Problems 13–14

Positive and negative charges of equal magnitude lie along the symmetry axis of a cylinder. The distance from the positive charge to the left-end cap of the cylinder is the same as the distance from the negative charge to the right-end cap.



13. What is the flux of the electric field through the closed cylinder?

(1) zero (2) $+Q/\varepsilon_0$

(3) $+2Q/\varepsilon_0$ (4) $-Q/\varepsilon_0$

14. What is the sign of the flux through the right-end cap of the cylinder?

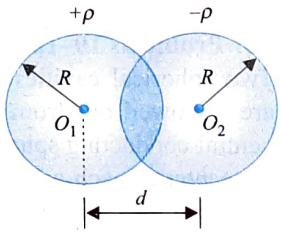
(1) Positive

(2) Negative

(3) There is no flux through the right-end cap.

For Problems 15–16

There are two nonconducting spheres having uniform volume charge densities ρ and $-\rho$. Both spheres have equal radius R. The spheres are now laid down such that they overlap as shown in figure. Take $\vec{d} = \overrightarrow{O_1O_2}$.



15. The electric field $\vec{E}$ in the overlapped region is

(1) non-uniform (2) zero

(3) $\frac{\rho}{3\varepsilon_0}\vec{d}$ (4) $\frac{-\rho}{3\varepsilon_0}\vec{d}$

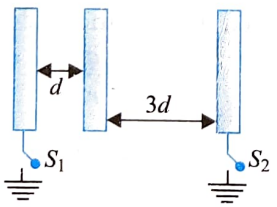
16. The potential difference ΔV between the centers of the two spheres for $d = R$ is

(1) $\frac{\rho}{3\varepsilon_0}d^2$ (2) $\frac{\rho}{\varepsilon_0}d^2$

(3) zero (4) $\frac{2\rho}{\varepsilon_0}d^2$

For Problems 17–19

Three metallic plates out of which the middle is given charge Q as shown in the figure. The outer plates can be earthed with the help of switches S_1 and S_2. The area of each plates is the same. Answer the following question based on the given passage.



17. The charge appearing on the outer surface of extreme left plate is

(1) $-(Q/2)$ (2) $(Q/2)$

(3) Q (4) $-Q$

18. The charge that will flow to the earth when only switch S_1 is connected to the earth is

(1) $-(Q/2)$ (2) $(Q/2)$

(3) Q (4) $-Q$

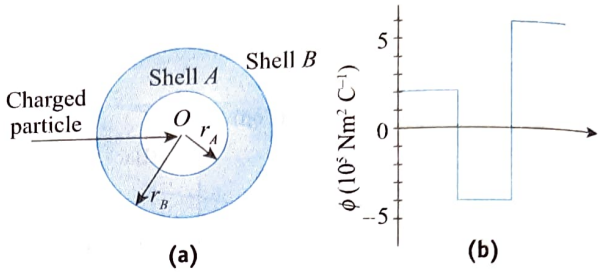
19. The charge that will flow to the earth through S_2 when both the switches S_1 and S_2 are grounded simultaneously is

(1) Q (2) $-Q$

(3) $3Q/4$ (4) $Q/4$

For Problems 20–22

A charged particle is suspended at the center of two thin concentric spherical charged shells, made of nonconducting material. Figure (a) shows cross section of the arrangement. Figure (b) gives the net flux ϕ through a Gaussian sphere centered on the particle, as a function of the radius r of the sphere.



20. What is the charge on the central particle?

(1) 0.2 μC (2) 2 μC

(3) 1.77 μC (4) 3.4 μC

21. What is the charge on shell A?

(1) 5.31 μC (2) –5.31 μC

(3) –3.54 μC (4) –1.77 μC

22. In which range of the values of r is the electric field zero

(1) 0 to r_A

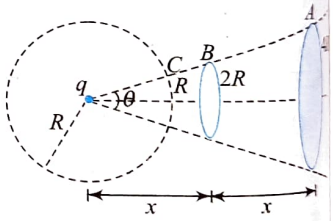
(2) r_A to r_B

(3) for $r > r_B$

(4) for no range of r, electric field is zero

For Problems 23–25

Two nonconducting plates A and B of radii $2R$ and $4R$, respectively, are kept at distances x and $2x$ from the point charge q. A surface cutout of a nonconducting shell A is kept such that its center coincides with the point charge. Each plate and spherical surface carries surface charge density σ.



23. If F_A, F_B, and F_C are the forces on plate A, plate B, and spherical surface C due to charge q, respectively, then

(1) $F_A = F_B = F_C$ (2) $F_C < F_B < F_A$

(3) $F_C > F_B > F_A$ (4) $F_A = F_B > F_C$

24. If ϕ_1 is flux through surface of B due to electric field of A and ϕ_2 is the flux through A due to electric field of B, then

(1) $\phi_1 = \phi_2$ (2) $\phi_1 > \phi_2$

(3) $\phi_1 < \phi_2$ (4) It depends on x and R

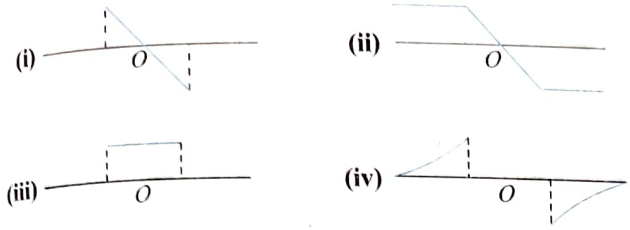
25. If distance x and radius R are doubled so that F_A becomes F_A', F_B becomes F_B', then the correct option is

(1) $F_B > F_B'$ (2) $F_B' > F_B$

(3) $F_A' > F_B'$ (4) none of these

For Problems 26–27

Two very large parallel disks of charge have their centers on the x-axis and their planes perpendicular to the x-axis. The disk that intersects $x = -R$ has uniform positive surface charge density $+\sigma$; the disk that intersects $x = +R$ has uniform negative surface charge density $-\sigma$.



26. Which graph best represents the plot of the x-component of the electric field vector on the x-axis?

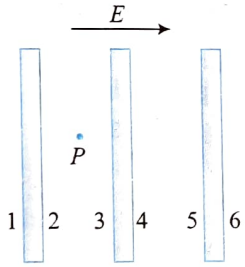
(1) (i) (2) (ii)
(3) (iii) (4) (iv)

27. Which graph best represents the plot of the electric potential (V) as a function of x (treating $V = 0$ at $x = 0$)?

(1) (i) (2) (ii)
(3) (iii) (4) (iv)

For Problems 28–30

Three uncharged conducting large plates are placed parallel to each other in a uniform electric field.



28. Find the induced charge density on surface 1.

(1) $\varepsilon_0 E$ (2) $-\varepsilon_0 E$
(3) zero (4) can't be found

29. Find the induced charge density on surface 4.

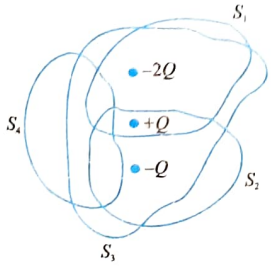
(1) $\varepsilon_0 E$ (2) $-\varepsilon_0 E$
(3) zero (4) can't be found

30. Find electric field at a point P in the region between the plates.

(1) E towards right (2) E towards left
(3) zero (4) can't be found

Matrix Match Type

1. Consider three point charges $-2Q$, $+Q$, and $-Q$ and four closed surfaces S_1, S_2, S_3, and S_4.

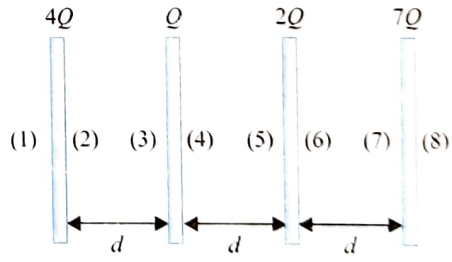


Column I	Column II
i. Net flux through S_1	a. $-\frac{Q}{\varepsilon_0}$
ii. Net flux through S_2	b. $-\frac{2Q}{\varepsilon_0}$
iii. Net flux through S_3	c. zero
iv. Net flux through S_4	d. nonzero

2. Inside a neutral metallic spherical shell, a charge Q_1 is placed, and outside the shell, a charge Q_2 is placed.

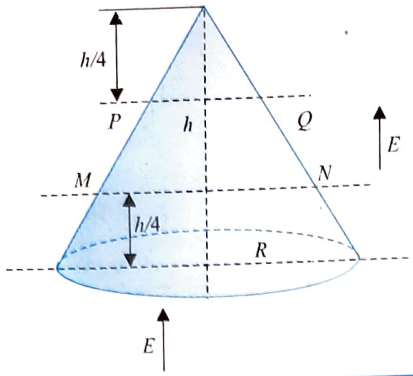
Column I	Column II
i. If Q_1 is at the center of the shell	a. Electric field inside the shell remains zero
ii. If Q_1 is not at the center of the shell	b. Electric field inside the shell remains nonzero
iii. If the position of Q_1 is changed within the shell keeping Q_2 fixed	c. Electric field inside changes
iv. If the position of Q_2 is changed outside keeping Q_1 fixed inside at any point	d. Electric field outside changes

3. Four large parallel identical conducting plates of area A are arranged as shown in figure. The charges on each plate are given in the figure and the separation between the plates is d (d is very small). The surfaces of the plates are numbered (1), (2), …, (8).



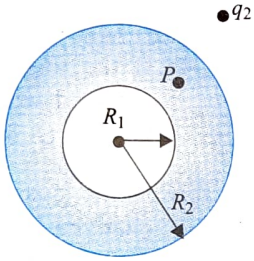
Column I	Column II
i. Surfaces having charges of the same magnitude and sign	a. 1 and 8
ii. Surfaces having positive charges	b. 3 and 5
iii. Uncharged surfaces	c. 2 and 3
iv. Charged surfaces	d. 6 and 7

4. The axis of a hollow cone as shown in figure is vertical. Its base radius is R. It is kept in a uniform electric field E parallel to its axis.



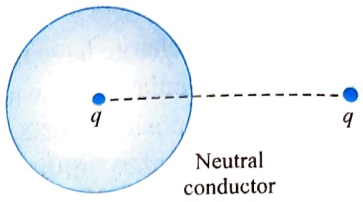
Column I	Column II
i. Magnitude of flux through base of cone	a. $\pi R^2 E$
ii. Magnitude of flux through curved part of cone	b. $\frac{\pi R^2 E}{2}$
iii. Magnitude of flux through curved part $MNQP$ of cone	c. zero
iv. Net flux through the entire cone	d. non zero

5. A conducting shell of inner radius R_1 and outer radius R_2 is given a charge $+Q$. A point charge q_1 is placed inside the shell and q_2 is placed outside the shell. Then for various locations of q_1 and q_2, match the entries of Column I with the entries of Column II.



Column I	Column II
i. If q_1 is at center and $q_2 = 0$, then $\vec{E}$ at center of shell due to charge on outer surface of shell is	a. $\frac{q_1}{4\pi\varepsilon_0 r^2}$
ii. If q_1 is not at center and q_2 is at distance r from the center, then $\vec{E}$ at the inner surface of shell (at a point closest to q_2) due to charge on outer surface of the shell is	b. $\frac{q_2}{4\pi\varepsilon_0 (r - R_1)^2}$
iii. If q_1 is at center and q_2 is at distance r from the center, then $\vec{E}$ at a point distant r_2 ($> r$) from the center of the shell due to outer surface charge is	c. zero
iv. If q_1 is not at center and $q_2 = 0$, then $\vec{E}$ at point P (P is at distance r from q_1 due to charge of the inner surface of shell is	d. cannot be determined

6. For the situation shown in the figure, match the entries of Column I with the entries of Column II.

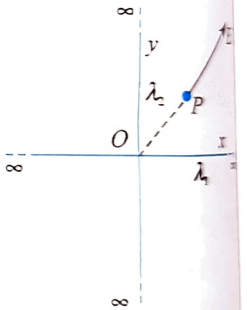


Column I	Column II
i. If we displace the inside charge,	a. distribution of charge on the inner surface of the conductor is uniform
ii. If we displace the outside charge keeping the inside charge at center,	b. distribution of charge on the inner surface of the conductor is nonuniform
iii. If both the charges are displaced,	c. distribution of charge on the outer surface of the conductor is uniform
iv. If outside charge is not present and inside charge is at center,	d. distribution of charge on the outer surface of the conductor is nonuniform

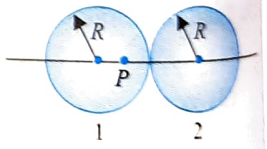
Numerical Value Type

1. The electric field in a region is radially outward with magnitude $E = ar$. If $a = 100\ \text{Vm}^{-2}$ and $R = 0.30$ m, then the value of charge contained in a sphere of radius R centred at the origin is $W \times 10^{-10}$ C. Find W.

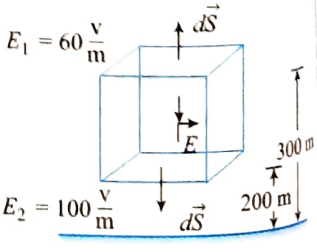
2. Two mutually perpendicular infinite wires along x-axis and y-axis carry charge densities λ_1 and λ_2. The electric line of force at P is along the line $y = x/\sqrt{3}$, where P is also a point lying on the same line. Find λ_1/λ_2.



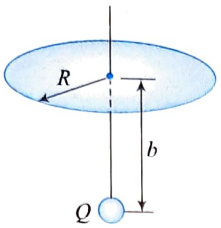
3. Figure shows, in cross section, two solid spheres with uniformly distributed charge throughout their volumes. Each has radius R. Point P lies on a line connecting the centers of the spheres, at radial distance $R/2$ from the center of sphere 1. If the net electric field at point P is zero, and Q_1 is 64 μC and $Q_2 = 8\ x\mu$C, what is the value of x.



4. It has been experimentally observed that the electric field in a large region of earth's atmosphere is directed vertically down. At an altitude of 300 m, the electric field is 60 Vm^{-1}. At an altitude of 200 m, the field is 100 Vm^{-1}. The net amount of charge contained in the cube of 100 m edge, located between 200 m and 300 m altitude is found to be $n \times 10^5\ \varepsilon_0$C. Find the value of n.

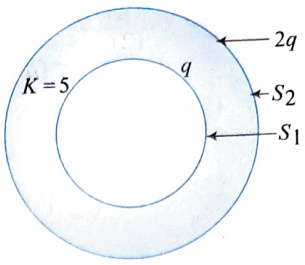


5. A point charge Q is located on the axis of disc of radius R at a distance b from the plane of the disk. If one fourth of the electric flux from the charge passes through the disk, then $R = \sqrt{\square}\, b$ (as shown in figure). Fill the space $\square$.



6. S_1 and S_2 are two hollow concentric spheres with charge q and $2q$. Space between S_1 and S_2 is filled with a dielectric of dielectric constants.

The ratio of flux through S_2 and flux through S_1 is K'. Then find the value of $\frac{9K'}{11}$.



7. Consider a uniform charge distribution with charge density 2 C/m^3 throughout in space. If a Gaussian sphere has a variable radius which changes at the rate of 2 m/s, then value of rate of change of flux is proportional to r_k (r = radius of sphere). Then, find the value of k.

Archives

JEE MAIN

Single Correct Answer Type

1. Let $P(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point p inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is

(1) 0 (2) $\frac{Q}{4\pi\varepsilon_0 r_1^2}$

(3) $\frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$ (4) $\frac{Qr_1^2}{3\pi\varepsilon_0 R^4}$ **(AIEEE 2009)**

2. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ up to $r = R$, and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance r ($r < R$) from the origin is given by

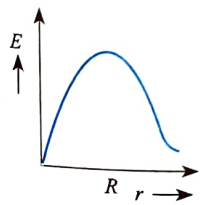
(1) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (2) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(3) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (4) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

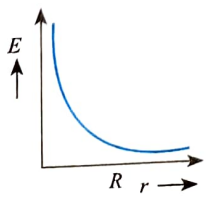
(AIEEE 2010)

3. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be

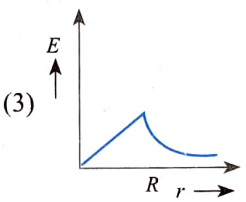
(1)



(2)



(3)



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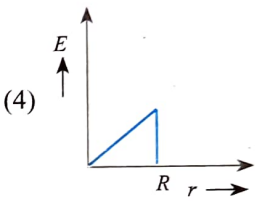


(AIEEE 2012)

4. This questions has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describe the two statements.

An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinite is zero.

Statement 1: When a charge 'q' is take from the centre of the surface of the sphere its potential energy changes by $q\rho/3\varepsilon_0$

Statement 2: The electric field at a distance r ($r < R$) from the centre of the sphere is $\rho r/3\varepsilon_0$

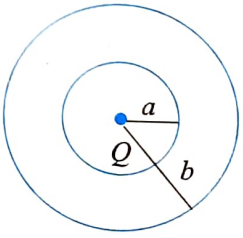
(1) Statement 1 is true, statement 2 is true; statement 2 is not the correct explanation of statement 1.

(2) Statement 1 is true, statement 2 is false.

(3) Statement 1 is false, statement 2 is true.

(4) Statement 1 is true, statement 2 is true, statement 2 is the correct explanation of statement 1. **(AIEEE 2012)**

5. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is



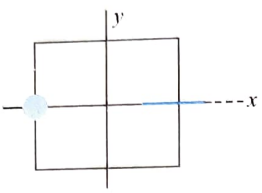
(1) $\frac{Q}{2\pi a^2}$ (2) $\frac{Q}{2\pi (b^2 - a^2)}$

(3) $\frac{2Q}{\pi (a^2 - b^2)}$ (4) $\frac{2Q}{\pi a^2}$ **(JEE Main 2016)**

JEE ADVANCED

Single Correct Answer Type

1. A disk of radius $a/4$ having a uniformly distributed charge 6 C is placed in the xy plane with its center at $(-a/2, 0, 0)$. A rod of length carrying a uniformly distributed charge 8 C is placed on the x-axis from $x = a/4$ to $x = 5a/4$. Two point charges –7 C and 3 C are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2$, $y = \pm a/2$, $z = \pm a/2$. The electric flux through this cubical surface is



(IIT-JEE 2009)

(1) $-2C/\varepsilon_0$ (2) $2C/\varepsilon_0$

(3) $10C/\varepsilon_0$ (4) $12C/\varepsilon_0$

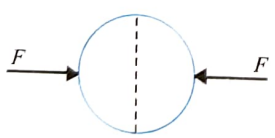
2. Three concentric metallic spherical shells of radii R, $2R$, and $3R$ are given charges Q_1, Q_2, and Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio $Q_1 : Q_2 : Q_3$ of the charges given to the shells is

(1) 1:2:3 (2) 1:3:5

(3) 1:4:9 (4) 1:8:18

(IIT-JEE 2009)

3. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F. F is proportional to

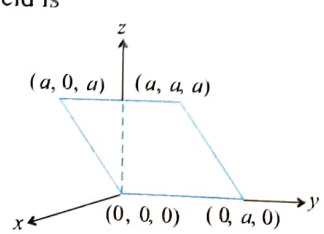


(1) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (2) $\frac{1}{\varepsilon_0}\sigma^2 R$

(3) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (4) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

(IIT-JEE 2010)

4. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is



(1) $2E_0a^2$ (2) $\sqrt{2}E_0a^2$

(3) E_0a^2 (4) $\frac{E_0a^2}{\sqrt{2}}$ **(IIT-JEE 2011)**

5. Which of the following statement(s) is/are correct?

(1) If the electric field due to a point charge varies as r^{-3} instead of r^{-2}, then Gauss's law will still be valid.

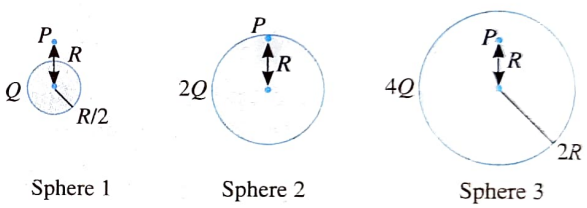
(2) Gauss's law can be used to calculate the field distribution around an electric dipole.

(3) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

(4) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$.

(IIT-JEE 2011)

6. Charges Q, $2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii $R/2$, R and $2R$ respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1, E_2 and E_3 respectively, then

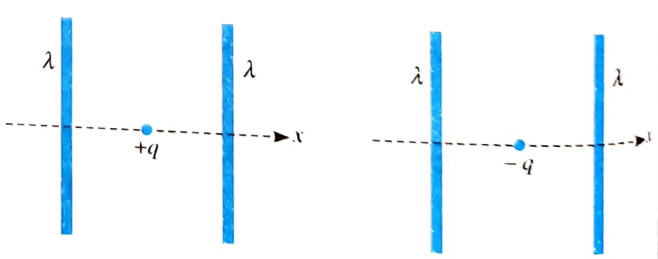


(1) $E_1 > E_2 > E_3$ (2) $E_3 > E_1 > E_2$

(3) $E_2 > E_1 > E_3$ (4) $E_3 > E_2 > E_1$

(JEE Advanced 2014)

7. The figure below depict two situations in which two infinitely long static line charges of constant positive linear charge density λ are kept parallel to each other. In the resulting electric field, point charges q and $-q$ are kept in equilibrium between them. The point charges are confined to move in the x-direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is (are)



(1) Both charges execute simple harmonic motion.

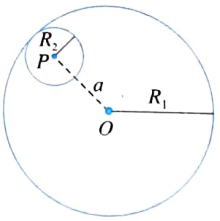
(2) Both charges will continue moving in the direction of their displacement.

(3) Charge $+q$ executes simple harmonic motion while charge $-q$ continues moving in the direction of its displacement.

(4) Charge $-q$ executes simple harmonic motion while charge $+q$ continues moving in the direction of its displacement.

(JEE Advanced 2015)

8. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is(are)



(1) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$

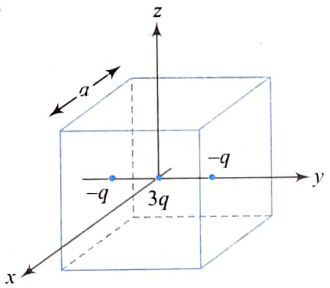
(2) $\vec{E}$ is uniform, its magnitude depends of R_2 and its direction depends on $\vec{r}$

(3) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$

(4) $\vec{E}$ is uniform, and both its magnitude and direction depends on $\vec{a}$ **(JEE Advanced 2015)**

Multiple Correct Answers Type

1. A cubical region of side a has its center at the origin. It encloses three fixed point charges, $-q$ at $(0, -a/4, 0)$, $+3q$ at $(0, 0, 0)$, and $-q$ at $(0, +a/4, 0)$. Choose the correct option(s).



(1) The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$.

(2) The net electric flux crossing the plane $y = +a/2$ is more than the net electric flux crossing the plane $y = -a/2$.

(3) The net electric flux crossing the entire origin is q/ε_0.

(4) The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x = +a/2$.

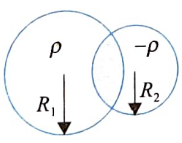
(IIT-JEE 2012)

2. Two nonconducting solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2, respectively, touch each other. The net electric field at a distance $2R$ from the center of the smaller sphere, along the line joining the center of the spheres is zero. The ratio ρ_1/ρ_2 can be

(1) -4 (2) $-32/25$

(3) $32/25$ (4) 4 **(IIT-JEE 2013)**

3. Two nonconducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region,



(1) the electrostatic field is zero

(2) the electrostatic potential is constant

(3) the electrostatic field is constant in magnitude

(4) the electrostatic field has same direction

(IIT-JEE 2013)

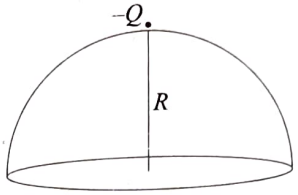
4. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

(1) $Q = 4\sigma\pi r_0^2$ (2) $r_0 = \dfrac{\lambda}{2\pi\sigma}$

(3) $E_1\left(\dfrac{r_0}{2}\right) = 2E_2\left(\dfrac{r_0}{2}\right)$ (4) $E_2\left(\dfrac{r_0}{2}\right) = 4E_3\left(\dfrac{r_0}{2}\right)$

(JEE Advanced 2014)

5. A point charge $+Q$ is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?



(1) The circumference of the flat surface is an equipotential

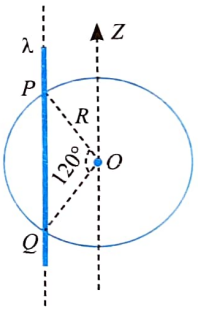
(2) The electric flux passing through the curved surface of the hemisphere is $-\dfrac{Q}{2\varepsilon_0}\left(1-\dfrac{1}{\sqrt{2}}\right)$

(3) Total flux through the curved and the flat surfaces is $\dfrac{Q}{\varepsilon_0}$

(4) The component of the electric field normal to the flat surface is constant over the surface

(JEE Advanced 2017)

6. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle $120°$ at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ϵ_0. Which of the following statements is (are) true?



(1) The electric flux through the shell is $\sqrt{3}R\lambda/\varepsilon_0$

(2) The z-component of the electric field is zero at all the points on the surface of the shell

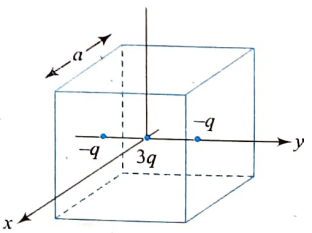
(3) The electric flux through the shell is $\sqrt{2}R\lambda/\varepsilon_0$

(4) The electric field is normal to the surface of the shell at all points

(JEE Advanced 2018)

Matrix Match Type

1. The electric field E is measured at a point $P(0,0,d)$ generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.



	List-I		List-II
P.	E is independent of d	1.	A point charge Q at the origin
Q.	$E \propto \frac{1}{d}$	2.	A small dipole with point charges Q at $(0,0,l)$ and $-Q$ at $(0,0,-l)$. Take $2l << d$
R.	$E \propto \frac{1}{d^2}$	3.	An infinite line charge coincident with the x-axis, with uniform linear charge density λ
S.	$E \propto \frac{1}{d^3}$	4.	Two infinite wires carrying uniform linear charge density parallel to the x-axis. The one along $(y = 0, z = l)$ has a charge density $+l$ and the one along $(y = 0, z = -l)$ has a charge density $-l$. Take $2l << d$
		5.	Infinite plane charge coincident with the xy-plane with uniform surface charge density

(1) P → 5; Q → 3,4; R → 1; S → 2

(2) P → 5; Q → 3; R → 1,4; S → 2

(3) P → 5; Q → 3; R → 1,2; S → 4

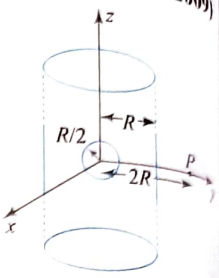
(4) P → 4; Q → 2,3; R → 1; S → 5

Numerical Value Type

1. A solid sphere of radius r has a charge Q distributed in its volume with a charge density $\rho = kr^a$ where k and a are constants and r is the distance from its centre. If the electric field at $r = R/2$ is 1/8 times that at $r = R$, find the value of a.

(IIT-JEE 2009)

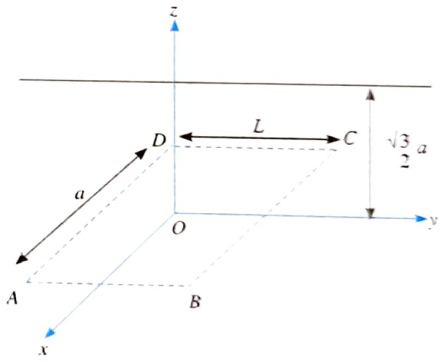
2. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its center on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $23\rho R/16k\varepsilon_0$. The value of k is



(IIT-JEE 2012)

3. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the yz plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field though the rectangular surface $ABCD$ lying in the $x - y$ plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is _____.

(JEE Advanced 2015)



Answers Key

EXERCISES

Single Correct Answer Type

1. (2)	2. (2)	3. (1)	4. (3)	5. (2)
6. (3)	7. (3)	8. (3)	9. (4)	10. (2)
11. (3)	12. (2)	13. (3)	14. (3)	15. (2)
16. (2)	17. (2)	18. (3)	19. (2)	20. (1)
21. (1)	22. (1)	23. (3)	24. (2)	25. (4)
26. (1)	27. (2)	28. (3)	29. (3)	30. (3)
31. (1)	32. (1)	33. (3)	34. (4)	35. (3)
36. (2)	37. (1)	38. (2)	39. (2)	40. (2)
41. (4)	42. (1)	43. (2)	44. (1)	45. (2)
46. (3)	47. (1)			

Multiple Correct Answers Type

1. (2),(3)	2. (2),(4)	3. (1),(2),(4)
4. (1),(4)	5. (1),(4)	6. (1),(2),(3)
7. (2),(3)	8. (3),(4)	9. (1),(3),(4)
10. (1),(3)	11. (2),(4)	12. (1),(2),(3)
13. (1),(2)	14. (1),(2),(3),(4)	15. (1),(3)
16. (1),(3),(4)		

Linked Comprehension Type

1. (4)	2. (1)	3. (1)	4. (4)	5. (1)
6. (4)	7. (4)	8. (2)	9. (2)	10. (2)
11. (3)	12. (1)	13. (1)	14. (1)	15. (3)
16. (1)	17. (2)	18. (3)	19. (3)	20. (3)
21. (2)	22. (4)	23. (4)	24. (1)	25. (4)
26. (3)	27. (2)	28. (2)	29. (1)	30. (1)

Matrix Match Type

1. i. → a., d.; ii. → c.; iii. → b. d., iv. → c.
2. i. → b.; ii. → b.; iii. → b. c., iv. → b., c.
3. i. → a.; ii. → a., b.; iii. → d.; iv. → a., b., c.
4. i. → a.; d., ii. → a.; d., iii. → b. d.; iv. → c.
5. i. → c.; ii. → b.; iii. → d.; iv → a.
6. i. → b., d.; ii. → a., d.; iii. → b., d.; iv → a., c.

Numerical Value Type

1. (3)	2. (3)	3. (9)	4. (4)	5. (3)
6. (9)	7. (2)			

ARCHIVES

JEE Main

Single Correct Answer Type

1. (3)	2. (2)	3. (3)	4. (3)	5. (1)

JEE Advanced

Single Correct Answer Type

1. (1)	2. (2)	3. (1)	4. (3)	5. (3)
6. (3)	7. (3)	8. (4)		

Multiple Correct Answers Type

1. (1),(3),(4)	2. (2),(4)	3. (3),(4)
4. (1),(2),(3)	5. (1),(2)	6. (1),(2)

Matrix Match Type

1. (2)

Numerical Value Type

1. (2)	2. (6)	3. (6)

3 Electric Potential

INTRODUCTION

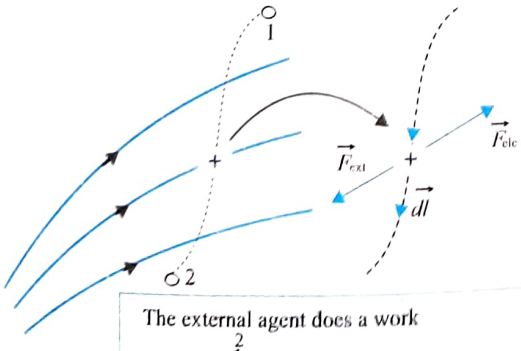
The concept of potential energy was introduced in mechanics in connection with conservative forces such as the gravitational force and the elastic force exerted by a spring. By using the law of conservation of energy, we can avoid working directly with forces when solving problems in mechanics. The concept of potential energy is also of great value in the study of electricity. Because electrostatic force is conservative, the electrostatic phenomena can be conveniently described in terms of electric potential energy. With this idea, we can define a scalar quantity known as electric potential. Since the electric potential at any point in an electric field is a scalar quantity, it can be used to describe electrostatic phenomena more simply than by relying only on electric field and electric forces.

The potential is characteristic of the field only, independent of a charged test particle that might be placed in the field. Potential energy is characteristic of the charge-field system due to an interaction between the field and a charged particle placed in the field.

WORK DONE TO MOVE A CHARGE IN AN ELECTROSTATIC FIELD

Let us consider an arbitrary electric field due to any charged object. If we move a test charge $+q$ from position 1 to position 2 (see figure) in this electrostatic field, at each position of the test charge, it will experience an electrostatic force $\vec{F}_{el} = q\vec{E}$. To move the test charge slowly, we must pull it against the electric force (field) with a force $\vec{F}_{ext} = -q\vec{E}$, opposite to the electric field $\vec{E}$. The work done by the external agent in shifting the test charge along the dashed line from 1 to 2 is

$$W_{ext} = \int_1^2 \vec{F}_{ext}\, \vec{dl} = \int_1^2 (-q\vec{E}) \cdot \vec{dl} = -q\int_1^2 \vec{E} \cdot \vec{dl}$$



The external agent does a work $W = -q\int_1^2 \vec{E}\cdot\vec{dl}$ in transporting the test charge q slowly from position 1 to position 2 in the static electric field E.

ILLUSTRATION 3.1

Find the work done by an external agent in slowly shifting a charge $q = 1\ \mu C$ in the electric field $\vec{E} = 10^3\hat{i}\ Vm^{-1}$ from the point $P(1, 2)$ to $Q(3, 4)$.

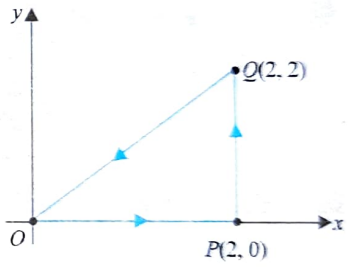
Sol. The work done by the external agent against the electric field is

$$W_{ext} = -q\int_1^2 \vec{E}\cdot\vec{dl} = -q\int (E\hat{i})\cdot(dx\hat{i} + dy\hat{j})$$

$$= -q\int_{x_1}^{x_2} E\,dx = -qE(x_2 - x_1) = -(10^{-6})(10^3)(3-1) = -2\times10^{-3}\,J$$

ILLUSTRATION 3.2

A charge particle $q = -10\ \mu C$ is carried along OP and PQ and then back to O along QO as shown in figure, in an electric field $\vec{E} = (x + 2y)\hat{i} + 2x\hat{j}$. Find the work done by an external agent in **(a)** each path and **(b)** the round trip.



Sol. Work done by an external agent in an electric field

(a) $W = -q\int \vec{E}\cdot\vec{dl} = -q(\int E_x dx + \int E_y dy)$

In Path $O \to P$: Since the displacement is along the x-axis for the path OP,

$$W_{O\to P} = -q\int E_x dx = -(-10\times10^{-6})\int_0^2 (x + 2y)\,dx,$$

where $y = 0$

or $W_{O\to P} = +20\times10^{-6}\,J$

In Path $P \to Q$: Since the displacement is along y-axis for path PQ,

$$W_{P\to Q} = -q\int E_y d_y = -q\left(2\int_0^2 x\,dy\right) \text{ where } x = 2$$

$$= -2(-10\times10^{-6})\int_0^2 2\,dy = 80\times10^{-6}\,J$$

In Path $Q \to O$: Since the displacement $dl = dx\hat{i} + dy\hat{j}$ along QO,

$$W_{Q\to O} = -q\int (E_x dx + E_y d_y) = -q\int_2^0 (x + 2y)\,dx + 2x\,dy$$

$$= -q\left[\int_2^0 (x + 2y)\,dx + 2\int_2^0 x\,dy\right]$$

where the equation of the path QO is given as $y = x$

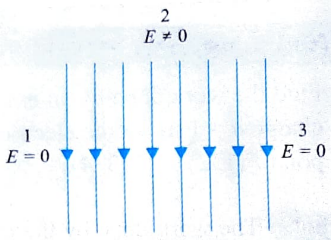
or
$$W_{Q\to O} = -q\left[\int_2^0 (x+2x)\,dx + 2\int_2^0 y\,dy\right]$$
$$= -q\left(\frac{3}{2}\left[x^2\right]_2^0 + \frac{2}{2}\left[y^2\right]_2^0\right) = -(-10\times10^{-6})\left\{\frac{-3\times4}{2} - 4\right\}$$
$$= -100\times10^{-6}\ \text{J}$$

(b) Then, the total work done in the round trip is
$$W = W_{O\to P} + W_{P\to Q} + W_{Q\to O} = 0$$

In the above example, $\oint \vec{E}\cdot\vec{dl} = 0$. Hence, the field is conservative and static.

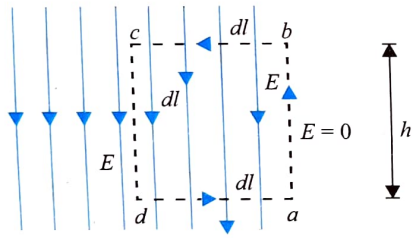
ILLUSTRATION 3.3

A uniform electric field is present in region 2, whereas it is dropped to zero abruptly in regions 1 and 3. Is this electric field conservative?



Sol. The closed line integral of E in the loop $abcda$ is

$$\oint \vec{E}.\vec{dl} = \int_a^b \vec{E}.\vec{dl} + \int_b^c \vec{E}.\vec{dl} + \int_c^b \vec{E}.\vec{dl} + \int_d^a \vec{E}.\vec{dl}$$



Since $\vec{E} \perp d\vec{l}$ from d to a and b to c.

$$\int_d^b \vec{E}\cdot d\vec{l} = \int_b^c \vec{E}\cdot d\vec{l} = 0$$

Since $E = 0$ in region 3, $\int_c^d \vec{E}\cdot d\vec{l} = \int_c^d E\,dl\cos\theta$

$$= \int_c^h E\,dl = E\,h\,(\neq 0)$$

Since $\oint \vec{E}\cdot d\vec{l} \neq 0$, the given field is non-conservative.

POTENTIAL AND POTENTIAL DIFFERENCE

POTENTIAL

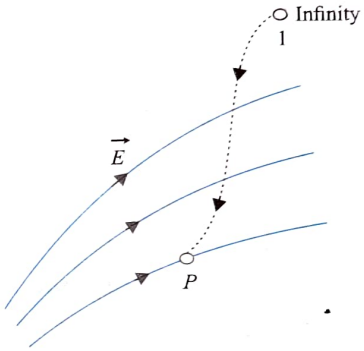
We know that in slowly bringing a point charge from point 1 to point 2, the work done by the external agent is independent of the path followed by the charge, which can be given by

$$W_{\text{ext}} = -q\int_1^2 \vec{E}\cdot\vec{dl}$$

If we choose the initial point at infinity and the final point at P, the above expression will be

$$W_{\text{ext}} = -q\int_\infty^P \vec{E}\cdot\vec{dl}$$

We can see that W_{ext} will be different for different values of the test charge. This means that even though you perform the same work in bringing a test charge from infinity to any point P following different paths, you will have to do different works in bringing different test charges. However, if we take the ratio W_{ext}/q, that is, work done per unit test charge, we will get a constant quantity, which can be defined as "potential at the point P" denoted by V. It is a constant quantity for a given point. Hence, it is a point function (like electric field) that may vary with positions. The "potential V" characterizes an electric field as a scalar function in addition to "field strength $\vec{E}$" as a vector function. Hence, potential field is a scalar field.



The potential at any point P is given by $V = -\int_\infty^P \vec{E}\,.\,\vec{dl}$, which is equal to the external work done per unit positive charge in shifting slowly from infinity to this point.

The potential at any point P is given by

$$V = \frac{W_{\text{ext}}(\infty\to P)}{q} = -\int_\infty^P \vec{E}\cdot d\vec{l}$$

Then, the field expression for potential is

$$V_P = -\int_\infty^P \vec{E}\cdot d\vec{l}$$

The potential at any observation point P of a static electric field is defined as the work done by the external agent (or negative of work done by the electrostatic field) in slowly bringing a unit positive point charge from infinity to the observation point. The potential at a point is more if the external agent does more work to shift the charge from infinity to the given point and vice versa. In this way, we define potential as the external work done by unit charge or roughly potential energy per unit charge.

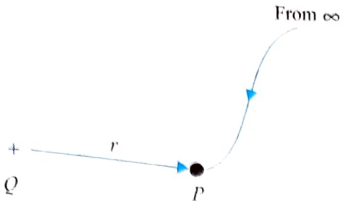
Potential Due to a Point Charge

The definition of a potential is given by the expression

$$V = \int_{\infty}^{P} \vec{E} \cdot \vec{dl} = \int_{\infty}^{P} E \cdot dl \cos\theta$$

$$= \int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r^2}(-dr)$$

$$= \frac{Q}{4\pi\varepsilon_0 r}$$



If $r \to 0$, $V \to \infty$.

If $r \to \infty$, $V \to 0$.

Hence, V varies hyperbolically. If Q is positive, V is positive and vice versa.

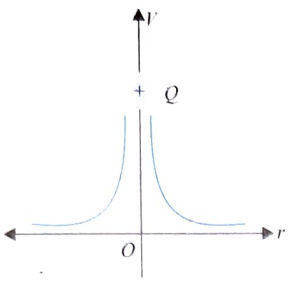


ILLUSTRATION 3.4

The electric field in a region is given by $\vec{E} = \frac{A}{x^3} i$. Write an expression for the potential in the region assuming the potential at infinity to be zero.

Sol. As $E = A/x^3$, potential in the region is

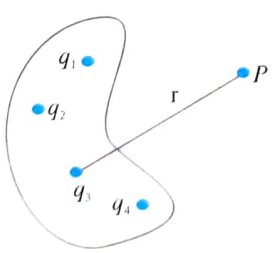
$$V = -\int_{\infty}^{x} \vec{E} \cdot \vec{dx} = -\int_{\infty}^{x} \left(\frac{A}{x^3} i\right) \cdot (dx i)$$

$$= -A\int_{\infty}^{x} x^{-3} dx = -A\left[\frac{x^{-2}}{-2}\right]_{\infty}^{x} = \frac{A}{2x^2}$$

Potential Due to Collection of Charges

Discrete Charges

Following the principle of superposition of potentials as described in the last section, let us find the potential V due to a collection of discrete point charges $q_1, q_2, \ldots, q_n$, at a point P.



As we know, the potential at point P is

$$V = \Sigma V_i$$

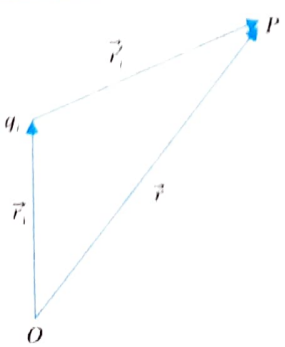
where $V_i = V = \frac{q_i}{4\pi\varepsilon_0 r_i}$ and r_i is the magnitude of the position vector P relative to q_i. Then

$$V = \frac{1}{4\pi\varepsilon_0} \Sigma \frac{q_i}{r_{pi}}$$

If the positions of the charges are given from a fixed origin O (see figure) put $\vec{r}_i' = \vec{r} - \vec{r}_i$ to obtain

The potential at P due to the system of point charges is given as the sum of their individual potentials at P,

$$V = \frac{1}{4\pi\varepsilon_0} \sum \frac{q_i}{r_i}$$



$$V = \frac{1}{4\pi\varepsilon_0} \Sigma \frac{q_i}{|\vec{r} - \vec{r}_i|}$$

CONTINUOUS CHARGES

For continuous charge distribution

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}$$

where r is the position of the point p from elementary charge dq. Put

$$dq = \rho dv \quad \text{(for volume charge)}$$
$$= \sigma dA \quad \text{(surface charge)}$$
$$= \lambda dl \quad \text{(line charge)}$$

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho dv}{r} \quad \text{(for volume charge distribution)}$$

$$= \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma dA}{r} \quad \text{(for surface charge distribution)}$$

$$= \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda dl}{r} \quad \text{(for line charge distribution)}$$

where ρ, σ, and λ may be a function of r.

ILLUSTRATION 3.5

Three charges $q_1 = 1\ \mu C$, $q_2 = -2\ \mu C$, and $q_3 = -1\ \mu C$ are placed at $A(0, 0, 0)$, $B(-1, 2, 3,)$ and $C(2, -1, 1)$. Find the potential of the system of three charges at $P(1, -2, -1)$.

Sol. If $\vec{r}_{Pi}$ is the position of P from the charge, its potential at P is

$$V_i = \frac{q_i}{4\pi\varepsilon_0 |\vec{r}_P - \vec{r}_i|}$$

Then, potential at P due to charge at A is

$$V_1 = \frac{q_1}{4\pi\varepsilon_0 |\vec{r}_P - \vec{r}_A|}$$

$$= \frac{10^{-6} \times 10^9 \times 9}{|(\hat{i} - 2\hat{j} - \hat{k}) - (0\hat{i} + 0\hat{j} + 0\hat{k})|} = \frac{9 \times 10^3}{\sqrt{6}} \text{ V}$$

Similarly,

$$V_2 = \frac{q_2}{4\pi\varepsilon_0 |\vec{r}_P - \vec{r}_B|}$$

$$= \frac{-2 \times 10^{-6} \times 9 \times 10^9}{|(\hat{i} - 2\hat{j} - \hat{k}) - (-\hat{i} + 2\hat{j} + 3\hat{k})|} = -3 \times 10^3 \text{ V}$$

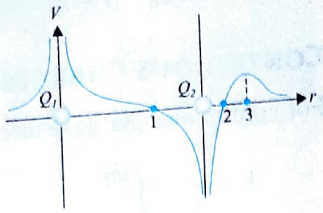
$$V_3 = \frac{q_3}{4\pi\varepsilon_0 |\vec{r}_P - \vec{r}_C|}$$

$$= \frac{-10^{-6} \times 9 \times 10^9}{|(\hat{i} - 2\hat{j} - \hat{k}) - (2\hat{i} - \hat{j} + \hat{k})|} = -\frac{9}{\sqrt{6}} \times 10^3 \text{ V}$$

Then, $V_P = \Sigma V_i = V_1 + V_2 + V_3 = -3 \times 10^3$ V

ILLUSTRATION 3.6

Two point charges Q_1 and Q_2 lie along a line at a distance from each other. Figure shows the potential variation along the line of charges. At which of the points 1, 2, and 3 is the electric field zero? What are the signs of the charges Q_1 and Q_2 and which of the two charges is greater in magnitude?

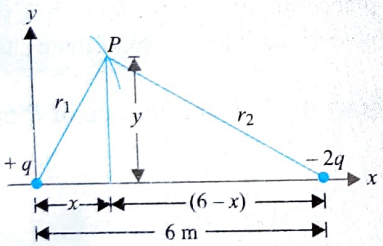


Sol. The electric field vector is zero at point 3. As $-dV/dr = E_r$, the negative of the slope of V verus r curve represents the component of electric field along r. Slope of curve is zero only at 3.

Near positive charge, net potential is positive and near a negative charge net potential is negative. Thus, charge Q_1 is positive and Q_2 negative. From the graph, it can be seen that net potential due to the two charges is positive everywhere in the region left of charge Q_1. Therefore, the magnitude of potential due to charge Q_1 is greater than that due to Q_2. Therefore, the absolute value of charge Q_1 is greater than that of Q_2. Secondly, point 1, where potential due to two charges is zero, is nearer to charge Q_2 thereby implying that Q_1 has greater absolute value. Also, potential is zero at 2, which is toward right of Q_2, as we know that potential is zero at an outside point toward the side of charge smaller in magnitude.

ILLUSTRATION 3.7

Two electric charges q and $-2q$ are placed at a distance 6 m apart on a horizontal plane. Find the locus of point on this plane where the potential has a value zero.



Sol. If point P is at a distance r_1 from point charge $+q$ and r_2 from charge $-2q$ (figure), then

$$V_P = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{+q}{r_1} + \frac{-2q}{r_2}\right]$$

According to the problem, $V_P = 0$, i.e.,

$$\frac{q}{r_1} - \frac{2q}{r_2} = 0$$

or $r_2 = 2r_1$ ...(i)

But if the charge q is assumed to be situated at the origin, then

$$r_1^2 = x^2 + y^2 \text{ and } r_2^2 = (6-x)^2 + y^2$$

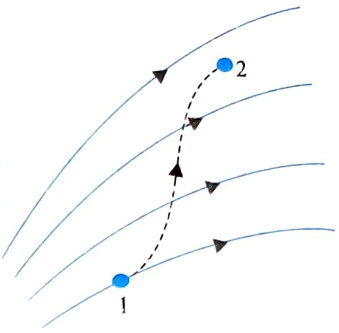
So substituting these values of r_1 and r_2 in Eq. (i), we get

$$(x+2)^2 + y^2 = 16$$

So the locus of the point P is a circle with radius 4 m and center (–2, 0).

POTENTIAL DIFFERENCE

The difference in potentials between any two points 1 and 2 can be defined as the work done by an external agent in slowly shifting a unit positive charge between these points (from 1 to 2).



$$\frac{W_{\text{ext}(1\to2)}}{q} = V_2 - V_1$$

Potential difference between any two points 1 and 2 is $\Delta V = \frac{W_{1\to2}}{q}$ which is equal to the work done by an external agent in slowly moving the unit test charge from 1 to 2.

where

$$W_{\text{ext}(1\to2)} = -\int_1^2 \vec{E}\cdot\vec{dl}$$

or $$V_2 - V_1 (= \Delta V) = -\int_1^2 \vec{E}\cdot\vec{dl}$$

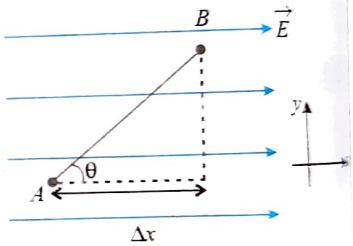
If $W_{\text{ext}} > 0$, $V_2 > V_1$; point 2 is at higher potential than point 1.
If $W_{\text{ext}} = 0$, $V_2 = V_1$; point 2 is at same potential as at point 1.
If $W_{\text{ext}} < 0$, $V_2 < V_1$; point 2 is at lesser potential than point 1.

POTENTIAL DIFFERENCE IN A UNIFORM ELECTRIC FIELD

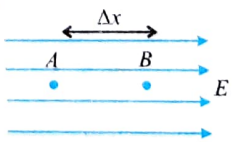
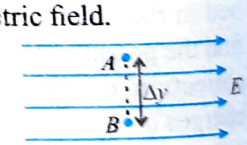
$$V_B - V_A = -\vec{E}\cdot\vec{AB}$$
$$\Rightarrow \quad = |E|\,|AB|\cos\theta$$
$$= -|E|\Delta x = -E\Delta x$$

where Δx is the distance between A and B along electric field.

We can also say that $|E| = \Delta V/\Delta x$.



Special Cases

Case I	Case II
Line AB is parallel to electric field.	Line AB is perpendicular to electric field.
	
$V_A - V_B = E\Delta x$ and $V_B - V_A = -E\Delta x$	$V_A - V_B = 0$ or $V_A = V_B$

Important Points:

Some important points on Gauss's law are as follows:

- In the direction of electric field, potential always decreases.
- The difference of potential between two points is called potential difference. It is also called voltage.
- Potential difference is a scalar quantity. Its S.I. unit is also volt.
- If V_A and V_B are the potential of two points A and B, then work done by an external agent in taking the charge q from A to B is

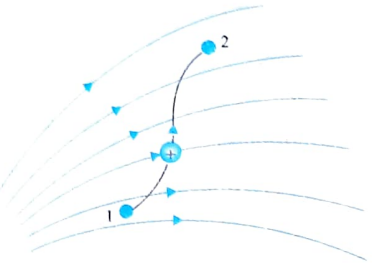
$$(W_{\text{ext}})_{AB} = q(V_B - V_A)$$

or $(W_{el})_{AB} = q(V_A - V_B)$ ...(i)

- Potential difference between two points is independent of reference point.
- Potential at a point is in fact potential difference between the potential at the point and infinity.
- In uniform electric field $\Delta V = V_2 - V_1 = -\vec{E} \cdot \vec{\Delta l}$
- The "potential at the point P" in electric field

$$V_P = \frac{W_{ext}(\infty \to P)}{q} = -\int_{\infty}^{P} \vec{E} \cdot d\vec{r}$$

- The "potential difference between two points" in electric field



$$V_2 - V_1 = \frac{W_{ext}(1 \to 2)}{q} = -\int_{1}^{2} \vec{E} \cdot d\vec{r}$$

$$V_2 - V_1 = \frac{W_{ext}(1 \to 2)}{q} = -\left(\int_{x_1}^{x_2} E_x\, dx + \int_{y_1}^{y_2} E_y\, dy + \int_{z_1}^{z_2} E_z\, dz \right)$$

ILLUSTRATION 3.8

1 μC charge is shifted from A to B and it is found that work done by an external force is 40 μJ in doing so against electrostatic forces then, find potential difference $V_B - V_A$.

Sol. The potential difference between two points in electric field

$$V_B - V_A = \frac{W_{ext}(A \to B)}{q} = \frac{40 \times 10^{-6}}{1 \times 10^{-6}} = 40\,\text{V}$$

Hence $V_B - V_A = -40\,\text{V}$

ILLUSTRATION 3.9

A uniform electric field is present in the positive x-direction. If the intensity of the field is 5 N/C then find the potential difference $(V_B - V_A)$ between two points A (0 m, 2 m) and B (5 m, 3 m).

Sol. The potential difference between two points in electric field

$$V_2 - V_1 = \frac{W_{ext}(1 \to 2)}{q} = -\int_{1}^{2} \vec{E} \cdot d\vec{r} = -\left(\int_{x_A}^{x_B} E_x\, dx + \int_{y_A}^{y_B} E_y\, dy \right)$$

$$\Rightarrow V_B - V_A = -E_x.\Delta x = -E_x.(x_B - x_A) = -5.(5-0) = -25\,\text{V}$$

Alternate: As electric field is constant and present in the positive x-direction only. Hence we can write directly

$$V_B - V_A = -E_x.\Delta x = -E_x.(x_B - x_A) = -5.(5-0) = -25\,\text{V}$$

ILLUSTRATION 3.10

An electric field is expressed as $\vec{E} = (2\hat{i} + 3\hat{j})\,\text{V/m}$. Find the potential difference $(V_A - V_B)$ between two points A and B whose position vectors are given by $\vec{r}_A = (\hat{i} + 2\hat{j})$ and $\vec{r}_B = (2\hat{i} + \hat{j} + 3\hat{k})\,\text{m}$.

Sol. The potential difference between two points in electric field

$$V_B - V_A = -\left(\int_{x_A}^{x_B} E_x\, dx + \int_{y_A}^{y_B} E_y\, dy + \int_{z_A}^{z_B} E_z\, dz \right)$$

$$= -\left(E_x.(x_B - x_A) + E_y.(y_B - y_A) + E_z.(z_B - z_A) \right)$$

$$= -\left(2(2-1) + 3(1-2)\right) = -(2-3) = 1\,\text{V}$$

Hence, $V_A - V_B = -1\,\text{V}$

ILLUSTRATION 3.11

Find the potential difference V_{AB} between A (0, 0, 0) and B (1 m, 1 m, 1 m) in an electric field:

(a) $\vec{E} = (y\hat{i} + x\hat{j})\,\text{Vm}^{-1}$ **(b)** $\vec{E} = (3x^2 y\hat{i} + x^3\hat{j})\,\text{Vm}^{-1}$

Sol.

(a) As $dV = -\vec{E} \cdot \vec{dr}$

Here $\vec{E} = (y\hat{i} + x\hat{j})\,\text{Vm}^{-1}$ and $\vec{dr} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

Hence, $dV = -(y\hat{i} + x\hat{j}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$

$$= -y\,dx + x\,dy = -d(xy)$$

[Using multiplication rule of derivative]

$$V_{AB} = -\int_{(1,1,1)}^{(0,0,0)} d(xy) = -[xy]_{(1,1,1)}^{(0,0,0)} = 1\,\text{V}$$

(b) As $dV = \int \vec{E} \cdot \vec{dr}$,

$\vec{E} = -(3x^3 y\hat{i} + x^3\hat{j})$ and $\vec{dr} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$\vec{dV} = -(3x^3 y\hat{i} + x^3\hat{j}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$

Hence, $dV = -(3x^2 y\,dx + x^3 dy) = -d(x^3 y)$

[Using multiplication rule of differentiation]

$$V_{AB} = -\int_{(1,1,1)}^{(0,0,0)} d(x^3 y) = -[x^3 y]_{(1,1,1)}^{(0,0,0)} = 1\,\text{V}$$

ILLUSTRATION 3.12

Uniform electric field of magnitude 100 Vm^{-1} in space is directed along the line $y = 3 + x$. Find the potential difference between points $A(3, 1)$ and $B(1, 3)$.

Sol. Equation of the line

$y = 3 + x$.

The slope of line

$\tan\theta = 1$ or $\theta = 45°$

We can express electric filed in vector form

$$\vec{E} = 100\cos\theta\,\hat{i} + 100\sin\theta\,\hat{j}\ (\text{Vm}^{-1})$$

Electric field $\vec{E} = |\vec{E}|(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

$$\vec{E} = \frac{100}{\sqrt{2}}\hat{i} + \frac{100}{\sqrt{2}}\hat{j}\,(\mathrm{Vm^{-1}})$$

$$\Delta\vec{r} = \overrightarrow{AB} = \vec{r}_B - \vec{r}_A = (\hat{i} + 3\hat{j}) - (3\hat{i} + \hat{j}) = -2\hat{i} + 2\hat{j}$$

$$\Delta V = -\vec{E}\cdot\Delta\vec{r} = -\left(\frac{100}{\sqrt{2}}\hat{i} + \frac{100}{\sqrt{2}}\hat{j}\right)\cdot\Delta\vec{r}$$

$$= -\left(\frac{100}{\sqrt{2}}\hat{i} + \frac{100}{\sqrt{2}}\hat{j}\right)\cdot(-2\hat{i} + 2\hat{j})$$

$$= -100\sqrt{2} + 100\sqrt{2}$$

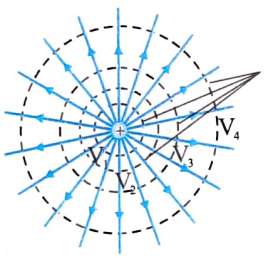
$$V_A - V_B = 0$$

EQUIPOTENTIAL SURFACE

For a given charge distribution, locus of all points or regions for which the electric potential has a constant value are called **equipotential regions**. Such equipotential can be surfaces, volumes or lines.

The easiest equipotential surfaces to visualize are those that surround an isolated point charge.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$



Equipotential surface

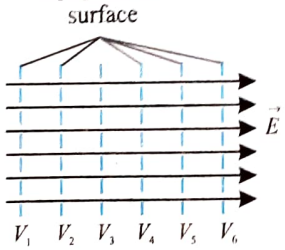
Thus, wherever r is the same, the potential is the same, and the equipotential surfaces are spherical surfaces centered on a point charge. There are an infinite number of such surfaces, one for every value of r, and figure illustrates equipotential surfaces of a positive point charge. The larger the distance r, the smaller is the potential of the equipotential surface. Here, it can be observed that $V_1 > V_2 > V_3 > V_4$, i.e., the electric potential decreases in the direction of electric field.

The equipotential surface must be perpendicular to the electric field

The potential difference between two points in electric field is given by $V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{r}$

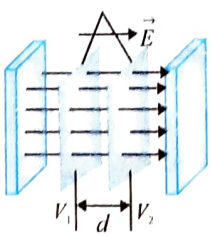
In an equipotential surface, $V_B = V_A$ hence, $\vec{E}\cdot d\vec{r} = 0$. It means the equipotential surface must be perpendicular to the electric field:

Equipotential surface



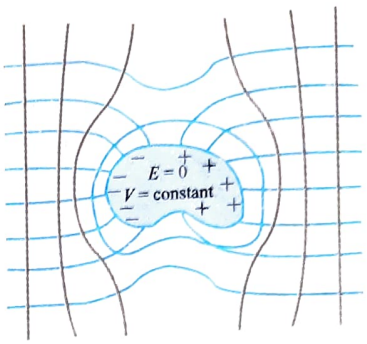
Electric potential decreases in the direction of electric field.
$V_1 > V_2 > V_3 > V_4 > V_5 > V_6$

Equipotential surface $V_1 > V_2$



Potential difference between two equipotential surface
$(V_1 > V_2) = E.d$

We know that the direction of the electric field just outside an electrical conductor is perpendicular to the conductor's surface, when the conductor is at equilibrium under electrostatic conditions. Thus, the surface of any conductor is an equipotential surface under such conditions. In fact, since the electric field is zero everywhere inside a conductor whose charges are in equilibrium, the entire conductor can be regarded as an equipotential volume.



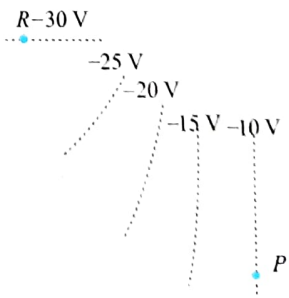
Important Points:

- Lines of force always intersect an equipotential perpendicularly.
- No two equipotential surfaces can intersect each other. Because if they do so, then at the point of intersection, here are two values of electric potential, which is not possible.
- A charged conductor of any shape is an equipotential surface. If it were not so, there would be a flow of charge from one end to another along the conductor.
- Equipotential surfaces are crowded together in a region of strong field whereas they are relatively far apart where the field is weak.
- If a charge is moved from one point to the other over an equipotential surface work done will be zero as

  $W_{AB} = q(V_B - V_A) = 0 \qquad \{\text{as } V_B = V_A\}$
- Work has to be done to move a charge from one equipotential surface to another.

ILLUSTRATION 3.13

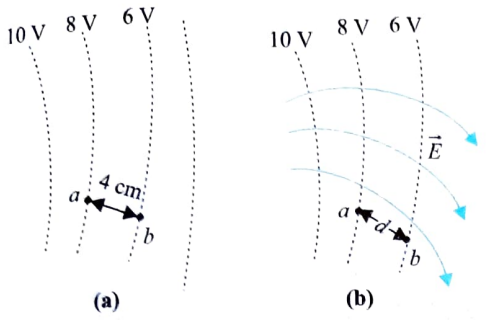
Following figure shows equipotential surfaces. What is the direction of electric field $\vec{E}$ at P and R?



Sol. Electric field lines are perpendicular to the equipotential surfaces and point in the direction of decreasing potential. At P electric field $\vec{E}$ is to the left and at R, $\vec{E}$ is upward.

ILLUSTRATION 3.14

Three equipotential surfaces are shown in Fig. (a). Draw the corresponding field lines and estimate the field strength at a point a where the distance between the surfaces is 4 cm.



Sol. The field lines are perpendicular to the equipotential surfaces as shown in Fig. (b).

In the vicinity of a, the surfaces are nearly flat, and so, in order to estimate the field strength, it is a reasonable approximation to use

$$V_a - V_b \approx E \cdot d$$

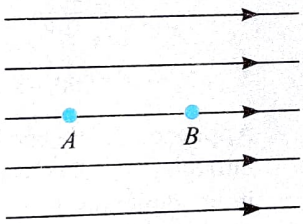
or $$E \approx \frac{V_a - V_b}{d} = \frac{8V - 6V}{4 \times 10^{-2}\,\text{m}} = 50\ \text{Vm}^{-1}$$

ILLUSTRATION 3.15

The figure shows two points A and B separated by 4 cm along the lines of a uniform electric field $E = 500$ V/m.

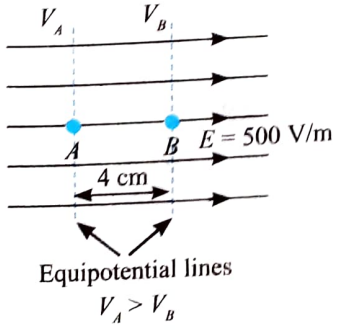
(a) The potential difference $(V_A - V_B) = ?$

(b) If the potential at A is 50 V, then the potential at $B = ?$



Sol.

(a) The electric field of lines and equipotential lines are perpendicular to each other. The figure shows the equipotential lines passing through A and B. Potential decreases in the direction of electric field, hence. Here $(V_A - V_B) = E.d$ is the distance between equipotential lines.



$$(V_A - V_B) = E.d = 500 \times \frac{4}{100} \Rightarrow V_A - V_B = 20\ \text{V}$$

(b) $50 - V_B = 20 \quad \Rightarrow \quad V_B = 30\ \text{V}$

ILLUSTRATION 3.16

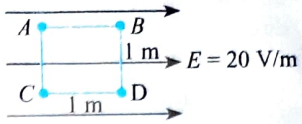
In the uniform electric field shown in figure, find

(a) The potential difference $V_A - V_C$.

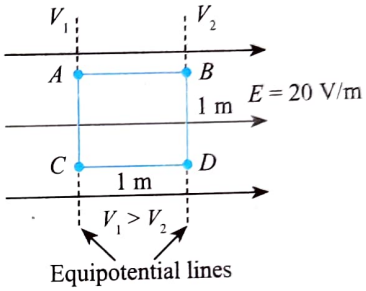
(b) The potential difference $V_B - V_D$.

(c) The potential difference $V_A - V_D$.

(d) The potential difference $V_C - V_D$.



Sol. The potential of point A and C should be equal as the points lies on the line which is perpendicular to electric field lines of force called equipotential line. And similar statement is true for the points B and D.



Hence, $V_A - V_C = V_B - V_D = 0$

As electric field is uniform, $V_D - V_C = V_B - V_A = 0 = -E\Delta x$

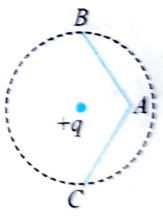
$$= -20 \times 1 = -20\text{V}$$

Hence, $V_A - V_D = V_C - V_D = 0 = 20\text{V}$

ILLUSTRATION 3.17

(a) A circle is drawn with centre as a charge $+q$. What is the work done in moving a charge $+q$ from B to C along the circumference of the circle?

(b) In the above question, if the charge $+q$ is first taken from B to A and then from A to C, on which path, the magnitude of work is greater.



Sol.

(a) Points B and C lie in equipotential surface.

Change in potential while moving from B to C along circumference will be zero.

(b) As $|V_B - V_A| = |V_C - V_A|$

and $(V_B - V_A) < 0$ and $(V_A - V_C) > 0$

And we know the work done by external agent,

$W_{ext} = q(V_f - V_i)$

Hence the work done from $B \to A$ is positive. And the work done from $A \to C$ is negative.

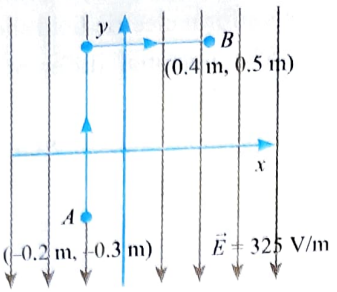
Also same amount of work will be done, $|W_{BA}| = |W_{AC}|$

ILLUSTRATION 3.18

A uniform electric field of magnitude 325 V/m is directed in the negative y direction in figure.

The coordinates of point A are (–0.2 m, –0.3 m) and those of point B are (0.4 m, 0.5 m).

Calculate the potential difference $V_B - V_A$, along the path shown in the figure.



Sol. The potential difference between two points A and B in electric field as, $V_B - V_A = -\int_A^B \vec{E}\cdot\vec{dr}$

Here the electric field is constant ,here we can write the potential difference between points A and B as

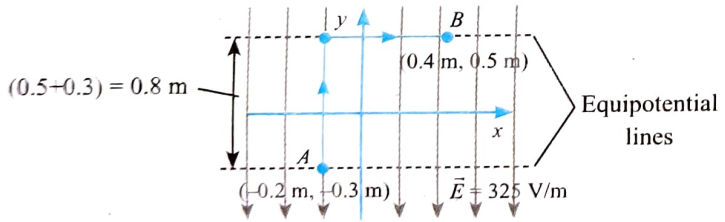
$$V_B - V_A = -\left[E_x(x_B - x_A) + E_y(y_B - y_A)\right]$$

$$= -\left[(-325)\{(0.5) - (-0.3)\}\right]$$

$$= 325 \times 0.8 = 260\,\text{V}$$

$$\Rightarrow V_B - V_A = +260\,\text{V}$$

Alternate: We can draw the equipotential lines passing through A and B as shown in figure. As potential decreases in the direction of electric field, it means the potential of point B must be greater than the potential of point A. Here the separation between the equipotential lines passing through A and B is 0.8 m (0.5 m + 0.3 m) and electric field is 325 V/m.



Hence the potential difference between A and B should be

$$V_B - V_A = E \times \Delta y = 325 \times 0.8 = 260\,V$$

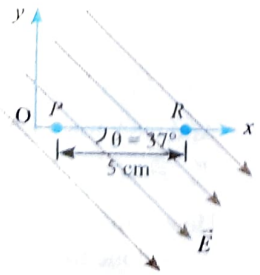
ILLUSTRATION 3.19

A uniform field of magnitude $\vec{E}$ = 2000 N/C is directed $\theta = 37°$ below the horizontal.

Find:

(a) The Potential difference between P and R ($V_P - V_R$).

(b) If we define the reference level of potential so that potential at R is V_R = 500 V, what is the potential at P?



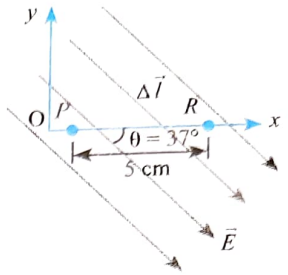
Sol.

(a) Approach 1: The potential difference between two points P and R in electric field as, $V_P - V_R = -\int_R^P \vec{E}.\vec{dl}$

As electric field is constant,

$$V_P - V_R = -\int_R^P \vec{E}.\vec{dl} = -\vec{E}.\int_R^P \vec{dl} = -\vec{E}.\left(\vec{r_P} - \vec{r_R}\right)$$

$$\Rightarrow V_P - V_R = -\vec{E}.\Delta\vec{l} = -E.\Delta l.\cos(180° - \theta)$$



Here, $\Delta l = PR = 5\text{ cm} = 5 \times 10^{-2}$ m

Hence, $V_P - V_R = -2000 \times 5 \times 10^{-2}(-\cos 37°)$

$$= 2000 \times 5 \times \frac{4}{5} = 80\text{ V}$$

Approach 2: Given,

$$\vec{E} = 2000\cos 37°\hat{i} - 2000\sin 37°\hat{j} = \left(1600\hat{i} - 1200\hat{j}\right)\text{N/C}$$

Hence,

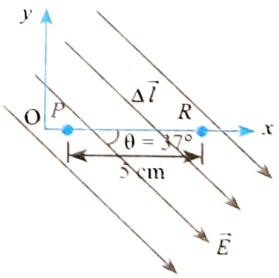
$$V_P - V_R = -\int_R^P \vec{E}.\vec{dl} = -\left[E_x(x_P - x_R) + E_y(y_P - y_R)\right]$$

$$V_P - V_R = -\left[1600(-5\times10^{-2})\right] = 80\text{ V}$$

Approach 3: We can draw the equipotential lines passing through P and R as shown in figure. As potential decreases in the direction of electric field, it means the potential of point P must be greater than the potential of point R.

The potential difference between two equipotential

$|V_P - V_R|$ = *Electric field* $\times \Delta l$



From figure $\Delta l = 5\cos 37° = 5 \times \frac{4}{5} = 4$ cm

Hence $V_P - V_R = 2000.(4\times10^{-2}) = 80$V

(b) As $V_P - V_R = 80$ V and we are given $V_R = 500$ V

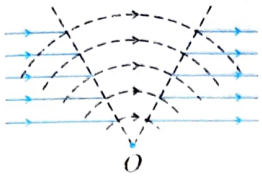
Hence, $V_P = 500\,\text{V} + 80\,\text{V} = 580\,\text{V}$

CONCEPT APPLICATION EXERCISE 3.1

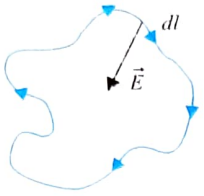
1. Are we free to call the potential of earth +100 V instead of zero? What effect would such an assumption have on measured values of

(a) potentials **(b)** potential difference?

2. Show that if at some part of a field the lines of force have the form of concentric circles whose centers are at point O (as given in figure), the field intensity at each point in this part of the field should be inversely proportional to the distance from the point O.



3. If you carry out the integral of the electric field $\int \vec{E} \cdot \vec{dl}$ for a closed path like that as shown in figure, the integral will always be equal to zero, independent of the shape of the path and independent of where charges may be located relative to the path. Explain why.

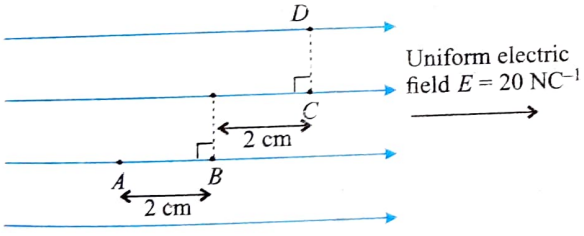


4. A charge 2 μC is taken from infinity to a point in an electric field, without changing its velocity. If work done against electrostatic forces is –40 μJ, then find the potential at that point.

5. When charge 10 μC is shifted from infinity to a point in an electric field, it is found that work done by electrostatic forces is 10 μJ. If the charge is doubled and taken again from infinity to the same point without accelerating it, then find the amount of work done by electric field and against electric field.

6. A charge 3 μC is released at rest from a point P where electric potential is 20 V. Find its kinetic energy when it reaches infinity.

7. Find out the following:



(a) $V_A - V_B$ (b) $V_B - V_C$ (c) $V_C - V_A$

(d) $V_D - V_C$ (e) $V_A - V_D$

(f) Arrange the order of potential for points A, B, C and D.

8. Find V_{ab} in an electric field $\vec{E} = (2\hat{i} + 3\hat{j} + 4\hat{k})\text{NC}^{-1}$.

where $\vec{r}_a = (\hat{i} - 2\hat{j} + \hat{k})\text{m}$ and $\vec{r}_b = (2\hat{i} + \hat{j} - 2\hat{k})\text{m}$

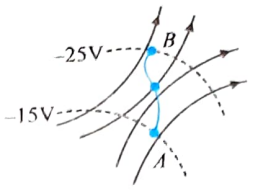
9. At a point due to a point charge, the values of electric field intensity and potential are 32 NC^{-1} and 16 JC^{-1}, respectively. Calculate the

(a) magnitude of the charge, and

(b) distance of the charge from the point of observation.

10. Two uniformly charged large plane sheets S_1 and S_2 having charge densities S_1 and $\sigma_2 (\sigma_1 > \sigma_2)$ are placed at a distance d parallel to each other. A charge q_0 is moved along a line of length a ($a < d$) at an angle 45° with the normal to S_1. Calculate the work done by the electric field.

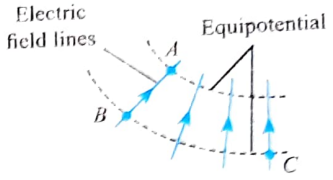
11. The figure shows field lines and equipotential (dashed) lines.



(a) Find the external work done to move a –2 μC charge at constant speed from A to B along the path shown.

(b) Find the work done by the electric field.

12. In moving from A to B along an electric field line, the electric field does 4.8×10^{-19} J of work on an electron in the field illustrated in figure. What are the differences in the electric potential? Find

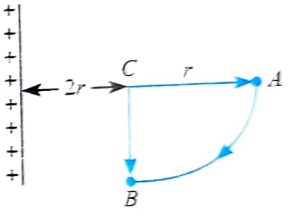


(a) The potential difference $V_B - V_A$

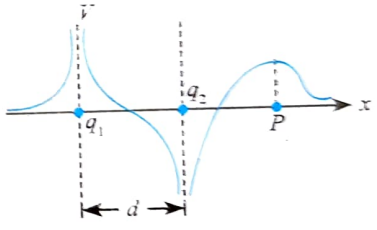
(b) The potential difference $V_C - V_A$

(c) The potential difference $V_C - V_B$

13. A charge q_0 is transported from point A to B along the arc AB with centre at C as shown in figure near a long charged wire with linear density λ lying in the same plane. Find the work done by external agent in doing so.



14. The graphical variation of electric potential due to point charge q_1 and q_2 lie on x axis at some separati on d. is shown in figure. If the origin is the point between the charges where potential is zero. Distance of q_2 from origin is d/4. Find the distance of point P (marked in figure) from charge q_2.



ANSWERS

4. –20 V 5. –20 μJ 6. 60 μJ

7. (a) 0.4 V (b) 0.4 V (c) – 0.8 V
(d) 0 (e) 0.8 (f) $V_A > V_B > V_C = V_D$

8. –1V 9. (a) $8/9 \times 10^{-9}$ C (b) 0.5 m

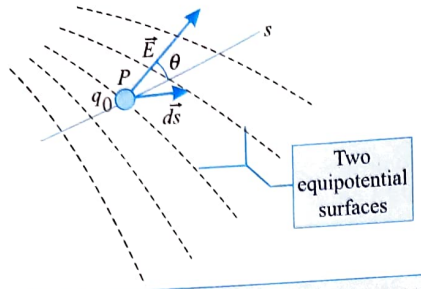
10. $W = \dfrac{q_0 a}{2\sqrt{2}\varepsilon_0}(\sigma_1 - \sigma_2)$ 11. –20 μJ

13. $\dfrac{q\lambda}{2\pi\varepsilon_0} ln\left(\dfrac{3}{2}\right)$ 14. $\dfrac{d}{\sqrt{3}-1}$

FINDING ELECTRIC FIELD FROM ELECTRIC POTENTIAL

The potential difference between two points in an electric field is given by

$$V_f - V_i = -\int_i^f \vec{E}\cdot\vec{ds} \qquad \ldots(i)$$



A test charge q_0 moves a distance ds from one equipotential surface to another. (The separation between the surfaces has been exaggerated for clarity.) The displacement $\vec{ds}$ makes an angle θ with the direction of the electric field $\vec{E}$.

We can write

$$dV = -\vec{E}\cdot\vec{ds} = E\cos\theta\cdot ds \qquad \ldots(ii)$$

$$E\cos\theta = -\frac{dV}{ds}$$

$E\cos\theta$ is the component of E in the direction of ds

$$E_s = -\frac{dV}{ds} \qquad \ldots(iii)$$

We write $\vec{E}$ and $\vec{ds}$ in terms of their components $\vec{E} = \hat{i}E_x + \hat{j}E_y + \hat{k}E_z$ and $\vec{ds} = \hat{i}dx + \hat{j}dy + \hat{k}dz$.

Then we have

$-dV = E_x dx + E_y dy + E_z dz$

Suppose the displacement is parallel to the x-axis, so $dy = dz = 0$. Then $-dV = E_x dx$

or $$E_x = -\left(\frac{dV}{dx}\right)_{y,z \text{ constant}}$$

where the subscript reminds us that only x varies in the derivative; recall that V is in general a function of x, y, and z. But this is just what is meant by the partial derivative $\frac{\partial V}{\partial x}$. The y- and z-components of E are related to the corresponding derivatives of V in the same way, so we have

$$E_x = -\frac{\partial V}{\partial x};\ E_y = -\frac{\partial V}{\partial y};\ E_z = -\frac{\partial V}{\partial z}$$

(components of E in terms of V)

The equation states: The negative of the rate of change of potential with position in any direction is the component of E in that direction. The minus sign implies that E points in the direction of decreasing V.

The electric field intensity in uniform electric field,

$$E = \frac{\Delta V}{\Delta d}$$

where ΔV is the potential difference between two points. Δd is the effective distance between the two points (projection of the displacement along the direction of electric field).

ILLUSTRATION 3.20

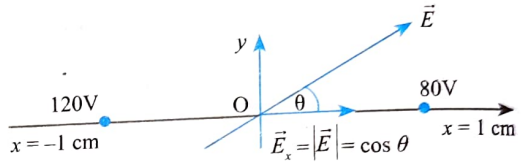
The electric potential decreases uniformly from 120 V to 80 V as one moves on the x-axis from $x = -1$ cm to $x = +1$ cm. What can be said about the electric field at the origin.

Sol. The relation between electric field and potential is given as, $E_r = -\frac{dV}{dr}$

We say $E_x = -\frac{dV}{dx}$, and $E_y = -\frac{dV}{dy}$

The electric potential along x-axis changing uniformly hence the component of electric field along x-axis.

$$E_x = -\frac{\Delta V}{\Delta x} = E_x = -\frac{(V_2 - V_1)}{(x_2 - x_1)} = -\frac{(80-120)}{(1-(-1))} = \frac{40}{2} = 20\ \text{V/cm}$$



$|E\cos\theta| = 20$ V/cm

As $|\cos\theta| \le 1$, hence E may be equal to or greater than 20 V/cm

ILLUSTRATION 3.21

The potential at any point is given by $V = x(y^2 - 4x^2)$. Calculate the Cartesian components of the electric field at the point.

Sol. Here $V = x(y^2 - 4x^2)$, so

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}[xy^2 - 4x^3]$$

$$= -[y^2 - 12x^2]$$

or $$E_x = [12x^2 - y^2]$$

Similarly,

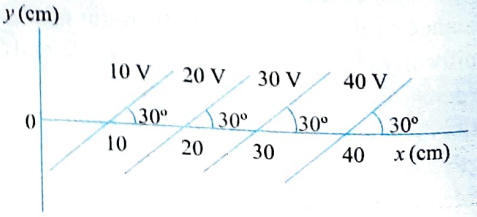
$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}[x(y^2 - 4x^2)]$$

or $$= -\frac{\partial}{\partial y}[xy^2 - 4x^3] = -2xy$$

$E_z = 0$ because V does not depend upon the z-coordinate.

ILLUSTRATION 3.22

Some equipotential surfaces are shown in figure. What can you say about the magnitude and the direction of the electric field?



Sol. Here $V = x(y^2 - 4x^2)$, so Here we can say that the electric field will be perpendicular to equipotential surfaces. Also

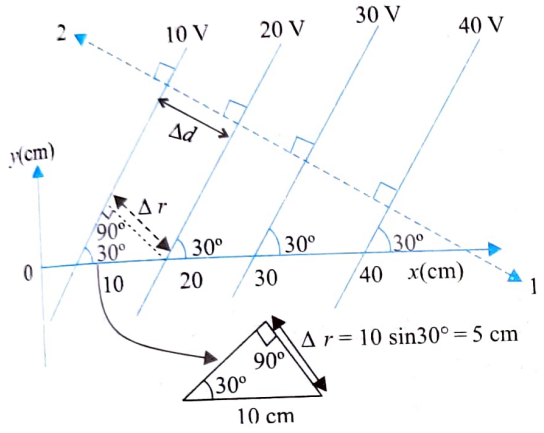
$$|\vec{E}| = \frac{\Delta V}{\Delta d}$$

where ΔV = potential difference between two equipotential surfaces.

Δd = perpendicular distance between two equipotential surfaces.

$$|\vec{E}| = \frac{10}{(10 \sin 30°) \times 10^{-2}} = 200 \text{ Vm}^{-1}$$

Now there are two perpendicular directions (1 or 2) as shown in figure, but since we know that electric potential decreases in the direction of electric field, so the correct direction is 2.



Hence, $E = 200 \text{ Vm}^{-1}$, making an angle 120° with the x-axis.

ILLUSTRATION 3.23

Determine the electric field strength vector if the potential of this field depends upon x- and y-coordinates as:
(a) $V = a(x^2 - y^2)$ and (b) $V = axy$

Sol.

(a) As $V = a(x^2 - y^2)$,

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\, a(x^2 - y^2) = -2ax$$

and $$E_y = -\frac{\partial V}{\partial y} = +2ay$$

$$\therefore \quad \vec{E} = E_x\hat{i} + E_y\hat{j} = -2ax\hat{i} + 2ay\hat{j} = -2a(x\hat{i} - y\hat{j})$$

(b) As $V = axy$,

$$E_x = -\frac{\partial V}{\partial x} = -ay$$

and $$E_y = -\frac{\partial V}{\partial y} = -ax$$

$$\therefore \quad \vec{E} = E_x\hat{i} + E_y\hat{j} = -ay\hat{i} - ax\hat{j} = -a(y\hat{i} + x\hat{j})$$

ILLUSTRATION 3.24

(a) Find the field intensity if potential $V = -kxy$ $(k > 0)$.
(b) Draw the field pattern of the field in (b).

Sol.

(a) The field intensity

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j}$$

$$= -\frac{\partial}{\partial x}(-kxy)\hat{i} - \frac{\partial}{\partial y}(-kxy)\hat{j} = k(y\hat{i} + x\hat{j})$$

(b) $\vec{E} = E_x\hat{i} + E_y\hat{j} = ky\hat{i} + kx\hat{j}$

or $E_x = ky$ and $E_y = kx$

Then,

$$\tan\theta = \frac{E_y}{E_x} = \frac{kx}{ky} = \frac{x}{y}$$

Since,

$$\tan\theta = \frac{dy}{dx}$$

(slope of lines of force)

$$\frac{dy}{dx} = \frac{x}{y}$$

or $ydy = xdx$

or $y^2 - x^2 = \text{constant}$

Hence, the field pattern is hyperbolic (as shown in figure).

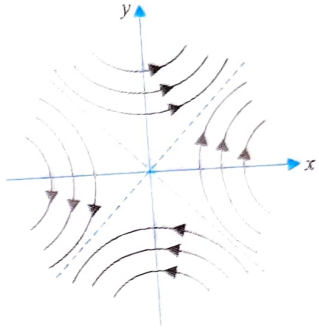
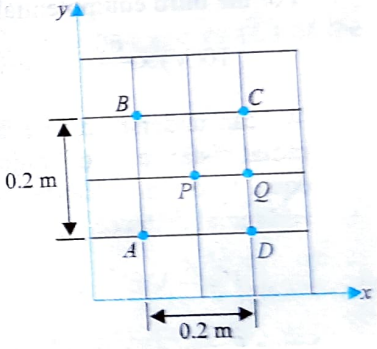


ILLUSTRATION 3.25

A, B, C, D, P, and Q are points in a uniform electric field. The potentials at these points are $V(A) = 2$ V, $V(P) = V(B) = V(D) = 5$ V, and $V(C) = 8$ V. Find the electric field at P.



Sol.

Method I: The component of electric field in x-direction is

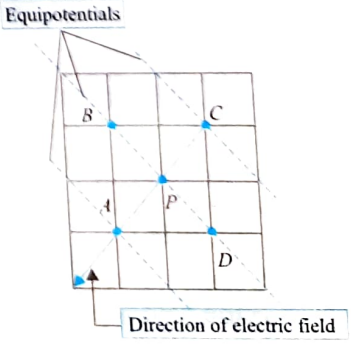
$$E_x = -\frac{\Delta V_x}{\Delta x} = \frac{-[V_D - V_A]}{(0.2)} = \frac{-[5-2]}{0.2} = -15 \text{ Vm}^{-1}$$

Similarly,

$$E_y = -\frac{\Delta V_y}{\Delta y} = \frac{-[V_B - V_A]}{0.2} = \frac{-[5-2]}{0.2} = -15 \text{ Vm}^{-1}$$

Hence, electric field $\vec{E} = E_x\hat{i} + E_y\hat{j} = -15\hat{i} - 15\hat{j}$ (Vm^{-1})

Method II: The points B, P, and D have same potential; hence, these points will lie on same equipotential surface. The separation between two equipotentials is



$$AP = 0.1 \times \sqrt{2} \text{ m}$$

$$|E| = \frac{(5-2)}{0.1\sqrt{2}} = 15\sqrt{2} \text{ Vm}^{-1}$$

$$E_{PA} = 15\sqrt{2} \text{ Vm}^{-1}$$

Here

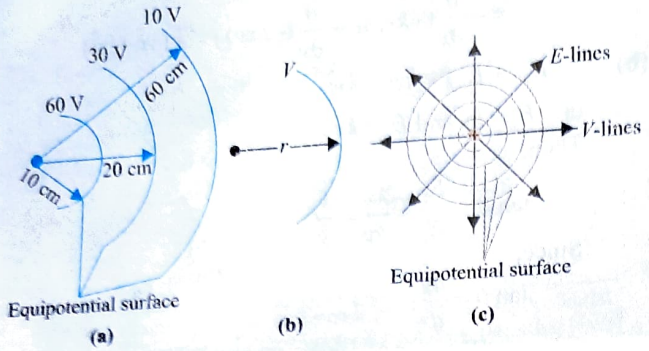
$$\vec{E} = 15\sqrt{2}\cos 45°\,(-\hat{i}) + 15\sqrt{2}\sin 45°\,(-\hat{j})$$

$$= -15\hat{i} - 15\hat{j} \text{ Vm}^{-1}$$

ILLUSTRATION 3.26

Referring to the spherical equipotential lines in Fig. (a), find

(a) $\vec{E} = f(r)$, (b) $\vec{E}$ – pattern.



Sol.

(a) For the first equipotential

$$60(\text{V}) \times \frac{10}{100}(\text{m}) = 6\,\text{Vm}$$

For the second equipotential

$$30(\text{V}) \times \frac{20}{100}(\text{m}) = 6\,\text{Vm}$$

For the third equipotential

$$10(\text{V}) \times \frac{60}{100}(\text{m}) = 6\,\text{Vm}$$

We can understand that the product of potential (V) and radial distance (r) of equipotential lines [as in Fig. (b)] is equal to 6 Vm^{-1}. Hence, general relation of potential (V) and radial distance (r) can be written as

$$V = \frac{6}{r}\text{Vm}^{-1}$$

Since $\vec{E} = -(\partial V)/(\partial r)\hat{r}$, substituting $V = 6/r$, we have

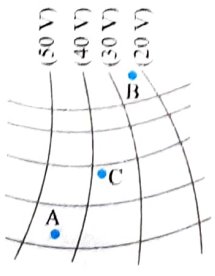
$$\vec{E} = -\frac{\partial}{\partial r}\left(-\frac{6}{r}\right)\hat{r}\ \text{Vm}^{-1}$$

or $$\vec{E} = \frac{6}{r^2}\hat{r}\ \text{Vm}^{-1}$$

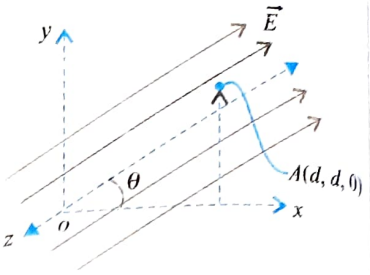
(b) Since $\vec{E}$ is directed in $\hat{r}$-direction and obeys inverse square law, E must be outward [as shown in Fig. (c)]. The above E-field must be caused by a positive point charge.

CONCEPT APPLICATION EXERCISE 3.2

1. The electric potential in a region is represented as $V = 2x + 3y - z$. Obtain expression for electric field strength.
2. Figure shows the lines of constant potential in region in which an electric field is present. The values of potentials are written in brackets. At which points A, B, or C, the electric field is greatest.



3. The equipotential curves in x, y plane are given by $V = x^2 + y^2 - 4x + 4$ where V is potential. Draw the rough sketch of electric field lines in x-y plane.
4. A uniform electric field exists in xy plane as shown in figure. Find the potential difference between origin O and $A(d, d, 0)$.



5. Electric potential in a 3-dimensional space is given by $V = \left(\frac{1}{x} + \frac{1}{y} + \frac{2}{z}\right)$ volt where x, y and z are in meter. A particle has charge $q = 10^{-12}$ C and mass $m = 10^{-9}$ g and is constrained to move in xy plane. Find the initial acceleration of the particle if it is released at (1, 1, 1) m.

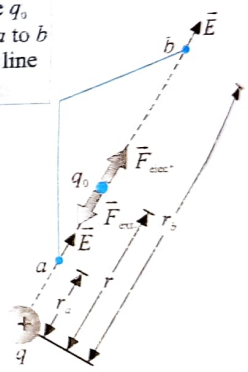
ANSWERS

1. $\vec{E} = -2\hat{i} - 3\hat{j} + \hat{k}$
2. At B
3. $Ed(\cos\theta + \sin\theta)$
4. $V = (x-2)^2 + y^2$
5. $(\hat{i} + \hat{j})\ \text{m/s}^2$

ELECTRIC POTENTIAL ENERGY

Let us move the charge q_0 from a to b in such a way that it is always maintained in equilibrium (i.e., it is moving with uniform velocity). This is possible if we apply a force F on the charge that is equal and opposite to q_0E at every point along its path as shown in figure. The force F prevents the charge q_0 from accelerating when moving from a to b. The work done by the force in the electric field is stored as potential energy (the situation is analogous to the motion of a particle in gravitational field). Thus, we may conclude that, a charged particle placed in an electric field has potential energy because of its interaction with the electric field. This is called electric potential energy. Let W_{ab} be the work done by the external force in carrying a positive charge q_0 from a to b while keeping the charge in equilibrium. The change in potential energy (ΔU) of charge q_0 is defined to be equal to the work done by the force in carrying the charge q_0 from one point to the other. That is,

Test charge q_0 moves from a to b along radial line from q.



Test charge q_0 moves along a straight line extending radially from charge q. As it moves from a to b, the distance varies from r_a to r_b.

$$\Delta U = W_{ab} \text{ or } U_b - U_a = (W_{ab})_{\text{ext}} = -(W_{ab})_{\text{elec.}}$$

where U_a and U_b designate the potential energies at points A and B, respectively.

Let us calculate the work done on a test charge q_0 moving in the electric field caused by a single, stationary point charge q. We will consider first a displacement along the radial line from point a to point b. The force on q_0 is given by Coulomb's law, and its radial component is

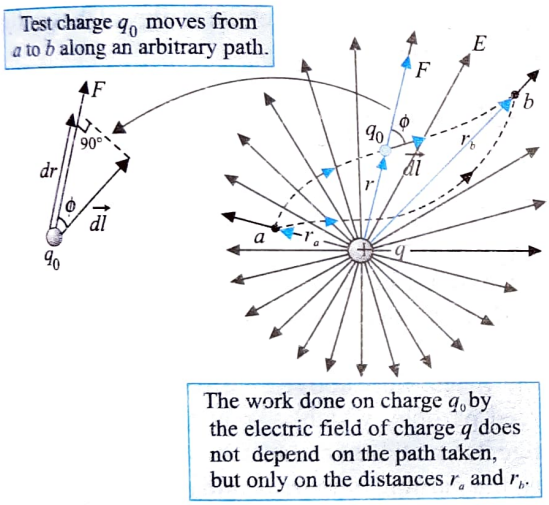
$$F_r = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2} \qquad ...(i)$$

The electric force is not constant during the displacement, and we have to integrate to calculate the work $W_{a\to b}$ done on q_0 by this force as q_0 moves from a to b. We find

$$W_{a\to b} = \int_{r_a}^{r_b} F_r\, dr = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\, dr = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) \qquad ...(ii)$$

The work done by the electric force for this particular path depends only on the end points. In fact, the work is the same for all possible paths from a to b. To prove this, we consider a more general displacement (as shown the figure). The work done on q_0 during this displacement is given by

$$W_{a\to b} = \int_{r_a}^{r_b} F\cos\phi\, dl = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}(\cos\phi\, dl) \qquad ...(iii)$$



But the figure shows that $\cos\phi\, dl = dr$. That is, work done during a small displacement $d\vec{l}$ depends only on the change dr in the distance r between the charges, which is the radial component of the displacement. Thus, Eq. (iii) is valid even for this more general displacement; the work done on q_0 by the electric field produced by q depends only on r_a and r_b and not on the details of the path. These are the needed characteristics for a conservative force, as we defined it in section. Thus the force on q_0 is a conservative force. Hence,

$$U_b - U_a = -W_{a\to b}$$

$$= -\frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$= \frac{qq_0}{4\pi\varepsilon_0 r_b} - \frac{qq_0}{4\pi\varepsilon_0 r_a} \qquad ...(iv)$$

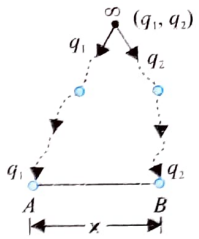
We see that Eq. (iv) is consistent if we define $qq_0/4\pi\varepsilon_0 r_a$ to be the potential energy U_a when q_0 is at point a, a distance r_a from q, and we define $qq_0/4\pi\varepsilon_0 r_b$ to be the potential energy U_b when q_0 is at point b, a distance r_b from q.

Thus, the potential energy U when the test charge q_0 is at any distance r from charge q is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r} \qquad ...(v)$$

(electric potential energy of two point charges q and q_0)

Another method of finding potential energy of two charges



- As initially there is no charge (i.e., electric field is zero), work done in bringing q_1 from ∞ to q_1 (A) is $W_1 = 0$.
- Potential at B due to charge q_1 at A is

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{x}$$

Work done in bringing q_2 from infinity to B is

$$W_2 = q_2(V_B - 0) = q_2\left(\frac{1}{4\pi\varepsilon_0}\frac{q_1}{x}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{x}$$

Total work in assembling q_1 and q_2 at A and B from infinity is

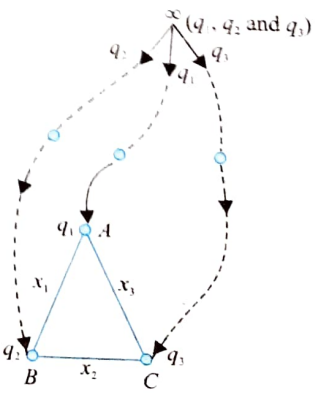
$$W = W_1 + W_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{x}$$

This work done becomes the potential energy (U) of the system of two charges. Thus,

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{x} \qquad ...(vi)$$

Note: Eq. (iv) is true for any sing of q_1 and q_2. If both q_1 and q_2 are either positive or negative (i.e., $q_1q_2 > 0$), U is positive. This implies that the system is free (as the electrostatic force between them is repulsive) and a positive amount of work is required to be done against this force to bring the charges from infinity to their present locations. But if q_1 and q_2 are of opposite sings (i.e., $q_1q_2 < 0$), U is negative. This means that the system is bound (as the electrostatic force between them is attractive).

POTENTIAL ENERGY OF A SYSTEM OF THREE CHARGES



Let us calculate the work done in building a configuration of three charges q_1, q_2, and q_3 by bringing them from infinity to the locations A, B, and C, respectively as shown in figure.

As initially there is no charge (i.e., electric field is zero), work done in bringing charge q_1 from infinity to A is $W_1 = 0$.

Potential at B due to charge q_1 at A is

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{x_1}$$

Work done in bringing charge q_2 from infinity to B is

$$W_2 = q_2(V_B - 0) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{x_1}$$

Potential at C due to charges q_1 and q_2, i.e.,

$$V_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{x_3} + \frac{q_2}{x_2}\right)$$

Work done in bringing charge q_3 from infinity to C is

$$W_3 = q_3(V_C - 0) = q_3\left[\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{x_3} + \frac{q_2}{x_2}\right) - 0\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3 q_1}{x_3} + \frac{q_2 q_3}{x_2}\right]$$

Total work done in assembling the three charges at A, B, and C from infinity is

$$W = W_1 + W_2 + W_3$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{x_1} + \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3 q_1}{x_3} + \frac{q_2 q_3}{x_2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{x_1} + \frac{q_2 q_3}{x_2} + \frac{q_3 q_1}{x_3}\right]$$

This work done becomes the potential energy (U) of the three charges q_1, q_2, and q_3, i.e.,

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{x_1} + \frac{q_2 q_3}{x_2} + \frac{q_3 q_1}{x_3}\right]$$

Note: In case of discrete distribution charges,

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \ldots\right] = \frac{1}{2}\frac{1}{(4\pi\varepsilon_o)}\sum_{i\neq j}\frac{q_i q_j}{r_{ij}}$$

[½ is used as each term in summation will appear twice]

POTENTIAL ENERGY OF CHARGES IN AN EXTERNAL ELECTRIC FIELD

POTENTIAL ENERGY OF SINGLE CHARGE IN AN EXTERNAL FIELD

Let V be the potential at a point P due to an external field. Work done by an external agent in bringing the charge q from infinity to P is

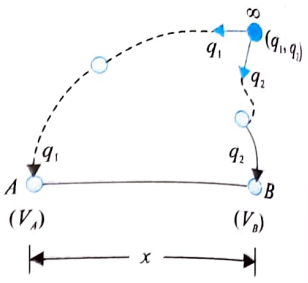
$$W = q(V - 0) = qV$$

This work done becomes the potential energy (U) of the charge (q) when placed at a point P in the external field, i.e.,

$$U = qV \qquad \ldots(i)$$

POTENTIAL ENERGY OF A SYSTEM OF TWO CHARGES IN AN EXTERNAL FIELD

Let V_A and V_B be the potentials at points A and B in an external field, where $AB = x$.



Work done in bringing the charge q_1 from infinity to A is

$$W_1 = q_1(V_A - 0) = q_1 V_A$$

Total potential at B due to the external field and the charge q_1 at A is

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{x}$$

Work done in bringing charge q_2 from infinity to B is

$$W_2 = q_2\left[\left(V_B + \frac{1}{4\pi\varepsilon_0}\frac{q_1}{x}\right) - 0\right] = q_2 V_B + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{x}$$

Total work done in assembling the configuration of two charges in an electric field is

$$W = W_1 + W_2 = q_1 V_A + \left(q_2 V_B + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{x}\right)$$

This work done becomes the potential energy of the configuration in the external electric field is

$$U = q_1 V_A + q_2 V_B + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{x} \qquad \ldots(i)$$

Note:

- $U = q_0 V$, we may define the potential energy of charge q_0 at any point in the field to be equal to the amount of work done by external agent in bringing the charge q_0 from infinity to that point.
- In the discussion above, we have assumed that the charged particle, which is moving in equilibrium, will change its kinetic energy as it moves from point to point. Thus, if a particle of mass m and q_0 has potential energy U_A and kinetic energy K_A at point A and U_B and K_B at B, the conservation of mechanical energy requires (in the absence of work done by any external force acting of the charge)

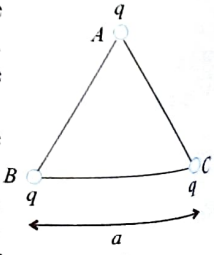
$$U_A + K_B = U_B + K_B$$

Since $U_A = q_0 V_A$ and $U_B = q_0 V_B$, we may write

$$(V_A - V_B) = K_B - K_A$$

ILLUSTRATION 3.27

Three equal charges q are placed at the corners of an equilateral triangle of side a.



(a) Find out potential energy of charge system.

(b) Calculate work required to decrease the side of triangle to $a/2$.

(c) If the charges are released from the shown position and each of them has same mass m, then find the speed of each particle when they lie on triangle of side $2a$.

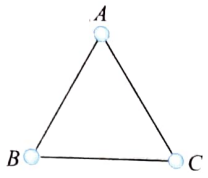
Sol.

Method I

(a) Assume all the charges are at infinity initially. Work done in putting charge q at corner A is

$$W_1 = q(V_f - V_i) = q(0-0) = 0$$

Since potential energy at A is zero in absence of charges, work done in putting q at corner B in presence of charge at A is



$$W_2 = q(V_B - 0) = q\left[\frac{Kq}{a} - 0\right] = \frac{Kq^2}{a}$$

Similarly, work done in putting charge q at corner C in presence of charge at A and B is

$$W_3 = q(V_f - V_i) = q\left[\left(\frac{Kq}{a} + \frac{Kq}{a}\right) - 0\right]$$

So net potential energy is

$$U = W_1 + W_2 + W_3$$

$$= 0 + \frac{Kq^2}{a} + \frac{2Kq^2}{a} = \frac{3Kq^2}{a} = \frac{3}{4\pi\varepsilon_0}\frac{q^2}{a}$$

Method II (using direct formula)

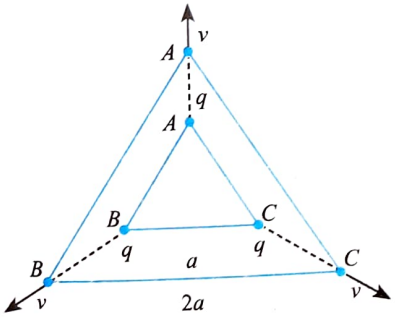
$$U = U_{12} + U_{13} + U_{23}$$

$$= \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a} = \frac{3Kq^2}{a} = \frac{3}{4\pi\varepsilon_0}\frac{q^2}{a}$$

(b) Work required to decrease the sides is

$$W = U_f - U_i = \frac{3Kq^2}{a/2} - \frac{3Kq^2}{a} = \frac{3Kq^2}{a} = \frac{3}{4\pi\varepsilon_0}\frac{q^2}{a}$$

(c) By conservation of mechanical energy



$$\Delta U + \Delta K = 0$$

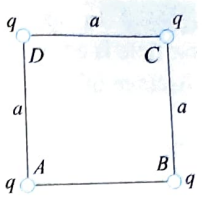
or $U_i - U_f = K_f - K_i$

or $$\frac{3Kq^2}{a} - \frac{3Kq^2}{2a} = 3\left(\frac{1}{2}mv^2\right) - 0$$

or $$v = \sqrt{\frac{Kq^2}{am}} = \frac{1}{2}\sqrt{\frac{q^2}{\pi\varepsilon_0 a m}}$$

ILLUSTRATION 3.28

Four identical charges q are placed at the corners of a square of side a. Find the potential energy of one of the charges due to the remaining charges.



Sol. The electric potential of point A due to the charges placed at B, C, and D is

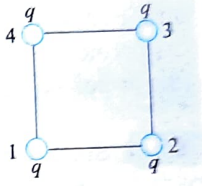
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{2}a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{a} = \frac{1}{4\pi\varepsilon_0}\left(2 + \frac{1}{\sqrt{2}}\right)\frac{q}{a}$$

Therefore, potential energy of the charge at A is

$$qV = \frac{1}{4\pi\varepsilon_0}\left(2 + \frac{1}{\sqrt{2}}\right)\frac{q^2}{a}$$

ILLUSTRATION 3.29

Four identical point charges q are placed at four corners of a square of side a. Find the potential energy of the charge system.



Sol. **Method I (using direct formula)**

$$U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

$$= \frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a}$$

$$= \left[\frac{4Kq^2}{a} + \frac{2Kq^2}{a\sqrt{2}}\right] = \frac{2Kq^2}{a}\left[2 + \frac{1}{\sqrt{2}}\right] = \frac{q^2}{2\pi\varepsilon_0 a}\left[2 + \frac{1}{\sqrt{2}}\right]$$

Method II [using $U = (U_1 + U_2 + \cdots)$]

U_1 is the total potential energy of charge at corner 1 due to all other charges; U_2 is the total potential energy of charge at corner 2 due to all other charges; U_3 is the total potential energy of charge at corner 3 due to all other charges; U_4 is the total potential energy of charge at corner 4 due to all other charges.

Since due to symmetry,

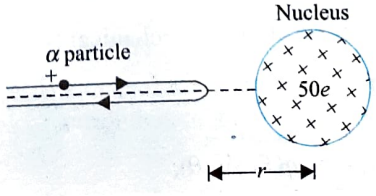
$$U_1 = U_2 = U_3 = U_4$$

$$U_{net} = \frac{U_1 + U_2 + U_3 + U_4}{2}$$

$$= \frac{1}{2}\cdot 4\cdot\left[\frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{\sqrt{2}a}\right] = \frac{2Kq^2}{a}\left[2 + \frac{1}{\sqrt{2}}\right]$$

ILLUSTRATION 3.30

An alpha particle with kinetic energy 10 MeV is heading toward a stationary tin nucleus of atomic number 50. Calculate the distance of closest approach (see figure). (Given charge of alpha particle is twice the charge of electron.)



Sol. In approaching the nucleus, kinetic energy of alpha particle is converted into electrical potential energy. So if r is the distance of the closest approach, then

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

or $$r = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(50e)}{(\text{Kinetic Energy})}$$

or $$r = (9\times10^9)\frac{(2\times1.6\times10^{-19})(1.6\times10^{-19}\times50)}{(10\times10^6\times1.6\times10^{-19})}$$

$$= 14.4\times10^{-15}\,\text{m} = 14.4\ \text{fm}$$

ILLUSTRATION 3.31

A proton moves with a speed u directly toward a free proton originally at rest. Find the distance of closest approach for the two protons. Given that mass of the proton is m and charge of the proton is $+e$.

Sol. Since the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion, the other particle will also start moving. So the velocity of the first particle will decrease while that of the other particle will increase, and at the closest approach both will move with the same velocity. So, if v is the common velocity of each particle at the closest approach, as no external forces are acting on them (the system of particles), the linear momentum of the system of particles will be conserved. By conservation of momentum, we get

$$mu = mv + mv,$$

i.e., $$v = \frac{1}{2}u$$

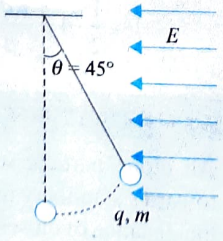
And by conservation of energy, we have

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

So, $$r = \frac{e^2}{\pi\varepsilon_0 mu^2} \quad (\text{as } v = u/2)$$

ILLUSTRATION 3.32

A horizontal electric field ($E = mg/q$) exist in space, as shown in figure. A particle of mass m, having charge q, is attached at the end of a light insulated rod. If the particle is released from the position shown in the figure, find the angular velocity of the rod when it passes through the bottom most position.



Sol. Using conservation of mechanical energy,

$$\Delta K + \Delta U = 0$$

$$\Delta U = \Delta U_{\text{gravity}} + \Delta U_{\text{electrical}}$$

$$\Delta U_{\text{electrical}} = q\Delta V = -q(El\sin\theta),$$

$$\Delta U_{\text{gravity}} = -mg(l - l\cos\theta)$$

$$\Delta K = \frac{1}{2}mv^2 - 0 \qquad \text{...(i)}$$

$$qEl\sin\theta + mg(l - l\cos\theta) = \frac{1}{2}mv^2 \text{ From Eq. (i)}$$

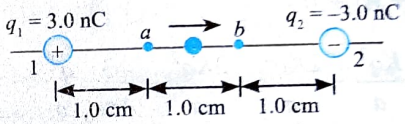
$$mgl\sin\theta + mgl - mgl\cos\theta = \frac{mv^2}{2}$$

$$\frac{gl}{\sqrt{2}} + gl - \frac{gl}{\sqrt{2}} = \frac{1}{2}mv^2 \text{ or } v = \sqrt{2gl}$$

$$\omega = \frac{v}{l} = \frac{\sqrt{2gl}}{l} = \sqrt{\frac{2g}{l}} \text{ or } \omega = \sqrt{\frac{2g}{l}}$$

ILLUSTRATION 3.33

Two-point charges q_1 and q_2 are fixed at a distance 3.0 cm as shown in figure. A dust particle with mass $m = 5.0\times10^{-9}$ kg and charge $q_0 = 2.0$ nC starts from rest at point 'a' and moves in a straight line to point 'b'. What is its speed v at point b?



Sol. Applying conservation of mechanical energy

$$\Delta K + \Delta U = 0$$

or $$(K_f - K_i) + (U_f - U_i) = 0$$

Here $K_i = 0$ and $K_f = \frac{1}{2}mv^2$ and $U_i = q_0V_i$ and $U_f = q_0V_f$

Hence, $$(K_f - K_i) + q_0(V_f - V_i) = 0$$

$$\left(\frac{1}{2}mv^2 - 0\right) + (q_0V_f - q_0V_i) = 0$$

And solving for v, we find, $v = \sqrt{\frac{2q_0(V_i - V_f)}{m}}$ ...(i)

Electric potential at 'a': $V_i = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1a}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2a}}$

$$V_i = (9.0\times10^9)\times\left(\frac{3.0\times10^{-9}}{0.010} + \frac{(-3.0\times10^{-9})}{0.020}\right) = 1350\,\text{V} \qquad \text{...(ii)}$$

Electric potential at 'b': $V_f = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1b}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2b}}$

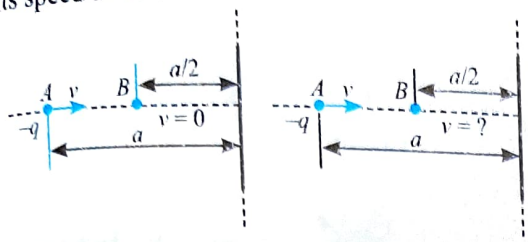
$$V_f = (9.0\times10^9)\times\left(\frac{3.0\times10^{-9}}{0.020} + \frac{(-3.0\times10^{-9})}{0.010}\right) = -1350\,\text{V} \qquad \text{...(iii)}$$

From (i),(ii) and (iii) we get

$$v = \sqrt{\frac{2(2.0\times10^{-9})[(1350) - (-1350)]}{5.0\times10^{-9}}} = 46\ \text{m/s}$$

ILLUSTRATION 3.34

A particle of mass m carrying charge 'q' is projected with velocity 'v' from point 'A' towards an infinite line of charge from a distance 'a'. Its speed reduces to zero momentarily at point 'B' which is at a distance $a/2$ from the line of charge. If another particle with mass m and charge '$-q$' is projected with the same velocity 'v' from 'A' towards the line of charge what will be its speed at 'B'.



Sol. Applying conservation of mechanical energy

$$\Delta K + \Delta U = 0 \text{ or } \left(K_f - K_i\right) + q\left(V_f - V_i\right) = 0 \quad \ldots(i)$$

When a positive charge is projected towards the wire

$$\left(0 - \frac{1}{2}mv^2\right) + q\Delta V = 0 \Rightarrow \frac{1}{2}mv^2 = q\left(V_B - V_A\right) \quad \ldots(ii)$$

When a negative charge is projected towards the wire

$$\left(\frac{1}{2}mv_B^2 - \frac{1}{2}mv^2\right) + (-q)\Delta V = 0 \Rightarrow \left(\frac{1}{2}mv_B^2 - \frac{1}{2}mv^2\right) = q\left(V_B - V_A\right)$$

$\ldots$ (iii)

From (ii) and (iii), we get

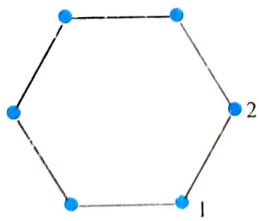
$$\frac{1}{2}mv^2 = \left(\frac{1}{2}mv_B^2 - \frac{1}{2}mv^2\right) \Rightarrow v_B^2 = 2v^2$$

or $\quad v_B = \sqrt{2}v$

ILLUSTRATION 3.35

Small identical balls with equal charges are fixed at the vertices of a right polygon with side l. At a certain instant, one of the balls is released, and after a sufficiently long-time interval later the ball adjacent to the previously released is freed. The kinetic energies of the released balls are found to differ by T at a sufficient long distance from the polygon. Determine the charge of each ball.

Sol. If ball '1' is taken to infinity, work will be done by electrostatic force this work done will be equal to charge in kinetic energy of the ball. Let potential at position 1 is V. Taking ball at 1 as a system.



$$\Delta U + \Delta K = 0 \Rightarrow qV_{\text{initial}} + K_{\text{initial}} = qV_{\text{final}} + K_{\text{final}}$$

$$qV = 0 + K_1 \quad \ldots(i)$$

Now second adjacent ball is removed. The potential at 2 will be same for ball other than 1. The potential at 2 will be (V – potential at 2 due to 1).

$$q\left(V - \frac{1}{4\pi\varepsilon_0}\frac{q}{l}\right) = K_2 \quad \ldots(ii)$$

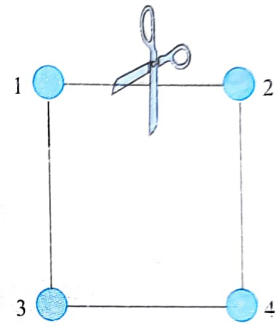
From (i) and (ii)

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{l} = K_1 - K_2 = T$$

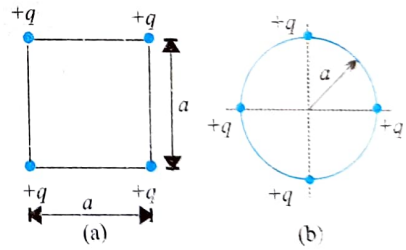
or $\quad q^2 = 4\pi\varepsilon_0 lT \Rightarrow q = \sqrt{4\pi\varepsilon_0 lT}$

CONCEPT APPLICATION EXERCISE 3.3

1. Four balls, each with mass m, are connected by four nonconducting strings to form a square with side a, as shown in figure. The assembly is placed on a horizontal nonconducting frictionless surface. Balls 1 and 2 each have charge q, and balls 3 and 4 are uncharged. Find the maximum speed of balls 1 and 2 after the string connecting them is cut.

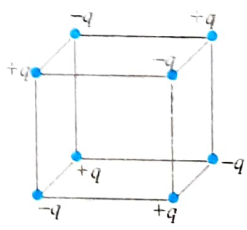


2. Consider the configuration of a system of four charges each of value $+q$. Find the work done by external agent in changing the configuration of the system from Fig. (a) to Fig. (b).



3. Four charges $+q$, $-q$, $+q$, and $-q$ are placed in order on the four consecutive corners of a square of side a. Find the work done in interchanging the positions of any two neighbouring charges of opposite sign.

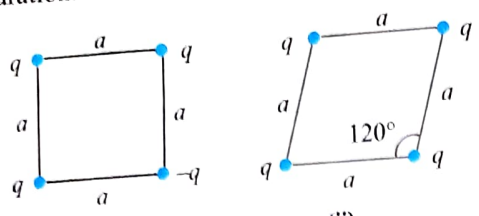
4. Charges $+q$ and $-q$ are located at the corners of a cube of side as shown in figure. Find the work done to separate the charges to infinite distance.



5. Two charged particles having charge $1\mu C$ and $-1\mu C$ and of mass 50 gm each are held at rest while their separation is 2 m. Find the speed of the particles when their separation is 1 m.

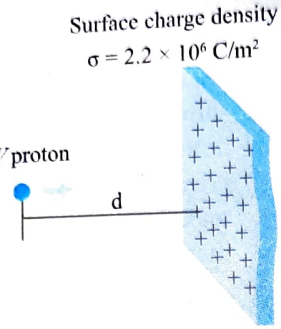
6. Four-point charges q, q, q and $-q$ are placed at the vertices of a square of side length a as shown in Fig. (i). The arrangement is changed and the charge are positioned at the vertices of a rhombus of side length a with $-q$ charge

at the vertex where angle is 120° as shown in Fig. (ii). Find the work done by the external agent in changing the configuration.



(i) (ii)

7. A 100 eV proton is projected directly towards a large metal plate that has surface charge density of 2.2×10^{-6} C/m^2. From what distance must the proton be projected, if it is to just fail to strike that plate?



8. A particle (A) having charge Q and mass m is at rest and is free to move. Another particle (B) having charge q and mass m is projected from a large distance towards the first particle with speed u.
   (a) Calculate the least kinetic energy of the system during the subsequent motion.
   (b) Find the final velocity of both the particles. Consider coulomb force only.

9. A particle (A) having charge Q and mass m and another particle (B) having charge q and mass m are initially held at a distance $r = \dfrac{qQ}{2\pi\varepsilon_0 mu^2}$ apart. Particle B is projected directly towards A with velocity u and particle A is released simultaneously. Find the velocity of particle A after a long time. Consider coulomb force only.

ANSWERS

1. $\sqrt{\dfrac{k_e q^2}{3am}}$
2. $-\dfrac{q^2}{4\pi\varepsilon_0 a}\left(3-\sqrt{2}\right)$
3. $\dfrac{q^2}{4\pi\varepsilon_0 a}\left(4-2\sqrt{2}\right)$
4. $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a} \times \dfrac{4}{\sqrt{16}}\left[3\sqrt{3}-3\sqrt{6}-\sqrt{2}\right]$
5. $\dfrac{3}{10}$ m/s
6. $-\dfrac{Kq^2}{a}\left(1-\dfrac{1}{\sqrt{3}}\right)$
7. 40 mm
8. (a) $\dfrac{1}{4}mu^2$ (b) A will move to right with velocity u and B will be at rest.
9. $\left(\dfrac{1+\sqrt{3}}{2}\right)u$

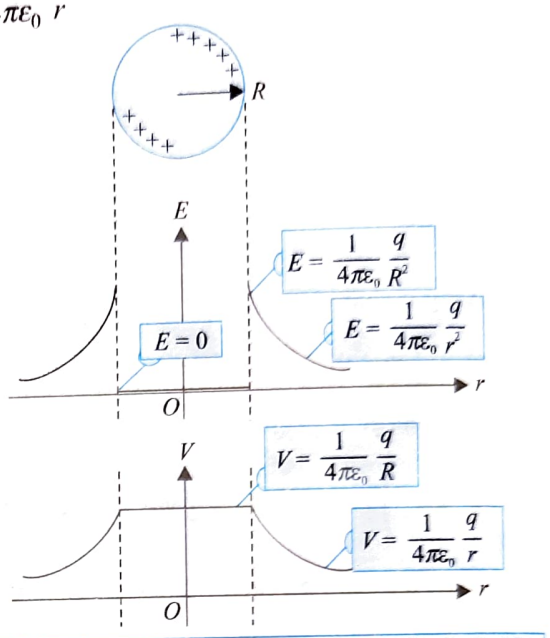
ELECTRIC POTENTIAL OF SOME CONTINUOUS CHARGE DISTRIBUTIONS

CHARGED CONDUCTING SPHERE

A solid conducting sphere of radius R has a total charge q. At all points outside the sphere the field is the same at equal distance from center of the sphere, as if the sphere were removed and replaced by a point charge q. We take $V = 0$ at infinity, as we did for a point charge.

Then the potential at a point outside the sphere at a distance r from its center is the same as the potential due to a point charge q at the center. We have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$



Electric field magnitude E and potential V at points inside and outside a positively charged spherical conductor.

The potential at the surface of the sphere is

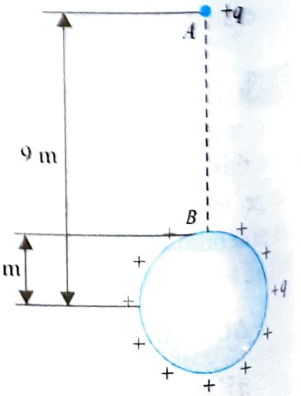
$$V_{\text{surface}} = \frac{q}{4\pi\varepsilon_0 R}$$

Inside the sphere, $\vec{E}$ is zero everywhere; otherwise, charge would move within the sphere. Hence, if a test charge moves from any point to any other point inside the sphere, no work is done on that charge. This means that the potential is the same at every point inside the sphere and is equal to its value $q/4\pi\varepsilon_0 R$ at the surface.

Note: The variation of electric field and potential for conducting shell is same as conducting sphere.

ILLUSTRATION 3.36

A very small sphere of mass 80 g having a charge q is held at a height of 9 m vertically above the center of a fixed conducting sphere of radius 1 m, carrying an equal charge q. When released, it falls until it is repelled back just before it comes in contact with the sphere as shown in figure. Calculate the charge q. [$g = 10$ ms^{-2}]



Sol. Here both electric and gravitational potential energies are changing, and for an external point, a charged sphere behaves as if whole of its charge were concentrated at its center. Applying conservation of energy between initial and final positions, we have

$$\frac{1}{4\pi\varepsilon_0}\frac{qq}{9}+mg\times 9=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{1}+mg\times 1$$

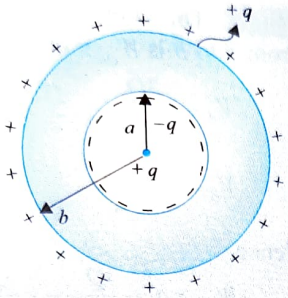
(as Kinetic Energy is zero at both locations)

or $$q^2=\frac{80\times10^{-3}\times10}{10^9}$$

or $$q=20\sqrt{2}\ \mu C$$

ILLUSTRATION 3.37

A hollow uncharged spherical conductor has inner radius a and outer radius b. A positive point charge $+q$ is in the cavity at the center of the sphere (figure). Find the potential $V(r)$ everywhere, assuming that $V=0$ at $r\to\infty$.



Sol. The conductor is an equipotential volume, so V is constant for $a\le r\le b$. The field lines inside the cavity must end on the inner surface of the cavity, so this surface has an induced charge $-q$. Since the shell is uncharged, a positive charge $+q$ is on the outer surface (figure).

The three charges q at the center, $-q$ on the inner surface, and $+q$ on the outer surface produce a field

$$E_r=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

for $r>b$, so the potential for $r>b$ is

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Outside the shell, $V(r)$ is the same as that due to a point charge q at the origin. Choosing $V=0$ at $r=\infty$, we have

$$V(r)=\frac{1}{4\pi\varepsilon_0}\frac{q}{r},\quad r\ge b$$

At $r=b$, the potential is $q/4\pi\varepsilon_0 b$, V remains at this constant value throughout the spherical shell from $r=b$ to $r=a$, so

$$V(r)=\frac{1}{4\pi\varepsilon_0}\frac{q}{b},\quad a\le r\le b$$

For any point inside the cavity $(r<a)$

$V(r)$ = Electric potential due to q at center + Electric potential due charge distributed on spherical surface of radius a + Electric potential due to charge distributed on spherical surface of radius b. So

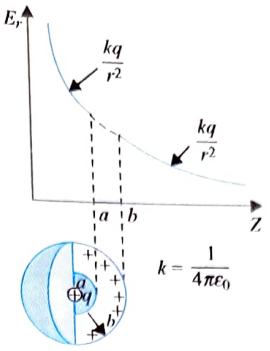
$$V(r)=V_q+V_{a,\text{ inside}}+V_{b,\text{ outside}}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}+\frac{1}{4\pi\varepsilon_0}\frac{(-q)}{a}+\frac{1}{4\pi\varepsilon_0}\frac{q}{b}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}+\frac{1}{4\pi\varepsilon_0}\frac{q}{b}-\frac{1}{4\pi\varepsilon_0}\frac{q}{a},\quad r\le a$$

Following figure shows the electric potential as a function of distance from the center of the cavity.

Electric field versus r



Electric potential versus r

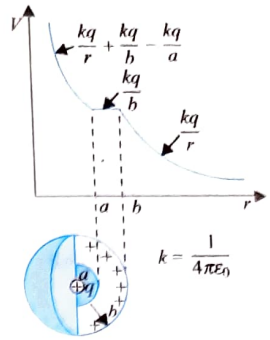
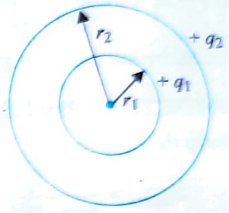


ILLUSTRATION 3.38

Figure shows two concentric conducting shells of radii r_1 and r_2 carrying uniformly distributed charges q_1 and q_2, respectively. Find an expression for the potential of each shell.



Sol. The potential of each sphere consists of two points: one due to its own charge, and the other due to the charge on the other sphere. Using the principle of superposition, we have

$$V_1=V_{r_1,\text{ surface}}+V_{r_2,\text{ inside}}$$

and $$V_2=V_{r_1,\text{ outside}}+V_{r_2,\text{ surface}}$$

Hence,

$$V_1=\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}+\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

and $$V_2=\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_2}+\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

ILLUSTRATION 3.39

A metal sphere A of radius a is charged to potential V. What will be its potential if it is enclosed by a spherical conducting shell B of radius b and the two are connected by a wire?

Sol. If the charge on the sphere of radius a is q, then

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{a}$$

i.e., $$q=(4\pi\varepsilon_0 a)V$$

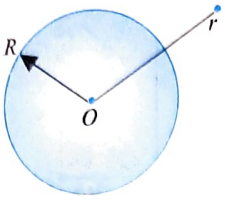
Now, when sphere A is enclosed by spherical conductor B and the two are connected by a wire, charge will reside on the outer surface of B and so the potential of B will be

$$V_B=\frac{1}{4\pi\varepsilon_0}\frac{q}{b}=\frac{1}{4\pi\varepsilon_0}\frac{4\pi\varepsilon_0 a}{b}V=\frac{a}{b}V$$

Now as sphere A is inside B, so its potential is

$$V_A=V_B=\frac{a}{b}(V)\qquad [V \text{ as } a<b]$$

NON-CONDUCTING SOLID SPHERE



OUTSIDE THE SPHERE

The field intensity outside the sphere is

$$E_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

or $\dfrac{dV_{\text{outside}}}{dr} = -E_{\text{outside}}$ or $dV_{\text{outside}} = -E_{\text{outside}}\,dr$

or $\displaystyle\int_0^V dV_{\text{outside}} = -\int_\infty^r \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}dr$ or $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

as $V_\infty = 0$

At $r = R$, $V = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{R}$

i.e., at the surface, potential is $V_S = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}$

INSIDE THE SPHERE

Inside the sphere,

$$E_{\text{inside}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}r$$

or $\dfrac{dV_{\text{inside}}}{dr} = -E_{\text{inside}}$ or $dV_{\text{inside}} = -E_{\text{inside}}\,dr$

or $\displaystyle\int_{V_S}^{V} dV_{\text{inside}} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}\int_R^r r\,dr$

or $\displaystyle V - V_S = -\frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}\left[\frac{r^2}{2}\right]_R^r$

Substituting $V_s = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}$ we get

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}\left[\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right] = \frac{V_S}{2}\left[3 - \frac{r^2}{R^2}\right] = \frac{q}{8\pi\varepsilon_0 R}\left[3 - \frac{r^2}{R^2}\right]$$

At the center $r = 0$ and

$$V_C = \frac{3}{2}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{R}\right) = \frac{3}{2}V_S$$

That is, potential at the center is 1.5 times the potential at surface. Thus, for a uniformly charged nonconducting sphere, we have the following formula for potential:

$$V_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r};\ V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R},\ V_{\text{inside}} = \frac{q}{8\pi\varepsilon_0 R}\left(3 - \frac{r^2}{R^2}\right)$$

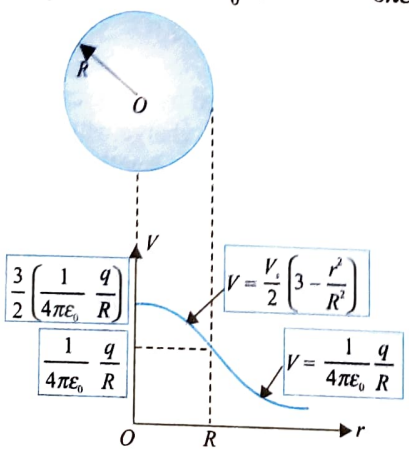
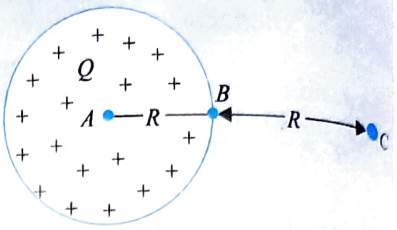


ILLUSTRATION 3.40

Find the electric work done in bringing a charge q from A to B in a sphere of charge Q distributed uniformly throughout its volume.



Sol. The work done by electric force in bringing charge q from A to B is $W_{\text{ele}} = q(V_A - V_B)$ where

$$V_A = \frac{3Q}{8\pi\varepsilon_0 R},$$

$$V_B = \frac{Q}{4\pi\varepsilon_0 R}$$

Hence,

$$W_{\text{ele}} = q\left(\frac{3Q}{8\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 R}\right) = \frac{Qq}{8\pi\varepsilon_0 R}$$

Alternative method:

The work done from A to B can also be given by $W_{\text{ele}} = q\int E\,dr$,

where

$$E = \frac{Qr}{4\pi\varepsilon_0 R^3};\ r \le R$$

or $\displaystyle W = q\int_0^R \left(\frac{Q}{4\pi\varepsilon_0 R^3}r\right)dr = \frac{Qq}{8\pi\varepsilon_0 R}$

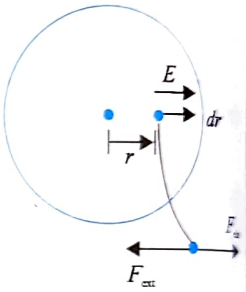
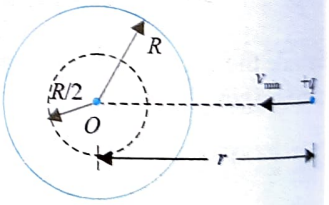


ILLUSTRATION 3.41

A positive charge Q is uniformly distributed throughout the volume of a dielectric sphere of radius R. A point mass having charge $+q$ and mass m is fired toward the center of the sphere with velocity v from a point at distance r ($r > R$) from the center of the sphere. Find the minimum velocity v so that it can penetrate $R/2$ distance of the sphere. Neglect any resistance other than electric interaction. Charge on the small mass remains constant throughout the motion.



Sol. Using conservation of mechanical energy $\Delta K + \Delta U = 0$.

$$\left(0 - \frac{1}{2}mv^2\right) + q(V_f - V_i) = 0$$

or $\dfrac{1}{2}mv^2 = q(V_f - V_i)$ ...(i)

$$V_i = \frac{Q}{4\pi\varepsilon_0 r} \text{ and } V_f = \frac{q}{8\pi\varepsilon_0 R}\left[3 - \frac{r^2}{R^2}\right]$$

where $r=\frac{R}{2}$; here, $V_f=\frac{11Q}{32\pi\varepsilon_0 R}$

Putting the values of V_i and V_f in Eq. (i)

$$\frac{1}{2}mv^2=\frac{11qQ}{32\pi\varepsilon_0 R}-\frac{qQ}{4\pi\varepsilon_0 r}$$

or $$mv^2=\frac{11qQ}{16\pi\varepsilon_0 R}-\frac{qQ}{2\pi\varepsilon_0 r}$$

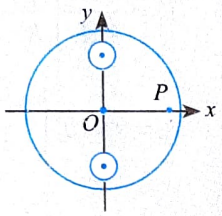
$$=\frac{qQ}{2\pi\varepsilon_0}\left[\frac{11}{8R}-\frac{1}{r}\right]$$

or $$v^2=\frac{qQ}{2m\pi\varepsilon_0 R}\left[\frac{11}{8}-\frac{R}{r}\right]$$

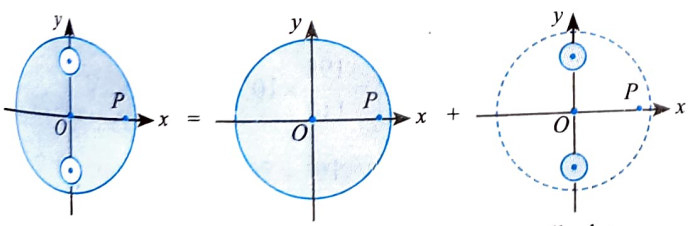
Hence $v=\sqrt{\frac{qQ}{2m\pi\varepsilon_0 R}\left[\frac{11}{8}-\frac{R}{r}\right]}$

ILLUSTRATION 3.42

A non-conducting sphere of radius $R=5$ cm has its centre at the origin O of coordinate system as shown in figure. It has two spherical cavities of radius $r=1$ cm, whose centres are at (0, 3 cm), (0, –3 cm) respectively. and Solid material of sphere has uniform positive charge density $\rho=1/\pi\,\mu C/m^3$. Calculate Electric potential at point P (4 cm, 0).



Sol Here we can use method of superposition. The given system can be considered as combination a complete sphere without cavities (say big sphere) and cavities as two spheres with negative charge density (say small spheres). The point under consideration is located in side the big sphere of positive charge density without cavities and outside the small spheres with negative charge density.



Big sphere without cavities
Charge density ρ

Small spheres
Charge density –ρ

Charge on the sphere (including cavities)

$$Q=\frac{4}{3}\pi R^3\rho=\frac{4}{3}\pi\left(5\times10^{-2}\right)^3\frac{1}{\pi}\times10^{-6}=\frac{500}{3}\times10^{-12}\,C$$

Charge in a volume equal to that of cavity

$$q=\frac{4}{3}\pi R^3\rho=\frac{4}{3}\pi\left(1\times10^{-2}\right)^3\frac{1}{\pi}\times10^{-6}=\frac{4}{3}\times10^{-12}\,C$$

Potential at P, $V_P=V_{\text{wholesphere}}-2V_{\text{cavity}}$

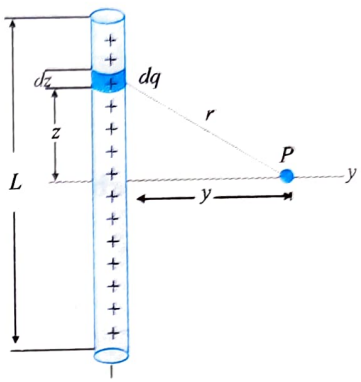
$$V_P=\frac{kQ}{2R}\left[3-\frac{x^2}{R^2}\right]-2\frac{kq}{5\times10^{-2}}$$

$$=\frac{9\times10^9\times(500/3)\times10^{-12}}{2\times5\times10^{-2}}\left[3-\left(\frac{4}{5}\right)^2\right]-\frac{2\times9\times10^9\times(4/3)\times10^{-12}}{5\times10^{-2}}$$

$$=\frac{873}{25}\,V=34.92\,V$$

UNIFORM LINE OF CHARGE

We can use the geometry of figure to find the potential due to a uniform line of positive charge at point P, a distance y from the rod on its perpendicular bisector. Using the charge element $dq=\lambda\,dz$ (where λ is the linear charge density), we have



$$dV=\frac{1}{4\pi\varepsilon_0}\frac{dq}{r}=\frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dz}{\sqrt{z^2+y^2}}$$

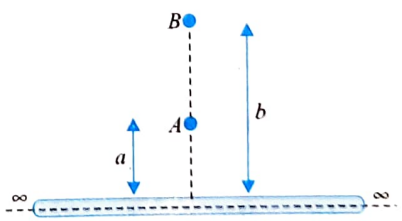
Carrying out the integration over the length L and noting that y is a constant, we obtain

$$V=\frac{1}{4\pi\varepsilon_0}\int_{-L/2}^{+L/2}\frac{\lambda\,dz}{\sqrt{z^2+y^2}}$$

$$=\frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(z+\sqrt{z^2+y^2}\right)\right]_{-L/2}^{+L/2}$$

$$=\frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{L/2+\sqrt{(L^2/4)+y^2}}{-L/2+\sqrt{(L^2/4)+y^2}}\right]$$

For an infinite rod, $L\to\infty$; hence, absolute potential of an infinite rod is not defined. For an infinite rod, the potential at any point near the rod comes out to be infinite. But the potential difference between two points situated at distances a and b can easily be calculated as follows:



$$V_b-V_a=\int_a^b-E_r\,dr$$

$$=\int_a^b\frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}\,dr$$

$$=\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

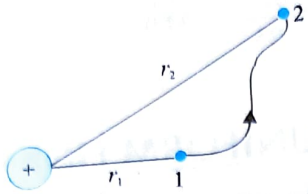
Important Point:
The absolute potential of an infinite rod and an infinite sheet of charge is not defined, but we can define the potential difference near the infinite charge distribution.

ILLUSTRATION 3.43

A charge particle q is shifted from point 1 to point 2 in the electric field of a straight long linear charge of λ.

(a) Find the electric work done if $r_1 = R$ and $r_2 = 2R$

(b) Find the potential difference between 1 and 2.



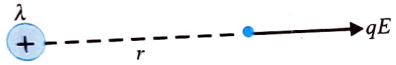
Sol.

(a) The work done by the electric field is

$$W_{el} = \int \vec{F}_{el} \cdot \vec{dl} = q\int \vec{E} \cdot \vec{dl}$$

$$q\int \left(\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}\right)\cdot(dr\hat{r}) = \frac{\lambda q}{2\pi\varepsilon_0}\int_{r=R}^{r=2R}\frac{dr}{r} = \frac{\lambda q}{2\pi\varepsilon_0}\ln 2$$

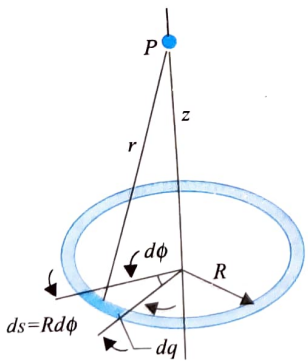
(b) $V_2 - V_1 = \dfrac{W_{ext}}{q} = \dfrac{\left(\dfrac{\lambda q}{2\pi\varepsilon_0}\ln 2\right)}{q} = \dfrac{\lambda}{2\pi\varepsilon_0}\ln 2$



A RING OF CHARGE

Figure shows a uniform ring of positive charge. The contribution to the potential at point P on its axis due to the charge element $dq = \lambda\, ds = \lambda R\, d\phi$ is

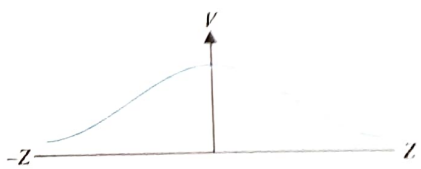
$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R\, d\phi}{\sqrt{R^2+z^2}}$$



Integrating around the ring, we note that R and z both remain constant. The variable of integration is ϕ, which ranges from 0 to 2π. Thus,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R}{\sqrt{R^2+z^2}}\int_0^{2\pi} d\phi = \frac{1}{4\pi\varepsilon_0}\frac{2\pi\lambda R}{\sqrt{R^2+z^2}}$$

Total charge on the ring is $q = 2\pi\lambda R$. Distance of point under consideration from the ring is $r = \sqrt{R^2+z^2}$.



$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

where r is the distance from ring to point under consideration.

At $z = 0$,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$; maximum potential is at the center of ring

and at $z \to \infty$, $V = 0$.

ILLUSTRATION 3.44

Two circular loops of radii 0.05 and 0.09 m, respectively, are put such that their axes coincide and their centers are 0.12 m apart. A charge of 10^{-6} C is spread uniformly on each loop. Find the potential difference between the centers of the loops.

Sol. The potential at the center of a ring will be due to charge on both the rings, and as every element of a ring is at a constant distance from the center, so

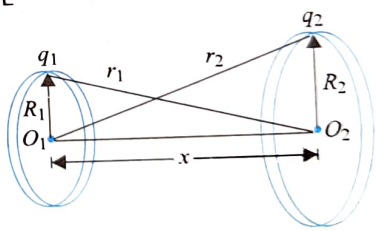
$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R_1} + \frac{q_2}{\sqrt{R_2^2+x^2}}\right]$$

$$= 9\times10^9\left[\frac{10^{-4}}{5} + \frac{10^{-4}}{\sqrt{9^2+12^2}}\right]$$

$$= 9\times10^5\left[\frac{1}{5}+\frac{1}{15}\right] = 2.40\times10^5\ \text{V}$$

Similarly,

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_2}{R_2} + \frac{q_1}{\sqrt{R_1^2+x^2}}\right]$$



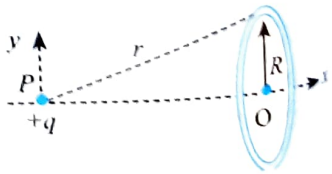
or $V_2 = 9\times10^5\left[\dfrac{1}{9}+\dfrac{1}{13}\right] = \dfrac{198}{117}\times10^5 = 1.69\times10^5$ V

So $V_1 - V_2 = (2.40-1.69)\times10^5 = 71$ kV

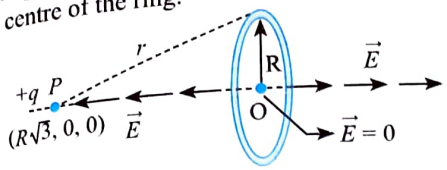
ILLUSTRATION 3.45

A circular ring of radius R with uniform charge density λ per unit length is located in the y-z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point $P(R\sqrt{3},0,0)$ on the positive x-axis directly towards 0, with an initial speed v.

Find the smallest (non-zero) value of speed v such that the particle does not return to P.

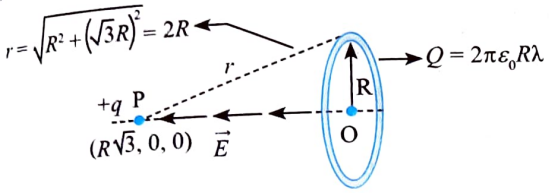


Sol. The electric field due to ring on left side of its centre is towards negative x-direction and right side of its centre is towards positive x-direction, the electric field at the centre of ring is zero. It means the particle will not come back due to repulsion if it crosses the centre of the ring.



Applying conservation of mechanical energy $\Delta K + \Delta U = 0$

or $(K_f - K_i) + (U_f - U_i) = 0$



$$\left(0 - \frac{1}{2}mv^2\right) + (qV_f - qV_i) = 0$$

$$\frac{1}{2}mv^2 = q(V_f - V_i) \Rightarrow v = \sqrt{\frac{2q(V_f - V_i)}{m}} \quad \ldots(i)$$

The potential at initial position,

$$V_i = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(2\pi R\lambda)}{2R} = \frac{\lambda}{4\varepsilon_0}$$

The potential at the centre of the ring,

$$V_f = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{1}{4\pi\varepsilon_0}\frac{(2\pi\varepsilon_0 R\lambda)}{R} = \frac{\lambda}{2\varepsilon_0}$$

Hence from (i)

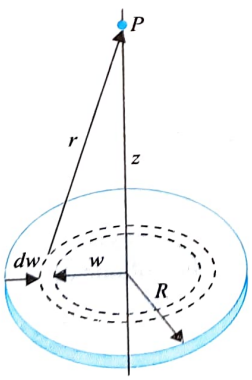
$$\Rightarrow \quad v = \sqrt{\frac{2q\left(\lambda/2\varepsilon_0 - \lambda/4\varepsilon_0\right)}{m}} \quad \text{or} \quad v_{\min} = \sqrt{\frac{\lambda q}{2\varepsilon_0 m}}$$

CHARGED DISK

With the geometry of figure, due to the ring of radius w and charge $dq = \sigma\, dA$ with area element $dA = 2\pi w dw$,

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{w^2 + z^2}}$$

$$= \frac{1}{4\varepsilon_0\pi}\frac{2\pi\sigma w dw}{\sqrt{w^2 + z^2}}$$



To sum the contributions from all the rings on the disk, we integrate as w ranges from 0 to R:

$$V = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{w\, dw}{\sqrt{w^2 + z^2}} = \frac{\sigma}{2\varepsilon_0}(\sqrt{R^2 + z^2} - |z|)$$

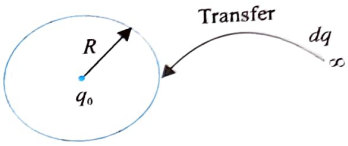
As $R \to \infty$ (i.e., for an infinite sheet of charge), $V \to \infty$. Hence, we cannot define absolute potential of an infinite sheet of charge, but we can define potential difference between two points near the infinite sheet of charge.

ENERGY FOR A CONTINUOUS DISTRIBUTION OF CHARGE

The energy required to assemble differential charges dq by bringing them from infinity to assemble a charged ball is the intrinsic energy of the charged balls.

SELF-ENERGY OF A CHARGED SPHERICAL SHELL SPHERE

Consider a spherical shell having charge Q and radius R. Let the instantaneous charge on the shell be q. Work done by an external agent in slowly bringing a charge from infinity and assembling on the surface of the shell is



$$dW = V\, dq = \frac{q}{4\pi\varepsilon_0 R}dq$$

Net work done in charging the shell is

$$W = \int_0^Q \frac{q}{4\pi\varepsilon_0 R}dq = \frac{Q^2}{8\pi\varepsilon_0 R}$$

This work done is the electrical potential energy or self-energy of the charged sphere.

SELF-ENERGY OF A UNIFORMLY CHARGED SPHERE

Let us consider that the charge Q being brought from infinity to form the sphere of radius R.

Let at any time, charge q has already been brought to from a sphere of radius x and further a very small charge dq is brought from infinity and assembled on the surface of sphere, which increase its radius by dx. Electrical potential energy of this charge dq is given by

$dU = dq \times$ potential at the position of charge dq.

$$= dq \times V = dq\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{x}\right) \Rightarrow dU = \frac{1}{4\pi\varepsilon_0}\frac{q}{x}dq$$

If Q is the total charge to be brought to form the complete sphere of radius R, then the charge density ρ is given by

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$dU = \frac{1}{4\pi\varepsilon_0 x}\left(\frac{4}{3}\pi x^3\rho\right)(4\pi x^2 dx\rho) = \frac{4\pi\rho^2}{3\varepsilon_0}x^4 dx$$

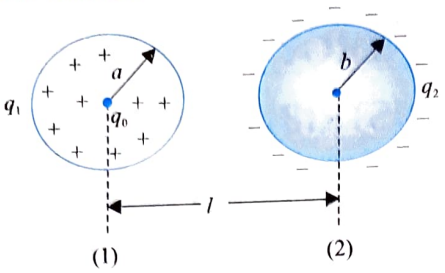
Total electrical potential energy is

$$U = \int dU = \int_0^R \frac{4\pi\rho^2}{3\varepsilon_0}x^4 dx = \frac{3}{5}\left(\frac{q^2}{4\pi\varepsilon_0 R}\right) = \frac{3q^2}{20\pi\varepsilon_0 R}$$

Note: Self or intrinsic energy is always positive. The interaction energy may be positive or negative.

Self and Interaction Energy of Two Spheres

Two spheres 1 and 2 are placed at a sufficient distance apart. Sphere 1 is uniformly charged, and sphere 2 is a charged conducting shell.



Total energy of the system is the sum of the self-energies of the spheres and interaction energy. Thus, total energy of system is

$$U = U_1 + U_2 + U_{12}$$

where U_1 is the intrinsic energy of ball 1, U_2 is the intrinsic energy of ball 2, U_{12} is the interaction energy of balls 1 and 2. For interaction energy, we can treat the two spheres as point charges in this case. Hence total energy is

$$U = \left(\frac{3q_1^2}{20\pi\varepsilon_0 a}\right) + \left(\frac{q_2^2}{8\pi\varepsilon_0 b}\right) + \frac{1}{4\pi\varepsilon_0}\frac{(-q_1 q_2)}{l}$$

Illustration 3.46

A spherical shell of radius R_1 with uniform charge q is expanded to a radius R_2. Find the work performed by the electric forces in this process.

Sol. Initial self-energy

$$U_i = \frac{q^2}{8\pi\varepsilon_0 R_1}$$

Final self-energy

$$U_f = \frac{q^2}{8\pi\varepsilon_0 R_2}$$

$$\Delta U = U_f - U_i = \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

Work done by electrical forces is

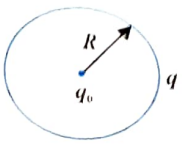
$$W = -\Delta U = \frac{q^2}{8\pi\varepsilon_0}\left[\frac{1}{R_2} - \frac{1}{R_1}\right]$$

Illustration 3.47

A spherical shell of radius R_1 with a uniform charge q has a point charge q_0 at its center. Find the work performed by the electric forces during the shell expansion from radius R_1 to radius R_2.

Sol. The electrical potential energy of the system is

U = self potential energy of shell + interaction energy of shell and point charge



$$= \frac{q^2}{8\pi\varepsilon_0 R} + \frac{qq_0}{4\pi\varepsilon_0 R}$$

Initial potential energy of system is

$$U_i = \frac{q^2}{8\pi\varepsilon_0 R_1} + \frac{qq_0}{4\pi\varepsilon_0 R_1}$$

Final potential energy of system is

$$U_f = \frac{q^2}{8\pi\varepsilon_0 R_2} + \frac{qq_0}{4\pi\varepsilon_0 R_2}$$

From work energy theorem,

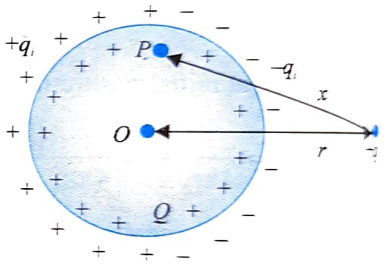
$$W = -\Delta U = -(U_f - U_i) = U_i - U_f \quad \ldots(i)$$

On substituting the values of U_i and U_f in Eq. (i), we get

$$W = \frac{q\left(q_0 + \frac{q}{2}\right)}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Electric Field and Potential Due to Induced Charges

A point charge $+q$ is placed at a distance r from a metal sphere of radius R and having charge Q. There is a point P in the sphere at a distance x from $+q$. Find the electric field and potential at point P due to the induced charges on the surface of sphere.



Material of a conductor is an equipotential region. Potential at the center of sphere is only due to the charge Q, $+q$, and induced charges. The potential at the center due to induced charges will be zero, as the net magnitude of induced charges is zero; also all the induced charges are equidistant from the center. Thus net potential at the center of sphere can be given by

$$V_0 = V_Q + V_q + V_{\text{induced}} = \frac{KQ}{R} + \frac{Kq}{r}$$

Sphere being equipotential at point P, the potential must be equal to that at point O. Note that at P, potential due to induced charges will be nonzero as all induced charges are not symmetrically from point P. Thus, net potential at point P can be gives by:

$$V_P = V_Q + V_q + V_{\text{induced}} = \frac{KQ}{R} + \frac{Kq}{x} + V_{\text{induced}}$$

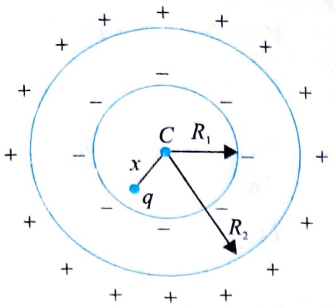
As the volume of solid conductor is equipotential volume.

$$V_o = V_P \Rightarrow \frac{KQ}{R} + \frac{Kq}{r} = \frac{KQ}{R} + \frac{Kq}{x} + V_{\text{induced}}$$

$$V_{\text{induced charge}} = \frac{Kq}{r} - \frac{Kq}{x}$$

ILLUSTRATION 3.48

Inside a conducting hollow sphere of inner radius R_1 and outer radius R_2, a point charge q is placed at a distance x from the center as shown in figure. Find (a) electric potential at C (b) electric field and potential at a distance r from the center outside the shell.



Sol.

(a) The electric potential at center due to this system is due to q, induced charge $-q$ on inner surface, and induced charge $+q$ on outer surface. Thus,

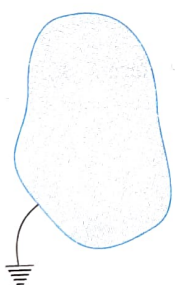
$$V_C = \frac{Kq}{x} - \frac{Kq}{R_1} + \frac{Kq}{R_2}$$

(b) Electric field and potential at a distance r from the center outside the shell will only be due to the charge on outer surface because outside cavity the field due all the cavity charges is always zero. As induced charge on inner surface of cavity always nullifies the effect of point charge inside it.

$$E_{out} = \frac{Kq}{r^2} \text{ and } V_{out} = \frac{Kq}{r}$$

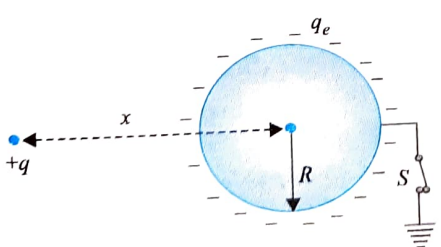
EARTHING OF A CONDUCTOR

Earthing means connecting a conductor with earth. Earth is an infinite resource and a sink of charges, so the potential of earth does not change and it is assumed to be zero. After earthing, the charges on conductors vary, and so the potential of the conductor becomes zero. Consider a solid uncharged conducting sphere shown in figure. A point charge q is placed at a distance x from the center of the sphere. Here due to q, the potential on the sphere is



$$V = \frac{Kq}{x}$$

The charge is induced on the sphere due to the point charge q, but the potential at the center due to the induced charges on the sphere is zero. If we close the switch S, earth supplies a charge q_e to the sphere to make the net potential zero. Thus, the final potential on the sphere can be taken as



$$V = \frac{Kq}{x} + \frac{Kq_e}{R} = 0 \quad \text{or} \quad q_e = -\frac{qR}{x}$$

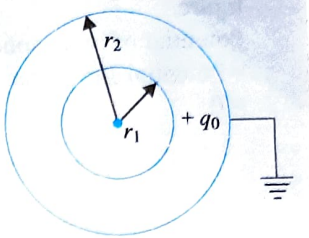
Earth supplied a negative charge to nullify positive potential on it due to q.

ILLUSTRATION 3.49

In figure, if the inner shell has charge $+q_0$ and the outer shell is earthed, then

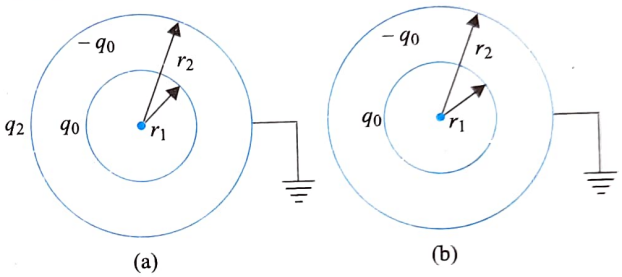
(a) determine the charge on the outer shell, and

(b) find the potential of the inner shell.



Sol.

(a) We know that charges on facing surfaces are equal and opposite. If the charge on the inner sphere is q_0, then the charge on the inner surface of the shell should be $-q_0$. Let the charge on the outer surface of the shell be q_2. As the shell is earthed, so its potential should be zero. Thus,



$$V_{shell} = \frac{k\,q_0}{r_2} + \frac{k\,(-q_0)}{r_2} + \frac{k\,q_2}{r_2} = 0 \text{ or } q_2 = 0$$

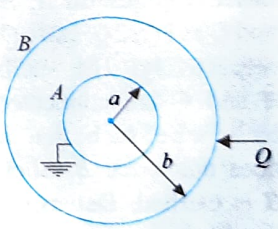
Hence, the charge on the outer surface of the shell is zero. Final charges appearing are shown in Fig. (b).

(b) Potential of inner sphere is

$$V_i = \frac{k\,q_0}{r_1} + \frac{k\,(-q_0)}{r_2} = \frac{q_0}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

ILLUSTRATION 3.50

Consider two concentric spherical metal shells of radii a and b, where $b > a$. The outer shell has charge Q, but the inner shell has no charge. Now, the inner shell is grounded. This means that the inner shell will come at zero potential and that electric field lines leave the outer shell and go to infinity, but other electric field lines leave the outer shell and end on the inner shell.

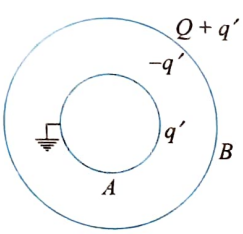


(a) Find the charge on the inner shell.

(b) Find the potential of the outer sphere.

Sol.

(a) When an object is connected to earth (grounded), its potential is reduced to zero. Let q' be the charge on A after it is earthed as shown in figure.



The charge q' on A induces $-q'$ on the inner surface of B and $+q'$ on the outer surface of B. In equilibrium, the charge distribution is as shown in figure.

Potential of inner sphere
= Potential due to charge on A + Potential due to charge on B
= 0

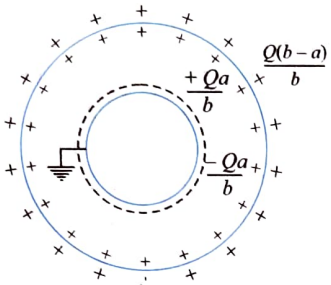
$$V_A = \frac{q'}{4\pi\varepsilon_0 a} - \frac{q'}{4\pi\varepsilon_0 b} + \frac{Q+q'}{4\pi\varepsilon_0 b} = 0$$

or $q' = -Q\left(\frac{a}{b}\right)$

This implies that a charge $+Q(a/b)$ has been transferred to the earth leaving negative charge on A.

Final charge distribution will be as shown in figure.

As $b > a$, so charge on the outer surface of the outer shell will be positive



$$\frac{Q(b-a)}{b} > 0$$

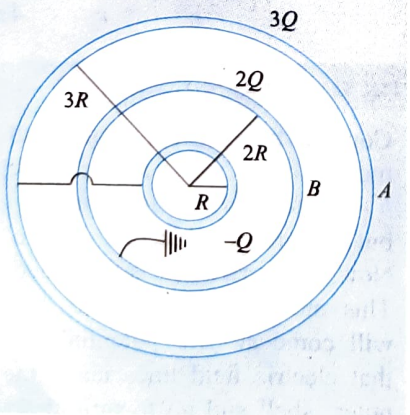
(b) Potential of outer surface V_B
= Potential due to charge on A
+ Potential due to charge on B

$$V_B = V_{a,\text{ out}} + V_{b,\text{ both surface}} = \frac{1}{4\pi\varepsilon_0}\frac{q'}{b} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\left(-Q\frac{a}{b}\right)}{b} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{b} = \frac{Q(b-a)}{4\pi\varepsilon_0 b^2}$$

ILLUSTRATION 3.51

Figure shows three thin concentric spherical shells A, B, and C with initial charges on A, B, and C as $3Q$, $2Q$, and $-Q$, respectively. The shells A and C are connected by a wire such that it does not touch B. Shell B is earthed. Determine the final charges q_A, q_B, and q_C



Sol. The first equation holds for conservation of charge on A and C.

$$q_A + q_C = 3Q - Q = 2Q \quad \text{...(i)}$$

The second equation holds for zero potential of earthed surface

$$(V_B)_{\text{surface}} + (V_C)_{\text{out}} + (V_A)_{\text{in}} = 0$$

or $$\frac{Kq_B}{2R} + \frac{Kq_C}{2R} + \frac{Kq_A}{3R} = 0$$

or $$\frac{q_B}{2} + \frac{q_C}{2} + \frac{q_A}{3} = 0 \quad \text{...(ii)}$$

Also the third equation holds for potential of A and C being equal $V_A = V_C$.

$$V_A = (V_A)_{\text{surface}} + (V_B)_{\text{out}} + (V_C)_{\text{out}}$$

$$V_C = (V_A)_{\text{in}} + (V_B)_{\text{in}} + (V_C)_{\text{surface}}$$

$$\therefore \quad \frac{Kq_A}{3R} + \frac{Kq_B}{3R} + \frac{Kq_C}{3R} = \frac{Kq_C}{R} + \frac{Kq_B}{2R} + \frac{Kq_A}{3R}$$

or $$\frac{q_A}{3} + \frac{q_B}{3} + \frac{q_C}{3} = q_C + \frac{q_B}{2} + \frac{q_A}{3} \quad \text{...(iii)}$$

Now on solving for q_A, q_B, and q_C, we get

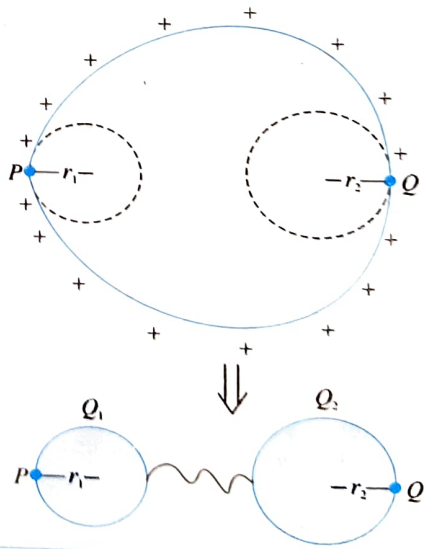
$$q_C = \frac{Q}{2};\ q_A = \frac{3Q}{2},\ \text{and}\ q_B = \frac{-3Q}{2}$$

CHARGE DISTRIBUTION ON A CONDUCTOR SURFACE (UNIQUENESS THEOREM)

If you inject a charge $+q$ (by taking out electrons) into a conductor, the excess (injected) charges will be very quickly redistributed at the surface of the conductor to ensure a zero electric field inside the conductor. Since $E = 0$ inside the conductor, V = constant. The same potential is felt at any point of the conductor.

Let us take two points P and Q at the surface of the conductor having radii of curvature r_1 and r_2. Cut two spheres of radii r_1 and r_2 passing through P and Q and connect them by a long conducting wire. To make this (two spheres) equivalent to the given system (conductor), we need to keep the spheres far apart to minimize their induction effects. Since the potentials at P and Q are same, $V_P = V_Q$

where $$V_P = \frac{Q_1}{4\pi\varepsilon_0 r_1} = \frac{\sigma_1 4\pi r_1^2}{4\pi\varepsilon_0 r_1} = \frac{\sigma_1 r_1}{\varepsilon_0}$$



The given charged conductor is made equivalent to the system of two spheres removed from the conductor connected with a long thin conducting wire.

and $$V_Q = \frac{Q_2}{4\pi\varepsilon_0 r_2} = \frac{\sigma_2 4\pi r_2^2}{4\pi\varepsilon_0 r_2} = \frac{\sigma_2 r_2}{\varepsilon_0}$$

Then, $\sigma_1 r_1 = \sigma_2 r_2$

The charge distribution takes place in a unique way at the surface of the conductor such that the product of surface charge density σ and radius of curvature r at any point of the conductor will be a constant; i.e., $\sigma r = c$.

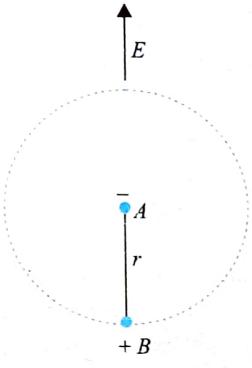
CIRCULAR MOTION OF A CHARGED PARTICLE IN ELECTRIC FIELD

When a charged particle performs circular motion in an electric field, an additional force due to the electric field acts on the charged particle. Also we have to consider the involvement of electrostatic potential energy due to electric field.

We will learn this concept through some illustrations.

ILLUSTRATION 3.52

Two small particles A and B having masses $m = 0.5$ kg each and charges $q_1 = (-155/18\ \mu C)$ and $q_2 = (+100\ \mu C)$, respectively, are connected at the ends of a nonconducting, flexible, and inextensible string of length $r = 0.5$ m.



Particle A is fixed and B is whirled along a vertical circle with center at A. If a vertically upward electric field of strength $E = 1.1 \times 10^5$ NC^{-1} exists in the space, calculate the minimum velocity of particle B required at the highest point so that it may just complete the circle ($g = 10$ ms^{-2}).

Sol. Let us analyse the forces acting on charge B.

(i) Force of attraction between charges A and B

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$= 9\times10^9 \times \frac{\left(\frac{155}{18}\times10^{-6}\right)(100\times10^{-6})}{(0.5)^2} = 31\ \text{N}$$

(acts in radial direction, always directed toward the center)

(ii) Force due to electric field

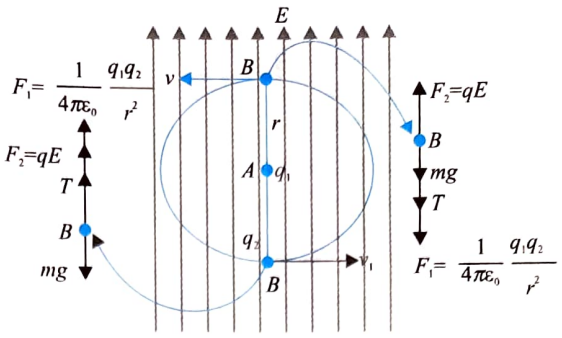
$$F_2 = q_2 E$$
$$= (100\times10^{-6})\times(1.1\times10^5)$$
$$= 11\ \text{N (directed vertically upward)}$$

(iii) Weight $W = mg = 0.5 \times 10 = 5$ N (vertically downward).

(iv) Tension T in the string, acting downward.

Since F_1 is always directed toward the center, the critical position depends on F_2 and W. Their resultant is 6 N (vertically upward). At the critical position, this resultant must be directed toward the center. Hence, the tension in the thread is minimum when particle B is at the lowest position. Considering free body diagram at this position, we have

$$\frac{mv_0^2}{r} = F_1 + F_2 + T - W$$



But for $T = 0$, we get $v_0 = \sqrt{37}$ ms^{-1}. When the particle moves from the lowest to the highest position, work is done on it by force F_2; however, gravitational potential energy increases and no work is done by F_1. Let minimum velocity required at the highest point be v. Using work energy theorem between the lowest and highest position on circle,

$$W_{\text{total}} = \Delta K$$

or $W_{\text{electric}} + W_{\text{gravity}} = \Delta K$ or $qE(2r) - mg(2r) = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

which gives $v = \sqrt{16}$ ms^{-1}

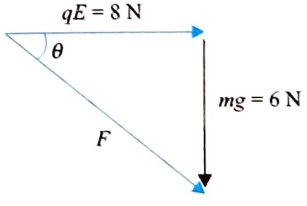
ILLUSTRATION 3.53

A small sphere of mass $m = 0.6$ kg carrying a positive charge $q = 80\ \mu C$ is connected with a light, flexible, and inextensible string of length $r = 30$ cm and whirled in a vertical circle. If a horizontal rightward electric field of strength $E = 10^5$ NC^{-1} exists in the space, calculate the minimum velocity of the sphere required at the highest point so that it may just complete the circle ($g = 10$ ms^{-2}).

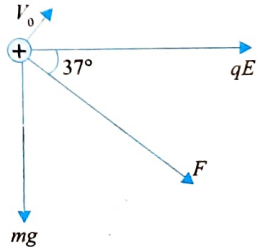
Sol. When a particle having no charge is whirled in a vertical circle, the only force (other than tension in thread) acting on the particle is its weight. It is vertically downward and tension is minimum at the highest point, which is vertically above the center of the circle.

In the previous illustration, there were two forces mg and qE (other than tension in thread) on the particle were responsible for analysis of critical condition. Their resultant was vertically upward. In that case, tension was minimum at the lowest point of vertical circle, which was vertically below the center of the circle. It means tension is minimum when the resultant force (other than tension) acting on the particle is toward center.

In the present illustration, weight is $mg = 0.6 \times 10 = 6$ N (downward) and $qE = (80 \times 10^{-6})(10^5) = 80$ N (horizontally rightward).



Resultant force F of these two forces is at $\theta = [= \tan^{-1}(6/8) = 37°]$, with the horizontal as shown in figure. Hence, tension is minimum at A, as shown in figure.

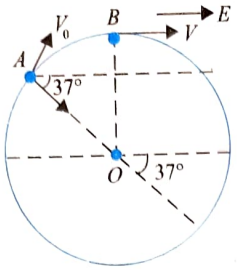


Let critical velocity at A be v_0. Considering free body diagram of sphere at A,

$qE \cos 37° + mg \sin 37° = \frac{mv_0^2}{r}$

or $v_0 = \sqrt{5}$ ms^{-1}

As the sphere moves from A to B, work qE ($r \cos 37°$) is done on the sphere by the electric field and the gravitational potential energy increases by mg ($r - r \sin 37°$). We can find the required minimum velocity v, at the highest point B, by using work energy theorem between points A and B.

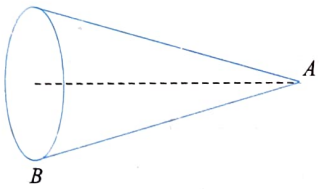


$$W_{total} = \Delta K \Rightarrow qE\, r \cos 37° - mgr(1 - \sin 37°) = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

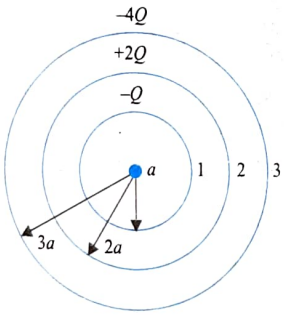
which gives $v = 3$ ms^{-1}

CONCEPT APPLICATION EXERCISE 3.4

1. A cone made of insulating material has a total charge Q spread uniformly over its sloping surface. Calculate the energy required to take a test charge q from infinity to apex A of cone. The slant length is L.



2. Three concentric spherical conductors of radii a, $2a$, and $3a$ have charges $-Q$, $+2Q$, and $-4Q$, respectively. If r is the distance of the point under consideration from the center of the spheres, then find the electric field and potential due to the given configuration, for the values



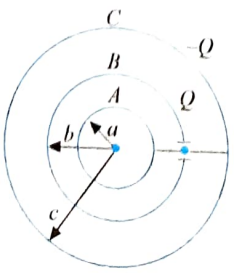
(a) $r < a$ **(b)** $a < r < 2a$

(c) $2a < r < 3a$ **(d)** $r > 3a$

3. Water from a tap, maintained at a constant potential V, is allowed to fall by drops of radius r through a small hole into a hollow conducting sphere of radius R standing on an insulating stand until it fills the entire sphere. Find the potential of the hollow conductor after it is completely filled with water.

4. Two identical thin rings, each of radius R m, are coaxially placed at a distance R m from each other. If Q_1 coulomb and Q_2 coulomb are the charges uniformly spread on the two rings, find the work done in moving a charge q from the center of one ring to that of the other.

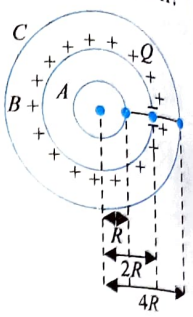
5. Three conducting spherical shells have radii a, b, and c such that $a < b < c$ (figure). Initially, the inner shell is uncharged, the middle shell has a positive charge Q, and the outer shell has a negative charge $-Q$.



(a) Find the electric potential of the three shells.

(b) If the inner and outer shells are now connected by a wire that is insulated as it passes through the middle shell, what is the electric potential of each of the three shells? Also, what is the final charge on each shell?

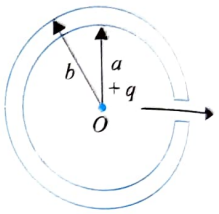
6. Figure shows three concentric spherical conductors A, B, and C with radii R, $2R$, and $4R$, respectively. A and C are connected by a conducting wire, and B is uniformly charged (charge = $+Q$). Find



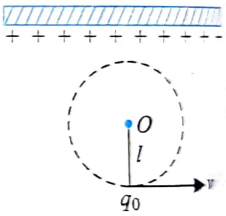
(a) charges on conductors A and C, and

(b) potentials of A and B.

7. A point charge q is located at the center O of an uncharged spherical capacitor provided with a small orifice. The inside and outside radii of the capacitor are a and b, respectively (figure). What amount of work has to be performed to slowly transfer the charge q bit by bit from the point O through the orifice to infinity?

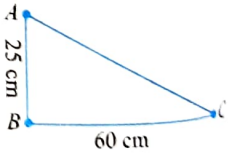


8. Figure shows a large conducting ceiling having uniform charge density σ below which a charge particle of charge q_0 and mass m is hung from point O, through a small string of length l. Calculate the minimum horizontal velocity v required for the string to become horizontal.



9. A small ball of mass 2×10^{-3} kg, having a charge 1 μC, is suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity that should be imparted to the lower ball so that it can make a complete revolution.

10. ABC is a right angled triangle, where AB and BC are 25 cm and 60 cm respectively. A metal sphere of 2 cm radius charged to a potential of 9×10^5 volt is placed at B.



Find the amount of work done in carrying a positive charge of 1 Coulomb from C to A.

ANSWERS

1. $\dfrac{qQ}{2\pi\varepsilon_0 L}$ **2.** (a) $\dfrac{Q}{3\pi\varepsilon_0 a}$ (b) $-\dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{1}{r} + \dfrac{1}{3a}\right]$

(c) $\dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{r} - \dfrac{4}{3a}\right]$ (d) $-\dfrac{3}{4\pi\varepsilon_0}\dfrac{Q}{r}$

3. $\left(\dfrac{R}{r}\right)^2 V$ **4.** $\dfrac{q}{4\pi\varepsilon_0}\dfrac{(Q_1 - Q_2)}{\sqrt{2}R}\left(\sqrt{2} - 1\right)$

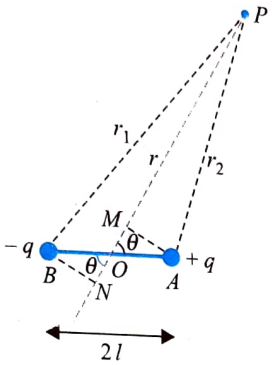
5. (a) 0 (b) $kQ\left[\frac{(b-a)-(c-b)}{b^2(c-a)}\right]$

6. (a) $q_2 = Q/3$ (b) $\frac{5Q}{48\pi\varepsilon_0 R}$ 7. $\frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$

8. $\frac{\sqrt{2Mgl+2q_0El}}{m}$ 9. 5.86 ms^{-1} 10. $\frac{21}{5}\times10^4$ Joule

POTENTIAL DUE TO AN ELECTRIC DIPOLE

Suppose an electric dipole consists of two charges $-q$ and $+q$ at B and A, respectively, separated by a distance $2l$ (as shown in figure). We have to calculate the electric potential at any point P where $OP = r$ and $\angle AOP = \theta$. Let $BP = r_1$ and $AP = r_2$.



Draw AM perpendicular to PO and BN perpendicular to PO produced backward, then $OM = ON$. In ΔOMA,

$$\frac{OM}{OA} = \cos\theta$$

$$\therefore \quad OM = OA\cos\theta = l\cos\theta = ON$$

Hence, $r_1 = BP \simeq NP = OP + ON = r + l\cos\theta$ ...(i)

and $r_1 = AP \simeq MP = OP - OM = r - l\cos\theta$ ...(ii)

Therefore, potential at point P due to both the charges $-q$ (at B) and $+q$ (at A) can be written as

$$V = \frac{-q}{4\pi\varepsilon_0}\frac{1}{r_1} + \frac{q}{4\pi\varepsilon_0}\frac{1}{r_2} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2}-\frac{1}{r_1}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-l\cos\theta} - \frac{1}{r+l\cos\theta}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r+l\cos\theta - r + l\cos\theta}{r^2 - l^2\cos^2\theta}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{2l\cos\theta}{r^2 - l^2\cos^2\theta}$$

i.e., $$= \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2 - l^2\cos^2\theta} \qquad \{\because \quad p = q(2l)\}$$

Special Cases

- If $l << r$, then electric dipole is very short and the potential is given by
$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$
- If P lies on the axial line, we have $\theta = 0°$, so
$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos 0°}{r^2 - l^2\cos^2 0°} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2 - l^2} \qquad (\because \cos 0° = 1)$$
- If $l << r$, then electric dipole is very short ; and
$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} \qquad \text{(for a point on axial line)}$$
- If P lies on the equatorial line, $\theta = 90°$, so that $V = 0$.

WORK DONE IN ROTATING AN ELECTRIC DIPOLE IN A UNIFORM ELECTRIC FIELD

Suppose an electric dipole of dipole moment $p(= q\,2l)$ is rotated in a uniform electric field E through an angle θ from its stable equilibrium position. The work done by an external agent in rotating the dipole further from this position through a very small angle $d\theta$ is

$$dW = \text{couple} \times \text{angular displacement} = (pE\sin\theta)\,d\theta$$

This work will be done on the dipole by external agent as it is being rotated against its natural tendency (which is to align itself along the direction of the electric field). Hence, the work done in rotating the dipole through the angle θ from its equilibrium position is

$$W = \int_0^\theta pE\sin\theta\,d\theta = pE[-\cos\theta]_0^\theta$$

$$= pE[-\cos\theta + \cos 0] = pE[1 - \cos\theta]$$

$$\Rightarrow \quad W_{\text{external}} = pE(1 - \cos\theta)$$

POTENTIAL ENERGY OF AN ELECTRIC DIPOLE IN A UNIFORM ELECTRIC FIELD

Let us assume that V_+ and V_- are the potentials at the points where $+q$ and $-q$ are placed. The potential energy possessed by the dipole is equal to the work done by an external agent to bring the charges from infinity to the given points.

$$U = W_{\text{ext}} = +qV_+ + (-q)(V_-)$$

$$= q(V_+ - V_-) = q\Delta V$$

$$\Delta V = \vec{E}\cdot\Delta\vec{l} = \vec{E}\cdot\vec{l}$$

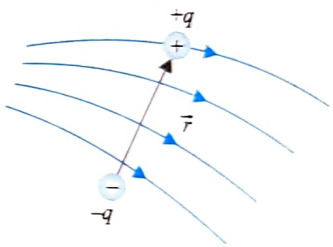
$$= q(-\vec{E}\cdot\vec{l}) = -q\vec{l}\cdot\vec{E}$$

where

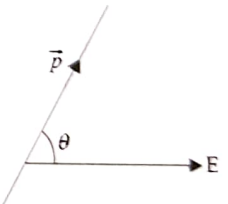
$$q\vec{l} = \vec{p}$$

$$U = -\vec{p}\cdot\vec{E} = -pE\cos\theta \quad ...(i)$$

where θ is the angle between $\vec{p}$ and $\vec{E}$.



A dipole $\vec{p}$ in an electric field possesses potential energy U.



θ is the angle between $\vec{p}$ and $\vec{E}$.

Note: When a dipole rotates from an initial orientation θ_i to another orientation θ_f, the work W done on the dipole by the electric field is

$$W = -\Delta U = -(U_f - U_i) = -pE(\cos\theta_f - \cos\theta_i)$$

where U_f and U_i are calculated with Eq. (i). If the change in orientation is caused by an applied torque (commonly said to be due to an external agent), then the work W_a done on the dipole by the applied torque is the negative of the work done on the dipole by the field, that is,

$$W_a = -W = (U_f - U_i) = -pE(\cos\theta_f - \cos\theta_i)$$

ILLUSTRATION 3.54

Two point charges of 3.2×10^{-19} C and -3.2×10^{-19} C are separated from each other by 2.4×10^{-10} m. The dipole is situated in a uniform electric field of intensity 4×10^5 Vm^{-1}. Calculate the work done in rotating the dipole by 180°.

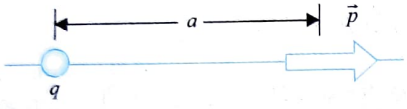
Sol. Work done in rotating the dipole by angle θ is

(here $\theta = 180°$)

$$W = pE(1-\cos\theta)$$
$$= pE[1-(-1)] = 2pE = 2qd\,E$$
$$= 2\times3.2\times10^{-19}\times2.4\times10^{-10}\times4\times10^{5}\,\text{J}$$
$$= 61.44\times10^{-24}\,\text{J}$$

ILLUSTRATION 3.55

What is the potential energy of the charge and dipole system shown in figure?



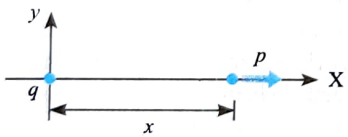
Sol. Electric field due to point charge at the centre of the dipole, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$

Hence potential energy of the system

$$U = -pE\cos\theta = -\frac{pq}{4\pi\varepsilon_0 a^2} \quad (\theta = 0°)$$

ILLUSTRATION 3.56

A short dipole is placed along the x-axis at $x = x$ as shown in figure.



(a) Find the force acting on the dipole due to a point charge q placed at the origin.

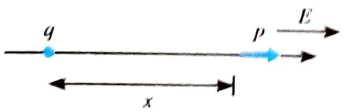
(b) Find the force on the dipole if the dipole is rotated by 180° about the z-axis.

Sol.

(a) Electric field due to point charge at the position of the dipole, $E_q = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$

The potential energy of the charge particle and dipole system, $U = -\vec{p}\times\vec{E}_q$

$$= -pE\cos0° = -\frac{qp}{4\pi\varepsilon_0 x^2}$$

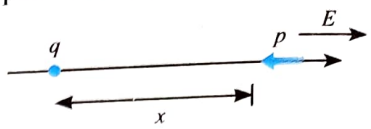


The force acting on the dipole due to a point charge,

$$F = -\frac{\partial U}{\partial x} = \frac{-pq}{2\pi\varepsilon_0 x^3}$$

Negative sign indicates that force on dipole is toward the positive x-direction or the force is attractive.

(b) Now the dipole is rotated by 180° about the z-axis.



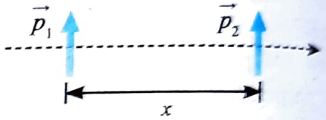
The potential energy of the charge particle and dipole system, $U = -\vec{p}\times\vec{E}_q = -pE\cos180° = \frac{qp}{4\pi\varepsilon_0 x^2}$

The force acting on the dipole $F = -\frac{\partial U}{\partial x} = \frac{pq}{2\pi\varepsilon_0 x^3}$

Positive sign indicates that force on dipole is toward the positive x-direction or the force is repulsive.

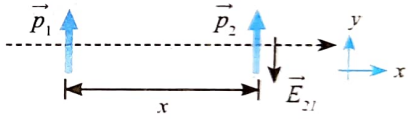
ILLUSTRATION 3.57

Figure shows two dipole moments parallel to each other and placed at a distance x apart. What is the magnitude of force of interaction? What is the nature of force, attractive or repulsive?



Sol. Potential energy of dipole system, $U = -\vec{p}_2\times\vec{E}_{21}$

E_{21} is the field due to p_1 at p_2, $E_{21} = \frac{1}{4\pi\varepsilon_0}\frac{p_1}{x^3}$



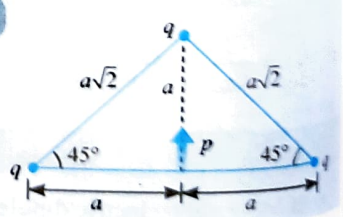
Hence, $U = -p_2\frac{1}{4\pi\varepsilon_0}\frac{p_1}{x^3}\cos\pi = \frac{1}{4\pi\varepsilon_0}\frac{p_1p_2}{x^3}$

The force acting on the dipole, $F = -\frac{\partial U}{\partial x} = \frac{3}{4\pi\varepsilon_0}\frac{p_1p_2}{x^4}$

F comes out to be positive, so it is a repulsive force.

ILLUSTRATION 3.58

For the electrostatic charge system as shown in figure, find the electrostatic energy of the system.



Sol. The total electric potential energy consists of interaction of all the three charges among themselves and interaction of these three charges with dipole. So,

PE of the system is

U = (3 pairs of charged particles) + (3 pairs of dipole and charged particles) = $U_1 + U_2$

The potential energy of interaction of all the three charges,

$$U_1 = 2\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\sqrt{2}a}\right) + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a}$$

The potential energy of interaction of the charges and dipole,

$$U_2 = -\vec{p}.\vec{E}_{up} - \vec{p}.\vec{E}_{left} - \vec{p}.\vec{E}_{right}$$

Here $\vec{E}_{up}$ = Electric field at the position of the dipole due to the charge particle placed above the dipole

$\vec{E}_{left}$ = Electric field at the position of the dipole due to the charge particle placed left of the dipole

and $\vec{E}_{right}$ = Electric field at the position of the dipole due to the charge particle placed right of the dipole

As $\vec{p}\cdot\vec{E}_{left} = \vec{p}\cdot\vec{E}_{right} = 0$

(Because electric fields produced by left and right charges are perpendicular to $\vec{p}$.)

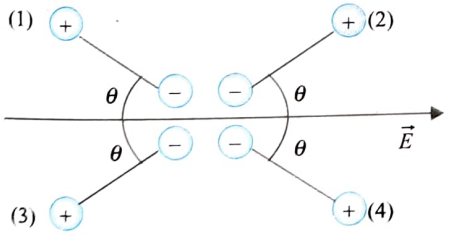
Hence $U_2 = -\vec{p}\cdot\vec{E}_{up} = -p\cdot\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\right)\cos\pi = \frac{1}{4\pi\varepsilon_0}\frac{pq}{a^2}$

Hence potential energy of the system,

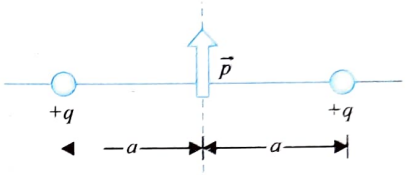
$$U = 2\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\sqrt{2}a}\right) + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a} + \frac{1}{4\pi\varepsilon_0}\frac{pq}{a^2}$$

CONCEPT APPLICATION EXERCISE 3.5

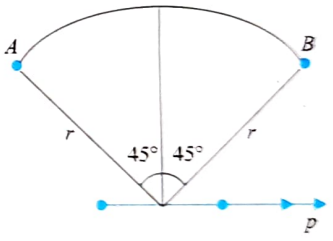
1. State the following statements as True or False.
   (a) Electric potential at any point on the bisector of a dipole is zero.
   (b) A dipole experiences maximum torque at the position where potential energy is zero.
2. Figure shows four orientations of an electric dipole in an external electric field. Rank the orientations according to the
   (a) magnitude of the torque on the dipole, and
   (b) potential energy of the dipole, greatest first.



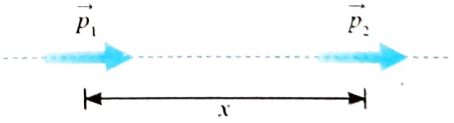
3. (a) In question 2, if the dipole rotates from orientation 1 to orientation 2, is the work done on the dipole by the field positive, negative, or zero?
   (b) If, instead, the dipole rotates from the orientation 1 to orientation 4, is the work done by the field more than, less than, or the same as in (a)?
4. A neutral water molecule (H_2O) in its vapour state has an electric dipole moment of magnitude 6.2×10^{-30} Cm.
   (a) If a molecule is placed in an electric field of 1.5×10^4 NC^{-1}, what maximum torque can the field exert on it? (Such a field can easily be set up in the laboratory.)
   (b) How much work must an external agent do to turn this molecule end for end in this field, starting from its fully aligned position, for which $\theta = 0$?
5. An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $4\sqrt{3}$ Nm. Calculate the
   (a) magnitude of the electric field, and
   (b) potential energy of the dipole, if the dipole has charges of ±8 nC.
6. An electric dipole consists of two opposite charges each of magnitude 1 mC separated by 2 cm. The dipole is placed in an external uniform field of 10^5 NC^{-1} intensity. Find the
   (a) maximum torque exerted by the field on the dipole, and
   (b) work done in rotating the dipole through 180° starting from the position $\theta = 0°$.
7. What is the potential energy of dipole with charge particles system as shown in figure?



8. The potential energies associated with four orientations of an electric dipole in an electric field are (1) $-5U_0$, (2) $-7U_0$, (3) $3U_0$, and (4) $5U_0$, where U_0 is positive. Rank the orientations according to the
   (a) angle between the electric dipole moment $\vec{P}$ and the electric field $\vec{E}$, and
   (b) magnitude of the torque on the electric dipole, greatest first.
9. A charge $+q$ is carried from point A $(r, 135°)$ to point B $(r, 45°)$ following a path, which is a quadrant of circle of radius r (see figure). If the dipole moment is P, find the work done by the external agent (assume short dipole).



10. Two dipoles p_1 and p_2 are placed along the same axis at a distance x apart as shown in figure. What is the magnitude of the force of interaction? What is the nature of force, attractive or repulsive?



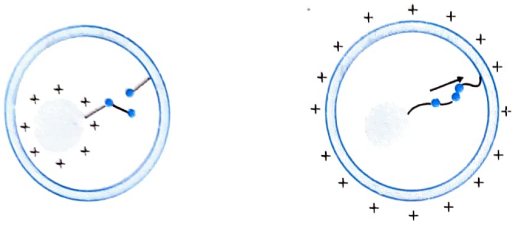
ANSWERS

1. (a) True (b) True
2. (a) Torque is same for all (b) (1, 3) and (2, 4)
3. (a) Electrical fields do positive work (b) Same as (a)
4. (a) 9.3×10^{-26} N-m (b) 1.86×10^{-25} J
5. (a) 2.5×10^{10} NC^{-1} (b) −4 J
6. (a) 0.002 Nm (b) 0.004 J
7. Zero 8. (a) 4, 3, 1 and 2 (b) 3 > 1 = 4 > 2
9. $\frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}pq}{r^2}$ 10. $\frac{-3}{2\pi\varepsilon_0}\frac{p_1p_2}{x^4}$ (attractive)

VAN DE GRAAFF GENERATOR

A Van de Graaff generator creates static electricity. The current generated by a Van De Graaff generator remains the same, while the voltage changes according to the applied load.

We provide charge to a metal body it will spread on the outer surface of it. We can transfer complete charge of a metal body to another hollow metal body, if we put the charged body inside the hollow metal body and the two are connected by a wire, whole of the charge of inner body will flow to the outer surface of the hollow body as shown in figure.



As we know, that whole of the charge given to a metal body spreads on its outer surface, no matter how large charge is. This concept can be used to develop very high charges in a Van de Graaff generator.

The Van de Graaff generator is a device that produces intense electric field (building up high voltage of a few million volt).

Consider a small conductor carrying a positive charge q kept inside the cavity of a large conductor. The electric field lines that leave the positive charge q must end on the inner surface of the large conductor irrespective of charge on the outer surface. The potential on the inner conductor must be higher because electric field lines begin from it and end on the larger conductor.

If the two conductors are now connected by a conducting wire, all the charged originally on the smaller conductor will flow to the larger one. The positive charge transferred to the larger sphere resides completely on the outside surface of the larger conductor. This process can be repeated indefinitely to produce large potential on the outer conductor.

Consider a shell of radius R and charge Q enclosing a smaller sphere of radius and charge q. The potentials of the two spheres are

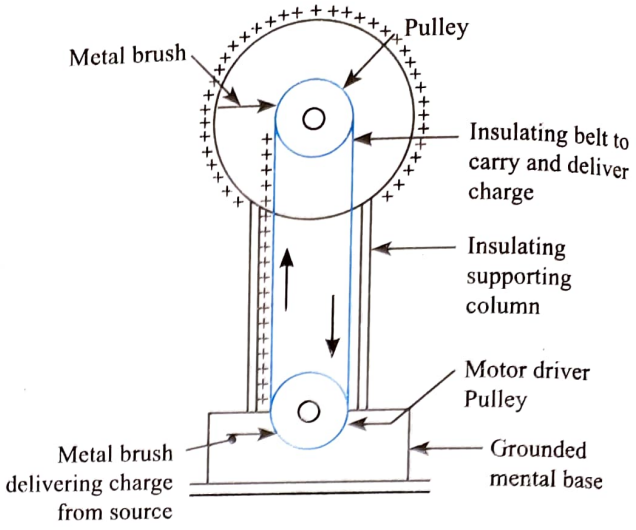
$$V(R) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right)$$

and $$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right)$$

The potential difference between the two inner and outer sphere is

$$V(r) - V(R) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$$

Thus for positive q, whatever be the magnitude and sign of Q, the small sphere is at a higher potential than the shell. When an electric contact is provided the charge would flow from the small sphere to the shell.



The construction of a simple Van De Graaff generator is given in figure. A motor is required to turn the belt at a constant speed around the two rollers. The lower roller is built of a material that has a stronger triboelectric property. Now when the motor starts turning the belt around the lower roller, electrons are captured from the insulated belt onto the lower roller. Slowly more and more charge becomes concentrated on the roller. This phenomenon of concentration of charge results in repelling the electrons from the tips of the brush assembly. It also starts to attract electrons from the air molecules between the lower roller and brush assembly. Due to this phenomenon, the positively charged air molecules get carried on the belt away from the negatively charged roller. The belt therefore gets charged positively and moves towards the upper rollers.

The upper roller is made from or coated with a material that is higher up in the triboelectric series such as nylon due to which it tries to repel the positive charge on the belt. The upper brush is directly connected to the inside of the output terminal or sphere at one end and almost touches the upper roller and belt at the other. The electrons in the brush become attracted to the positive charges on the belt. The air particles break down too and the free electrons move towards the belt. The sphere takes up all of the charge and the excess charge gets spread to the outside of the terminal output or sphere. It is this simple electrostatic effect that allows the Van De Graaff generator to output very high voltages continuously.

Solved Examples

EXAMPLE 3.1

A small sphere is charged uniformly and placed at point $A(u, v)$ so that at point B (8 m, 7 m) electric field strength is $\vec{E} = (54\hat{i} + 72\hat{j})$ NC and potential is + 900 volt. Calculate:

(a) magnitude of charge.

(b) co-ordinates of point A, and

(c) if di-electric strength of air 3×10^6 Vm^{-1}, minimum possible radius of the sphere.

Sol.

(a) Since, potential due to sphere is positive, therefore, it is positively charged. Let magnitude of charge on sphere be q and let distance AB be equal to r.

Potential at B, $V = 9\times10^9 \frac{q}{r} = 900$ volt ...(i)

Magnitude of Electric field at B is

$$E = 9\times10^9 \frac{q}{r^2} = \sqrt{54^2 + 72^2}\ \text{NC}^{-1}$$

$$9\times10^9 \frac{q}{r^2} = 90\ \text{NC}^{-1} \quad \ldots(ii)$$

Dividing Eq. (i) by (ii), $r = 10$ m

Subtracting this value in Eq. (i), we get $q = 10^{-6}$ coulomb or $1\mu C$

(b) Since q is a positive charge, therefore $\overrightarrow{AB} \perp \vec{E}$ or $\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{\vec{E}}{|\vec{E}|}$

$$\frac{(8-u)\hat{i} + (7-v)\hat{j}}{r} = \frac{54\hat{i} + 72\hat{j}}{90}$$

$$(8-u)\hat{i} + (7-v)\hat{j} = 6\hat{i} + 8\hat{j}$$

$u = 2$ m and $v = -1$ m

or coordinates of point A are (2 m, –1 m)

(c) Since, minimum radius of sphere corresponds to electric field strength at surface of sphere to be equal to dielectric strength of air.

Therefore radius R of sphere is given by,

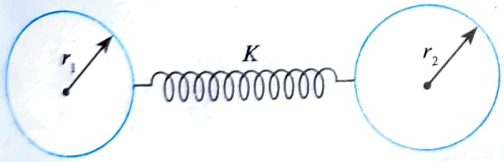
$$9\times10^9 \frac{q}{R^2} = 3\times10^6\ \text{Vm}^{-1}$$

$$\Rightarrow R = \sqrt{3\times10^{-3}}\ \text{m} = \sqrt{30}\ \text{cm} = 5.48\,\text{cm}$$

EXAMPLE 3.2

Two conducting spheres of radii r_1 and r_2 are connected by a metallic spring of stiffness k and natural length l ($>> r_1$ and r_2). A positive charge $+Q$ is slowly delivered to any sphere.

(a) Find the charge on each sphere.



(b) What is the value of Q, if the equilibrium separation between the sphere is $2l$?

Sol. Both the sphere will have same potential. $V_1 = V_2$

$$K\frac{Q-x}{r_1} = K\frac{x}{r_2} \Rightarrow x = \frac{Q\cdot r_2}{(r_1+r_2)}$$

$$(Q-x) = Q - \frac{Q\cdot r_2}{r_1+r_2} = \frac{Q r_1}{r_1+r_2}$$

Free body diagram of sphere 1 at equilibrium

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{(2l)^2} = Kl \Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{Q\cdot r_1}{r_1+r_2}\right)\cdot\frac{Q\cdot r_2}{(r_1+r_2)}}{4l^2} = Kl$$

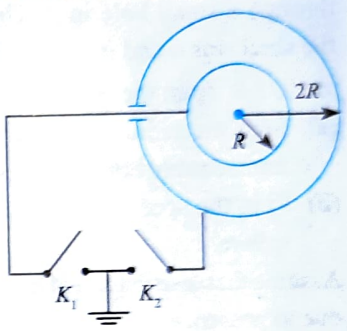
$$\frac{Q^2}{4\pi\varepsilon_0\cdot 4l^2}\left[\frac{r_1 r_2}{(r_1+r_2)^2}\right] = Kl$$

$$Q^2 = \frac{16K\pi\varepsilon_0 l^3\cdot(r_1+r_2)^2}{r_1 r_2}$$

$$\Rightarrow Q = 4(r_1+r_2)l\sqrt{\frac{l\pi\varepsilon_0 K}{r_1 r_2}}$$

EXAMPLE 3.3

Two concentric shells of radii R and $2R$ are shown in figure. Initially a charge q is imparted to the inner shells. Now key K_1 is closed and opened and then key K_2 is closed and opened. After the keys K_1 and K_2 are alternately closed n times each, find the potential difference between the shells. Note that finally the key K_2 remains closed.



Sol. When K_1 is closed first time, outer sphere is earthed and the potential on it becomes zero. Let the charge on it be q_1'. Potential due to charge on the inner sphere and that due to charge on the outer sphere is

$$V_1' = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{2R} + \frac{q_1'}{2R}\right] = 0 \quad \text{or} \quad q_1' = -q$$

When K_2 is closed first time, the potential V_2' on the inner sphere becomes zero as it is earthed. Let the new charge on inner sphere be q_2'.

$$0 = \frac{1}{4\pi\varepsilon_0}\frac{q_2'}{R} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{(2R)} \quad \text{or} \quad q_2' = \frac{q}{2}$$

Now when K_1 will be closed second time, charge on the outer sphere will be $-q_2'$ i.e., $-q/2$. After one event involving closure and opening of K_1 and K_2, charge is reduced to half its initial value.

Similarly, when K_1 will be closed n^{th} time, charge on the outer sphere will be $-\frac{q}{2^{n-1}}$ as each time charge will be reduced to half the previous value.

After closing K_2 nth time, charge on inner shell will be negative of half the charge on the outer shell, i.e., $(-q/2^n)$ and potential on it will be zero.

For potential of the outer shell,

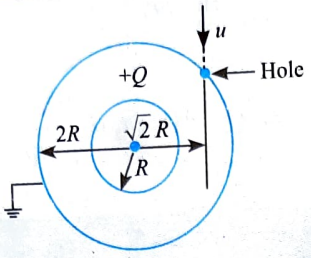
$$V_0 = \frac{1}{4\pi\varepsilon_0}\frac{(+q/2^n)}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{(-q/2^{n-1})}{2R}$$

$$= \frac{-q[-1+2]}{4\pi\varepsilon_0 2^{n+1} R} = \frac{-q}{4\pi\varepsilon_0 2^{n+1} R}$$

Potential difference, $V_0 - V_i = \frac{-q}{4\pi\varepsilon_0 2^{n+1} R} - 0 = \frac{-q}{4\pi\varepsilon_0 2^{n+1} R}$

EXAMPLE 3.4

Two concentric spherical shells have radii R and $2R$. The outer shell is grounded and the inner one is given a charge +Q small particle having mass m and charge $-q$ enters the outer shell through a small hole in it. The speed of the charge entering the shell was u and its initial line of motion was at a distance $a = \sqrt{2}R$ from the centre.



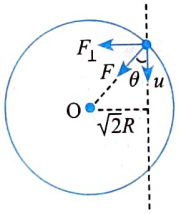
(a) Find the radius of curvature of the path of the particle immediately after it enters the shell.

(b) Find the speed with which the particle will hit the inner sphere.

Assume that distribution of charge on the spheres do not change due to presence of the charge particle.

Sol.

(a) Charge $-Q$ is induced on the inner surface of the outer shell. There is no charge on the outer surface of the outer shell as it is grounded.



An electric field exists in the space between the two shells in radial outwards direction.

$$E = K\frac{Q}{x^2} \quad \text{for} \quad R < x < 2R$$

Just after the charge enters, it experience a force due to this electric field directed towards the centre.

$$F = K\frac{Qq}{(2R)^2}$$

Component of this force perpendicular to the direction of instantaneous velocity u is

$$F_\perp = F\sin\theta = F\frac{\sqrt{2}R}{2R} = \frac{F}{\sqrt{2}} = K\frac{Qq}{4\sqrt{2}R^2}$$

If radius of curvature of the path is r then, $\frac{mu^2}{r} = F_\perp$

$$\therefore \quad \frac{mu^2}{r} = \frac{KqQ}{4\sqrt{2}R^2} \Rightarrow r = \frac{16\sqrt{2}\,\pi\varepsilon_0 R^2 mu^2}{Qq}$$

(b) The potential difference between the two spheres is

$$V_{inner} - V_{outer} = \left(K\frac{Q}{R} - \frac{KQ}{2R}\right) - \left(\frac{KQ}{2R} - \frac{KQ}{2R}\right) = \frac{KQ}{2R}$$

Applying conservation of energy for the charge entering the shell gives:

$$\Delta K + \Delta U = 0 \Rightarrow K_i + U_i = K_f + U_f$$

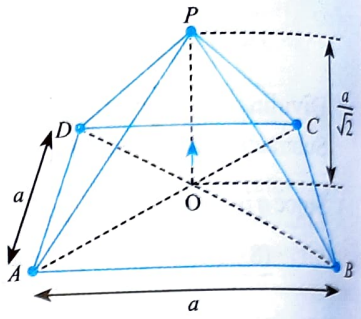
$$\frac{1}{2}mv^2 + (-q)V_{inner} = \frac{1}{2}mu^2 + (-q)V_{outer}$$

$$\therefore \quad \frac{1}{2}mv^2 = \frac{1}{2}mu^2 + q(V_{inner} - V_{outer})$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + \frac{Qq}{8\pi\varepsilon_0 R} \Rightarrow v = \sqrt{u^2 + \frac{Qq}{4\pi\varepsilon_0 mR}}$$

EXAMPLE 3.5

Four charge the particles each having charge Q are fixed at the corners of the base (at A, B, C, and D) of a square pyramid with slant length a ($AP = BP = DP = PC = a$). A charge $-Q$ is fixed at point P. A dipole with dipole moment P is placed at the center of base and perpendicular to its plane as shown in figure. Find:



(a) the force on dipole due to charge particles, and

(b) the potential energy of the system.

Sol.

(a) Charges at A, B, C, and D are placed at an equilateral position of dipole. Hence, the force on each of them due to dipole is

$$F_1 = \frac{Qp}{4\pi\varepsilon_0\left(a/\sqrt{2}\right)^3}$$

This force is downward on charges. Hence, force due to these charges on dipole is $4F_1$ (upward). Force on dipole due to charge at P is

$$F_2 = \frac{2pQ}{4\pi\varepsilon_0\left(a/\sqrt{2}\right)^3}(upward)$$

Net force on dipole is

$$U = 4\left[\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a}\right] + 2\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\sqrt{2}a} - 4\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a}$$

(b) PE of the system is

U = (10 pairs of charged particles) + (5 pairs of dipole and charged particles)

As potential energy of the dipole with four charges at A, B, C, and D will be zero,

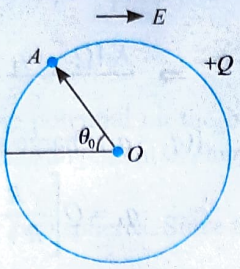
$$U = 4\left[\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a}\right] + 2\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\sqrt{2}a} - 4\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a}$$

$$-\frac{1}{4\pi\varepsilon_0}\frac{pQ}{\left(a/\sqrt{2}\right)^2}$$

$$U = \frac{Q^2}{2\sqrt{2}\,\pi\varepsilon_0 a} - \frac{pQ}{2\pi\varepsilon_0 a^2}$$

EXAMPLE 3.6

A conducting sphere of radius R having charge Q is placed in a uniform external field E. O is the centre of the sphere and A is a point on the surface of the sphere such that AO makes an angle of $\theta_0 = 60°$ with the opposite direction of external field. Calculate the potential at point A due to charge on the sphere only.



Sol. The sphere is conducting hence the potential at A should be equal to potential at O, $V_A = V_0$

The net potential at points A and O should be due to electric field (E) and due to charge (Q)

$$V_{AE} + V_{AQ} = V_{OE} + V_{OQ}$$

Here, V_{AE} and V_{AQ} are the electric fields at A due to electric field (E) and charge (Q) respectively. And V_{OE} and V_{OQ} are the electric fields at O due to electric field (E) and charge (Q) respectively

$$(V_{AE} - V_{OE}) + V_{AQ} = V_{OQ}$$

$$ER\cos\theta_0 + V_{AQ} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

$$\Rightarrow V_{AQ} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} - \frac{ER}{2}$$

EXAMPLE 3.7

A nonconducting disk of radius a and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disk, from a height H with zero initial velocity. The particle has $q/m = 4\varepsilon_0 g/\sigma$.

(a) Find the value of H if the particle just reaches the disk.

(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

Sol.

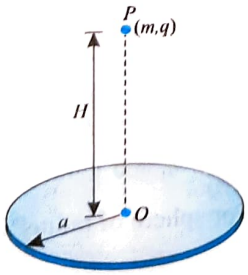
(a) Given that a is the radius of disk, σ is the surface charge density and $q/m = 4\varepsilon_0 g/\sigma$. The kinetic energy of the particle, when it reaches the disk, can be taken as zero. Potential due to a charged disk at any axial point situated at a distance x from O is

$$V(x) = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{a^2 + x^2} - x\right]$$

Hence,

$$V(H) = \frac{\sigma}{2\varepsilon_0}[\sqrt{a^2 + H^2} - H]$$

and $V(O) = \dfrac{\sigma a}{2\varepsilon_0}$



According to the law of conservation of energy, we have the loss of gravitation potential energy is equal to the gain in electric potential energy.

$$mg\,H = q\Delta V = q[V(0) - V(H)]$$

$$= q[a - \{\sqrt{(a^2 + H^2)} - H\}]\frac{\sigma}{2\varepsilon_0} \quad \ldots(i)$$

We are given

$$\frac{\sigma}{2\varepsilon_0} = 2\,mg$$

Putting this in Eq. (i), we get

$$mg\,H = 2mg[a - \{\sqrt{(a^2 + H^2)} - H\}]$$

or $H = 2[a + H - \sqrt{(a^2 + H^2)}]$

$$= 2a + 2H - 2\sqrt{(a^2 + H^2)}$$

or

$$2\sqrt{(a^2 + H^2)} = H + 2a$$

or $4a^2 + 4H^2 = H^2 + 4a^2 + 4aH$

or $3H^2 = 4aH$

or $H = \dfrac{4a}{3}$ (Since $H = 0$ is not valid)

(b) The total potential energy of the particle at height x is

$$U(x) = mgx + qV(x) = mgx + \frac{q\sigma}{2\varepsilon_0}(\sqrt{a^2 + x^2} - x)$$

$$= mgx + 2mg + \left[\sqrt{(a^2 + x^2)} - x\right]$$

$$= mg\,[2\sqrt{(a^2 + x^2)} - x] \quad \ldots(ii)$$

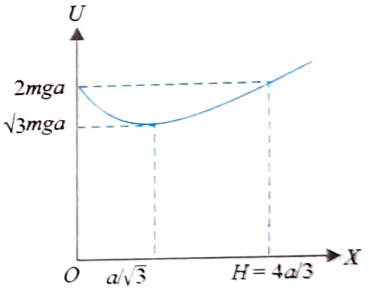
For equilibrium

$$\frac{dU}{dx} = 0$$

This gives

$$x = a/\sqrt{3}$$

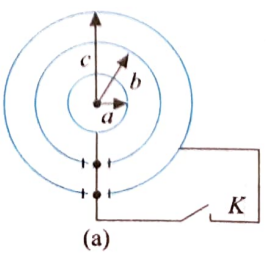
From Eq. (ii), graph between $U(x)$ and x can be plotted as shown in figure.



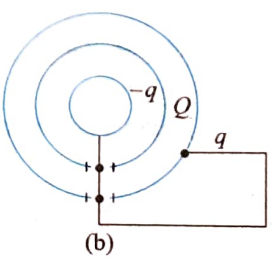
EXAMPLE 3.8

Three concentric conducting shells of radii a, b, and c are shown in figure. Charge on the shell of radius b is Q. If the key K is closed, find the charges on the innermost and outermost shells and the ratio of charge densities of the shells. Given that $a : b : c = 1 : 2 : 3$.

Sol. After closing the key, the innermost and outermost shells will be at the same potential. Let the charge on the outer shell be q and that on the inner shell be $-q$, then the total charge on inner and outer shells is zero.



(a)



(b)

Potential on the innermost shell

V_a = Sum of potentials due to $-q$, Q, and q

$$= -\frac{q}{a} + \frac{Q}{b} + \frac{q}{c}$$

Similarly, the potential on the outermost shell

$$V_c = -\frac{q}{c} + \frac{Q}{c} + \frac{q}{c}$$

As $V_a = V_c$, we have

$$-\frac{q}{a} + \frac{Q}{b} + \frac{q}{c} = -\frac{q}{c} + \frac{Q}{c} + \frac{q}{c}$$

From the given conditions, $c = 3a$, $b = 2a$.

Equation (i) now becomes

$$-\frac{q}{a} + \frac{Q}{2a} = -\frac{q}{3a} + \frac{Q}{3a}$$

or $\quad q = \frac{Q}{4}$

Thus, the charge on the outermost shell = $Q/4$, and the charge on the innermost shell is $-(Q/4)$.

$$\sigma_a = \frac{1}{4\pi a^2}\left(-\frac{Q}{4}\right)$$

$$\sigma_b = \frac{+Q}{4\pi b^2} = \frac{Q}{4\pi(4a^2)}$$

$$\sigma_c = \frac{1}{4\pi c^2}\left(\frac{Q}{4}\right) = \frac{+Q}{16\pi(9a^2)}$$

EXAMPLE 3.9

A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand. S_2 is initially uncharged.
S_1 is given a charge Q, brought into contact with S_2 and removed. S_1 is recharged such that the charge on it is again Q, and it is again brought into contact with S_2 and removed. This procedure is repeated n times.

(a) Find the electrostatic energy of S_2 after n such contacts with S_1.

(b) What is the limiting value of this energy as $n \to \infty$?

Sol.

(a) When S_1 and S_2 come in contact, there is transfer of charges till the potentials of the two spheres become equal. During first contact,

$V_1 = V_2$ (q_1 charge shifts from S_1 to S_2)

$$\Rightarrow \quad \frac{K(Q - q_1)}{r} = \frac{Kq_1}{R} \text{ or } q_1 = Q\left(\frac{R}{R+r}\right)$$

During second contact, again

$V_1 = V_2$

$$\Rightarrow \quad \frac{K[Q - (q_2 - q_1)]}{r} = \frac{Kq_2}{R}$$

[$(q_2 - q_1)$ charge shifts from S_1 to S_2]

$$\therefore \quad q_2 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2\right]$$

On third contact, again

$V_1 = V_2$

$$\Rightarrow \quad \frac{K[Q - (q_3 - q_2)]}{r} = \frac{Kq_3}{R}$$

[$(q_3 - q_2)$ charge shifts from S_1 to S_2]

$$\therefore \quad q_3 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \left(\frac{R}{R+r}\right)^3\right]$$

On nth contact, by symmetry

$V_1 = V_2$

$$\Rightarrow \quad \frac{K[Q - (q_n - q_{n-1})]}{r} = \frac{Kq_n}{R}$$

[$(q_n - q_{n-1})$ charge shift from S_1 to S_2]

$$q_n = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \cdots + \left(\frac{R}{R+r}\right)^n\right]$$

$$= \frac{QR}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right]$$

The electrostatic energy of S_2 after n contacts is

$$U_n = \frac{1}{2}\frac{q_n^2}{C} = \frac{1}{2} \times \frac{1}{4\pi\varepsilon_0 R} \times \left\{\frac{QR}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right]\right\}^2$$

(b) The limiting value is

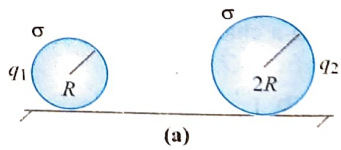
$$\underset{n\to\infty}{\text{Lt}}\ U_n = \underset{n\to\infty}{\text{Lt}}\left[\frac{1}{2} \times \frac{1}{4\pi\varepsilon_0 R}\left\{\frac{QR}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right]\right\}^2\right]$$

$$= \frac{Q^2 R}{2(4\pi\varepsilon_0) r^2}$$

EXAMPLE 3.10

Two isolated metallic solid spheres of radii R and $2R$ are charged such that both of these have same charge density σ. The spheres are located far away from each other and connected by a thin conducting wire. Find the new charge density on the bigger sphere.

Sol. For sphere of radius R [see Fig. (a)],

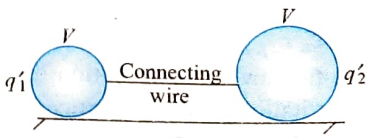


(a)

$$\sigma = \frac{q_1}{4\pi R^2}$$

or $\quad q_1 = \sigma \times 4\pi R^2$

For sphere of radius $2R$ [see Fig. (b)],



(b)

$$\sigma = \frac{q_2}{4\pi(2R)^2}$$

or $q_2 = \sigma \times 16\pi R^2$

When the two spheres are connected, the potential on the two spheres will be the same. There will be a rearrangement of charge for this to happen.

Let q_1' and q_2' be the new charges on the two spheres. Since the total charge remains the same, we get

$$q_1' + q_2' = q_1 + q_2 = \sigma \times 20\pi R^2 \quad ...(i)$$

Also, since $V_1 = V_2$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1'}{R} = \frac{1}{4\pi\varepsilon_0}\frac{q_2'}{2R}$$

$$q_1' = \frac{q_2'}{2} \quad ...(ii)$$

Substituting the value of q_1' from Eq. (ii) in Eq. (i)

$$\frac{q_2'}{2} + q_2' = \sigma \times 20\pi R^2$$

or $\frac{3q_2'}{2} = \sigma \times 20\pi R^2$

or $\frac{q_2'}{4\pi(2R)^2} = \frac{\sigma}{3} \times \frac{5}{2}$

New charge density on the bigger sphere is

$$\frac{q_2'}{4\pi(2R)^2} = \frac{5\sigma}{6}$$

EXAMPLE 3.11

The potential in the electric field varies as $V = -ax^2 + b$ with respect to x-coordinate, where a and b are constants. Find the charge density $\rho(x)$ in a space.

Sol. The electric field E is present along x-axis because $\frac{\partial V}{\partial y} = \frac{\partial V}{\partial z} = 0$.

Then,

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} = 2ax\hat{i} \quad ...(i)$$

According to Gauss's law, the net flux of $\vec{E}$ out of the cubical Gaussian surface is

$$E(x + dx)\,A - E(x)\,A = \frac{\delta q}{\varepsilon_0}$$

or $\frac{\partial E}{\partial x} \cdot A\delta x = \frac{\delta q}{\varepsilon_0}$

or $\frac{\partial E}{\partial x} = \frac{1}{\varepsilon_0}\left(\frac{\delta q}{A\delta x}\right) = \frac{\rho(x)}{\varepsilon_0}$ ...(ii)

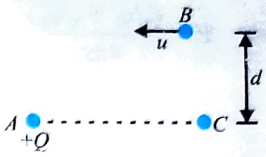
Using Eqs. (i) and (ii),

$$\frac{\delta}{\delta x}(2ax)\hat{i} = \frac{\rho(x)}{\varepsilon_0}$$

or $\rho(x) = 2a\varepsilon_0$

EXAMPLE 3.12

A positive charge $+Q$ is fixed at a point A. Another positively charged particle of mass m and charge $+q$ is projected from a point B with velocity u as shown in figure. Point B is at a large distance from A and at distance d from the line AC. The initial velocity is parallel to the line AC. The point C is at a very large distance from A. Find the minimum distance (in meter) of $+q$ from $+Q$ during the motion. Take $Qq = 4\pi\varepsilon_0 mu^2 d$ and $d = (\sqrt{2} - 1)$ m.



Sol. The path of the particle will be as shown in figure. At the point of minimum distance (D), the velocity of the particle will be $\perp$ to its position vector with respect to $+Q$. Applying conservation of angular momentum, we get

$$mud = mvr_{min} \quad \text{or} \quad vr_{min} = ud$$

Now by conservation of energy

$$\frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + \frac{KQq}{r_{min}}$$

or $\frac{1}{2}mu^2\left(1 - \frac{d^2}{r_{min}^2}\right) = \frac{mu^2 d}{r_{min}}$

$[\because KQq = mu^2 d \text{ (given)}]$

or $r_{min}^2 - 2r_{min}d - d^2 = 0$

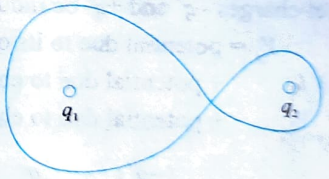
or $r_{min} = \frac{2d \pm \sqrt{4d^2 + 4d^2}}{2} = d(1 \pm \sqrt{2})$

Since distance cannot be negative

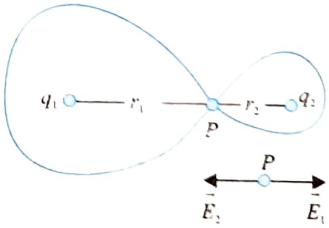
$$r_{min} = d(1 + \sqrt{2})$$

EXAMPLE 3.13

Two point like charges q_1 and q_2 are fixed in free space. At every point on the curve shown, the net electrostatic potential created by these charges is V. Find the separation r between the charges.



Sol. The electric field is zero where equipotential surface cuts. At P



$$|E_1| = |E_2|$$

or $k\frac{q_1}{r_1^2} = k\frac{q_2}{r_2^2}$

or $\left(\frac{q_1}{q_2}\right)^{1/2} = \frac{r_1}{r_2}$ ...(i)

Also $V = k\frac{q_1}{r_1} + k\frac{q_2}{r_2}$ ...(ii)

and $r = r_1 + r_2$ ...(iii)

From (i) and (iii),

$$r_1=\frac{r\sqrt{q_1}}{\sqrt{q_1}+\sqrt{q_2}} \text{ and } r_2=\frac{r\sqrt{q_2}}{\sqrt{q_1}+\sqrt{q_2}} \quad ...(iv)$$

From (ii) and (iv),

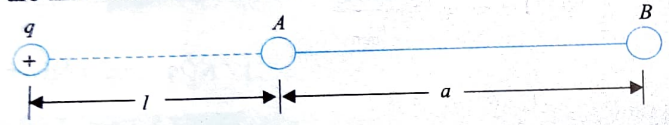
$$V=\frac{(\sqrt{q_1}+\sqrt{q_2})}{4\pi\varepsilon_0}\left[\frac{q_1}{r\sqrt{q_1}}+\frac{q_2}{r\sqrt{q_2}}\right]$$

$$=\frac{(\sqrt{q_1}+\sqrt{q_2})}{4\pi\varepsilon_0 r}[\sqrt{q_1}+\sqrt{q_2}]$$

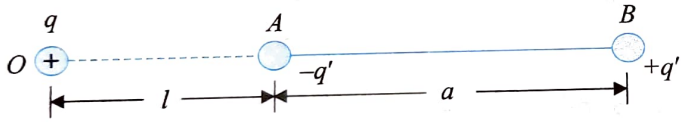
or $$r=\frac{(\sqrt{q_1}+\sqrt{q_2})^2}{4\pi\varepsilon_0 V}$$

EXAMPLE 3.14

Two small metal spheres A and B, each of radius r and supported on insulating stands, located at a distance a ($a >> r$) from each other are connected by a thin conducting wire. A point charge q is brought near the spheres at distance l ($l >> r$) on the line joining the centers of the spheres. What are the moduli of charge induced on the spheres?



Sol. As A and B are connected, $V_A = V_B$.



Let charges $-q'$ and $+q'$ be induced on A and B, respectively.

V_A = potential due to its own charge
+ potential due to charge placed at O
+ potential due to charge placed at B

$$=(K)\frac{(-q')}{r}+(K)\frac{q}{l}+(K)\frac{(q')}{a} \quad ...(i)$$

Similarly, potential of sphere B is

V_B = potential due to its own charge
+ potential due to q
+ potential due to $-q$

$$=(K)\frac{q'}{r}+(K)\frac{q}{(l+a)}+(K)\frac{(-q')}{a} \quad ...(ii)$$

As $V_A = V_B$,

$$K\left(\frac{-q'}{r}\right)+K\left(\frac{q}{l}\right)+K\left(\frac{q'}{a}\right)$$

$$=K\left(\frac{q'}{r}\right)+K\left(\frac{q}{l+a}\right)+K\left(\frac{-q'}{a}\right)$$

or $$\frac{2q'}{r}-\frac{2q'}{a}=\frac{q}{l}-\frac{q}{l+a} \text{ or } q'=\frac{a}{2}\left[\frac{-a^2 r}{l(l+a)(r-a)}\right]$$

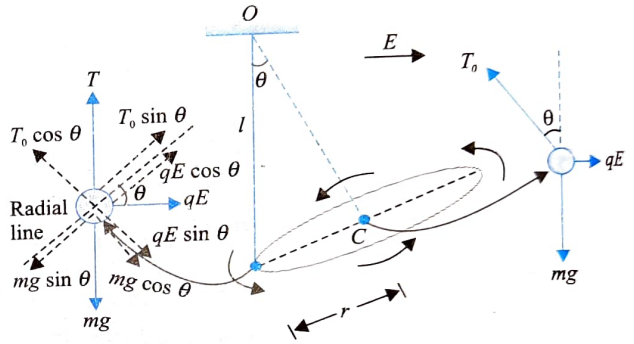
As $r << a$,

$$q'=\frac{a}{2}\left[\frac{-a^2 r}{l(l+a)(-a)}\right]=-\frac{q\,a\,r}{l(l+a)}$$

EXAMPLE 3.15

A particle of mass $m = 0.1$ kg and having positive charge $q = 75\ \mu C$ is suspended from a point by a thread of length $l = 10$ cm. In the space, a uniform horizontal electric field $E = 10^4\ NC^{-1}$ exists. The particle is drawn aside so that the thread becomes vertical and then it is projected horizontally with velocity v such that the particle starts to move along a circle with the same constant speed v. Calculate the radius of the circle and speed v ($g = 10\ ms^{-2}$).

Sol. Weight mg of the particle acts vertically downward and force qE horizontally. Therefore, in equilibrium position, the thread is inclined to the vertical. Let, in static equilibrium of the particle, inclination of the thread with the vertical be q and tension be T_0.



Considering free body diagram,

$$T_0 \sin\theta = qE \quad \text{and} \quad T_0 \cos\theta = qE$$

$$\Rightarrow \quad \tan\theta=\frac{qE}{mg} \quad \text{or} \quad \theta = 37°$$

The particle moves along a circle with constant speed. This is possible only when the thread traces the surface of a cone whose axis is inclined at an angle θ with the vertical. This arrangement is now called conical pendulum.

Note: The axis of a conical pendulum is always along the thread when the particle is in static equilibrium position.

Horizontal velocity imparted to the particle must be perpendicular to the direction of electric field. Radius of the circular path is $r = l \sin\theta = 6$ cm.

Now considering free body diagram of the revolving particle and resolving the forces normal to the direction of resultant force (centripetal force),

$$T\cos\theta = mg\cos\theta + qE\sin\theta$$
$$= 1.5625\ N$$

Now resolving the forces along the direction of resultant force,

$$qE\cos\theta + T\sin\theta - mg\sin\theta = \frac{mv^2}{r}$$

or $$v = 0.75\ ms^{-1}$$

Exercises

Single Correct Answer Type

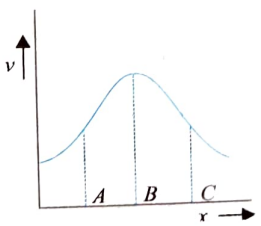
1. Mark the correct statement:
   (1) If E is zero at a certain point, then V should be zero at that point.
   (2) If E is not zero at a certain point, then V should not be zero at that point.
   (3) If V is zero at a certain point, then E should be zero at that point.
   (4) If V is zero at a certain point, then E may or may not be zero.

2. An uncharged conductor A is brought near a positively charged conductor B. Then
   (1) the charge on B will increase, but the potential of B will not change
   (2) the charge on B will not change, but the potential of B will decrease
   (3) the charge on B will decrease, but the potential of B will not change
   (4) the charge on B will not change, but the potential of B will increase

3. The electric field lines are closer together near object A than they are near object B. We can conclude that
   (1) the potential near A is greater than the potential near B
   (2) the potential near A is less than the potential near B
   (3) the potential near A is equal to the potential near B
   (4) nothing about the relative potentials near A and B

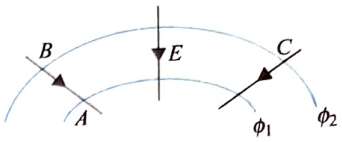
4. Mark the correct statement:
   (1) An electron and a proton when released from rest in a uniform electric field experience the same force and the same acceleration.
   (2) Two equipotential surfaces may intersect.
   (3) A solid conducting sphere holds more charge than a hollow conducting sphere of the same radius.
   (4) No work is done in taking a positive charge from one point to another inside a negatively charged metallic sphere.

5. Variation of electrostatic potential along the x-direction is shown in figure. The correct statement about electric field is
   (1) x-component at point B is maximum
   (2) x-component at point A is toward positive x-axis
   (3) x-component at point C is along negative x-axis
   (4) x-component at point C is along positive x-axis



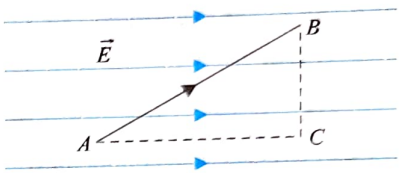
6. When the positively charged hanging pendulum bob is made fixed, the work done in slowly shifting a unit positive charge from infinity to P is V. If the pendulum is free to move, the corresponding work done is V'. Then
   (1) $V = V'$
   (2) $V > V'$
   (3) $V < V'$
   (4) $V \leq V'$

7. In moving from A to B along an electric field line, the work done by the electric field on an electron is 6.4×10^{-19} J. If ϕ_1 and ϕ_2 are equipotential surfaces, then the potential difference $V_C - V_A$ is



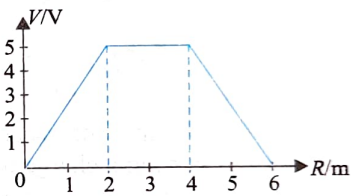
   (1) −4 V
   (2) 4 V
   (3) zero
   (4) 6.4 V

8. An electron is taken from point A to point B along the path AB in a uniform electric field of intensity E = 10 Vm^{-1}. Side AB = 5 m, and side BC = 3 m.



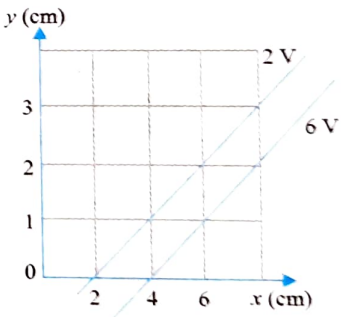
   Then, the amount of work done on the electron by us is
   (1) 50 eV
   (2) 40 eV
   (3) −50 eV
   (4) −40 eV

9. The variation of potential with distance R from the fixed point is shown in figure.



   The electric field at R = 5 m is
   (1) 2.5 Vm^{-1}
   (2) −2.5 Vm^{-1}
   (3) 0.4 Vm^{-1}
   (4) −0.4 Vm^{-1}

10. Figure shows two equipotential lines in the xy plane for an electric field. The scales are marked. The x-component and y-component of the field in the space between these equipotential lines are, respectively



   (1) +100 Vm^{-1}, −200 Vm^{-1}
   (2) −100 Vm^{-1}, +200 Vm^{-1}
   (3) +200 Vm^{-1}, +100 Vm^{-1}
   (4) −200 Vm^{-1}, −400 Vm^{-1}

11. A large insulated sphere of radius r charged with Q units of electricity is placed in contact with a small insulated uncharged sphere of radius r' and is then separated. The charge on the smaller sphere will now be
   (1) $\frac{Q(r'+r)}{r'}$
   (2) $\frac{Q(r'+r)}{r}$
   (3) $\frac{Qr}{r'+r}$
   (4) $\frac{Qr'}{r'+r}$

12. When a 2 μC charge is carried from point A to point B, the amount of work done by the electric field is 50 μJ. What is the potential difference and which point is at a higher potential?

(1) 25 V, B (2) 25 V, A

(3) 20 V, B (4) both are at same potential

13. An electric field is expressed as $\vec{E} = 2\hat{i} + 3\hat{j}$. Find the potential difference $(V_A - V_B)$ between two points A and B whose position vectors are given by $r_A = \hat{i} + 2\hat{j}$ and $r_B = 2\hat{i} + \hat{j} + 3\hat{k}$.

(1) −1 V (2) 1 V

(3) 2 V (4) 3 V

14. The potential function of an electrostatic field is given by $V = 2x^2$. Determine the electric field strength at the point (2 m, 0, 3 m).

(1) $\vec{E} = 4\hat{i}$ (NC^{-1}) (2) $\vec{E} = -4\hat{i}$ (NC^{-1})

(3) $\vec{E} = 8\hat{i}$ (NC^{-1}) (4) $-\vec{E} = -8\hat{i}$ (NC^{-1})

15. Two spherical conductors of radii R_1 and R_2 are separated by a distance much larger than the radius of either sphere. The spheres are connected by a conducting wire as shown in figure. If the charges on the spheres in equilibrium are q_1 and q_2, respectively, what is the ratio of the field strength at the surfaces of the spheres?

(1) R_2/R_1 (2) R_2^2/R_1^2

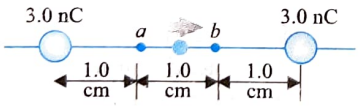
(3) R_1/R_2 (4) R_1^2/R_2^2

16. There is an electric field E in the x-direction. If the work done by the electric field in moving a charge of 0.2 C through a distance of 2 m along a line making an angle 60° with the x-axis is 4 J, then what is the value of E?

(1) $\sqrt{3}$ NC^{-1} (2) 4 NC^{-1}

(3) 5 NC^{-1} (4) 20 NC^{-1}

17. As shown in figure, a dust particle with mass $m = 5.0 \times 10^{-9}$ kg and charge $q_0 = 2.0$ nC starts from rest at point a and moves in a straight line to point b. What is its speed v at point b?



(1) 26 ms^{-1} (2) 34 ms^{-1}

(3) 46 ms^{-1} (4) 14 ms^{-1}

18. Two charged particles having charges 1 and −1 μC and of mass 50 g each are held at rest while their separation is 2 m. Now the charges are released. Find the speed of the particles when their separation is 1 m.

(1) $\frac{1}{5}$ ms^{-1} (2) $\frac{3}{5}$ ms^{-1}

(3) $\frac{3}{10}$ ms^{-1} (4) $\frac{2}{7}$ ms^{-1}

19. A 100 eV electron is projected directly toward a large metal plate that has surface charge density of -2.0×10^{-6} Cm^{-2}. From what distance must the electron be projected, if it is to just fail to strike that plate?

(1) 0.44 mm (2) 0.20 mm

(3) 1 mm (4) 0.30 mm

20. A point charge q is placed inside a conducting spherical shell of inner radius $2R$ and outer radius $3R$ at a distance of R from the center of the shell. Find the electric potential at the center of the shell.

(1) $\frac{1}{4\pi\varepsilon_0}\frac{q}{2R}$ (2) $\frac{1}{4\pi\varepsilon_0}\frac{4q}{3R}$

(3) $\frac{1}{4\pi\varepsilon_0}\frac{5q}{6R}$ (4) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{3R}$

21. n charged drops, each of radius r and charge q, coalesce to form a big drop of radius R and charge Q. If V is the electric potential and E is the electric field at the surface of a drop, then

(1) $E_{big} = n^{2/3}E_{small}$ (2) $V_{big} = n^{1/3}V_{small}$

(3) $E_{small} = n^{2/3}E_{big}$ (4) $V_{big} = n^{2/3}V_{small}$

22. Four identical charges are placed at the points (1, 0, 0), (0, 1, 0), (−1, 0, 0), and (0, −1, 0). Then,

(1) the potential at the origin is zero

(2) the electric field at the origin is not zero

(3) the potential at all points on the z-axis, other than the origin, is zero

(4) the field at all points on the z-axis, other than the origin, acts along the z-axis

23. Two point charges Q and $-Q/4$ placed along the x-axis are separated by a distance r. Take $-Q/4$ as origin and it is placed at the right of Q. Then the potential is zero

(1) at $x = r/3$ only

(2) at $x = -r/5$ only

(3) both at $x = r/3$ and at $x = -r/5$

(4) there exist two points on the axis where the electric field is zero

24. The electric potential decreases uniformly from 120 V to 80 V as one moves on the x-axis from $x = -1$ cm to $x = +1$ cm. The electric field at the origin

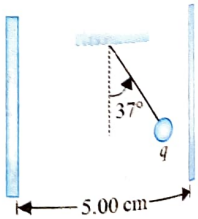
(1) must be equal to 20 Vcm^{-1}

(2) must be equal to 20 Vm^{-1}

(3) may be greater than 20 Vcm^{-1}

(4) may be less than 20 Vcm^{-1}

25. A small sphere with mass 1.2 g hangs by a thread between two parallel vertical plates 5.00 cm apart. The plates are insulating and have uniform surface charge densities $+\sigma$ and $-\sigma$. The charge on the sphere is $q = 9 \times 10^{-6}$ C. What potential difference between the plates will cause the thread to assume an angle of 37° with the vertical as shown in figure?



(1) 30 V (2) 12 V

(3) 50 V (4) 25 V

26. A particle of mass m carrying charge q is projected with velocity v from point P toward an infinite line of charge from a distance a. Its speed reduces to zero momentarily at point Q, which is at a distance $a/2$ from the line of charge. If another particle with mass m and charge $-q$ is projected with the same velocity v from P toward the line of charge, what will be its speed at Q?

(1) $\sqrt{2}v$ (2) $2v$

(3) $\sqrt{v}$ (4) $\sqrt{2}$

27. A solid conducting sphere of radius 10 cm is enclosed by a thin metallic shell of radius 20 cm. A charge $q = 20\ \mu C$ is given to the inner sphere. Find the heat generated in the process when the inner sphere is connected to the shell by a conducting wire.

(1) 12 J (2) 9 J

(3) 24 J (4) zero

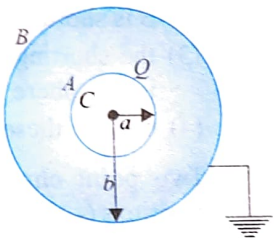
28. Find the potential V of an electrostatic field $\vec{E} = a(y\hat{i} + x\hat{j})$, where a is a constant.

(1) $axy + C$ (2) $-axy + C$

(3) axy (4) $-axy$

29. A conducting sphere A of radius a, with charge Q, is placed concentrically inside a conducting shell B of radius b. B is earthed. C is the common center of A and B. Study the following statements.

(i) The potential at a distance r from C, where $a \le r \le b$, is $\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r}\right)$



(ii) The potential difference between A and B is $\frac{1}{4\pi\varepsilon_0}Q\left(\frac{1}{a} - \frac{1}{b}\right)$

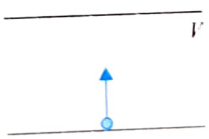
(iii) The potential at a distance r from C, where $a \le r \le b$, is $\frac{1}{4\pi\varepsilon_0}Q\left(\frac{1}{r} - \frac{1}{b}\right)$

Which of the following statements are correct?

(1) Only (i) and (ii) (2) Only (ii) and (iii)

(3) Only (i) and (iii) (4) All

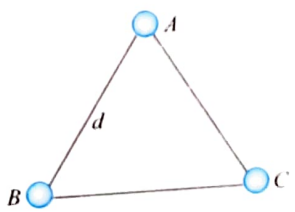
30. An electron having charge e and mass m starts from the lower plate of two metallic plates separated by a distance d. If the potential difference between the plates is V, the time taken by the electron to reach the upper plate is given by



(1) $\sqrt{\frac{2md^2}{eV}}$ (2) $\sqrt{\frac{md^2}{eV}}$

(3) $\sqrt{\frac{md^2}{2eV}}$ (4) $\frac{2md^2}{eV}$

31. Three identical metallic uncharged spheres A, B, and C, each of radius a, are kept at the corners of an equilateral triangle of side d ($d >> a$) as shown in figure. The fourth sphere (of radius a), which has a charge q, touches A and is then removed to a position far away. B is earthed and then the earth connection is removed. C is then earthed. The charge on C is



(1) $\frac{qa}{2d}\left(\frac{2d-a}{2d}\right)$ (2) $\frac{qa}{2d}\left(\frac{2d-a}{d}\right)$

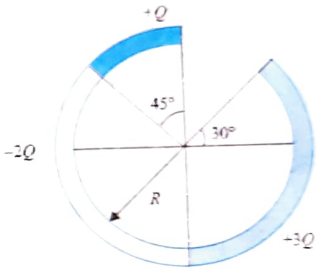
(3) $-\frac{qa}{2d}\left(\frac{d-a}{d}\right)$ (4) $\frac{2qa}{d}\left(\frac{d-a}{2d}\right)$

32. Two concentric conducting spherical shells of radii a_1 and a_2 ($a_2 > a_1$) are charged to potentials V_1 and V_2, respectively. Find the charge on the inner shell.

(1) $q_1 = 4\pi\varepsilon_0\left(\frac{V_1 - V_2}{a_2 - a_1}\right)a_1a_2$ (2) $q_1 = 4\pi\varepsilon_0\left(\frac{V_1 + V_2}{a_2 + a_1}\right)a_1a_2$

(3) $q_1 = \pi\varepsilon_0\left(\frac{V_1 - V_2}{a_2 + a_1}\right)a_1a_2$ (4) $q_1 = 2\pi\varepsilon_0\left(\frac{V_1 + V_2}{a_2 - a_1}\right)a_1a_2$

33. Figure shows three circular arcs, each of radius R and total charge as indicated. The net electric potential at the center of curvature is



(1) $\frac{Q}{2\pi\varepsilon_0 R}$

(2) $\frac{5Q}{12\pi\varepsilon_0 R}$

(3) $\frac{3Q}{32\pi\varepsilon_0 R}$

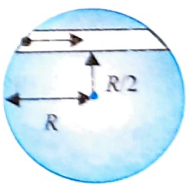
(4) none of these

34. The point charge q is within the cavity of an electrically neutral conducting shell whose outer surface has spherical shape. Find the potential V at a point P lying outside the shell at a distance r from the center O of the outer sphere.

(1) $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ (2) $V = \frac{1}{2\pi\varepsilon_0}\frac{q}{r}$

(3) $V = \frac{1}{\pi\varepsilon_0}\frac{q}{r}$ (4) $V = \frac{3}{4\pi\varepsilon_0}\frac{q}{r}$

35. A unit positive point charge of mass m is projected with a velocity V inside the tunnel as shown. The tunnel has been made inside a uniformly charged nonconducting sphere. The minimum velocity with which the point charge should be projected such that it can reach the opposite end of the tunnel is equal to



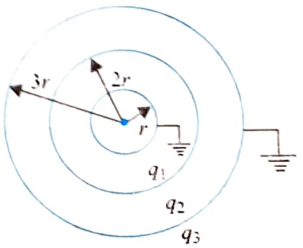
(1) $[\rho R^2/4m\varepsilon_0]^{1/2}$

(2) $[\rho R^2/24m\varepsilon_0]^{1/2}$

(3) $[\rho R^2/6m\varepsilon_0]^{1/2}$

(4) zero because the initial and the final points are at same potential

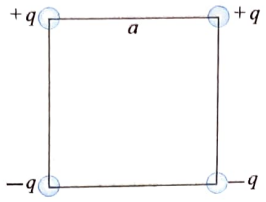
36. Three concentric conducting spherical shells have radii r, $2r$, and $3r$ and charges q_1, q_2, and q_3, respectively. Innermost and outermost shells are earthed as shown in figure. The charges shown are after earthing. Select the correct alternative.



(1) $q_1 + q_3 = -q_2$ (2) $q_1 = -q_2$

(3) $\frac{q_3}{q_2} = -\frac{1}{3}$ (4) None of these

37. Four charges are placed at four corners of a square as shown figure. The side of the square is a. Two charges are positive and two are negative, but their magnitudes are the same. Now, an external agent starts decreasing all the sides of the square slowly and at the same rate. What happens to the electrical potential energy of the system and what will be the nature of work done by the agent?

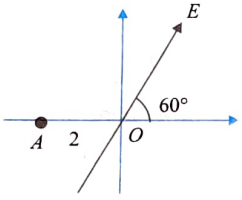


(1) increases, positive (2) increases, negative
(3) decreases, negative (4) increases, positive

38. At a distance r from a point located at origin in space, the electric potential varies as $V = 10r$. Find the electric field at $\vec{r} = 3\hat{i} + 4\hat{j} - 5\hat{k}$.

(1) $\sqrt{2}(3\hat{i} + 4\hat{j} - 5\hat{k})$ (2) $-\sqrt{2}(3\hat{i} + 4\hat{j} - 5\hat{k})$
(3) $-\sqrt{3}(3\hat{i} + 4\hat{j} - 5\hat{k})$ (4) None of these

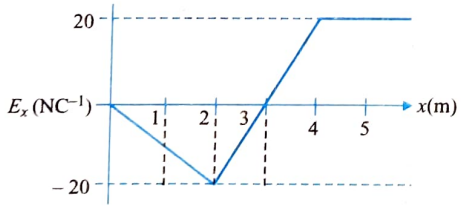
39. A uniform electric field of 100 Vm^{-1} is directed at 60° with the positive x-axis as shown in figure. If $OA = 2$ m, the potential difference $V_O - V_A$ is



(1) –50 V
(2) 50 V
(3) 100 V
(4) –100 V

40. A graph of the x-component of the electric field as a function of x in a region of space is shown in figure. The y- and z-components of the electric field are zero in this region. If the electric potential is 10 V at the origin, then the potential at $x = 2.0$ m is

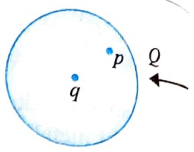
(1) 10 V (2) 40 V
(3) –10 V (4) 30 V



41. An uncharged conductor A is brought near a positively charged conductor B. The size of the conductor A is much greater than the size of conductor B. Then,

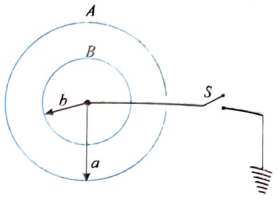
(1) the charge on B will increase, but the potential of B will not change
(2) the charge on B will not change, but the potential of B will decrease
(3) the charge on B will decrease, but the potential of B will not change
(4) the charge on B will not change, but the potential of B will increase

42. Inside a hollow conducting sphere, which is uncharged, a charge q is placed at its center. Let electric field at a distance x from center at point p be E and potential at this point be V. Now, some positive charge Q is given to this sphere, then



(1) E will remain the same (2) E will increase
(3) V will decrease (4) V will remain the same

43. Consider two concentric metal spheres. The outer sphere is given a charge $Q > 0$, then

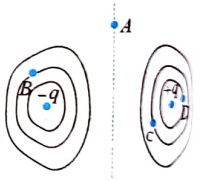


(1) the electrons will flow from earth to inner sphere if S is shorted
(2) the electrons will flow from inner sphere to the earth if S is shorted
(3) the shorting of S will produce a charge of $-Q$ on the inner sphere
(4) none of these

44. Two point charges $+Q$ each have been placed at the positions $(-a/2, 0, 0)$ and $(a/2, 0, 0)$. The locus of the points where $-Q$ charge can be placed such that total electrostatic potential energy of the system can become equal to zero, can be represented by which of the following equations?

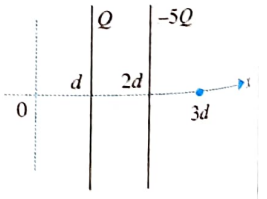
(1) $z^2 + (y - a)^2 = 2a$ (2) $z^2 + (y - a)^2 = 27a^2/4$
(3) $z^2 + y^2 = 15a^2/4$ (4) None of these

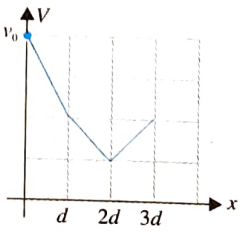
45. Figure shows equipotential surfaces for a two charges system. At which of the labeled points will an electron have the highest potential energy?

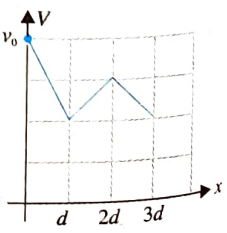


(1) point A
(2) point B
(3) point C
(4) point D

46. Two large identical plates are placed in front of each other at $x = d$ and $x = 2d$ as shown in the figure. If charges on plates are Q and $-5Q$, the potential versus distance graph for region $x = 0$ to $x = 3d$ is (d is very small and potential at $x = 0$ is v_0)



(1)


(2)


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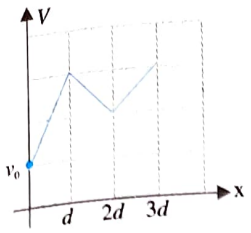
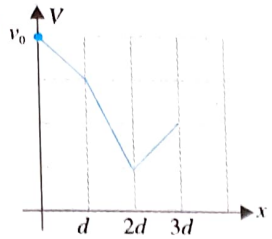
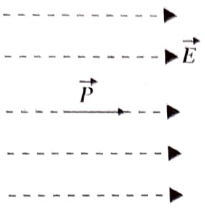


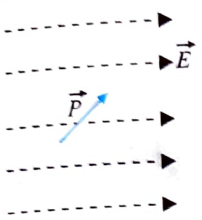
(4)



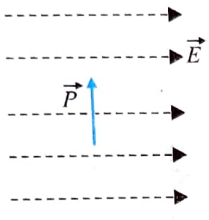
47. An electric dipole of dipole moment $\vec{p}$ is oriented parallel to a uniform electric field $\vec{E}$, as shown. It is rotated to one of the four orientations shown below. Rank the final orientations according to the change in the potential energy of the dipole–field system, most negative to most positive.



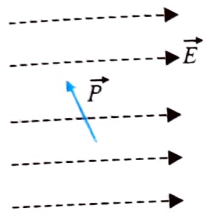
(i)



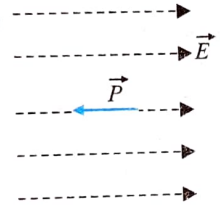
(ii)



(iii)

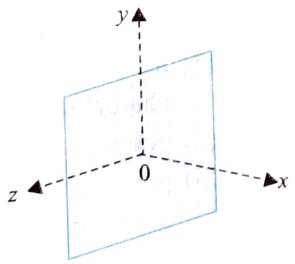


(iv)



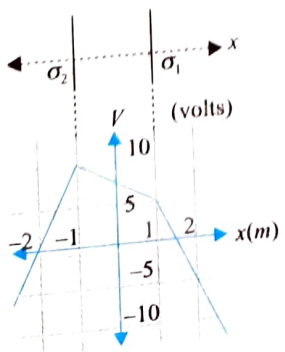
(1) i., ii., iv., iii.
(2) iv., iii., ii., i.
(3) i., ii., iii., iv.
(4) iii., ii., and iv. tie, then i.

48. A plane sheet has a uniformly distributed charge. Origin is selected at the middle of the sheet (in yz plane). A charge $+Q$ is moved from point A $(10a, 2a, 0)$ to point B $(10a, 0, 2a)$ along a circle and then to point C $(10a, 0, 0)$. If the initial electrostatic potential energy is U_0, what will be final electrostatic potential energy?



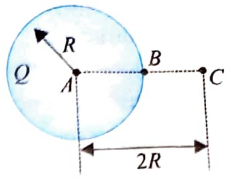
(1) zero
(2) U_0
(3) $U_0\sqrt{2}$
(4) $\frac{U_0}{2}$

49. Figure shows the potential due to two similarly charged infinite sheets with charge per unit area σ_2 and σ_1. From examining this plot we can deduce that



(1) $\sigma_2 > \sigma_1$
(2) $\sigma_2 < \sigma_1$
(3) $\sigma_2 = \sigma_1$
(4) None of these

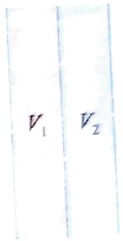
50. Find the ratio of electric work done in bringing a charge q from A to B (W_{AB}) and then from B to C (W_{BC}) in a sphere of charge Q distributed uniformly throughout its volume. $(W_{AB}/W_{BC} = ?)$



(1) 1
(2) 1.5
(3) 0.75
(4) None of these

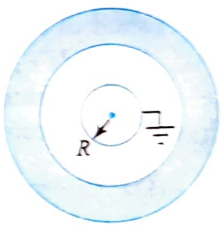
51. A nonuniformly charged ring is kept near an uncharged conducting solid sphere. The distance between their centres (which are on the same line normal to the plane of the ring) is 3 m and their radius is 4 m. If total charge on the ring is 1 μC, then the potential of the sphere will be
(1) 2.25 kV
(2) 3 kV
(3) 1.8 kV
(4) cannot be determined

52. An electron traveling in a uniform electric field passes from a region of potential V_1 to a region of higher potential V_2. Then



(1) no change takes place in velocity component parallel to interface of two regions
(2) direction of its motion remains unchanged but speed increases
(3) direction of its motion may change but speed must be decreased
(4) decrease in kinetic energy is proportional to $\sqrt{V_2 - V_1}$

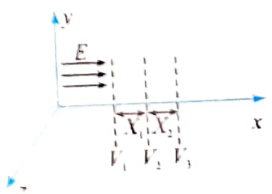
53. A conducting sphere of radius R and a concentric thick spherical shell of inner radius $2R$ and outer radius $3R$ is shown in figure. A charge $+10Q$ is given to the shell and inner sphere is earthed. Then charge on inner sphere is



(1) $-4Q$
(2) $-10Q$
(3) zero
(4) none of these

54. Let V_0 be the potential at the origin in an electric field $\vec{E} = 4\hat{i} + 5\hat{j}$. The potential at the point (x, y) is
(1) $V_0 - 4x - 5y$
(2) $V_0 + 4x + 5y$
(3) $4x + 5y - V_0$
(4) $-4x - 5y$

55. In an electric field shown in figure, three equipotential surfaces are shown. If function of electric field is $E = 2x^2$ Vm^{-1}, and given that $V_1 - V_2 = V_2 - V_3$, then we have



(1) $x_1 = x_2$
(2) $x_1 > x_2$
(3) $x_2 > x_1$
(4) data insufficient

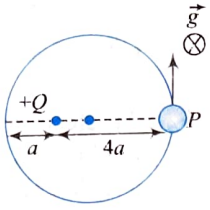
56. The potential of a point B (−20 m, 30 m) taking the potential of a point A (30 m, −20 m) to be zero in an electric field $\vec{E} = 10x\hat{i} - 20\hat{j}$ NC^{-1} is
(1) 350 V
(2) −100 V
(3) 300 V
(4) 3500 V

57. A positive charge $+q_1$ is located to the left of a negative charge $-q_2$. On a line passing through the two charges, there are two places where the total potential is zero. The reference is assumed to be at infinity. The first place is between the charges and is 4.00 cm to the left of the negative charge. The second place is 7.00 cm to the right of the negative charge. If $q_2 = -12\ \mu C$, what is the value of charge q_1.

(1) 44 μC (2) 12 μC

(3) 32 μC (4) 64 μC

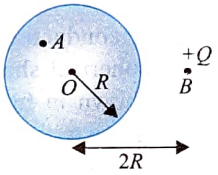
58. The diagram shows a small bead of mass m carrying charge q. The bead can freely move on the smooth fixed ring placed on a smooth horizontal plane. In the same plane a charge $+Q$ has also been fixed as shown. The potential at the point P due to $+Q$ is V. The velocity with which the bead should projected from the point P so that is can complete a circle should be greater than



(1) $\sqrt{\frac{6qV}{m}}$ (2) $4\sqrt{\frac{qV}{m}}$

(3) $\sqrt{\frac{3qV}{m}}$ (4) $\sqrt{\frac{7qV}{2m}}$

59. A point charge $+Q$ is placed at point B at a distance $2R$ from the center O of an uncharged thin conducting shell of radius R as shown in the figure. If V_A be the potential at point A, which is at a radial distance of $R/2$ from the center of the shell, then



(1) $V_A > \frac{Q}{8\pi\varepsilon_0 R}$ (2) $V_A < \frac{Q}{8\pi\varepsilon_0 R}$

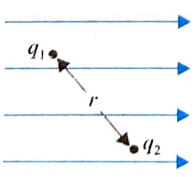
(3) $V_A = \frac{Q}{8\pi\varepsilon_0 R}$ (4) $V_A = 0$

60. Uniform electric field exists in a region and is given by $\vec{E} = E_0\hat{i} + E_0\hat{j}$. There are four points $A(-a, 0)$, $B(0, -a)$, $C(a, 0)$, and $D(0, a)$ in the xy plane. Which of the following is the correct relation for the electric potential?

(1) $V_A = V_C > V_B = V_D$ (2) $V_A = V_B > V_C = V_D$

(3) $V_A > V_C > V_B = V_D$ (4) $V_A < V_C < V_B < V_D$

61. Two point charges q_1 and q_2 are placed in an external uniform electric field as shown in figure. The potential at the location of q_1 and q_2 are V_1 and V_2, respectively. The effect of q_1 and q_2 is not included in V_1 and V_2, i.e., V_1 and V_2 are potentials at location of q_1 and q_2 due to external unspecified charges only. Then electric potential energy for this configuration of two charged particle is



(1) $\frac{q_1V_1 + q_2V_2}{2}$ (2) $q_1V_1 + q_2V_2$

(3) $q_1V_1 + q_2V_2 + \frac{q_1q_2}{4\pi\varepsilon_0 r}$ (4) $\frac{q_1q_2}{4\pi\varepsilon_0 r}$

62. There are two uncharged identical metallic spheres 1 and 2 of radius r separated by a distance d ($d >> r$). A charged metallic sphere of same radius having charge q is touched with one of the sphere. After some time it is moved away from the system. Now the uncharged sphere is earthed. Charge on earthed sphere is

(1) $q/2$ (2) $-qr/d$

(3) $-qr/2d$ (4) $-q/2$

63. Eight point charges of charge q each are placed on the eight corners of a cube of side a. A solid neutral metallic sphere of radius $a/3$ is placed with its center at the center of the cube. As a result, charges are induced on the sphere, which form certain patterns on its surface. What is the potential of the sphere?

(1) $\frac{8}{\sqrt{3}}\frac{kq}{a}$ (2) $\frac{16}{\sqrt{3}}\frac{kq}{a}$

(3) $\frac{-8}{\sqrt{3}}\frac{kq}{a}$ (4) zero

Multiple Correct Answers Type

1. Two point charges of different magnitudes and of opposite signs are separated by some distance. There can be

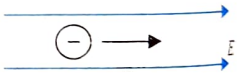
(1) only one point in space where net electric field intensity is zero

(2) only two points in space where net electric potential is zero

(3) infinite number of points in space where net electric field intensity is 0

(4) infinite number of points in space where net electric potential is zero

2. A negative charge is moved by an external agent in the direction of electric field. Then



(1) potential energy of the charge increases

(2) potential energy of the charge decreases

(3) positive work is done by the electric field

(4) negative work is done by the electric field

3. If a charged conductor is enclosed by a hollow charged conducting shell (assumed concentric and spherical in shape), and they are connected by a conducting wire, then which of the following statement(s) would be correct?

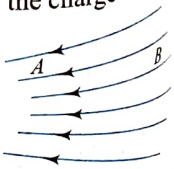
(1) Potential difference between the two conductors becomes zero.

(2) If charge on the inner conductor is q and on the outer conductor is $2q$, then finally charge on the outer conductor will be $3q$.

(3) The charge on the inner conductor is totally transferred to the outer conductor.

(4) If charge on the inner conductor is q and charge on the outer conductor is zero, then finally the charge on each conductor will be $q/2$.

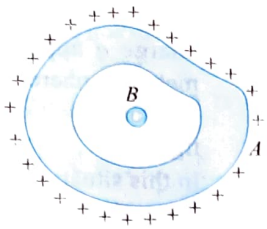
4. Which of the following is true for the figure showing electric lines of force? (E is electrical field, and V is potential.)



(1) $E_A > E_B$ (2) $E_B > E_A$

(3) $V_A > V_B$ (4) $V_B > V_A$

5. An uncharged conducting ball B lies inside a charged conductor A as shown in figure. B is isolated from A.



(1) When B touches the inner surface of A, then potential of B will change.

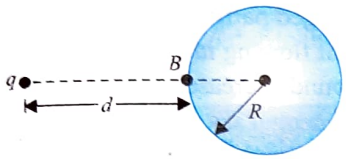
(2) No net charge is inside the cavity.

(3) There is an induced charge on B.

(4) Potentials of A and B are same.

6. For the situation shown in figure, mark the correct statement(s).

Hollow neutral conductor



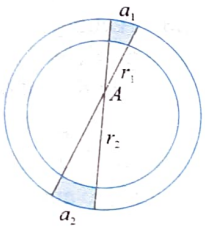
(1) Potential of the conductor is $q/[4\pi\varepsilon_0(d+R)]$.

(2) Potential of the conductor is $q/4\pi\varepsilon_0 d$.

(3) Potential of the conductor cannot be determined as the nature of distribution of induced charges is not known.

(4) Potential at point B due to the induced charges is $-qR/[4\pi\varepsilon_0(d+R)d]$.

7. A wire having a uniform linear charge density λ is bent in the form of a ring of radius R. Point A as shown in figure is in the plane of the ring but not at the center. Two elements of the ring of lengths a_1 and a_2 subtend very small same angle at point A. They are at distances r_1 and r_2 from point A, respectively ($r_2 > r_1$).



(1) The ratio of charges of element a_1 to that of element a_2 is r_1/r_2.

(2) The element a_1 produced greater magnitude of electric field at A than element a_2.

(3) The elements a_1 and a_2 produce same potential at A.

(4) The direction of the net electric field produced by the elements only at A is toward element a_2.

8. At a distance of 5 cm and 10 cm outward from the surface of a uniformly charged solid sphere, the potentials are 100 V and 75 V, respectively. Then

(1) potential at its surface is 150 V

(2) the charge on the sphere is $(5/3) \times 10^{-10}$ C

(3) the electric field on the surface is 1500 Vm^{-1}

(4) the electric potential at its center is 0 V

9. Two infinite, parallel, nonconducting sheets carry equal positive charge density σ. One is placed in the yz plane and the other at distance $x = a$. Take potential $V = 0$ at $x = 0$. Then

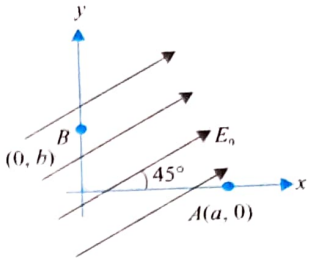
(1) for $0 \le x \le a$, potential $V = 0$

(2) for $x \ge a$, potential $V = \dfrac{\sigma}{\varepsilon_0}(x-a)$

(3) for $x \ge a$, potential $V = -\dfrac{\sigma}{\varepsilon_0}(x-a)$

(4) for $x \le 0$, potential $V = \dfrac{\sigma}{\varepsilon_0}x$

10. A uniform electric field E_0 exists in a region at angle 45° with the x-axis. There are two point A $(a, 0)$ and B $(0, b)$ having potential V_A and V_B, respectively, then



(1) $a < b$

(2) $a > b$

(3) $a = b$

(4) None of these

11. A small sphere is charged uniformly and placed at some point $A(x_0, y_0)$ so that at point B(9 m, 4 m) electric field strength is $\vec{E} = (54\hat{i} + 72\hat{j})$ NC^{-1} and potential is 1800 V. Then

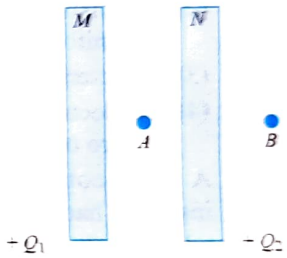
(1) the magnitude of charge on the sphere is 4 μC

(2) the magnitude of charge on the sphere is 2 μC

(3) coordinates of A are $x_0 = -3$ m, $y_0 = -12$ m

(4) coordinates of A are $x_0 = 4$ m, $y_0 = -1$ m

12. Two conducting plates M and N, each having large surface area A (on one side), are placed parallel to each other figure. The plate M is given charge Q_1 and N, charge Q_2 ($<Q_1$). Then



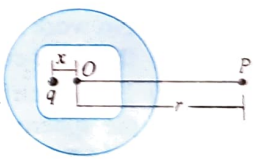
(1) electric field at point A is $(Q_1 - Q_2)/2A\varepsilon_0$ toward right

(2) electric field at point B is $(Q_1 + Q_2)/2A\varepsilon_0$ toward right

(3) electric potential of N is greater than M

(4) all of the above

13. A point charge q is placed within the cavity of an electrically neutral conducting shell whose outer surface has spherical shape figure. Then,



(1) the potential V at point P lying outside the shell at a distance r from the centre O of the outer surface depends upon the value of x

(2) potential at P does not depend upon the value of x

(3) a total charge q will be induced on the outer surface of the shell which will be distributed uniformly on the outer surface

(4) a total charge $-q$ will be induced on the inner surface of the shell, which will be distributed nonuniformly on the inner surface

14. The electric potential at a certain distance from a point charge is 600 V and the electric field is 200 NC^{-1}. Which of the following statements will be true?

(1) The work done in moving a point charge of 1 μC from the given point to a point at a distance of 9 m will be 4×10^{-4} J.

(2) The distance of the given point from the charge is 3 m.

(3) The potential at a distance of 9 m will be 200 V.
(4) The magnitude of charge is 0.2×10^{-3} C.

15. An electric charge 2×10^{-8} C is placed at the point (1, 2, 4). At the point (3, 2, 1), the electric
(1) field will increase by a factor K if the space between the points is filled with a dielectric of dielectric constant K
(2) field will be along the y-axis
(3) potential will be 49.9 V
(4) field will have no y-component

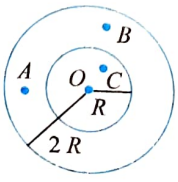
16. The electric potential in a region along the x-axis varies with x according to the relation $V(x)=4 + 5x^2$. Then
(1) potential difference between the points $x = 1$ and $x = -2$ is 15 V
(2) the force experienced by the above charge will be toward the $+x$-axis
(3) a uniform electric field exists in this region along the $+x$-axis
(4) force experienced by a 1 C charge at $x = -1$ m will be 10 N

17. A conducting sphere of radius R has a charge. Then,
(1) the charge is uniformly distributed over its surface, if there is no external electric field
(2) the distribution of charge over its surface will be nonuniform if an external electric field exists in the space
(3) the potential at every point of the sphere must be the same
(4) the electric field strength inside the sphere will be equal to zero only when no external electric field exists

18. A small conducting sphere of radius a mounted on an insulated handle and a positive charge q is inserted through a hole in the wall of a hollow conducting sphere of inner radius b and outer radius c. The hollow sphere is supported on an insulating stand and is initially uncharged. The small sphere is placed at the center of the hollow sphere. Neglect any effect of the hole. Which of the following statements will be true for this system?
(1) No work will be done in carrying a small charge from the inner conductor to the outer conductor.
(2) The electric field at a point in the region between the spheres at a distance r from the center is equal to $q / 4\pi\varepsilon_0 r^2$.
(3) The electric field at a point outside the hollow sphere at a distance r from the center is $q / 4\pi\varepsilon_0 r^2$.
(4) The potential of the inner sphere with respect to the outer sphere is given by $V_{ab} = \dfrac{q}{4\pi\varepsilon_0}\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$.

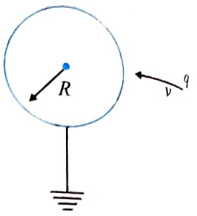
19. Mark the correct statements.
(1) A given conducting sphere cannot be charged to a potential greater than a certain value.
(2) A given conducting sphere can be charged to a potential less than a certain minimum value.
(3) A given conducting sphere can be charged to any extent.
(4) None of the above

20. A hollow conducting sphere of inner radius r and outer radius $2R$ is given charge Q as shown in the figure, then the



(1) potential at A and B is same
(2) potential at O and B is same
(3) potential at O and C is same
(4) potential at A, B, C and O is same

21. A charged particle having a positive charge q approaches a grounded metallic spheres of radius R with a constant small speed v as shown in figure.



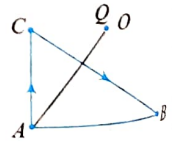
In this situation,
(1) as the charge draws nearer to the surface of the sphere, a current flows into the ground
(2) as the charge draws nearer to the surface of the sphere, a current flows out of the ground into the sphere
(3) as the charged particle draws nearer, the magnitude of the current flowing in the connector joining the shell to the ground increases
(4) as the charged particle draws nearer, the magnitude of the current flowing in the connector joining the sphere to the ground decreases

22. The electric potential at a point certain distance from a point charge is 600 V and the electric field is 200 NC^{-1}. Which of the following statements will be true?
(1) The magnitude of the charge is 20×10^{-3} C.
(2) The distance of the given point from the charge is 3 m.
(3) The potential at a distance of 9 m will be 200 V.
(4) The work done by an external agent in moving a point charge of 1 mC from the given point to a point at a distance of 9 m will be 4×10^{-4} J.

23. A charged particle X moves directly toward another charged particle Y. For the X and Y system, the total momentum is p and the total energy is E.
(1) If Y is fixed, E is conserved but not p.
(2) If Y is fixed, neither E nor p is conserved.
(3) p and E are conserved if both X and Y are free to move.
(4) (a) is true only if X and Y have similar charges.

24. Consider the following conclusions regarding the components of an electric field at a certain point in space given by $E_x = -Ky$, $E_y = Kx$, $E_z = 0$.
(1) The field is conservative.
(2) The field is nonconservative.
(3) The lines of force are straight lines.
(4) The lines of force are circles.

25. There is a fixed positive charge Q at O, and A and B are points equidistant from O. A positive charge $+q$ is taken slowly by an external agent from A to B along the line AC and then along the line CB.



(1) The total work done on the charge is zero.
(2) The work done by the electrostatic force from A to C is negative.
(3) The work done by the electrostatic force from C to B is positive.
(4) The work done by electrostatic force in taking the charge from A to B is dependent on the actual path.

26. The electric potential in a region is given by the relation $V(x) = 4 + 5x^2$. If a dipole is placed at position (–1, 0) with

dipole moment $\vec{P}$ pointing along positive y-direction, then

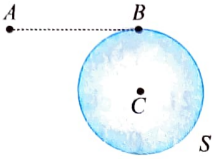
(1) net force on the dipole is zero

(2) net torque on the dipole is zero

(3) net torque on the dipole is not zero and it is in clockwise direction

(4) Net torque on the dipole is not zero and it is in anticlockwise direction

27. S is a solid neutral conducting sphere. A point charge q of 1×10^{-6} C is placed at point, C is the center of sphere, and AB is a tangent. $BC = 3$ m and $B = 4$ m.



(1) The electric potential of the conductor is 1.8 kV

(2) The electric potential of the conductor is 2.25 kV

(3) The electric potential at B due to induced charges on the sphere is –0.45 kV.

(4) The electric potential at B due to induced charges on the sphere is 0.45 kV.

Linked Comprehension Type

For Problems 1–4

Four charges $+q$, $+q$, $-q$, and $-q$ are placed, respectively, at the corners A, B, C, and D of a square of side a, arranged in the given order. E and F are the midpoints of sides BC and CD, respectively. O is the center of square.

1. The electric field at O is

(1) $\dfrac{q}{\sqrt{2}\pi\varepsilon_0 a^2}$ (2) $\dfrac{q}{\sqrt{3}\pi\varepsilon_0 a^2}$

(3) $\dfrac{\sqrt{3}q}{\pi\varepsilon_0 a^2}$ (4) $\dfrac{\sqrt{2}q}{\pi\varepsilon_0 a^2}$

2. The electric potential at O is

(1) $\dfrac{\sqrt{2}q}{\pi\varepsilon_0 a}$ (2) $\dfrac{\sqrt{3}q}{\pi\,\varepsilon_0 a}$

(3) $\dfrac{q}{\pi\varepsilon_0 a}$ (4) zero

3 The work done in carrying a charge e from O to E is

(1) $\dfrac{\sqrt{2}q\,e}{\pi\,\varepsilon_0 a}$ (2) $\dfrac{q\,e}{\pi\,\varepsilon_0 a}\left[\dfrac{1}{\sqrt{5}}-1\right]$

(3) $\dfrac{q\,e}{\pi\,\varepsilon_0 a}\left[\dfrac{1}{\sqrt{5}}+1\right]$ (4) zero

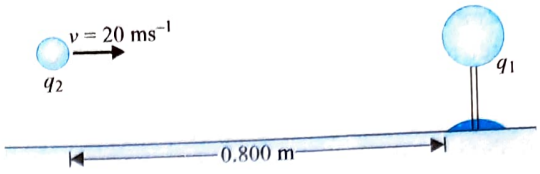
4. The work done in carrying a charge e from O to F is

(1) $\dfrac{\sqrt{2}q\,e}{\pi\,\varepsilon_0 a}$ (2) $\dfrac{q\,e}{\pi\,\varepsilon_0 a}\left[\dfrac{1}{\sqrt{5}}-1\right]$

(3) $\dfrac{q\,e}{\pi\,\varepsilon_0 a}\left[\dfrac{1}{\sqrt{5}}+1\right]$ (4) zero

For Problems 5–6

A small metal sphere, carrying a net charge $q_1 = -2$ μC, is held in a stationary position by insulating supports. A second small metal sphere, with a net charge of $q_2 = -8$ μC and mass 1.50 g, is projected toward q_1. When the two spheres are 0.800 m apart, q_2 is moving toward q_1 with speed 20 ms^{-1} as shown in figure. Assume that the two spheres can be treated as point charges. You can ignore the force of gravity.



5. The speed of q_2 when the spheres are 0.400 m apart is

(1) $2\sqrt{10}$ ms^{-1} (2) $2\sqrt{6}$ ms^{-1}

(3) $4\sqrt{10}$ ms^{-1} (4) $4\sqrt{6}$ ms^{-1}

6. How close does q_2 get to q_1?

(1) 0.20 m (2) 0.30 m

(3) 0.10 m (4) 0.15 m

For Problems 7–8

A charge Q is distributed over two concentric hollow spheres of radii r and R ($R > r$) such that their surface densities are equal.

7. The charges on smaller and bigger shells are

(1) $\dfrac{Qr^2}{r^2+R^2}$ and $\dfrac{QR^2}{r^2+R^2}$, respectively

(2) $Q\left(1+\dfrac{r^2}{R^2}\right)$ and $Q\left(1+\dfrac{R^2}{r^2}\right)$, respectively

(3) $Q\left(1-\dfrac{r^2}{R^2}\right)$ and $Q\left(1-\dfrac{R^2}{r^2}\right)$, respectively

(4) $\dfrac{QR^2}{r^2+R^2}$ and $Q\dfrac{Qr^2}{r^2+R^2}$, respectively

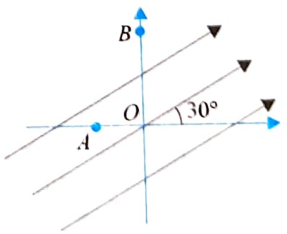
8. The potential at the common centre is

(1) $\dfrac{\sqrt{2}}{\pi\varepsilon_0}\dfrac{Q(R+r)}{(R^2+r^2)}$ (2) $\dfrac{1}{2\pi\varepsilon_0}\dfrac{Q(R+r)}{(R^2+r^2)}$

(3) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q(R+r)}{(R^2+r^2)}$ (4) $\dfrac{1}{\pi\varepsilon_0}\dfrac{Q(R-r)}{(R^2+r^2)}$

For Problems 9–10

A uniform electric field of 100 Vm^{-1} is directed at 30° with the positive x-axis as shown in figure. $OA = 2$ m and $OB = 4$ m.



9. The potential difference $V_O - V_A$ is

(1) $100\sqrt{3}$ V (2) $200\sqrt{3}$ V

(3) $-100\sqrt{3}$ V (4) $-200\sqrt{3}$ V

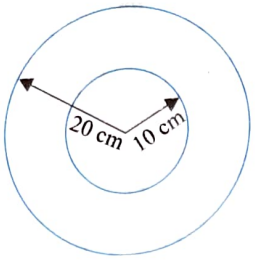
10. The potential difference $V_B - V_A$ is

(1) $-100\left[2+\sqrt{3}\right]$ V (1) $100\left[2+\sqrt{3}\right]$ V

(3) $100\left[2-\sqrt{3}\right]$ V (4) $-100\left[2-\sqrt{3}\right]$ V

For Problems 11–14

We have an isolated conducting spherical shell of radius 10 cm. Some positive charge is given to it so that the resulting electric field has a maximum intensity of 1.8×10^6 NC^{-1}. The same amount of negative charge is given to another isolated conducting spherical shell of radius 20 cm. Now, the first shell is placed inside the second so that both are concentric as shown in figure.



11. The electric potential at any point inside the first shell is
(1) 18×10^4 V (2) 9×10^4 V
(3) 4.5×10^4 V (4) 1.8×10^4 V

12. The electric field intensity just inside the outer sphere is
(1) 4.5×10^5 NC^{-1} (2) 9×10^5 NC^{-1}
(3) 4.5×10^4 NC^{-1} (4) 5×10^4 NC^{-1}

13. The electrostatic energy stored in the system is
(1) 1.0 J (2) 0.045 J
(3) 0.09 J (4) 1.8 J

14. If both the spheres are connected by a conducting wire, then
(1) nothing will happen
(2) some part of the energy stored in the system will convert into heat
(3) charge on both spheres will be positive
(4) entire amount of the energy stored in the system will convert into heat

For Problems 15–18

The electric potential varies in space according to the relation $V = 3x + 4y$. A particle of mass 0.1 kg starts from rest from point (2, 3.2) under the influence of this field. The charge on the particle is +1 μC. Assume V and (x, y) are in SI units.

15. The component of electric field in the x-direction (E_x) is
(1) -3 Vm^{-1} (2) 4 Vm^{-1}
(3) 5 Vm^{-1} (4) 8 Vm^{-1}

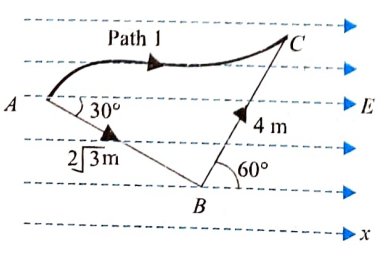
16. The component of electric field in the y-direction (E_y) is
(1) 3 Vm^{-1} (2) -4 Vm^{-1}
(3) 5 Vm^{-1} (4) 8 Vm^{-1}

17. The time taken to cross the x-axis is
(1) 20 s (2) 40 s
(3) 200 s (4) 400 s

18. The velocity of the particle when it crosses the x-axis is
(1) 20×10^{-3} ms^{-1} (2) 40×10^{-3} ms^{-1}
(3) 30×10^{-3} ms^{-1} (4) 50×10^{-3} ms^{-1}

For Problem 19–21

In a certain region, electric field E exists along the x-axis which is uniform. Given $B = 2\sqrt{3}$ m and $BC = 4$ m. Points A, B, and C are in xy plane.



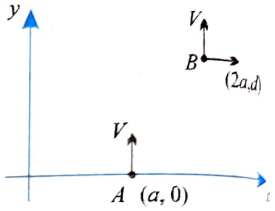
19. Find the potential difference $V_A - V_B$ between the points A and B.
(1) $3E$ (2) $4E$
(3) E (4) $2E$

20. Find the potential difference $V_C - V_B$ between the points C and B.
(1) $2E$ (2) $3E$
(3) $-E$ (4) $-2E$

21. A charged particle q is moved from A to C as shown in path 1. What is the work done by electric field in this process?
(1) $2qE$ (3) $5qE$
(3) qE (4) $4qe$

For Problems 22–24

An uniform electric field of strength E exists in a region. An electron of mass m enters a point A perpendicular to x-axis with velocity V. It moves through the electric field and exits at point B. The components of velocity at B are shown in figure. At B the y-component of velocity remain unchanged.



22. Find electric field
(1) $\frac{2maV^2}{ed^2}$ (2) $\frac{maV^2}{ed^2}$
(3) $\frac{maV^2}{2ed^2}$ (4) $\frac{2maV^3}{ed^3}$

23. Find velocity at B
(1) $V\sqrt{1-\left(\frac{2a}{d}\right)^2}$ (2) $V\sqrt{1+\left(\frac{2a}{d}\right)^2}$
(3) $V\sqrt{1+\left(\frac{a}{d}\right)^2}$ (4) $V\sqrt{2+\left(\frac{2a}{d}\right)^2}$

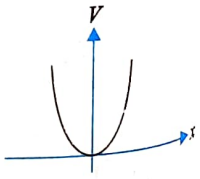
24. Find the rate of work done by the field at B
(1) $\frac{2ma^2V^3}{d^3}$ (2) $\frac{ma^2V^3}{2d^3}$
(3) $\frac{4ma^2V^3}{d^3}$ (4) $\frac{ma^2V^3}{d^3}$

For Problems 25–27

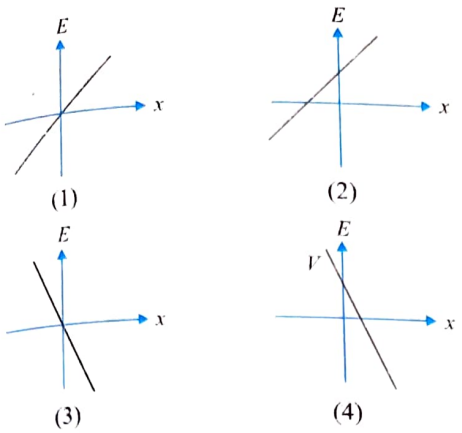
We know that electric field (E) at any point in space can be calculated using the relation

$$\vec{E} = -\frac{\delta V}{\delta x}\hat{i} - \frac{\delta V}{\delta y}\hat{j} - \frac{\delta V}{\delta z}\hat{k}$$

if we know the variation of potential (V) at that point. Now let the electric potential in volt along the x-axis vary as $V = 2x^2$, where x is in meter. Its variation is as shown in figure.



25. A charge particle of mass 10 mg and charge 2.5 μC is released from rest at $x = 2$ m. The variation of electric field (E) along the x-axis is expressed by



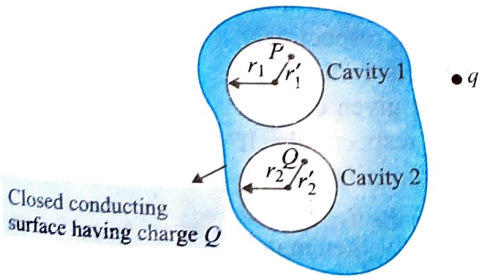
26. The velocity of the particle when it crosses origin is
(1) 0.5 ms^{-1} (2) 1 ms^{-1}
(3) 2 ms^{-1} (4) 4 ms^{-1}

27. Will the particle perform a simple harmonic motion? The time period of its oscillations of the particle should be
(1) Yes, time period = 2π s
(2) No, time period = 2π s
(3) Yes, time period = 4π s
(4) The particle will perform SHM, but time period cannot be found from the given data.

For Problems 28–32

A point charge q_1 is placed inside cavity 1 and another point charge q_2 is inside cavity 2. A point charge q is placed outside the conductor. For the situation described above, answer the following questions.

28. The charge on outer surface of the conductor would be
(1) $Q+q_1+q_2$ and nonuniformly distributed
(2) $Q+q_1+q_2$ and its uniform or nonuniform distribution depends upon locations of q_1 and q_2
(3) $Q+q_1+q_2$ and would be distributed uniformly
(4) $Q+q_1+q_2$ and the distribution depends upon the locations of q_1, q_2, and q



29. If q_1 is at the center of cavity 1, then $\vec{E}$ at point S, at a distant r from the center of cavity 1 ($r > r_1$), due to the induced charge on the surface of cavity 1 is
(1) $q_1/4\pi\varepsilon_0 r^2$ away from the center of cavity 1
(2) $q_1/4\pi\varepsilon_0 r_1^2$ away from the center of cavity 1
(3) zero
(4) $q_1/4\pi\varepsilon_0 r^2$ toward the center of cavity 1

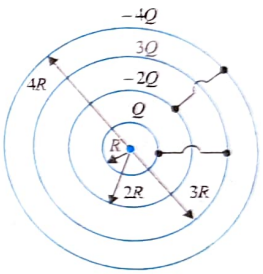
30. $\vec{E}$ inside the conductor at point S distant r from point charge q, due to charge on outer surface of the conductor, would be
(1) $(Q+q_1+q_2)/4\pi\varepsilon_0 r^2$ away from charge q
(2) $q/4\pi\varepsilon_0 r^2$ toward charge q
(3) zero
(4) cannot be determined

31. If charge q_2 is at point Q (inside cavity 2), then $\vec{E}$ at the center of cavity 2 due to induced charge on the surface of cavity 2 would be
(1) $q_2/4\pi\varepsilon_0 r_2'^2$ toward q_2 (2) $q_2/4\pi\varepsilon_0 r_2'^2$ away from q_2
(3) zero (4) cannot be determined

32. If the potential of the conductor is V_0 and charge q_2 is placed at the center of cavity 2, then potential at point Q is
(1) $\dfrac{q_2}{4\pi\varepsilon_0 r_2'}+V_0$ (2) $\dfrac{q_2}{4\pi\varepsilon_0}\left(\dfrac{1}{r_2'}+\dfrac{1}{r_2}\right)+V_0$
(3) $\dfrac{q_2}{4\pi\varepsilon_0}\left(\dfrac{1}{r_2'}-\dfrac{1}{r_2}\right)+V_0$ (4) V_0

For Problems 33–36

Four concentric hollow spheres of radii R, $2R$, $3R$, and $4R$ are given the charges as shown in figure. Then the conductors 1 and 3, 2 and 4 are connected by conducting wires (both the connections are made at the same time).



33. The charge on the inner surface of the third conductor is
(1) $-6Q/5$ (2) $6Q/5$
(3) $-2Q$ (4) $+2Q$

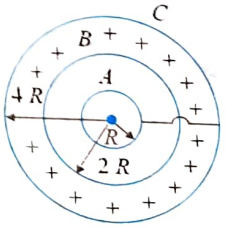
34. The charge on the fourth conductor is
(1) $22Q/5$ (2) $+11Q/3$
(3) $-11Q/3$ (4) $-22Q/5$

35. The potential of conductor 1 is
(1) $3Q/40\pi\varepsilon_0 R$ (2) $-19Q/40\pi\varepsilon_0 R$
(3) $-3Q/40\pi\varepsilon_0 R$ (4) $19Q/40\pi\varepsilon_0 R$

36. The potential of conductor 2 is
(1) $-Q/8\pi\varepsilon_0 R$ (2) $Q/8\pi\varepsilon_0 R$
(3) $Q/32\pi\varepsilon_0 R$ (4) $-Q/32\pi\varepsilon_0 R$

For Problems 37–39

Three concentric spherical conductors A, B, and C of radii r, $2R$, and $4R$, respectively. A and C is shorted and B is uniformly charged (charge+Q).



37. Charge on conductor A is
(1) $Q/3$ (2) $-Q/3$
(3) $2Q/3$ (4) $-2Q/3$

38. Potential at A is

(1) $Q/4\pi\varepsilon_0 R$ (2) $Q/16\pi\varepsilon_0 R$

(3) $Q/20\pi\varepsilon_0 R$ (4) $5Q/48\pi\varepsilon_0 R$

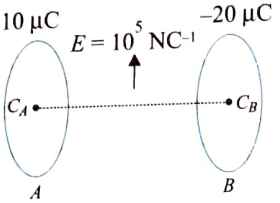
39. Potential at B is

(1) $Q/4\pi\varepsilon_0 R$ (2) $Q/16\pi\varepsilon_0 R$

(3) $Q/48\pi\varepsilon_0 R$ (4) $5Q/48\pi\varepsilon_0 R$

For Problem 40–41

Two circular rings A and B each of radius $a = 30$ cm are placed coaxially with their axis horizontal in a uniform electric field $E = 10^5$ NC^{-1} directed vertically upward as shown in figure. Distance between centers of the rings A and B (C_A and C_B) is 40 cm. Ring A has positive charge $q_A = 10\,\mu$C and B has a negative charge $q_B = -20\,\mu$C. A particle of mass m and charge $q = 10\,\mu$C is released from rest at the center of ring A. If particle moves along C_AC_B, then



40. Work done by electric field, when particle moves from C_A to C_B is

(1) –1.2 J (2) 1.2 J

(3) –3.6 J (4) 3.6 J

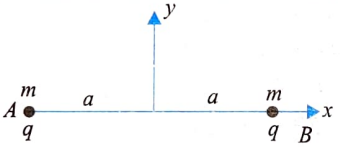
41. Speed of particle when it reaches at center of B is

(1) $6\sqrt{2}$ m/s (2) $12\sqrt{2}$ m/s

(3) $2\sqrt{6}$ m/s (4) $4\sqrt{6}$ m/s

For Problems 42–44

Two point charges are placed on x-axis as shown. $a = 1$ cm, $q = 1\,\mu$C, mass of each particle $m = 6$ g. The charges are tied at the end of an inextensible string of length $2a$. The whole system is free to move on a horizontal frictionless surface.



42. What is the tension in the string?

(1) 11.25 N (2) 22.5 N

(3) 45 N (4) 90 N

43. Now suppose a third charge of equal value $+q$ is fixed at the point $(a/2, 0)$ and the system of charges A and B is released. There is no friction between third charge and string. Find the maximum velocity of the system of charges A and B.

(1) 2 ms^{-1} (2) 10 ms^{-1}

(3) 10 cm s^{-1} (4) 5 ms^{-1}

44. What is the tension in the string when velocity is maximum.

(1) 45 N (2) 67.5 N

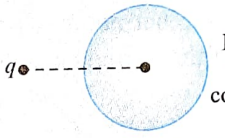
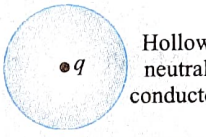
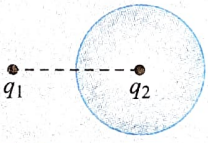
(3) 90 N (4) 112.5 N

Matrix Match Type

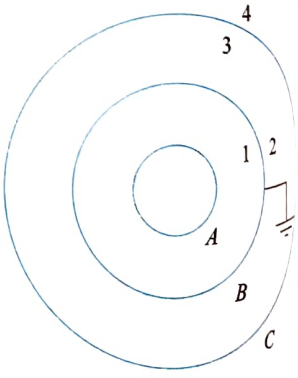
1. Match the columns

Column I	Column II
i. Electrically neutral thick conducting spherical shell, with a point charge at its center.	a. Electric potential at the center of the cavity due to charges induced on the inner and outer surfaces of the conductor is zero.
ii. Electrically neutral thick conducting spherical shell, with a point charge inside it and to the right of its center	b. Electric field everywhere in the cavity due to charges induced on the inner surface of conductor is zero.
iii. Electrically neutral thick conducting spherical shell, with a point charge at its center. Shell is earthed.	c. Electric field everywhere inside the cavity due to charges induced on the outer surface of the conductor is zero or induced charge on the outer surface is zero.
iv. Electrically neutral thick conducting spherical shell, with a point charge inside it and to the right of its center. Shell is earthed.	

2. Match the entries of Column I with the entries of Column II.

Column I	Column II
i. 	a. $\vec{E}$ inside the conductor is zero
ii. 	b. $\lvert\vec{E}\rvert$ inside the conductor is constant but not zero
iii. 	c. $\lvert\vec{E}\rvert$ inside the conductor is varying
	d. Potential inside the conductor is same as that of conductor
	e. Potential inside the conductor varies

3. Figure shows three concentric thin spherical shells A, B, and C of radii R, $2R$, and $3R$, respectively. The shell B is earthed and A and C are given charges q and $2q$, respectively. If the charge on surfaces 1, 2, 3, and 4 are q_1, q_2, q_3, and q_4, respectively, then match the following columns.



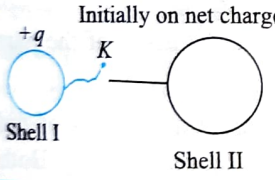
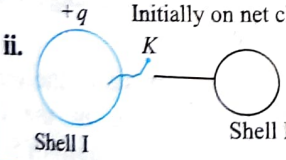
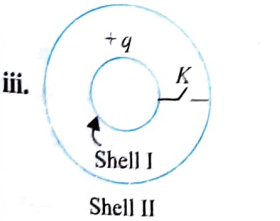
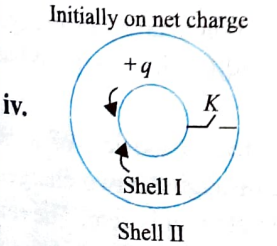
Column I	Column II
i. q_1	a. $\frac{2}{3}q$
ii. q_2	b. $\frac{4}{3}q$

iii. q_3	c. $-\frac{4}{3}q$
iv. q_4	d. $-q$

4. A solid conducting sphere of radius a is placed inside a conducting shell of radius b so that both are concentric. Now, the shell is given a charge Q. Match the following.

Column I	Column II
i. Charge appearing on the inner sphere	a. zero
ii. Charge appearing on the inner sphere after it is earthed	b. Q
iii. Electric field intensity (E) inside the inner sphere (before earthing)	c. Qa/b
iv. Electric field intensity (E) inside the inner sphere after it is earthed	d. $\frac{1}{4\pi\varepsilon_0}\frac{Q}{ab}$

5. In the following table, Column I gives certain situations involving two thin conducting shells connected by a conducting wire via key K. In all situations, one sphere has net charge $+q$ and other sphere has no net charge. After key K is pressed, Column II gives some resulting effect. Match the figures in Column I with the statements in Column II.

Column I	Column II
i. 	a. Charge flows through the connecting wire
ii. 	b. Potential energy of system of spheres decreases
iii. 	c. No heat is produced
iv. 	d. Sphere I has no charge after equilibrium is reached

6. An electric dipole is placed in a uniform external electric field. θ is the angle between the dipole moment and the field direction. In general, the dipole rotates under a torque. With reference to the behavior of the dipole in an electric field, match Column I with Column II.

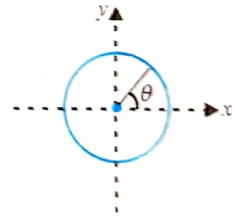
Column I	Column II
i. Potential energy of the dipole is maximum	a. conserved
ii. Angular acceleration of the dipole is maximum	b. not conserved
iii. Angular momentum of the dipole	c. $\theta = 180°$
iv. Kinetic energy of the dipole	d. $\theta = 90°$

7. A certain electric field is given as

$$E = -[(2xy + z^2)\hat{i} + (2yz + x^2)\hat{j} + (2zx + y^2)\hat{k}]$$

Column I	Column II
i. Work done by electric field in taking a unit charge from (0, 0, 0) to (3, 4, 0) along a straight line is	a. −36 units
ii. Work done by electric field in taking a charge from (3, 4, 0) to (0, 0, 0) along a straight line is	b. +36 units
iii. Work done by external agent in taking a charge from (3, 4, 0) to (6, 8, 0) without change in KE is	c. zero unit
iv. Net charge enclosed in a sphere of radius 5 units centered at the origin is	d. +252 units

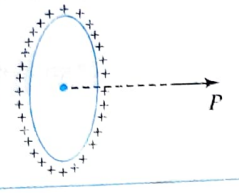
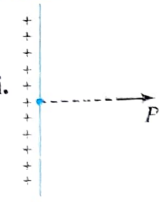
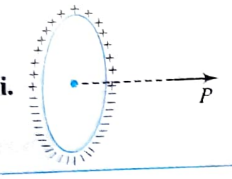
8. A nonconducting ring has linear charge density λ. Match the following column regarding this ring.



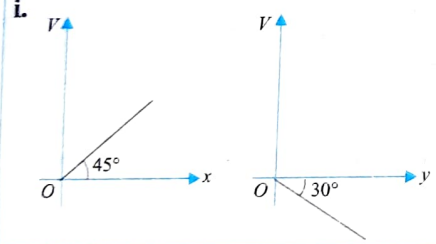
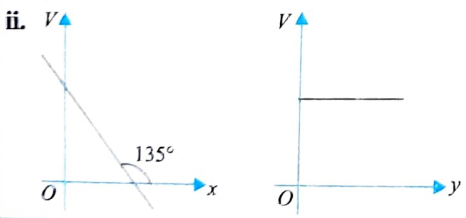
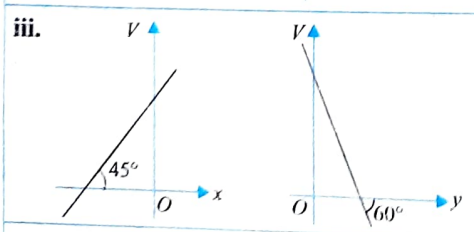
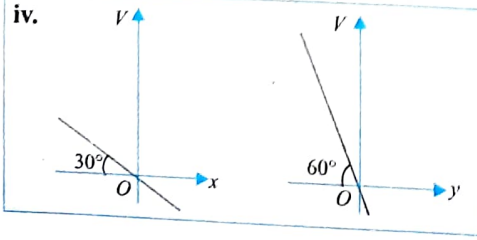
Column I	Column II
i. $\lambda = \lambda_0$ Cm^{-1}	a. Electric field at center of the ring is in the direction of $(-\hat{j})$.
ii. $\lambda = \lambda_0 \cos\theta$ Cm^{-1}; $0 \le \theta \le \pi$	b. Electric field at center of the ring is zero.
iii. $\lambda = \lambda_0 \sin\theta$ Cm^{-1}; $0 \le \theta \le 2\pi$	c. Electrostatic potential at center of the ring is zero.
iv. $\lambda = \lambda_0 (1 - \cos 2\theta)$ Cm^{-1}; $0 \le \theta \le 2\pi$	d. Electric field at center of the ring is in the direction $(-\hat{i})$.

9. A short dipole of dipole moment P is kept near different sources of electric field (an infinite line charge, a point charge, a ring charge, etc) in Column I. The force F

experienced by the dipole, the torque t acting on the dipole and the potential energy U of dipole are given in Column II.

Column I	Column II
i. 	a. $F = 0$
ii. 	b. $\tau = 0$
iii. 	c. $F \neq 0$
iv.	d. $U < 0$

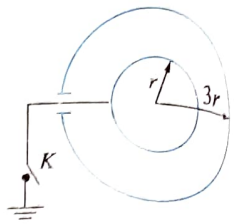
10. Column I shows graphs of electric potential V versus x and y in a certain region for four situations. Column II gives the angle that the electric field vector makes with positive x-direction.

Column I	Column II
i. 	a. 0°
ii. 	b. $\tan^{-1}(3)$
iii. 	c. 120°
iv. 	d. 150°

Numerical Value Type

1. There are 27 drops of a conducting fluid. Each drop has radius r, and each of them is charged to the same potential V_1. They are then combined to form a bigger drop. The potential of the bigger drop is V_2. Find the ratio V_2/V_1. Ignore the change in density of the fluid on combining the drops.

2. Figure shows two conducting thin concentric shells of radii r and $3r$. The outer shell carries charge q and the inner shell is neutral. The amount of charge that flows from the inner shell to the earth after the key K is closed is equal to $(1/n)^{th}$ of the charge on the outer shell. What is the value of n?

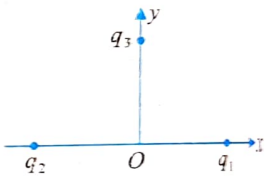


3. A charge of 5 μC is placed at the center of a square $ABCD$ of side 10 cm. Find the work done (in μJ) in moving a charge of 1 μC from A to B.

4. The linear charge density on a dielectric ring of radius R varies with θ as $\lambda = \lambda_0 \cos\theta/2$, where λ_0 is constant. Find the potential at the center O of the ring [in volt].

5. The point charges $-2q$, $-2q$, and $+q$ are put on the vertices of an equilateral triangle of side a. Find the work done by some external force in increasing the separation to $2a$ (in joules).

6. Two points charges $q_1 = q_2 = 2$ μC are fixed at $x_1 = +3$ m and $x_2 = -3$ m as shown. A third particle of mass 1 g and charge $q_3 = -4$ μC is released from rest at $y = 4$ m. Find the speed (in terms of the nearest integer in ms^{-1}) of the particle as it reaches the origin.



7. Two concentric spherical conducting shells of radii R and $2R$ are carrying charges q and $2q$, respectively. Both are now connected by a conducting wire. Find the change in electric potential (in V) on the outer shell.

8. A uniform electric field of 10 NC^{-1} exists in the vertically downward direction. Find the increase in the electric potential (in V) as one goes up through a height of 50 cm.

9. The kinetic energy of a charged particle decreases by 10 J as it moves from a point at potential 100 V to a point at potential 200 V. Find the charge on the particle (in × 10^{-1} C).

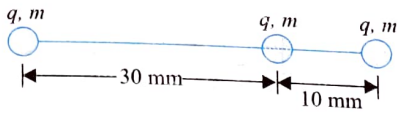
10. The electric field strength depends only on the x, y, and z coordinates according to the law $E = \dfrac{a(x\hat{i} + y\hat{j} + z\hat{k})}{(x^2 + y^2 + z^2)^{3/2}}$, where $a = 280$ Nm^2C^{-1} is a constant. If the potential difference between (3, 2, 6) and (0, 3, 4) is x^2. What is the value of x.

11. A positive charge $+q_1$ is located to the left of a negative charge $-q_2$. On a line passing through the two charges, there are two places where the total potential is zero. The reference is assumed to be at infinity. The first place is between the charges and is 4.00 cm to the left of the negative charge. The

second place is 7.00 cm to the right of the negative charge. If $q_2 = -12\ \mu C$ and $q_1 = 11 \times x\ \mu C$, what is the value of charge x.

12. A positively charged particle starts at rest 25 cm from a second positively charged particle, which is held stationary throughout the experiment. The first particle is released and accelerates directly away from the second particle. When the first particle has moved 25 cm, it has reached a velocity of $10\sqrt{2}\ ms^{-1}$. What is the maximum velocity (in $\times 10\ ms^{-1}$) that the first particle will reach?

13. Two small spheres are attached to the ends of a long light nonconducting rod

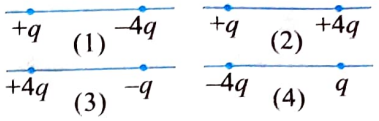


at a distance 40 mm from each other. A third, middle sphere can slide along the rod without friction as shown. All three spheres are nonconducting, have identical masses $m = 1$ g, and a positive charge $q = 1$ C is distributed evenly on the surface of each sphere. The whole system is placed on a horizontal frictionless nonconducting surface. Initially, all three spheres are at rest and the middle sphere is located a distance 30 mm from one of the ends of the rod and a distance10 mm from the other. Find the maximum speed v (in ms^{-1}) of the middle sphere after the system is released.

14. Three point charges of 0.1 C each are placed at the corners of an equilateral triangle with side $L = 1$ m. If this system is supplied energy at the rate of 1 kW, how much time (in hour) will be required to move one of the charges to the midpoint of the line joining the other two?

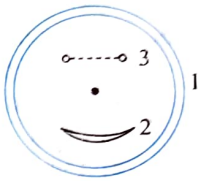
15. A radioactive source in the form of a metal sphere of radius 10^{-2} m emits beta particles (electrons) at the rate of 5×10^{10} particles per second. The source is electrically insulated. How long will it take (in μs) for its potential to be raised by 2 V assuming that 40% of the emitted beta particles escape from the source.

16. The figure shows four situations in which charges as indicated ($q > 0$) are fixed on an axis. How many situations is there a point to the left of the charges where an electron would be in equilibrium?



17. An electric field is given by $\vec{E} = (y\hat{i} + x\hat{j})\frac{N}{C}$. Find the work done (in J) in moving a 1C charge from $\vec{r}_A = (2\hat{i} + 2\hat{j})$m to $\vec{r}_B = (4\hat{i} + \hat{j})$m

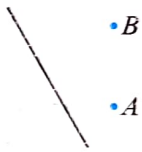
18. The arrangement shown consists of three elements.



   a. A thin rod of charge −3.0 μC that forms a full circle of radius 6.0 cm.
   b. A second thin rod of charge 2.0 μc that forms a circular arc of radius 4.0 cm and concentric with the full circle, subtending an angle of 90° at the centre of the full circle.
   c. An electric dipole with a dipole moment that is perpendicular to a radial line and has magnitude 1.28×10^{-21} C –m

   Find the net electric potential in volts at the centre.

19. An infinite plane of charge with $\sigma = 2\epsilon_0 \frac{C}{m^2}$ is tilted at a 37° angle to the vertical direction as shown below. Find the potential difference, $V_A - V_B$ in volts, between points A and B at 5 m distance apart. (where B is vertically above A).



20. A small sphere of mass m = 0.5 kg carrying a positive charge $q = 110\ \mu C$ is connected with a light, flexible and inextensible string of length $r = 60$ cm and whirled in a vertical circle. If a vertically upward electric field of strength $E = 105\ NC^{-1}$ exists in the space, calculate minimum velocity of sphere(in m/s) required at highest point so that it may just complete the circle. ($g = 10\ ms^{-2}$).

Archives

JEE Main

Single Correct Answer Type

1. This question contains Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement 1: For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement 2: The net work done by a conservative force on an object moving along a closed loop is zero.

(a) Statement 1 is true, statement 2 is false.
(b) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.
(c) Statement 1 is true, statement 2 is true; statement 2 is not the correct explanation for statement 1.
(d) Statement 1 is false, statement 2 is true.

(AIEEE 2009)

2. Two points P and Q are maintained at the potentials of 10 V and – 4 V, respectively. The work done in moving 100 electrons from P to Q is

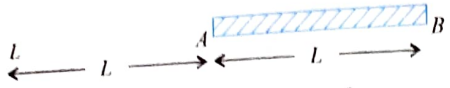
(1) -19×10^{-17} J (2) 9.60×10^{-17} J
(3) -2.24×10^{-16} J (4) 2.24×10^{-16} J

(AIEEE 2009)

3. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is

(1) $-24\pi a\epsilon_0 r$ (2) $-6\pi a\epsilon_0 r$
(3) $-24\pi a\epsilon_0$ (4) $-6a\epsilon_0$ **(AIEEE 2011)**

4. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is



(1) $\frac{3Q}{4\pi\varepsilon_0 L}$ (2) $\frac{Q}{4\pi\varepsilon_0 L \ln 2}$

(3) $\frac{Q \ln 2}{4\pi\varepsilon_0 L}$ (4) $\frac{Q}{8\pi\varepsilon_0 L}$ **(JEE Main 2013)**

5. Assume that an electric field $\vec{E} = 30x^2\,\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at $x = 2$ m is

(1) –80 J (2) 80 J

(3) 120 J (4) –120 J **(JEE Main 2014)**

6. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

(1) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

(2) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

(3) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

(4) $2R < R_4$ **(JEE Main 2015)**

7. Three concentric metal shells A, B and C of respective radii a, b and c $(a < b < c)$ have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is

(1) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2 - c^2}{c} + a\right]$ (2) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{a} + c\right]$

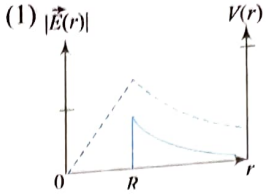
(3) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$ (4) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2 - c^2}{b} + a\right]$

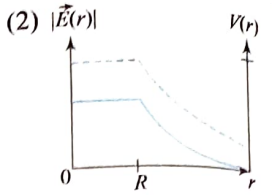
(JEE Main 2018)

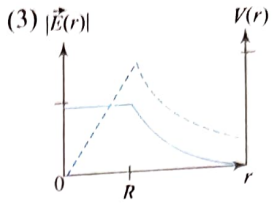
JEE ADVANCED

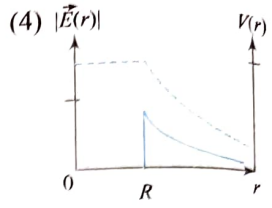
Single Correct Answer Type

1. Consider a thin spherical shell of radius R with its center at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential $V(r)$ with the distance r from the center, is best represented by which graph? **(IIT-JEE 2012)**

(1) 

(2) 

(3) 

(4) 

2. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical just after release. Then X is nearly **(IIT-JEE 2012)**

(1) 1×10^{-5} V (2) 1×10^{-7} V

(3) 1×10^{-9} V (4) 1×10^{-10} V

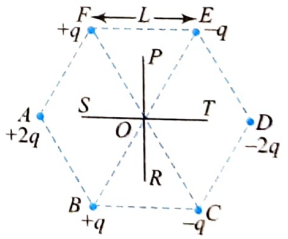
Multiple Correct Answers Type

1. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B $(< R_A)$ are kept far apart and each is given charge Q. Now they are connected by a thin metal wire. Then **(IIT-JEE 2011)**

(1) $E_A^{\text{inside}} = 0$ (2) $Q_A > Q_B$

(3) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$ (4) $E_A^{\text{on surface}} < E_B^{\text{on surface}}$

2. Six point charges are kept at the vertices of a regular hexagon of side L and center O, as shown in figure. Given that $K = q/4\pi\varepsilon_0 L^2$, which of the following statement(s) is are correct? **(IIT-JEE 2012)**



(1) The electric field at O is $6K$ along OD.

(2) The potential at O is zero.

(3) The potential at all points on the line PR is same.

(4) The potential at all points on the line ST is same.

Answers Key

EXERCISES

Single Correct Answer Type

1. (4)	2. (2)	3. (4)	4. (4)	5. (4)
6. (2)	7. (2)	8. (2)	9. (1)	10. (2)
11. (4)	12. (2)	13. (1)	14. (4)	15. (1)
16. (4)	17. (3)	18. (3)	19. (1)	20. (3)
21. (4)	22. (4)	23. (3)	24. (3)	25. (3)
26. (1)	27. (2)	28. (2)	29. (2)	30. (1)
31. (3)	32. (1)	33. (1)	34. (1)	35. (1)
36. (1)	37. (3)	38. (2)	39. (4)	40. (4)
41. (2)	42. (1)	43. (1)	44. (3)	45. (2)
46. (4)	47. (3)	48. (2)	49. (1)	50. (1)
51. (3)	52. (1)	53. (1)	54. (1)	55. (2)
56. (4)	57. (1)	58. (1)	59. (3)	60. (2)
61. (3)	62. (3)	63. (2)		

Multiple Correct Answers Type

1. (1),(4)	2. (1),(4)
3. (1),(2),(3)	4. (1),(4)
5. (2),(4)	6. (1),(4)
7. (1),(2),(3),(4)	8. (1),(3)
9. (1),(3),(4)	10. (1),(2),(3)
11. (1),(3)	12. (1),(2)
13. (2),(3),(4)	14. (1),(2),(3)
15. (3),(4)	16. (1),(3),(4)
17. (1),(2),(3)	18. (2),(3),(4)
19. (1),(2)	20. (1),(2),(3),(4)
21. (1),(3)	22. (2),(3)
23. (1),(3)	24. (2),(4)
25. (1),(2),(3)	26. (1),(3)
27. (1),(3)	

Linked Comprehension Type

1. (4)	2. (4)	3. (4)	4. (2)	5. (3)
6. (2)	7. (1)	8. (3)	9. (3)	10. (1)
11. (2)	12. (1)	13. (3)	14. (4)	15. (1)
16. (2)	17. (4)	18. (1)	19. (1)	20. (4)
21. (2)	22. (1)	23. (2)	24. (3)	25. (3)
26. (3)	27. (1)	28. (1)	29. (4)	30. (2)
31. (4)	32. (3)	33. (2)	34. (4)	35. (3)
36. (2)	37. (2)	38. (2)	39. (4)	40. (4)
41. (1)	42. (2)	43. (2)	44. (4)	

Matrix Match Type

1. i. → b., c.; ii. → c.; iii. → b., c.; iv. → c.
2. i. → a., d.; ii. → c., e.; iii. → c., e.
3. i. → d.; ii. → c.; iii. → b.; iv. → a.
4. i. → a.; ii. → c.; iii. → a.; iv. → a.
5. i. → a., b.; ii. → a., b.; iii. → a., b., d.; iv. → c., d.
6. i. → c.; ii. → d.; iii. → b.; iv. → b.
7. i. → a.; ii. → b.; iii. → d.; iv. → c.
8. i. → b.; ii. → c, d.; iii. – a, c.; iv. → b.
9. i. → b, c, d.; ii. → b, c, d.; iii. → c.; iv. → c.
10. i. → d.; ii. → a.; iii. → c.; iv. → b.

Numerical Value Type

1. (9)	2. (3)	3. (0)	4. (0)	5. (0)
6. (6)	7. (0)	8. (5)	9. (1)	10. (4)
11. (4)	12. (2)	13. (20)	14. (50)	15. (700)
16. (2)	17. (0)	18. (0)	19. (3)	20. (6)

ARCHIVES

JEE Main

Single Correct Answer Type

1. (3)	2. (4)	3. (4)	4. (3)	5. (None)
6. (3, 4)	7. (3)			

JEE Advanced

Single Correct Answer Type

1. (4) 2. (3)

Multiple Correct Answers Type

1. (1),(2),(3),(4) 2. (1),(2),(3)

4 Capacitor and Capacitance

INTRODUCTION

Capacitor (or condensor) is a device that stores (or condenses) electrostatic field energy. Capacitors provide temporary storage of energy in circuits and can be made to release the energy when required. The property of a capacitor that characterizes its ability to store energy is called its capacitance. When energy is stored in a capacitor, an electric field exists within the capacitor. The stored energy can be associated with the electric field. Indeed, energy can be associated with the existence of any electric field. The study of capacitors and capacitance leads us to an important aspect of an electric field, that is, its energy.

CAPACITANCE

ISOLATED CONDUCTOR

An electrical conductor (such as metals) holds its charge at its surface. When an isolated spherical conductor of radius R is given with a charge Q, its potential is given by

$$V = \frac{KQ}{R}$$

This tells us that the potential of an isolated spherical conductor is directly proportional to its charge. This holds good for an isolated conductor of any shape and size.

Since $V \propto Q$, the ratio of the charge given to an isolated conductor and its potential rise, that is, Q/V is a constant, which is defined as the capacitance of the conductor denoted by C. So

$$C = Q/V$$

If $V = 1$ V, $C = Q$. Hence, we define the capacitance of an isolated conductor as the charge required to rise the potential of the conductor by 1 V. In the SI system, the unit of capacitance is farad.

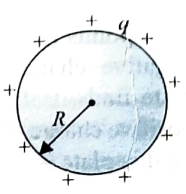
$$1(\text{F}) = \frac{1 \text{ coulomb (C)}}{1 \text{ volt (V)}} = 1 \text{ CV}^{-1}$$

In practice, the following smaller units of capacitance are also used:

1 microfarad (μF) = 10^{-6} farad
1 micro microfarad (μμF) = 10^{-12} farad
1 μμF is also known as 1 picofarad (pF)

CAPACITANCE OF A SPHERICAL CONDUCTOR OR CAPACITOR

As we have already said that a single conductor can also act as a capacitor, here we will find the capacitance of a single isolated sphere. For this, let a charge q be given to a spherical conductor of radius R, then potential on it is



$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

The other conductor is supposed to be at infinity, whose potential will be taken as zero. So the potential difference between the sphere and the conductor at infinity becomes $V - 0 = V$.

Then capacitance is

$$C = \frac{q}{V} = 4\pi\varepsilon_0 R$$

Thus, the capacitance of a spherical conductor is $C = 4\pi\varepsilon_0 R$.

Note: The earth is a spherical conductor of radius $R = 6.4 \times 10^6$ m. The capacitance of earth is

$$C = \left(\frac{1}{9 \times 10^9}\right)(6.4 \times 10^6) \approx 711 \times 10^{-6} \text{ F} = 711 \text{ μF}$$

Therefore, we can say that farad is a big unit. Thus, no body in the universe can have a capacitance of 1 F.

ILLUSTRATION 4.1

Two uniformly charged spherical drops at potential V coalesce to form a larger drop. If capacity of each smaller drop is C then find capacity and potential of larger drop.

Sol. When drops coalesce to form a larger drop then total charge and volume remains conserved. If r is radius and q is charge on smaller drop then $C = 4\pi\varepsilon_0 r$ and $q = CV$

Equating volume we get $\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3} r$

Capacitance of larger drop $C' = 4\pi\varepsilon_0 R = 2^{1/3} C$

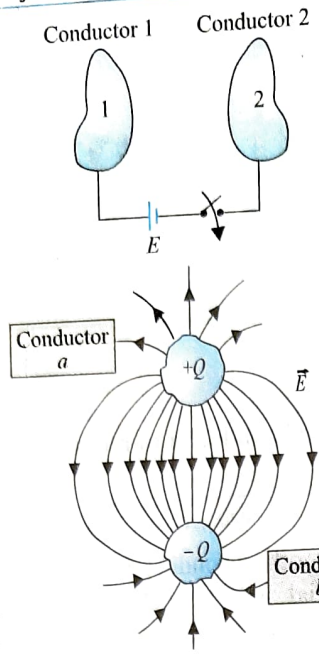
Charge on larger drop $Q = 2q = 2CV$

Potential of larger drop $V' = \frac{Q}{C'} = \frac{2CV}{2^{1/3}C} = 2^{2/3}V$

SYSTEM OF CONDUCTORS

Let us bring two conductors (called plates or electrodes) and connect them with the two terminals of a battery. The battery pulls some electrons from conductor 1 and injects these to conductor 2 simultaneously. As a result, conductor 1 will get a positive charge $+Q$, say, and hence conductor 2 will receive a negative charge $-Q$. Then, all E lines coming from conductor 1 will terminate at conductor 2. In this way, the total field and field energy will be confined in the space between the conductors.

When we bring a negatively charged conductor 1 near a positively charged conductor 2, its potential decreases. It means that more charge is required to raise the potential of the conductor 1 by 1 V. In other words, the capacitance of each conductor increases in the presence of the others. Hence, in a system of two conductors, each conductor can have (store) more charge than when they are isolated.



The potential difference between the conductors is $V = V_+ - V_-$, where V_+ and V_- are the potentials of the positive and the negative conductors, respectively. Since, $|V_+|$ and $|V_-|$ vary linearly with the charge Q, we have $V \propto Q$.

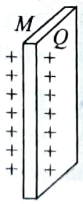
Then the ratio of Q and V, that is, Q/V is a constant, which can be defined as the capacitance of the system of two conductors. The ratio of the amount of charge supplied by the battery to each conductor and the potential difference between the conductors is defined as the capacitance of the system of two conductors. Thus,

$$C = \frac{Q}{V_+ - V_-} = \frac{\text{Charge flown through the battery}}{\text{Rise in potential difference}}$$

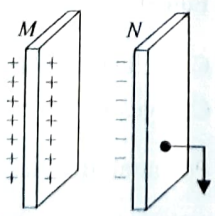
Note: The capacitance can be increased by reducing potential keeping the charge constant. Consider a conducting plate M which is given a charge Q such that its potential rises to V then the capacitance of the plate is given by

$$C = \frac{Q}{V}$$

If we place another identical conducting plate N parallel to it such that charge is induced on plate N (as shown in figure) and if N is earthed from the outer side (see figure) then the potential difference across the plates $V' = V_+ - V_-$. Then new capacitance of the system of plates,



$$C' = \frac{Q}{V'} = \frac{Q}{V_+ - V_-} \Rightarrow C' >> C$$

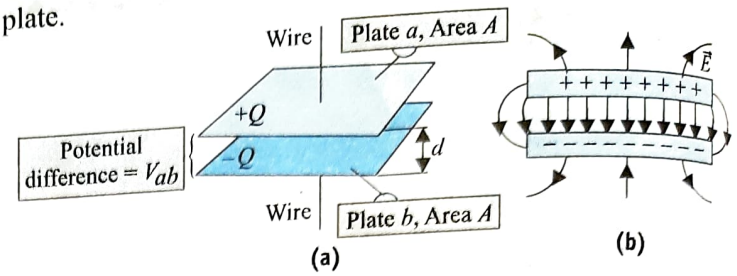


It means if an identical earthed conductor is placed in the vicinity of a charged conductor then the capacitance of the charged conductor increases appreciable. This is the principle of a parallel plate capacitor.

PARALLEL PLATE CAPACITOR

A parallel plate capacitor consists of two large plates placed parallel to each other with a separation d smaller in comparison to the length and breadth of the plates.

In an ideal capacitor, electric field resides in the region within the plates. No electric field is outside the plates (neglecting fringing effect for ideal case). So the entire energy resides within the capacitor and no energy is, therefore, outside the capacitor. Electric field is directed from the positive plate to the negative plate in such a way that the lines emerge perpendicularly from the positive plate and terminate perpendicularly to the negative plate.



(a) A charged parallel plate capacitor.
(b) When the separation of the plates is small compared to their size, the fringing of the electric field E at the edges is slight.

Electric field between the plates is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

Therefore, potential difference between the plates is

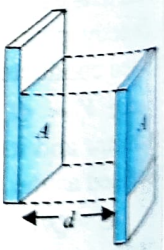
$$V = Ed = \frac{Qd}{A\varepsilon_0}$$

and capacitance is

$$C = \frac{Q}{V_+ - V_-} = \frac{\varepsilon_0 A}{d}$$

Important Points:

- If we place a dielectric slab completely fills the space between the plates of parallel plate capacitor the capacitance becomes $C = \frac{\varepsilon_0 KA}{d}$; where K = dielectric constant of the slab.
- If the plates of a parallel plate capacitor are not facing each other completely or if one of the plates of parallel plate capacitor slides relatively than the capacitance decrease as overlapping area decreases as shown in figure.



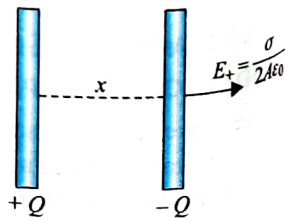
- The electric field between the plates of a capacitor is shown in figure. Non-uniformity of electric field at the boundaries of the plates is negligible if the separation between the plates is very small as compared to the length of the plates.

FORCE BETWEEN THE PLATES OF A PARALLEL PLATE CAPACITOR

Consider a parallel plate capacitor with plate area A. Suppose a positive charge $+Q$ is given to one plate and a negative charge $-Q$ to the other plate. The electric field due to the positive plate is

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

at all points if the plate is large. The negative charge $-Q$ on the other plate finds itself in the field of this positive charge. Therefore, the force on this plate is

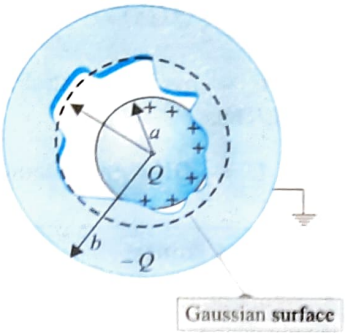


$$F = EQ = \frac{Q^2}{2A\varepsilon_0}$$

This force will be attractive.

SPHERICAL CAPACITOR

A spherical capacitor consists of two concentric spherical conducing shells of radii a and b, say $b > a$. The outer shell is earthed. Place a charge $+Q$ on the inner shell. It will reside on the outer surface of the shell. A charge $-Q$ will be induced on the inner surface of the outer shell. A charge $+Q$ will flow from the outer shell to earth.



Consider a Gaussian spherical surface of radius r such that $a < r < b$. From Gauss's law, the electric field at distance $r > a$ is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

The potential difference is

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{r} = -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

Since $V_b = 0$, we have

$$V_a = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

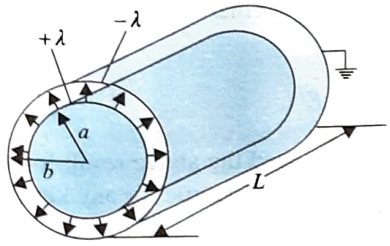
$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q(b-a)}{4\pi\varepsilon_0 ab}$$

Therefore, capacitance is

$$C = \frac{Q}{V_a - V_b} = \frac{Q}{V_a} = \frac{4\pi\varepsilon_0 ab}{b-a}$$

CYLINDRICAL CAPACITOR

A cylindrical capacitor consists of two coaxial cylinders of radii a and b, say $b > a$. The outer one is earthed. The cylinders are long enough so that we can neglect fringing of electric fields at the ends. The electric field at a point between the cylinders will be radial, and its magnitude will depend on the distance from the central axis. Consider a Gaussian surface of length y and radius r such that $a < r < b$. Flux through the plane surface is zero because the electric field and the area vector are perpendicular to each other.



In the long cylindrical capacitor, the linear charge density λ is assumed to be positive in this figure. The magnitude of charge in a length of either cylinder is λL.

For the curved part

$$\phi = \int \vec{E}\cdot d\vec{s} = \int E\,ds = E\int ds = E\,2\pi ry$$

Charge inside the Gaussian surface is

$$q = \frac{Qy}{L}$$

From Gauss's law

$$\phi = E2\pi ry = \frac{Qy}{L\varepsilon_0} \quad \text{or} \quad E = \frac{Q}{2\pi\varepsilon_0 Lr}$$

Potential difference is

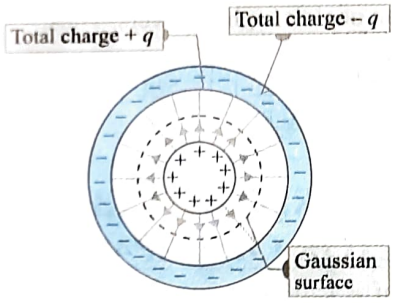
$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{r}$$

$$= -\int_a^b \frac{Q}{2\pi\varepsilon_0 Lr}\,dr = -\frac{Q}{2\pi\varepsilon_0 L}\int_a^b \frac{1}{r}\,dr$$

or $$V_a = \frac{Q}{2\pi\varepsilon_0 L}\ln\frac{b}{a} \qquad (\text{since } V_b = 0)$$

Capacitance is

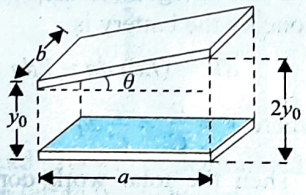
$$C = \frac{Q}{V_a - V_b} = \frac{Q}{V_a} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$



A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r (that encloses the positive plate).

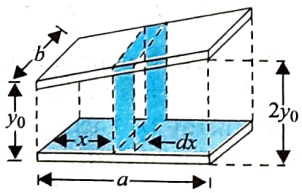
ILLUSTRATION 4.2

A capacitor has rectangular plates of length a and width b. The top plate is inclined at a small angle as shown in figure. The plate separation varies from $d = y_0$ at the left to $d = 2y_0$ at the right, where y_0 is much less than a or b. Calculate the capacitance of the system.



Sol. We consider a differential strip of width dx and length b to approximate a differential capacitor of area bdx and separation

$$d = y_0 + \left(\frac{y_0}{a}\right)x.$$



All such differential capacitors are in parallel arrangement. Thus,

$$dC = \frac{\varepsilon_0(bdx)}{y_0 + \left(\frac{y_0}{a}\right)x} \quad \text{or} \quad C = \int dC$$

or $$C = \varepsilon_0 b \int_0^a \frac{dx}{y_0 + \frac{y_0}{a}x}$$

$$= \frac{\varepsilon_0 b}{(y_0/a)}\left[\ln\left(\frac{y_0 + \frac{y_0}{a}\times a}{y_0}\right)\right] = \frac{\varepsilon_0 ab}{y_0}\ln 2$$

We can determine the expression for capacity in terms of θ as

$$d = (y_0 + x\tan\theta)$$

$$C = \int dC = \int_0^a \frac{\varepsilon_0 b\, dx}{(y_0 + x\tan\theta)}$$

$$= \frac{\varepsilon_0 b}{\tan\theta}\ln\frac{(y_0 + x\tan\theta)}{y_0}$$

For small θ,

$$\tan\theta \approx \theta \text{ or } C = \frac{\varepsilon_0 b}{\theta}\ln\left(1 + \frac{a\theta}{y_0}\right)$$

Now, we can use the expansion

$$\log(1+x) = x - \frac{1}{2}x^2 + \cdots$$

For $x < 1$, we can neglect higher powers. Thus,

$$C = \frac{\varepsilon_0 b}{\theta}\left[\frac{a\theta}{y_0} - \frac{1}{2}\left(\frac{a\theta}{y_0}\right)^2\right] = \frac{\varepsilon_0 ab}{y_0}\left[1 - \frac{a\theta}{2y_0}\right]$$

ENERGY STORED IN A CHARGED CONDUCTOR OR CAPACITOR

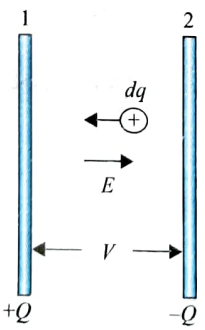
The charging of a capacitor is associated with a continuous pumping of positive charge from conductor 2 to conductor 1 by the battery (figure). If a charge $+dq$ is brought from conductor 2 to conductor 1 against the electric field E of the charged conductors, the work done by the battery is

$$dW = (Ed)\,dq = V dq$$

where $V = \frac{q}{C} = \frac{q}{C}dq$

Then the total work done by the battery in sending a charge Q is

$$W = \int dW = \int_0^Q \frac{q\,dq}{C} = \frac{Q^2}{2C}$$



A work $dW = q/C\,dq$ is done in shifting the charge dq from plate 2 to plate 1

The work is done at the expense of the chemical energy of the battery, which is stored in the capacitor in the form of electrostatic field energy U. Then, $U = Q^2/2C$. Putting $Q = CV$, the other two expressions of energy can be written as

$$U = \frac{1}{2}QV = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

ALTERNATIVE METHOD

A charged capacitor is a group of two charged conductors. The charge of one conductor is $Q_1 = +Q$ and potential $V_1 = V_+$, and the charge of the other conductor is $Q_2 = -Q$ and potential $V_2 = V_-$. As we know, the energy possessed by a charged conductor is

$$U = \frac{1}{2}qV$$

where q and V are the charge and potential of the conductor, respectively. Then the total energy possessed by the two conductors (plates) is

$$U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2$$

$$= \frac{1}{2}(+Q)(V_+) + \frac{1}{2}(-Q)(V_-) = \frac{Q}{2}(V_+ - V_-)$$

where $V_+ - V_- = V$ (potential difference between the conductor plates). Thus,

$$U = \frac{1}{2}QV \quad \text{(as obtained earlier)}$$

Important Points:

- Work done by battery, W_{Battery} = Charge supplied by battery × E.M.F. of the battery $QV = CV^2 = \frac{Q^2}{C}$

  But the energy stored in capacitor $U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

  it means 50% energy supplied by the battery is lost in form of heat.

  Hence heat developed in the circuit $H = \frac{Q^2}{2C}$ or $\frac{(\Delta Q)^2}{2C}$
- If many capacitors are connected in the circuit then heat developed in the circuit $H = \sum\frac{(\Delta Q)^2}{2C}$

ENERGY DENSITY IN A PARALLEL PLATE CAPACITOR

As derived earlier, the electrostatic energy stored in a parallel plate capacitor is $U = \frac{1}{2}CV^2$

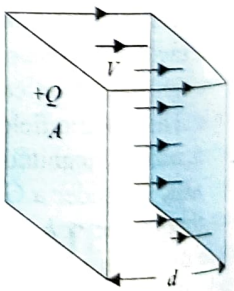
where $C = (\varepsilon_0 A/d)$ and $V = Ed$. So

$$U = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2 (Ad)$$

where $Ad = V$ (volume of the capacitor).

Then, $U/V = dU/dV$, that is, the energy stored per unit volume can be given by

$$\frac{dU}{dV} = u = \frac{1}{2}\varepsilon_0 E^2$$



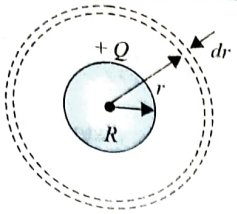
Field energy density inside the capacitor is $\frac{\varepsilon_0 E^2}{2}$

Although we have proved the above result for a parallel plate capacitor, but in general, this is true for any kind of capacitor or any other kind of electric field.

The potential energy of a spherical capacitor made of a single sphere can be found using the concept of energy density. Energy density is given by

$$u = \frac{1}{2}\varepsilon_0 E^2$$

$$\therefore\ dU = u\,dV = \frac{1}{2}\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\right)^2 4\pi r^2 dr$$



$$\text{or}\ U = \frac{1}{8\pi\varepsilon_0}Q^2\int_R^\infty \frac{dr}{r^2} = \frac{Q^2}{8\pi\varepsilon_0 R}$$

Note: It is a common misconception that electric field energy is a new kind of energy, different from the electric potential energy described before. It is simply a different way of interpreting electric potential energy. We can regard the energy of a given system of charges as being a shared property of all the charges, or we can think of the energy as being a property of the electric field that the charges create. Either interpretation leads to the same value of the potential energy.

ILLUSTRATION 4.3

A capacitor of capacitance C, which is initially uncharged, is connected with a battery of emf ε. Find the heat dissipated in the circuit during the process of charging.

Sol. Initial charge in the capacitor is $Q = 0$. When the capacitor is connected across battery, the final potential difference across the capacitor will be E. Final charge in the capacitor is $Q_f = C\varepsilon$. Initial potential energy stored is $U_i = 0$. Final potential energy stored is $U_f = (1/2)C\varepsilon^2$. So the charge supplied by the battery is

$$\Delta Q = Q_f - Q_i = (C\varepsilon - 0) = C\varepsilon$$

Work done by the battery in charging the capacitor is

$$W_{\text{battery}} = (\Delta Q)\,\varepsilon = (C\varepsilon)\varepsilon = C\varepsilon^2$$

We know that $W_{\text{battery}} = \Delta U$ + heat developed. So

$$H = H_{\text{batttery}} - \Delta U = C\varepsilon^2 - \frac{1}{2}C\varepsilon^2$$

$$\therefore\ \text{Heat developed} = \frac{1}{2}C\varepsilon^2$$

ILLUSTRATION 4.4

A capacitor of capacitance C, which is initially charged up to a potential difference ε, is connected with a battery of emf $\varepsilon/2$ such that the positive terminal of the battery is connected with the positive plate of the capacitor. After a long time

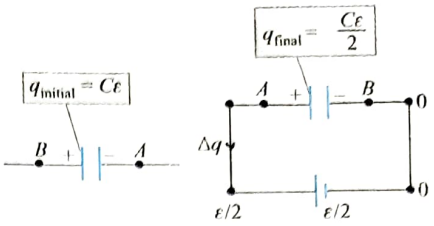
(i) find the total charge flow through the battery
(ii) find the total work done by the battery
(iii) Find the heat dissipated in the circuit during the process of charging

Sol.

(i) When the capacitor is connected to the battery of emf $\varepsilon/2$, the final charge in the capacitor is $C\varepsilon/2$. Charge flowing through the capacitor is the charge supplied by the battery

$$\Delta q = (q_{\text{final}} - q_{\text{initial}})$$
$$= (C\varepsilon/2 - C\varepsilon) = -C\varepsilon/2$$

(The charge is flowing from the capacitor to the battery.)



(ii) Work done by the battery is

$$W_{\text{battery}} = (\Delta q)\frac{\varepsilon}{2} = \left(-\frac{C\varepsilon}{2}\right)\frac{\varepsilon}{2} = -\frac{C\varepsilon^2}{4}$$

Change in the potential energy of the capacitor is $U_{\text{final}} - U_{\text{initial}}$. So

$$\Delta U = \frac{1}{2}C\left(\frac{\varepsilon}{2}\right)^2 - \frac{C\varepsilon^2}{2} = \frac{1}{8}C\varepsilon^2 - \frac{1}{2}C\varepsilon^2 = -\frac{3C\varepsilon^2}{8}$$

(iii) Work done by the battery = Change in potential energy of the capacitor + heat produced. So heat produced is

$$H = W_{\text{battery}} - \Delta U = \frac{3C\varepsilon^2}{8} - \frac{C\varepsilon^2}{4} = \frac{C\varepsilon^2}{8}$$

Approch 2: The heat produced, $H = \dfrac{(\Delta Q)^2}{2C} = \dfrac{(-C\varepsilon/2)^2}{2C} = \dfrac{C\varepsilon^2}{8}$

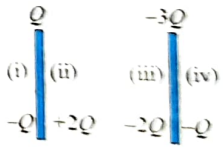
ILLUSTRATION 4.5

A parallel plate capacitor has capacitance C. If the charges of the plates are Q and $-3Q$, find the

(i) charges at the inner surfaces of the plates
(ii) potential difference between the plates
(iii) charge flown if the plates are connected
(iv) energy lost by the capacitor in (iii.)
(v) charge flown if any plate is earthed

Sol. The charge on surface (ii), $q_2 = \dfrac{Q - (-3Q)}{2} = 2Q$

(i) Hence the charges on the inner surfaces are $2Q$ and $-2Q$.



(ii) $V = \dfrac{\text{Charge of capacitor}}{\text{Capacitance}} = \dfrac{2Q}{C}$

(iii) If the plates are connected, $2Q$ and $-2Q$ will be neutralized, so $2Q$ charge will flow from the left to the right plate.

(iv) Energy lost = initial energy stored in capacitor

$$= \frac{(2Q)^2}{2C} = \frac{2Q^2}{C}$$

(v) If the left plate is earthed, $2Q$ charge flows from the ground to the left plate. If the right plate is earthed, $2Q$ charge flows from the ground to right plate.

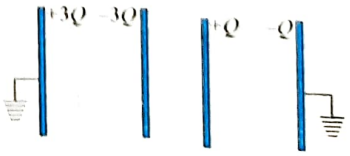
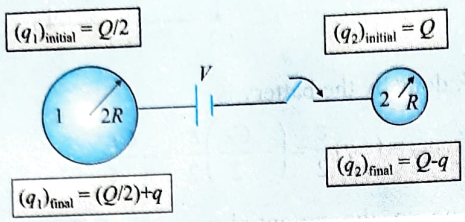


ILLUSTRATION 4.6

There are two spheres of radii R and $2R$ having charges Q and $Q/2$, respectively. These two spheres are connected with a cell of emf V volts as shown in figure. When the switch is closed, find the final charge on each sphere.



Sol. When the switch is closed, the potential difference between the spheres should be V. Let q charges flow from the sphere of radius R.

$$\frac{(q_1)_{final}}{C_1} = \frac{(q_2)_{final}}{C_2} = V$$

$$C_1 = 4\pi\varepsilon_0(2R),\ C_2 = 4\pi\varepsilon_0 R$$

Then

$$\frac{\left(\frac{Q}{2}+q\right)}{4\pi\varepsilon_0(2R)} - \frac{(Q-q)}{4\pi\varepsilon_0(R)} = V$$

or $$\frac{Q}{2} + q - 2Q + 2Q = 4\pi\varepsilon_0(2R)V$$

or $$q = \frac{8\pi\varepsilon_0 RV}{3} + \frac{Q}{2}$$

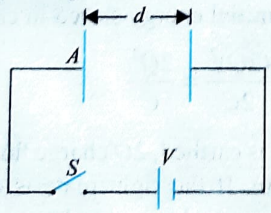
So the final charges on each of the spheres are

$$Q_1 = Q - q = \frac{-8\pi\varepsilon_0 RV}{3} + \frac{Q}{2}$$

and $$Q_2 = \frac{Q}{2} + q = Q + \frac{8\pi\varepsilon_0 RV}{3}$$

ILLUSTRATION 4.7

A parallel plate capacitor is connected across a battery of emf V as shown in figure. The plates of the capacitor have area A and separation between the plates is d. Now switch S is closed to connect the capacitors with battery. Now the distance between the plates is slowly reduced to $d/2$. Calculate the work done by the external agent in the process.



Sol. As the plates are moved slowly the charge flow from battery to the capacitor plates is very slow. Hence the energy dissipated must be zero.

The initial charge in capacitor, $q_{initial} = CV$

Where the capacitance is $C_{initial} = \frac{\varepsilon_0 A}{d} = C$

Initial energy stored in capacitor $U_{initial} = \frac{1}{2}CV^2$

Now the distance between the plates is slowly reduced to $\frac{d}{2}$

The capacitance will become $C_{final} = \frac{\varepsilon_0 A}{d/2} = 2C$

Final energy stored in capacitor $U_{final} = \frac{1}{2}(2C)V^2 = CV^2$

The final charge in capacitor, $q_{final} = 2CV$

Hence work done by battery, $W_{battery}$ = Charge supplied × E.M.F of the battery

$$\Rightarrow \quad W_{batrery} = (\Delta q)\cdot V = (2CV - CV)V = CV^2$$

Now applying conservation of energy principle

$$W_{battery} + W_{external} = \Delta U$$

$$CV^2 + W_{external} = \left(CV^2 - \frac{1}{2}CV^2\right)$$

$$\Rightarrow \quad W_{external} = -\frac{1}{2}CV^2$$

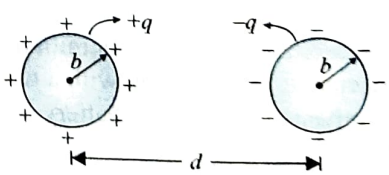
ILLUSTRATION 4.8

Two conducting sphere of radii a and b are placed at separation d ($d >> a$ and b) such that charge distribution on both the spheres remains spherically symmetric. If a charge $+q$ is provided to the sphere of radius a and $-q$ is given to the sphere of radius b. Find the electrostatic energy (U) of the system and calculate the capacitance of the system.

Sol. The electrostatic self-energy of two spheres,

$$U_1 = \frac{q^2}{8\pi\varepsilon_0 a} \text{ and } U_2 = \frac{q^2}{8\pi\varepsilon_0 b}$$

It is to be noted that electrostatic self-energy is always positive. We can write the interaction energy (U_{12}) of two spheres assuming them to be point charges.



$$U_{12} = \frac{q(-q)}{4\pi\varepsilon_0 d} = \frac{-q^2}{4\pi\varepsilon_0 d}$$

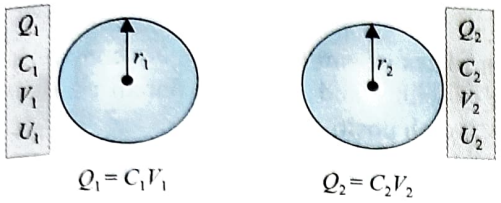
$\therefore$ Total energy of the system, $U = U_1 + U_2 + U_{12}$

$$\Rightarrow \quad U = \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right)$$

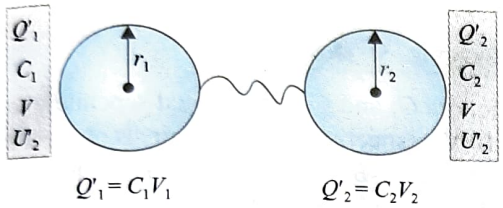
As, $$U = \frac{q^2}{2C} \text{ hence, } C = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right)}$$

SHARING OF CHARGE

When two conductors are joined together through a conducting wire, charge begins to flow from one conductor to another till both have the same potential. Due to the flow of charge, loss of energy also takes place in the form of heat.



(a)



(b)

Suppose there are two spherical conductors of radii r_1 and r_2 having charges Q_1 and Q_2, potentials V_1 and V_2, energies U_1 and U_2, and capacitances C_1 and C_2, respectively, as shown in figure. If these two spheres are connected through a conducting wire, then alteration of charge, potential, and energy takes place.

New charge: According to the conservation of charge $Q_1 + Q_2 = Q_1' + Q_2' = Q$ (say). Also

$$\frac{Q_1'}{Q_2'} = \frac{C_1V}{C_2V} = \frac{4\pi\varepsilon_0 r_1}{4\pi\varepsilon_0 r_2} \quad \text{or} \quad \frac{Q_1'}{Q_2'} = \frac{r_1}{r_2}$$

or $$1 + \frac{Q_1'}{Q_2'} = 1 + \frac{r_1}{r_2} \quad \text{or} \quad \frac{Q_1' + Q_2'}{Q_2'} = \frac{r_1 + r_2}{r_2}$$

or $$Q_2' = Q\left[\frac{r_2}{r_1 + r_2}\right]$$

Similarly

$$Q_1' = Q\left[\frac{r_1}{r_1 + r_2}\right]$$

Common potential:

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{Q_1' + Q_2'}{C_1 + C_2}$$

$$= \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Energy loss: As the electrical energies stored in the system before and after connecting the spheres are

$$U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

and $$U_f = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(C_1 + C_2)\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2$$

So energy loss is $$\Delta U = U_i - U_f = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

Capacity of a conductor is a constant term; it does not depend upon the charge (Q), potential (V), and nature of the material of the conductor.

ILLUSTRATION 4.9

Two conducting spheres of radii 6 cm and 12 cm each, having the same charge 3×10^{-8} C, are kept very far apart. If the spheres are connected to each other by a conducting wire, find

(i) the direction and amount of charge transferred and

(ii) final potential of each sphere

Sol.

(i) As equal charge is given to both the spheres, the potential $V (\propto q/R)$ of the smaller sphere will be higher and, hence, charge will flow from the smaller to the larger sphere when they are connected. Now, as charge is shared in proportion to capacity, and the capacity of the spherical conductor is proportional to its radius,

$$q_1' = \frac{R_1}{R_1 + R_2}q = \frac{6}{6+12}(3+3)\times 10^{-8} = 2\times 10^{-8}\ \text{C}$$

and $$q_2' = \frac{R_2}{R_1 + R_2}q = \frac{12}{6+12}(3+3)\times 10^{-8} = 4\times 10^{-8}\ \text{C}$$

So charge transferred is

$$(q_1 - q_1') = (q_2' - q_2) = 1\times 10^{-8}\ \text{C}$$

(ii) After sharing the final potential of each sphere,

$$V_1' = \frac{q_1'}{C_1} = \frac{q_1'}{4\pi\varepsilon_0 R_1} = \frac{9\times 10^9 \times 2\times 10^{-8}}{6\times 10^{-2}} = 3\ \text{kV}$$

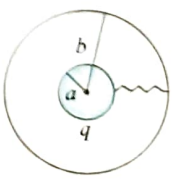
and $$V_2' = \frac{q_2'}{C_2} = \frac{q_2'}{4\pi\varepsilon_0 R_2} = \frac{9\times 10^9 \times 4\times 10^{-8}}{12\times 10^{-2}} = 3\ \text{kV}$$

So after sharing, both the conductors are at the same potential, i.e., $V_1' = V_2' = V = 3$ kV

ILLUSTRATION 4.10

A solid conducting sphere of radius 10 cm is enclosed by a thin metallic shell of radius 20 cm. A charge $q = 20\ \mu\text{C}$ is given to the inner sphere. Find the heat generated in the process. The inner sphere is connected to the shell by a conducting wire.

Sol. Before connection with the wire, the total electrical energy is $U_i = q^2/8\pi\varepsilon_0 a$; after connection with the wire, all charges are transferred to the outer sphere. So



$$U_f = \frac{q^2}{8\pi\varepsilon_0 b}$$

$$\therefore \quad \Delta H = U_i - U_f = \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$= \frac{(20\times 10^{-6})^2 \times 9\times 10^9}{2} \times \left(\frac{1}{10\times 10^{-2}} - \frac{1}{20\times 10^{-2}}\right)$$

$$= 9\ \text{J}$$

ILLUSTRATION 4.11

An isolated conductor, initially free from charge, is charged by repeated contacts with a plate, which after each contact has a charge Q due to some mechanism. If q is the charge on the conductor after the first operation, prove that the maximum charge that can be given to the conductor in this way is $Qq/(Q-q)$.

Sol. At each contact, the potential of both becomes same. At the first contact, charges on the conductor and the plate are $Q-q$ and q, respectively. So

$$V_1 = \frac{Q-q}{C_1} = \frac{q}{C_2} \quad \text{or} \quad \frac{C_1}{C_2} = \frac{Q-q}{q} \qquad ...(i)$$

In the second contact, the charge transferred to the plate is q_2. Then

$$V_2 = \frac{Q-q_2}{C_1} = \frac{q+q_2}{C_2} \text{ or } \frac{Q-q}{q} = \frac{Q-q_2}{q+q_2}$$

or $(q+q_2)(Q-q) = q(Q-q_2)$

or $qQ - q^2 + q_2Q - q_2q = qQ - qq_2$

or $q_2Q = q^2$

$\therefore \quad q_2 = \frac{q^2}{Q}$

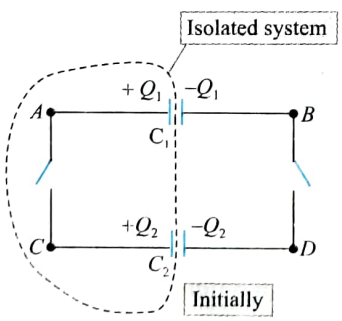
The total charge transferred in a large number of contacts is

$$q_{max} = q + \frac{q^2}{Q} + \frac{q^3}{Q^2} + \frac{q^4}{Q^3} + \cdots + \infty$$

$$= \frac{q}{1-\frac{q}{Q}} = \frac{qQ}{Q-q}$$

DISTRIBUTION OF CHARGES ON CONNECTING TWO CHARGED CAPACITORS

Two capacitors C_1 and C_2 are connected as shown in figure.



Common potential: By charge conservation of plates A and C before and after connection,

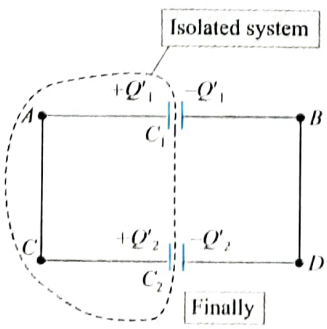
$$Q_1 + Q_2 = C_1V + C_2V$$

So common potential is

$$V = \frac{Q_1+Q_2}{C_1+C_2} = \frac{C_1V_1 + C_2V_2}{C_1+C_2}$$

$$= \frac{\text{Total charge}}{\text{Total capacitance}}$$

Also



$$Q_1' = C_1V = \frac{C_1}{C_1+C_2}(Q_1+Q_2)$$

$$Q_2' = C_2V = \frac{C_2}{C_1+C_2}(Q_1+Q_2)$$

Heat loss during redistribution:

$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$$

The loss of energy is in the form of heating in the wire.

Notes:

(i) When plates of similar charges are connected with each other (+ with + and − with −), put all values (Q_1, Q_2, V_1 and V_2) with positive sign.

(ii) When plates of opposite polarity are connected with each other (+ with −), take the charge and potential of one of the plates to be negative.

ILLUSTRATION 4.12

Two capacitors C_1 and C_2 are charged seperately to potentials 20 V and 10 V, respectively. The terminals of capacitors C_1 and C_2 are marked as (A-B) and (C-D), respectively. A is connected with C and B is connected with D.

(i) Find the final potential difference across each capacitors.

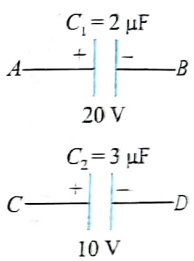
(ii) Find the final charge in both capacitors.

(iii) How much heat is produced in the circuit

Sol.

(i) Initial charge in capacitor C_1 is

$$(Q_1)_{initial} = C_1V_1$$
$$= 2 \times 20$$
$$= 40\ \mu C$$

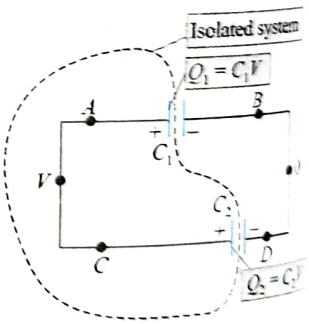


Initially capacitors C_1 and C_2 are separately charged

Initial charge in capacitor C_2 is

$$(Q_2)_{initial} = C_2V_2$$
$$= 3 \times 10$$
$$= 30\ \mu C$$

Let the potential of B and D be zero and the common potential difference across the capacitors be V, then the potentials at A and C will be V.

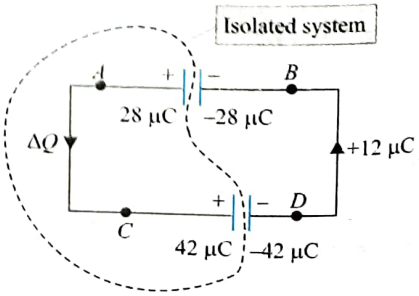


Finally capacitors C_1 and C_2 are connected together

From figure, it is clear that the left plates of capacitors C_1 and C_2 are forming an isolated system, i.e., they are not connected from outside. From charge conservation,

$$C_1V + C_2V = 3V + 2V = 40 + 30$$

$$5V = 70 \text{ or } V = 14\ V$$



Final charge in capacitor C_1 is

$(Q_1)_{final} = 2 \times 14 = 28\ \mu C$

Final charge in capacitor C_2 is

$(Q_2)_{final} = 3 \times 14 = 42\ \mu C$

The charge flowing in the circuit in the direction from A to C is

$\Delta Q = 40 - 28 = 12\ \mu C$

Now final charges on each plate are shown in figure.

(ii) Heat produced in the circuit is

$$H = \left[\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right] - \left[\frac{1}{2}(C_1 + C_2)V^2\right]$$

$$= \left[\frac{1}{2}\times 2\times(20)^2 + \frac{1}{2}\times 3\times(10)^2\right] - \left[\frac{1}{2}\times 5\times(14)^2\right]$$

$$= 400 + 150 - 490 = 550 - 490 = 60\ \mu J$$

ILLUSTRATION 4.13

If A is connected with D and B is connected with C, find the potential difference across each capacitor and the final charge in each capacitor.

Sol. Let the potential of B and C be zero and the common potential on the capacitors be V, then at A and D, it will be V. From charge conservation,

$C_1V + C_2V = 3V + 2V$

$= 40 - 30 = 10$

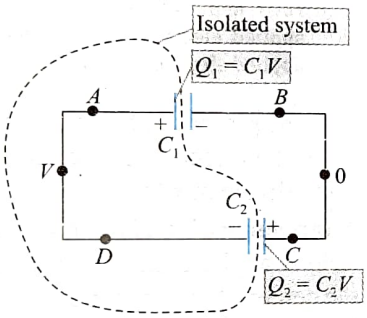
or $V = 2$ V

Final charge in capacitor C_1 is

$(Q_1)_{final} = 2 \times 2 = 4\ \mu C$

Final charge in capacitor C_2 is

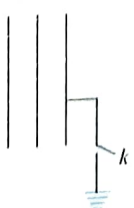
$(Q_2)_{final} = 3 \times 2 = 6\ \mu C$



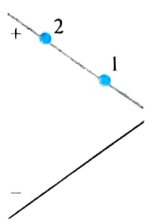
Finally capacitors C_1 and C_2 are connected together

CONCEPT APPLICATION EXERCISE 4.1

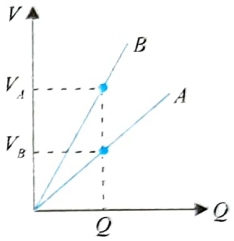
1. (a) How many excess electrons must be added to one plate and removed from the other to give a 5.00 nF parallel plate capacitor 25.0 J of stored energy?
   (b) How could you modify the geometry of this capacitor so that it can store 50.0 J of energy without changing the charge on its plates?
2. A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge $+Q$ is now given to its positive plate. Find the potential difference across the capacitor.
3. Three identical large metallic plates are placed parallel to each other at a very small separation as shown in figure. The central plate is given a charge Q. What amount of charge will flow to earth when the key is pressed?



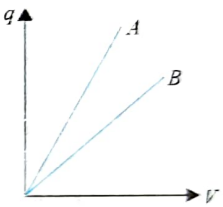
4. The plates of a plane capacitor are drawn apart keeping them connected to a battery. Next, the same plates are drawn apart from the same initial condition keeping the battery disconnected. In which case is more work done?
5. A parallel plate air capacitor is connected to a battery. If the plates of the capacitor are pulled farther apart, then state whether the following statements are true or false.



   (a) Strength of the electric field inside the capacitor remains unchanged, if the battery is disconnected before pulling the plates.
   (b) During the process, work is done by the external force applied to pull the plates irrespective of whether the battery is disconnected or not.
   (c) Electrostatic potential energy in the capacitor decreases if the battery remains connected.
6. Figure shows the variation of voltage V across the plates of two capacitors A and B versus increase in charge Q stored in them. Which of the capacitors has higher capacitance? Give reason for your answer.



7. Figure shows the variation of charge q versus the potential difference V for two capacitors C_1 and C_2. The two capacitors have the same plate separation, but the plate area of C_2 is double than that of C_1. Which of the lines in the figure corresponds to C_1 and C_2 and why?



8. A capacitor of capacitance C is charged to a potential difference V_0. The terminals of the charged capacitor are then connected to those of an uncharged capacitor of capacitance $C/2$.
   (a) Compute the original charge of the system.
   (b) Find the final potential difference across each capacitor.
   (c) Find the final energy of the system.
   (d) Calculate the decrease in energy when the capacitors are connected.
   (e) Where did the "lost" energy go?
9. A parallel plate vacuum capacitor with plate area A and separation x has charges $+Q$ and $-Q$ on its plates. The capacitor is disconnected from the source of charge, so the charge on each plate remains fixed.
   (a) What is the total energy stored in the capacitor?
   (b) The plates are pulled apart an additional distance dx. What is the change in the stored energy?
   (c) If F is the force with which the plates attract each other, then the change in the stored energy must equal the work $dW = Fdx$ done in pulling the plates apart. Find an expression for F.
   (d) Explain why F is not equal to QE, where E is the electric field between the plates?
10. A capacitor of capacitance C, which is initially charged up to a potential difference ε, is connected with a battery of emf ε such that the positive terminal of the battery is connected with the positive plate of the capacitor. Find the heat loss in the circuit during the process of charging.

11. In the above question, if the positive terminal of the battery is connected with the negative plate of the capacitor, find the heat loss in the circuit during the process of charging.

12. A radioactive source in the form of a metal sphere of diameter 10^{-3} m emits β particles at a constant rate of 6.25×10^{10} particles per second. If the source is electrically insulated, how long will it take (in μs) for its potential to rise by 1.0 volt, assuming that 80% of emitted β particles escape from the surface.

13. The plates of a capacitor are charged to a potential difference of 100 V and then connected across a resister. The potential difference across the capacitor decays exponentially with respect to time. After one second the potential difference between the plates of the capacitor is 80 V. What is the fraction of the stored energy which has been dissipated?

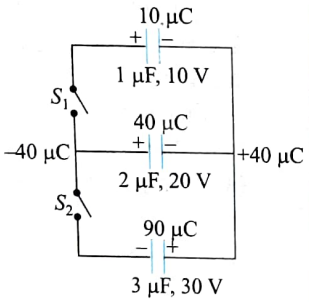
14. A parallel plate capacitor of capacitance C_0 is charged using a battery of emf V_0. Calculate the work done by external agent in reducing the separation between the plates to half its original value if

(a) The battery is disconnected before you start decreasing the plate separation.

(b) The battery remains connected while you are reducing the separation.

(Assume that the plates were moved very slowly)

15. Three capacitors of capacities 1 μF, 2 μF and 3 μF are charged by 10 V, 20 V and 30 V respectively. Now, positive plates of first two capacitors are connected with the negative plate of third capacitor on one side and negative plates of first two capacitors are connected with positive plate of third capacitor on the other side. Find



(a) common potential V

(b) final charges on different capacitors

ANSWERS

1. (a) 3.125×10^5 (b) Half the plate area or double the separation between the plates

2. $V + \frac{Q}{2C}$ **3.** $+Q$ **4.** When battery disconnected (second case)

5. (a) True (b) True (c) True **6.** Capacitor 'A'

7. Line A: Capacitor C_2, Line B: Capacitor C_1

8. (a) CV_0 (b) $\frac{2}{3}V_0$ (c) $\frac{1}{3}CV_0^2$ (d) $\frac{1}{6}CV_0^2$

9. (a) $\frac{xQ^2}{2\varepsilon_0 A}$ (b) $\left(\frac{Q^2}{2\varepsilon_0 A}\right)dx$ (c) $\frac{Q^2}{2\varepsilon_0 A}$ **10.** Zero

11. $2\varepsilon^2 C$ **12.** 6.95 μs **13.** $\frac{9}{25}$ **14.** (a) $-\frac{1}{4}C_0V_0^2$ (b) $-\frac{1}{2}C_0V_0^2$

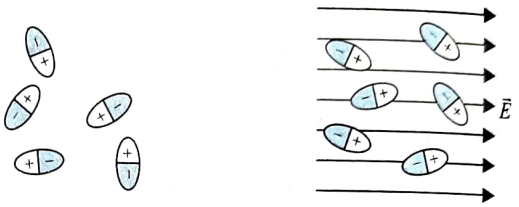
15. (a) $\frac{20}{3}$ volt (b) $q_1 = \frac{20}{3}\mu C, q_2 = \frac{40}{3}\mu C, q_3 = 20\ \mu C$

DIELECTRIC

A dielectric is a nonconductor up to a certain value of the field depending upon its nature. If the field exceeds the limiting value, called dielectric strength, a dielectric loses its insulating property and begins to conduct. Dielectrics are of two types:

Polar dielectrics: In polar molecules when no electric field is applied, the center of positive charges does not coincide with the center of negative charges.

A polar molecule has a permanent electric dipole moment ($\vec{p}$) in the absence of an electric field. But a polar dielectric has a net dipole moment equal to zero in the absence of an electric field because polar molecules are randomly oriented as shown in figure.

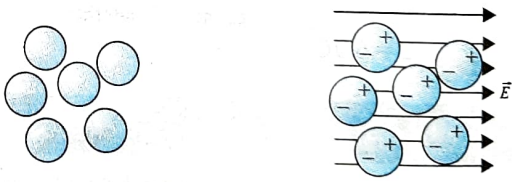


(a) Polar molecules with no applied electric field **(b) Polar molecules with applied electric field**

Polar molecules have random orientations when there is no applied electric field. The molecules tend to line up with an applied electric field $\vec{E}$.

In the presence of an electric field, polar molecules tend to line up in the direction of the electric field, and the substance has a finite dipole moment.

Nonpolar dielectrics: In nonpolar molecules, when no electric field is applied, the center of the positive charge coincides with the center of the negative charge in the molecule. Each molecule has zero dipole moment in its normal state.



(a) Nonpolar molecules with no applied electric field **(b) Nonpolar molecules with applied electric field**

Nonpolar molecules have their positive and negative charge centers at the same point. These centres becomes separated slightly by an applied electric field $\vec{E}$.

When an electric field is applied, the positive charge experiences a force in the direction of the electric field and negative charge experiences a force in the direction opposite to the field, i.e., molecules become induced electric dipole.

Note: In general, any nonconducting material can be called a dielectric, but nonconducting materials having nonpolar molecules are referred to as dielectric because an induced dipole moment is created in the nonpolar molecule.

POLARIZATION VECTOR

We know that under the action of an electric field, a net dipole moment appears in the dielectrics. Microscopically (for an atom or molecule) this effect may be very small but for the whole specimen this effect is significant inducing a considerable dipole

moment because millions of dipoles orient in same direction. As a result, equal positive and negative bound charges appear at the surface of the dielectric This effect is called polarization which is given by a vector

$$\vec{P} = \frac{n\vec{p}}{V},$$

where n = total number of dipoles and p = dipole moment of each atom.

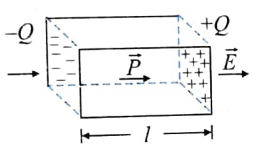
When a dielectric is subjected to an electric field a net dipole moment appears in it. As a result, an equal and opposite bound charge appears at the opposite surfaces of the dielectric which is called polarization. Polarization is defined as a vector $\vec{P} = \frac{\text{net dipole moment}}{\text{volume of the dielectric}}$ that is, dipole moment per unit volume.

Relation Between Polarisation Vector and Surface Charge Density

When a rectangular dielectric slab is placed in a uniform electric field it gets polarized. In consequence, equal and opposite bound charges $-Q$ and Q appear on the faces lying perpendicular to the field $\vec{E}$.

The net dipole moment of the slab is given as $p_{net} = np$, where n = total number of dipoles and p = dipole moment of each dipole. Substituting $p_{net} = Ql$, we have $Ql = np$. Dividing both sides with the volume of the specimen, we have

$$\frac{Ql}{Al} = \frac{np}{Al} \text{ or } \frac{Q}{A} = \left(\frac{np}{Al}\right)$$



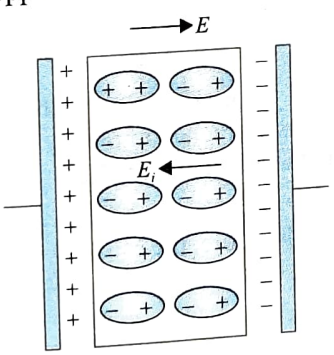
Here $\frac{np}{Al} = \frac{\text{total dipole moment}}{\text{total volume}} = P$

and $\frac{Q}{A} = \sigma_b$ (surface bond charge density)

Then, $P = \sigma_b$

Hence the polarization vector $\vec{P}$ is defined as dipole moment per unit volume which is numerically equal to surface bound charge density.

Polarization of a dielectric slab: A dielectric slab can be polarized by inducing equal and opposite charges on the two faces of the dielectric by the application of an electric field. Suppose a dielectric slab is inserted between the plates of a capacitor as shown in figure.



Induced electric field inside the dielectric is E_i; hence, this induced electric field decreases the main field E to $E - E_i$ i.e. new electric field between the plates will be $E' = E - E_i$.

Dielectric constant: After placing a dielectric slab in an electric field, the net field is decreased in that region. If E is the original electric field and E' is the reduced electric field, $E/E' = K$, where K is called dielectric constant. K is also known as relative permittivity (ε_r) of the material. The value of K is always greater than one. For vacuum, there is no polarization and hence $E = E'$ and $K = 1$.

Dielectric breakdown and dielectric strength: If a very high electric field is created in a dielectric, the outer electrons may get detached from their parent atoms. The dielectric then behaves like a conductor. This phenomenon is known as dielectric breakdown.

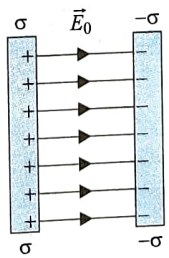
The maximum value of an electric field (or potential gradient) that a dielectric material can tolerate without electric breakdown is called its dielectric strength. SI unit of the dielectric strength of a material is Vm^{-1}.

INDUCED CHARGE ON THE SURFACE OF DIELECTRIC

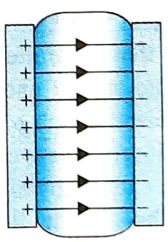
Consider an isolated, charged parallel plate capacitor with air or vacuum between its plates.Now we insert a dielectric slab, dielectric constant K, completely filling the space between the plates of the capacitor. The capacitor is not connected to the battery so charge cannot flow to or from the plates. This capacitor has a charge $+Q$ on one plate and $-Q$ on the other.

In the absence of electric field the dipole moments of polar molecules are randomly oriented and the net charge on all the dielectric's surfaces is zero. When the dielectric is placed in the electric field of the capacitor, the polar molecules experience a torque and tend to align with the electric field.

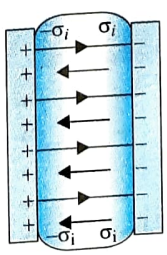
The overall effect of the dipole alignment is to make the left surface of the dielectric negative (dielectric surface facing the capacitor plate with charge $+Q$) and the right surface positive (dielectric surface facing the capacitor plate with charge $-Q$). These unbalanced charges are not free to move (charges are free in conductors only) they are tied to the molecule. These charges on the surface of the dielectric are called **bound charges**, **induced surface charges**. This bound charge density, σ_i creates an electric field of magnitude σ_i/ε_0, while that due to the free charge on the capacitor plates creates a field of magnitude σ/ε_0. Note that free charge is that which we place on the conducting capacitor plates by some external source (Battery) whereas bound charges appear due to polarization effects.



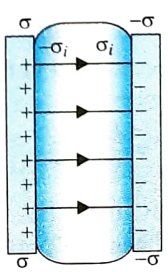
(a) Electric field of magnitude E_0 between two charged plates.



(b) Introduction of a dielectric constant K.



(c) Induced surface charges and their field (thinner lines).



(d) Resultant field of magnitude E_0/K when a dielectric is between charged plates.

The field produced by the bound charges is opposite to the original field between the plates. The total field is the superposition of the field from the charges on the conducting plates and the field caused by the bound charge on the surface of the dielectric.

Let K be the dielectric constant, E_0 be the original field in vacuum if the dielectric slab was not there, E_i be the electric field induced in the dielectric slab, and E be the net electric field in the dielectric slab. Therefore,

$$E = E_0 - E_i \quad \text{...(i)}$$

and $\frac{E_0}{E} = K$ (by definition of K of ε_r)

or $E = \frac{E_0}{K}$ ...(ii)

From (i) and (ii),

$$E_0 - E_i = \frac{E_0}{K}$$

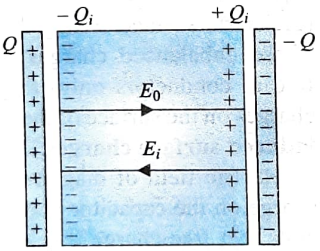
or $E_0K - E_iK = E_0$ or $E_0K - E_0 = E_iK$

or $E_i = \frac{K-1}{K}E_0$ ...(iii)

or $\frac{\sigma_i}{\varepsilon_0} = \frac{K-1}{K}\frac{\sigma}{\varepsilon_0}$ or $\sigma_i = \frac{K-1}{K}\sigma$

or $\frac{Q_i}{A} = \frac{K-1}{K}\frac{Q}{A}$

or $Q_i = Q\left(1 - \frac{1}{K}\right)$ ...(iv)



This is irrespective of the thickness of the dielectric slab, i.e., whether it fills up the entire space between the charged plates or any part of it.

CAPACITY OF PARALLEL PLATE CAPACITOR WITH DIELECTRIC

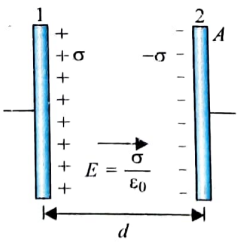
Suppose that a parallel plate capacitor has a plate area A and a separation d, and a dielectric slab of thickness t and area A is inserted between the plates. Let Q be the charge given to the capacitor plates. The electric field between the plates of the capacitor is given by

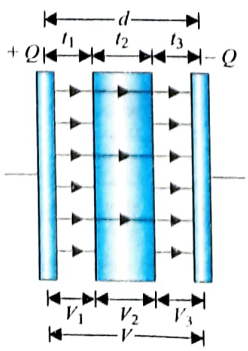
$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

The electric field in the region of the dielectric slab is

$$E' = \frac{E}{K} = \frac{Q}{K\varepsilon_0 A}$$

We know that if the electric field is constant, then the potential difference between two points separated by distance d along the field line is Ed. The potential difference between the two plates is, therefore,





$$V = V_1 + V_2 + V_3 = Et_1 + E't_2 + Et_3$$

$$= \frac{Q}{\varepsilon_0 A}t_1 + \frac{Q}{K\varepsilon_0 A}t_2 + \frac{Q}{\varepsilon_0 A}t_3$$

$$= \frac{Q}{\varepsilon_0 A}(t_1 + t_3) + \frac{Q}{K\varepsilon_0 A}t_2 = \frac{Q}{\varepsilon_0 A}(d - t) + \frac{Q}{K\varepsilon_0 A}t$$

$$\therefore \quad C = \frac{Q}{V_+ - V_-} = \frac{1}{\frac{(d-t)}{\varepsilon_0 A} + \frac{t}{k\varepsilon_0 A}} = \frac{\varepsilon_0 A}{(d-t) + \frac{t}{K}}$$

Hence capacitance of the system will be

$$C = \frac{\varepsilon_0 A}{(d-t) + \frac{t}{K}}$$

Note:

- The capacitance in the above situation is independent of the position of the dielectric slab with respect to the plates. The capacitance depends upon the thickness of the dielectric slab and the dielectric constant.
- The dielectric constant of the conducting slab (metal plate) is infinity; therefore, the term t/K reduces to zero. If we insert a metal plate of thickness t between the plates of the capacitor having area A and separation d, the capacitance will become $C = \varepsilon_0 A/(d-t)$. Also the capacitance will be independent of the position of the metal plate between the plates of the capacitor.
- If $t << d$, then $C = \varepsilon_0 A/d$. Hence if we place a thin metal plate parallel to the plate of a capacitor, the capacitance of the capacitor remains unchanged.
- If the slab completely fills the space between the plates, then $t = d$, and therefore,
$$C = \frac{\varepsilon_0 A}{d/K} = \frac{K\varepsilon_0 A}{d}$$
- If the space between the plates is completely filled with a conductor, then $t = d$ and $K = \infty$.

If we place many dielectric slabs parallel to the plate of a capacitor as shown in figure, then the capacitance is given by

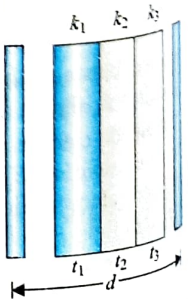
$$C = \frac{\varepsilon_0 A}{d - (t_1 + t_2 + \ldots) + \left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \ldots\right)}$$

If we introduce a number of dielectric slabs, which completely fill the space between the plates, then

$$d = t_1 + t_2 + t_3 + \ldots t_n$$

Therefore, the capacity of the capacitor will be

$$C = \frac{\varepsilon_0 A}{\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \ldots\right)}$$



EFFECT OF DIELECTRIC ON DIFFERENT PARAMETERS

Let the entire space between the plates of a capacitor be filled with a dielectric of dielectric constant K under two conditions:

(i) when the battery remains connected and (ii) after the battery is disconnected.

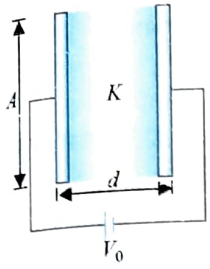
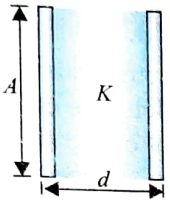
If battery remains connected	When battery is disconnected
In this case, the potential difference across the plates will remain same. 	In this case, the charge on the plates will remain same, i.e., $q = q_0$, since in an isolated system, charge is conserved. 
• Capacitance increases, i.e., $C = KC_0$, since capacitance depends upon geometrical factors only.	• Capacitance increases, i.e., $C = KC_0$, since capacitance depends upon geometrical factors only.
• Charge on capacitor is $q = CV = KC_0V_0 = Kq_0$ (Since initially $q_0 = C_0V_0$) Thus, charge increases and becomes K times of previous charge.	• Potential difference between the plates is $V = \frac{q}{C} = \frac{q_0}{KC_0} = \frac{V_0}{K}$ So the potential difference decreases.
• Electric field is $E = \left[\frac{V}{d}\right] = \frac{V_0}{d} = E_0$ $\left(\text{as } V = V_0 \text{ and } \frac{V_0}{d} = E_0\right)$ Thus, electric field remains same.	• Field between the plates is $E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K}$ $\left(\text{as } V = \frac{V_0}{K} \text{ and } E_0 = \frac{V_0}{d}\right)$ So, the electric field decreases.
• Energy stored in the capacitor is $U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)(V_0)^2$ $= K\frac{1}{2}C_0V_0^2 = KU_0$ $\left(\text{as } C = KC_0 \text{ and } U_0 = \frac{1}{2}C_0V_0^2\right)$	• Energy stored in the capacitor is $U = \frac{q^2}{2C} = \frac{q_0^2}{2KC_0} = \frac{U_0}{K}$ (as $q = q_0$ and $C = KC_0$) Thus, energy decreases and becomes $1/K$ times of previous energy.
• Thus, energy increases and becomes K times of previous energy.	

ILLUSTRATION 4.14

The electric field between the plates of a parallel plate capacitor is E_0. The space between the plates is filled completely with a dielectric. There are n molecules in unit volume of the dielectric and each molecule is like a dumb-bell of length L with its ends carrying charge $+q$ and $-q$. Assume that all molecular dipoles get aligned along the field between the plates. Find the electric field between the plates after insertion of the dielectric.

Sol. Due to electric field between the plates the molecules of the dielectric get polarized, the dipole moment per unit volume (after perfect alignment) in the dielectric = qLn

Net dipole moment induced in dielectric slab = $qLn.(Ad)$

∴ Charge induced on walls of the dielectric

$$Q_{in} = \pm\frac{qLnAd}{d} = \pm qLnA$$

∴ The electric field due to induced charge $E_{in} = \frac{Q_{in}}{\varepsilon_0 A} = \frac{qLn}{\varepsilon_0}$

The electric field between the plates, $E = E_0 - E_{in} = E_0 - \frac{qLn}{\varepsilon_0}$

ILLUSTRATION 4.15

You have been given a parallel plate air capacitor having capacitance C_0, a battery of emf $V$and three dielectric blocks having dielectric constants K_1, $K_2$and K_3such that $K_1 > K_2 > K_3$. Each dielectric block can fill completely the space between the plates. Describe a sequence of steps such as connecting or disconnecting the capacitor to the battery, inserting or taking out of one of the dielectrics etc-so that the capacitor ends up having maximum possible energy stored in it.

Sol. The potential energy of a capacitor is given by, $U = \frac{Q^2}{2C}$

For maximum value of U, we should have maximum value of charge(Q)and minimum value of capacitance (C). Q will be maximum when battery is connected with capacitance at maximum possible value. The capacitance of parallel plate air capacitor with completely filled with dielectric is given by $C = KC_0$

For maximum capacitance K should be maximum. Hence dielectric with constant K_1 should be inserted.

Hence, $C_{max} = K_1C_0$

If battery is connected to this capacitor, charge on it will be maximum

$$Q_{max} = VC_{max} = K_1VC_0$$

Now battery is disconnected and dielectric is removed so that the capacitance of the capacitor becomes minimum.

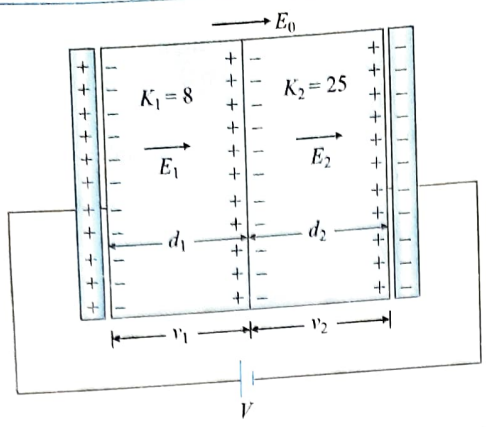
Hence the maximum possible energy stored in capacitor,

$$U_{max} = \frac{Q_{max}^2}{2C} = \frac{K_1^2V^2C_0^2}{2C_0} = \frac{1}{2}K_1^2V^2C_0$$

ILLUSTRATION 4.16

A parallel plate capacitor contains a mica-sheet (thickness 10^{-3} m) and a sheet of fibre (thickness 0.5×10^{-3} m). The dielectric constant of mica is 8 and that of fibre is 2.5. Assuming that the fibre breaks down when subjected to an electric field of 6.4×10^6 V/m. Find the maximum safe voltage that can be applied to the capcitor.

Sol. Let E_0 be the electric field due to charges on plates (not due to induced charges).



The potential difference across the plates of the capacitor,

$$V = V_1 + V_2$$

As the sheet of fibre breaks down when it is subjected to an electric field of 6.4×10^6 V/m

$$\Rightarrow \quad V = E_1 d_1 + E_2 d_2 = \frac{E_0}{K_1} d_1 + \frac{E_0}{K_2} d_2$$

$$= \frac{6.4\times10^6 \times 10^{-3}}{8} + \frac{6.4\times10^6 \times 0.5\times10^{-3}}{2.5}$$

$$= 2.08\times10^3 \text{ V} = 2.08 \text{ kV}$$

ILLUSTRATION 4.17

A parallel plate capacitor completely filled with a dielectric slab, dielectric constant $K = 3.4$, is fully charged with a 100 V battery. When the capacitor is fully charged, the battery is removed. The area of plates $A = 4.0$ m^2 and separation between plates $d = 4.0$ mm. **(a)** Find the capacitance, the electric field strength and the energy stored in the capacitor. **(b)** The dielectric slab is slowly removed without changing the plate separation and any charge transfer from capacitor. Find the new capacitance, electric field strength, voltage between plates and the energy stored in the capacitor.

Sol.

(a) In presence of dielectric slab the capacitance of a capacitor will increase K times, $C = KC_0$

$$C = \frac{K\varepsilon_0 A}{d} = \frac{(3.4)(8.85\times10^{-12})(4)}{4\times10^{-3}} = 3.0\times10^{-8} \text{ F}$$

The electric field between the plates,

$$E = \frac{V}{d} = \frac{100}{4.0\times10^{-3}} = 25 \text{ kV/m}$$

The total energy stored in the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(3\times10^{-8})(100)^2 = 1.5\times10^{-4} \text{ J}$$

(b) The capacitance without dielectric is

$$C_0 = \frac{C}{K} = \frac{3\times10^{-8}}{3.4} = 8.8\times10^{-9} \text{ F}$$

There is no transfer of charge, hence

$$CV = C_0V_0 \Rightarrow (KC_0)V = C_0V_0 \Rightarrow V_0 = KV$$

So potential difference between plates increases by a factor $K = 3.4$

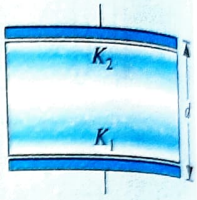
The electric field is $E = \frac{V}{d} = \frac{340}{4.0\times10^{-3}} = 85 \text{ kV/m}$

The energy of the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(8.8\times10^{-9})(340)^2 = 5.1\times10^{-4} \text{ J}$$

ILLUSTRATION 4.18

A dielectric slab fills the entire space of a parallel plate capacitor. The dielectric constant of the slab varies linearly with distance, varies from K_1 to K_2, from one plate to other. Find the equivalent capacity of the system. Separation between plates is d and area of plates A.



Sol. It is given that the dielectric constant varies linearly hence we can write the expression for dielectric constant as:

$$K(y) = ay + b \quad \ldots(i)$$

where a and b are constants and y is measured from lower plate these values can be determined from boundary conditions

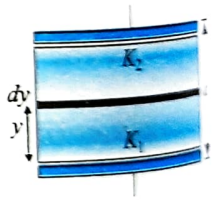
$$K = K_1 \text{ at } y = 0 \quad \text{hence} \quad K_1 = b \quad \ldots(ii)$$

$$K = K_2 \text{ at } y = d \quad \text{hence} \quad K_2 = ad + b \quad \ldots(iii)$$

From (ii) and (iii) $a = \frac{K_2 - K_1}{d}$, $b = K_1$

Let charge Q be given to the system, electric field at a distance from lower plate is

$$E(y) = \frac{Q}{K(y)\varepsilon_0 A}$$



So the potential difference across a differential element dy at adistance y is

$$dV = -E(y)dy$$

Thus $$-\int_{V_1}^{V_2} dV = \int_0^d \frac{Q}{K(y)\varepsilon_0 A} dy$$

$$= \frac{Q}{\varepsilon_0 A}\int_0^d \frac{dy}{ay+b} = \frac{Q}{\varepsilon_0 A}\left[\frac{1}{a}\ln(ay+b)\right]_0^d$$

$$V_1 - V_2 = \frac{Q}{\varepsilon_0 Aa}\ln\left(\frac{ad+b}{b}\right)$$

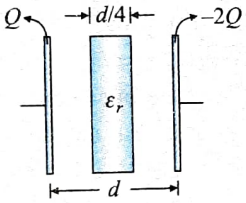
Thus $$C = \frac{Q}{V_1 - V_2} = \frac{\varepsilon_0 Aa}{\ln\left(\frac{ad+b}{b}\right)}$$

On substituting values of a and b, we get,

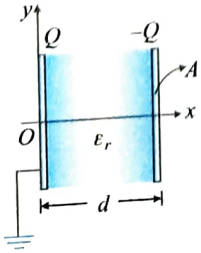
$$C = \frac{\varepsilon_0 A}{d}\frac{K_2 - K_1}{\ln(K_2/K_1)}$$

CONCEPT APPLICATION EXERCISE 4.2

1. A large thin conducting plate is kept in an external uniform electric field E_0. Find the induced surface charge density.
2. An isolated parallel plate capacitor stores an energy $U_0 = 10^{-3}$ J in the absence of a dielectric. If the dielectric of $\varepsilon_r = 100$ is introduced,
   (i) find the energy in the capacitor.
   (ii) where does the extra energy go?
3. A charged capacitor has energy $U_0 = 2 \times 10^{-4}$ J and $C_0 = 4 \times 10^{-8}$ F, in the absence of a dielectric. When the potential difference is kept constant by keeping the capacitor connected to the supply, assuming $\varepsilon_r = 150$,
   (i) find the new energy stored when the dielectric slab is completely filled with the capacitor. Account for this change in energy.
   (ii) how much excess charge flows from the supply
   (iii) find the ratio of the charge on the plate of the capacitor and the charge induced on the dielectric;
4. A parallel plate capacitor with plate area 100 cm^2, separated by a distance of 1mm. A dielectric of dielectric constant 5.0 and dielectric strength 1.9×10^7 V/m is filled between the plates. Find the maximum charge that can be stored on the capacitor without causing any dielectric breakdown.
5. A dielectric slab of thickness $d/4$ and relative permittivity $\varepsilon_r = 2$ is inserted between two parallel conducting plates each of area A kept at a separation d. Find the:



   (a) The charge density in the inner side of conductors
   (b) The charge density on the dielectric slab
   (c) The electric field in the dielectric
   (d) The potential difference between the conductor
   (e) The capacitance
6. The separation between the plates of a parallel-plate capacitor is 0.05 m. A field of 3×10^4 V/m is established between the plates. It is disconnected from the battery and an uncharged metal plate of thickness 0.01 m is inserted between the plates of the capacitor.
   (a) What would be the potential difference across the capacitor, (i) before the introduction of the metal plate and (ii) after its introduction.
   (b) What would be the potential difference if a plate of dielectric constant $K = 2$ is introduced in place of metal plate?
7. A dielectric slab is filled in the space between the parallel plate capacitor whose relative permittivity varies as



   $\varepsilon_r = a + bx$
   Find the (a) capacitance of the system (b) potential at a point situated at a distance x from the origin O.

ANSWERS

1. $\sigma = \varepsilon_0 E_0$ **2.** (i) 10^{-5} J

3. (i) 0.03 J (ii) 5.96×410^{-6} C (iii) $\frac{149}{150}$ **4.** 8.4×10^{-6} C

5. (a) $\frac{3Q}{2A}$ (b) $\frac{3Q}{4A}$ (c) $\frac{3Q}{4\varepsilon_0 A}$ (d) $\frac{21Qd}{16\varepsilon_0 A}$ (e) $\frac{8\varepsilon_0 A}{7d}$

6. (a) (i) 1.5 kV (ii) 1.20 kV (b) 1.35 kV

7. (a) $\frac{\varepsilon_0 Ab}{\ln\left(\frac{a+bd}{a}\right)}$ (b) $\frac{Q}{\varepsilon_0 Ab}\ln\left(\frac{a+bx}{a}\right)$

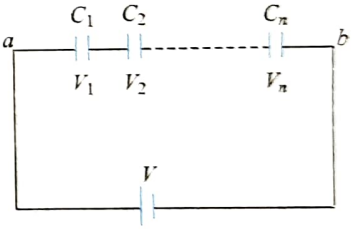
COMBINATION OF CAPACITORS

Sometimes to obtain a desired value of capacitance, we group two or more than two capacitors together. In this section we will learn how to find the equivalent capacitance when two or more than two capacitors are grouped together?

The equivalent capacitance of a group of capacitors is that value of capacitance of a single capacitor that will allow to flow (or store) the same amount of charge for a given potential difference as done by the combination. In other words, that single capacitor will perform the same task as done by the combination in all respects. Capacitors can be grouped in many ways, but the two common types of combinations are (i) series combination and (ii) parallel combination.

CAPACITORS CONNECTED IN SERIES

Suppose n capacitors are connected in series as shown in figure. C_{eq} is the equivalent capacitance of this system.

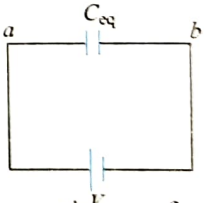


Let a battery V be applied across the combination, then

$$V = V_1 + V_2 + \cdots + V_n$$

or $$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \cdots + \frac{Q}{C_n}$$

or $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$



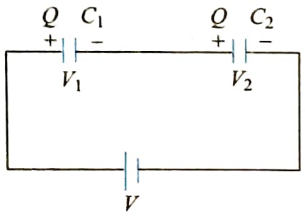
In general

$$\frac{1}{C_{eq}} = \sum_{n=1}^{n} \frac{1}{C_n}$$

Thus, the reciprocal of the equivalent capacitance of a series combination equals the sum of the reciprocals of the individual capacitances.

FINDING POTENTIAL DIFFERENCE ACROSS THE CAPACITORS CONNECTED IN SERIES

Suppose two capacitors C_1 and C_2 are connected in series and a potential difference V is applied across them as in figure.



Potentials appearing on the capacitors are V_1 and V_2, respectively. Q is the net charge that flows in circuit, then

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2} \quad \text{or} \quad C_{eq}=\frac{C_1C_2}{C_1+C_2}$$

$$V_1+V_2=V \qquad ...(i)$$

$$Q=C_1V_1=C_2V_2 \quad \text{or} \quad \frac{V_1}{V_2}=\frac{C_2}{C_1} \qquad ...(ii)$$

It means potentials on the capacitors will be divided in the inverse ratio of their capacitances.

From (i) and (ii),

$$V_1=\frac{C_2V}{C_1+C_2}, \quad V_2=\frac{C_1V}{C_1+C_2}$$

Note:

- In a series combination, the equivalent capacitance is always less than any of the individual capacitance.
- In a series combination, charge on each capacitor is same but potential is different. From $V=q/C$, it can be said that larger is the capacitance, lesser is the potential. We can say that the potential difference across the capacitor is inversely proportional to its capacitance in series combination.

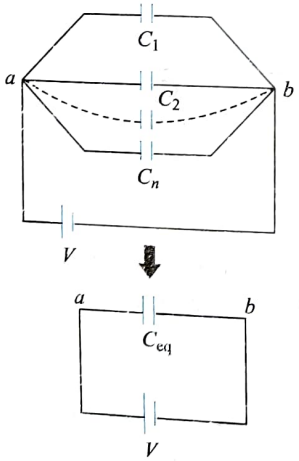
$$V_1:V_2:V_3=\frac{1}{C_1}:\frac{1}{C_2}:\frac{1}{C_3}$$

$$V_1=\frac{\frac{1}{C_1}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots}V; \quad V_2=\frac{\frac{1}{C_2}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots}V$$

where $V=V_1+V_2+V_3$

CAPACITORS CONNECTED IN PARALLEL

Let n capacitors be connected in parallel as shown in figure. C_{eq} is the equivalent capacitance of this system.



Let the total charge flowing through the battery be q, then

$$q=q_1+q_2+\ldots+q_n$$

or $$C_{eq}V=C_1V+C_2V+\ldots+C_nV$$

or $$C_{eq}=C_1+C_2+\ldots+C_n$$

In general

$$C_{eq}=\sum_{n=1}^{n}C_n$$

That is, the equivalent capacitance of a parallel combination equals the sum of the individual capacitances.

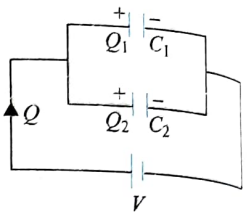
FINDING POTENTIAL DIFFERENCE ACROSS THE CAPACITORS CONNECTED IN PARALLEL

Let two capacitors C_1 and C_2 be connected in parallel and a potential difference V is applied across them. Then

$$C_{eq}=C_1+C_2$$

$$Q_1+Q_2=Q \qquad ...(i)$$

$$V=\frac{Q_1}{C_1}=\frac{Q_2}{C_2} \qquad ...(ii)$$



From (ii)

$$\frac{Q_1}{Q_2}=\frac{C_1}{C_2} \qquad ...(iii)$$

It means charge will be divided in the direct ratio of the capacitances. From (i) and (iii),

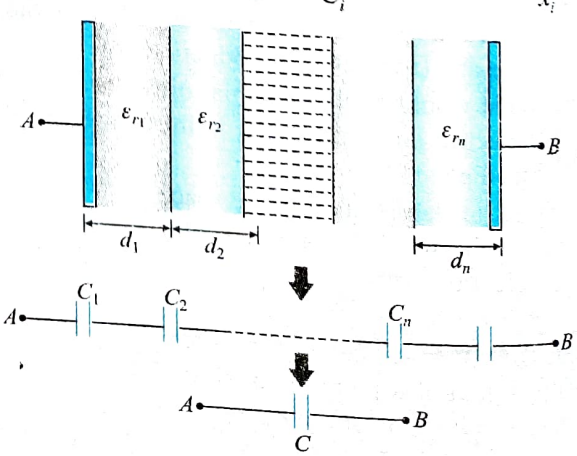
$$Q_1=\frac{C_1Q}{C_1+C_2}, \quad Q_2=\frac{C_2Q}{C_1+C_2}$$

Note:

- In a parallel combination, the equivalent capacitance is always greater than any individual capacitance.
- In a parallel combination, the potential on each capacitor is same but charge may be different. From $q=CV$, it can be said that greater is the capacitance, greater is the charge.
- Charge is distributed on the capacitors in the ratio of their capacitances, i.e., $q_1:q_2:\ldots:q_n=C_1:C_2:\ldots:C_n$

Important Points:

- When the dielectric are placed so that the field intensity is perpendicular to their interfaces, we can treat each dielectric forming a parallel plate capacitor with two thin conducting plates identical with that of the given capacitor. As a result, the given combination can be reduced to a series combination of n number of capacitors having the effective capacitance C given as $\frac{1}{C}=\Sigma\frac{1}{C_i}$, where $C_i=\frac{\varepsilon_0\varepsilon_r A}{x_i}$.



- When the dielectrics are connected so that their interfaces are parallel to the applied field, we can treat each dielectric forming a parallel plate capacitor corresponding to the area of its contact with the conducting plates. Then, taking all such capacitors in parallel, the equivalent capacitance of the combination can be given as $C = \Sigma C_i$, where $C_i = \dfrac{\varepsilon_0 \varepsilon_{ri} A_i}{d}$

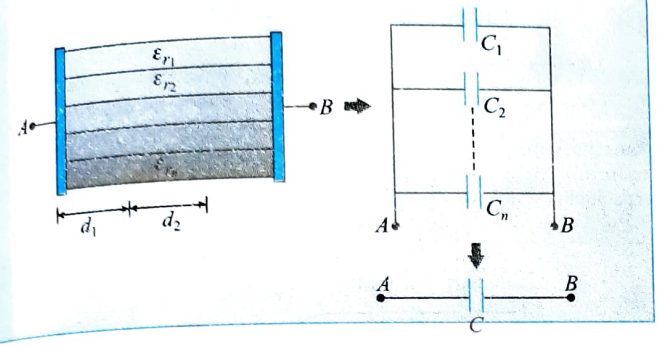
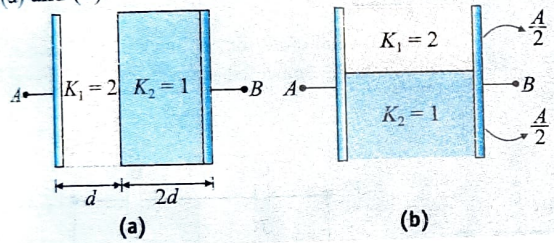


ILLUSTRATION 4.19

Find the equivalent capacitance of an air capacitor of capacitance C_0 when the two dielectric slabs of dielectric constants $K_1 = 2$ and $K_2 = 1$ are introduced as shown in Figs. (a) and (b).



Sol.

(i) The given system can be made equivalent to two capacitors C_1 and C_2 in series.

For left part $C_1 = \dfrac{\varepsilon_0 K_1 A}{d} = \dfrac{2\varepsilon_0 A}{d}$

For right part $C_2 = \dfrac{\varepsilon_0 K_2 A}{2d} = \dfrac{\varepsilon_0 A}{2d}$

As both capacitors are connected in series, hence the equivalent capacitance

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2\varepsilon_0 A} + \frac{2d}{\varepsilon_0 A} = \frac{5d}{2\varepsilon_0 A}$$

As $C_0 = \dfrac{\varepsilon_0 A}{d}$ hence $C = \dfrac{2C_0}{5}$

2nd approach:

$$C = \frac{\varepsilon_0 A}{\dfrac{x_1}{K_1} + \dfrac{x_2}{K_2}}, \text{ where } x_1 = d,\ x_2 = 2d, K_1 = 2 \ \&\ K_2 = 1$$

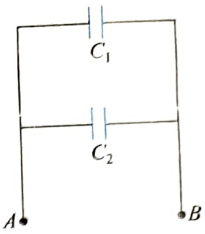
$$C = \frac{\varepsilon_0 A}{\dfrac{d}{2} + \dfrac{2d}{1}} = \frac{\varepsilon_0 A}{5d/2},$$

As $C_0 = \dfrac{\varepsilon_0 A}{d}$ hence $C = \dfrac{2C_0}{5}$

(ii) The given system can be made equivalent to two capacitors C_1 and C_2 in parallel.

For upper part $C_1 = \dfrac{\varepsilon_0 K_1 \left(\frac{A}{2}\right)}{d} = \dfrac{\varepsilon_0 A}{d}$

For lower part $C_2 = \dfrac{\varepsilon_0 K_2 \left(\frac{A}{2}\right)}{d} = \dfrac{\varepsilon_0 A}{2d}$



Then, $C_{AB} = C_1 + C_2$, (because C_1 and C_2 are in parallel).

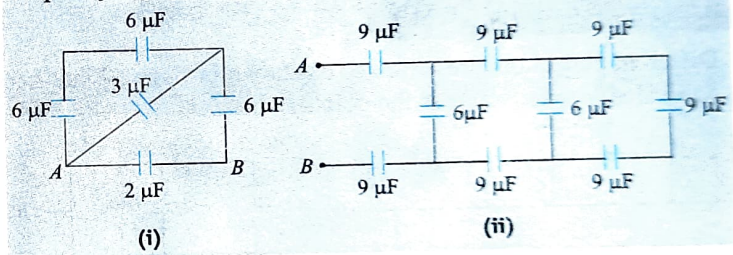
$$= \frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{2d} = \frac{3\varepsilon_0 A}{2d} = \frac{3}{2} C_0$$

FINDING EQUIVALENT CAPACITANCE

Equivalent capacitance can be found by using the successive reduction method. Let us learn this method through the following illustrations.

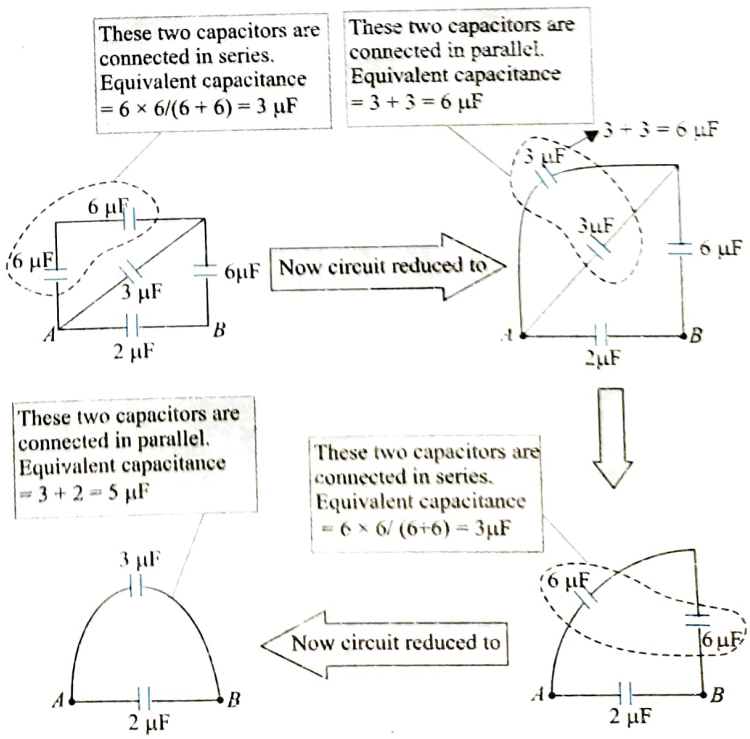
ILLUSTRATION 4.20

In figure, different capacitors are arranged. Find the equivalent capacity across the points A and B.



Sol.

(i)



(ii)

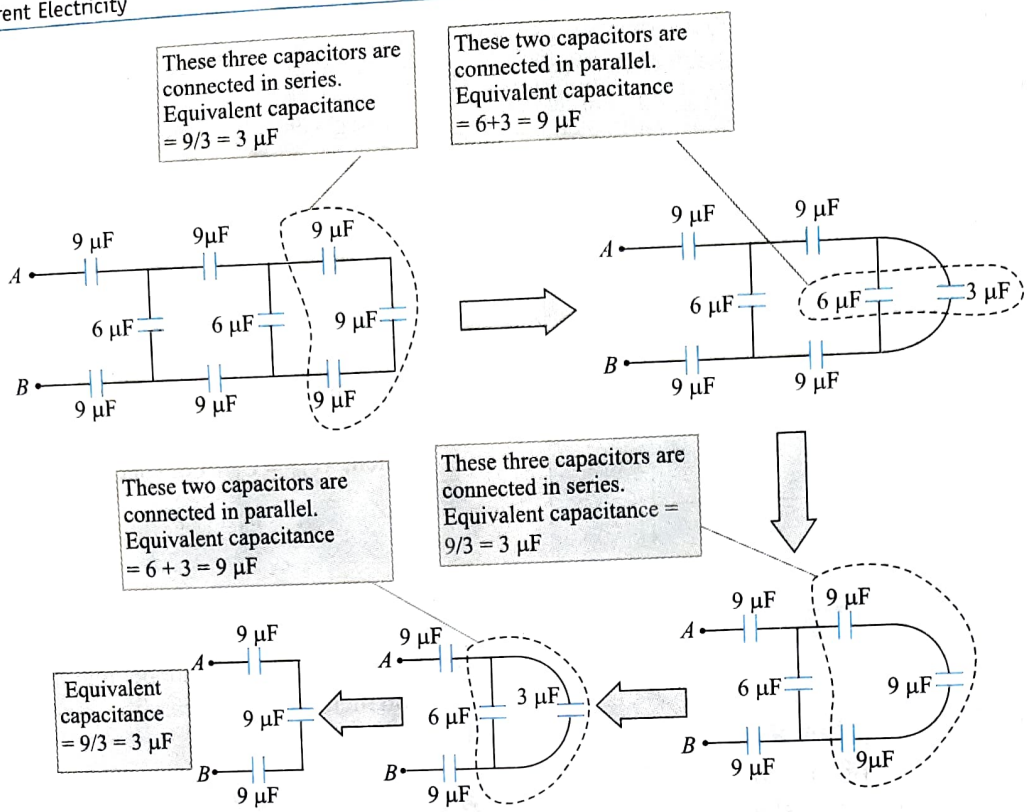
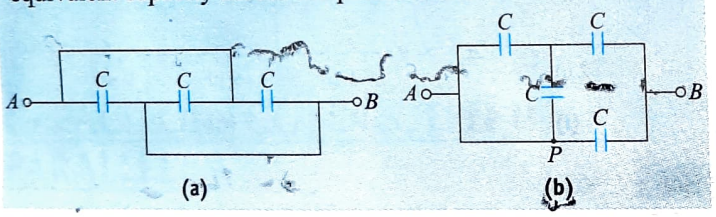


ILLUSTRATION 4.21

Different capacitors are arranged as shown in figure. Find the equivalent capacity across the points A and B.



Sol.

(a)

The potential of all four points will be V_A as they are directed connected through conducting wire.

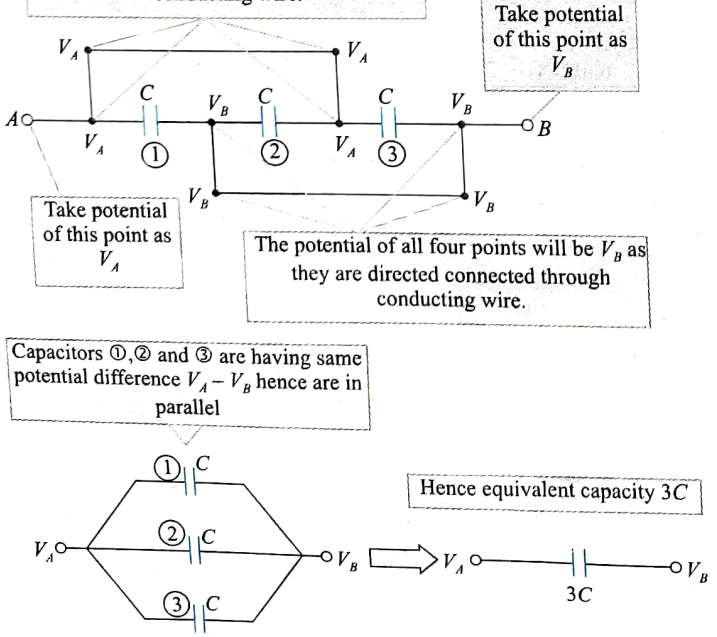
Take potential of this point as V_B

Take potential of this point as V_A

The potential of all four points will be V_B as they are directed connected through conducting wire.

Capacitors ①,② and ③ are having same potential difference $V_A - V_B$ hence are in parallel

Hence equivalent capacity $3C$



(b)

The potential of all four points will be V_A as they are directed connected through conducting wire.

These two capacitors are connected in parallel. Equivalent capacitance $= C + C = 2C$

These two capacitors are connected in series. Equivalent capacitance $= 2C \times C/(2C + C) = 2/3C$

Hence equivalent capacity $= \frac{2C}{3} + C = \frac{5C}{3}$

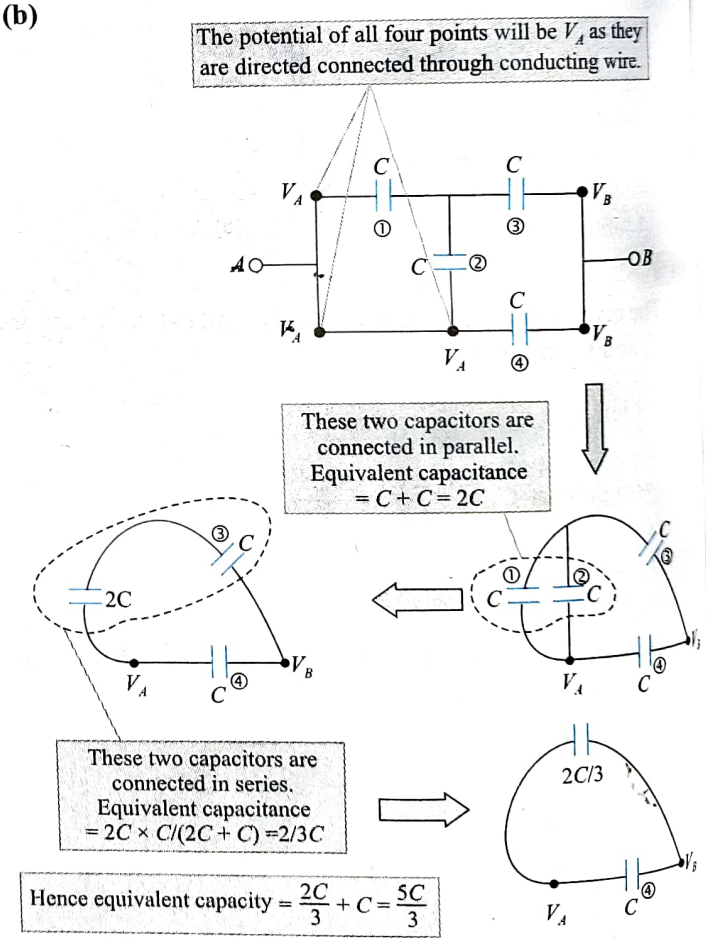
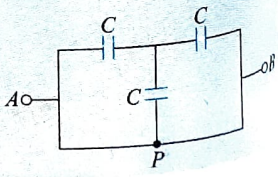


ILLUSTRATION 4.22

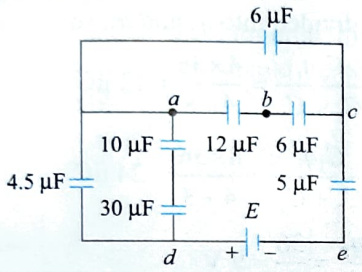
Three capacitors are arranged as shown in figure. Find the equivalent capacity across the points A and B.



Sol. Since there is no capacitor in the path APB, the points A, P, and B are electrically same, i.e., the input and output points are directly connected (short circuited).
Thus, the entire charge will prefer to flow along the path APB. It means that the capacitors connected in the circuit will not receive any charge for storing. Thus, the equivalent capacitance of this circuit is zero.

ILLUSTRATION 4.23

In the circuit shown in figure, the potential difference between the points a and b is 4 V. Find the emf ε of the battery. Assume that before connecting the battery in the circuit, all the capacitors were uncharged.

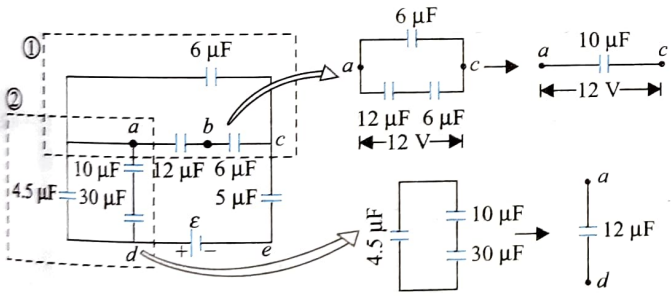


Sol. The capacitors 12 μF and 6 μF are in series arrangement. The charge in each will be equal.

$$q_1 = CV = 12 \times 4 = 48\ \mu C$$

The potential difference across the 6 μF capacitors is

$$V_{bc} = \frac{12 \times 4}{6}\ V = 8\ V$$



The potential difference between points a and c is $V_a - V_c = 4 + 8 = 12$ V. The equivalent capacity in portion (1) is 10 μF. The charge on it is $(10 \times 12)\ \mu C = 120\ \mu C$. Portions (1) and (2) of the circuit are in series combination.

Hence, $12 \times (V_d - V_a) = 120\ \mu C$ or $V_d - V_a = 10$ V. The capacitor 5 μF is also in series with 10 μF (see figure). Hence, charge on it is 120 μC. Thus, $V_c - V_e = 120/5 = 24$ V. Now $E = (12 + 10 + 24)$ V $= 46$ V.

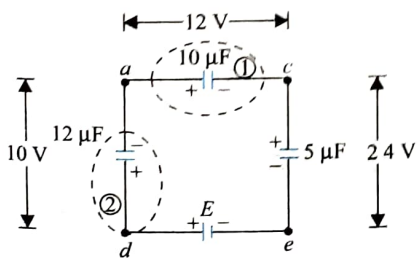
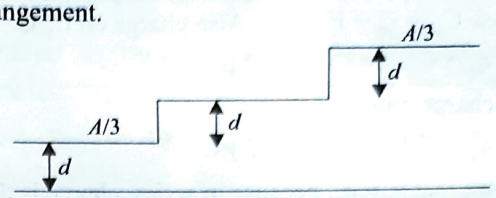


ILLUSTRATION 4.24

A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in figure. The area of each stair is $A/3$, and the height is d. Find the capacitance of this arrangement.

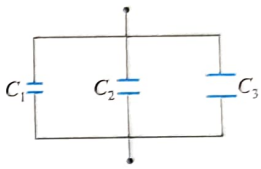


Sol. The arrangement is a parallel combination of three capacitors. Each capacitor has a plate area $A/3$, and the separations between the plates are d, $2d$, and $3d$, respectively. So

$$C_1 = \frac{\varepsilon_0 A/3}{3d} = \frac{\varepsilon_0 A}{3d}$$

$$C_2 = \frac{\varepsilon_0 A/3}{2d} = \frac{\varepsilon_0 A}{6d}$$

$$C_3 = \frac{\varepsilon_0 A/3}{3d} = \frac{\varepsilon_0 A}{9d}$$

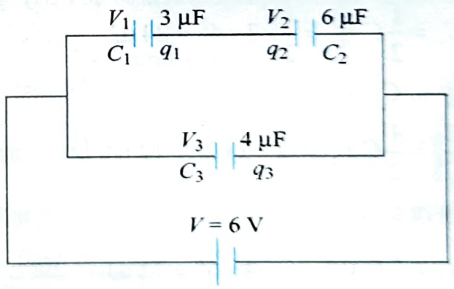


As these three capacitances are in parallel, their equivalent capacitance is given by

$$C = C_1 + C_2 + C_3 = \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{6d} + \frac{\varepsilon_0 A}{9d} = \frac{11\varepsilon_0 A}{18d}$$

ILLUSTRATION 4.25

Three capacitors of capacitances 3 μF, 6 μF, and 4 μF are connected as shown across a battery of emf 6 V.



(i) Find the equivalent capacitance.
(ii) Find the potential difference and charge on each capacitor.
(iii) Find the energy stored in each capacitor and the total energy stored in the system of capacitors.

Sol.

(i) C_1 and C_2 are in series, so their equivalent capacitance is

$$C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{3 \times 6}{3 + 6} = 2\ \mu F$$

This C' will be in parallel with C_3, so

$$C_{eq} = C' + C_3 = 2 + 4 = 6\ \mu F$$

(ii) 6 V will be divided across C_1 and C_2 in the inverse ratio of the capacitances. So potential difference across C_1 is

$$V_1 = \frac{C_2 V}{C_1 + C_2} = \frac{6 \times 6}{3 + 6} = 4\ V$$

and potential difference across C_2 is

$$V_2 = \frac{C_1 V}{C_1 + C_2} = \frac{3\times 6}{3+6} = 2 \text{ V}$$

Because C_3 is connected directly across the battery without any other capacitor in between, so potential difference across C_3 is $V_3 = V = 6$ V. Also charge on C_1 is

$$q_1 = C_1 V_1 = 3 \times 4 = 12 \ \mu\text{C}$$

and charge on C_2 is

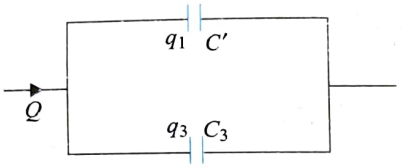
$$q_2 = C_2 V_2 = 6 \times 2 = 12 \ \mu\text{C}$$

We see that $q_1 = q_2$, because in series, charge is the same.

Charge on C_3 is

$$q_3 = C_3 V_3 = 4 \times 6 = 24 \ \mu\text{C}$$

Alternative method to find charge:



Total charge that will flow through the battery is

$$Q = C_{eq} V = 6\times 6 = 36 \ \mu\text{C}$$

It will be divided in the direct ratio of C' and C_3, so

$$q_1 = \frac{C'Q}{C' + C_3} = \frac{2\times 36}{2+4} = 12 \ \mu\text{C}$$

$$q_3 = \frac{C_3 Q}{C' + C_3} = \frac{4\times 36}{2+4} = 24 \ \mu\text{C}$$

(iii) Energy in C_1 is

$$U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 3 \times 4^2 = 24 \ \mu\text{J}$$

Energy in C_2 is

$$U_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2} \times 6 \times 2^2 = 12 \ \mu\text{J}$$

Energy in C_3 is

$$U_3 = \frac{1}{2} C_3 V^2 = \frac{1}{2} \times 4 \times 6^2 = 72 \ \mu\text{J}$$

Total energy is

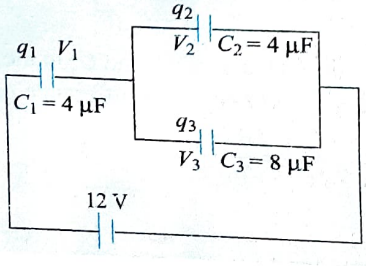
$$U = U_1 + U_2 + U_3 = 24 + 12 + 72 = 108 \ \mu\text{J}$$

Alternatively

$$U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2} \times 6 \times 6^2 = 108 \ \mu\text{J}$$

ILLUSTRATION 4.26

Three capacitors of capacitances 4 μF, 4 μF, and 8 μF are connected as shown across a battery of emf 12 V.



(i) Find the equivalent capacitance.

(ii) Find the potential difference and charge on each capacitor.

(iii) Find the energy stored in each capacitor and the total energy stored in the system of capacitors.

Sol.

(i) C_2 and C_3 are in parallel, so their equivalent capacitance is

$$C' = C_2 + C_3 = 4 + 8 = 12 \ \mu\text{F}$$

This C' will be in series with C_1, so the net equivalent capacitance is

$$C_{eq} = \frac{C_1 C'}{C_1 + C'} = \frac{4\times 12}{4+12} = 3 \ \mu\text{F}$$

(ii) $q_1 = C_{eq} V = 3\times 12 = 36 \ \mu\text{C}$.

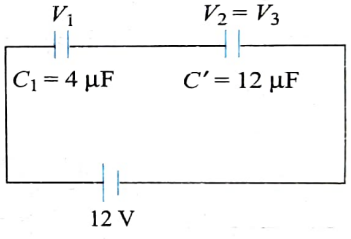
q_1 will be divided into q_2 and q_3, so

$$q_2 = \frac{C_2 q_1}{C_2 + C_3} = \frac{4\times 36}{4+8} = 12 \ \mu\text{C}$$

$$q_3 = \frac{C_3 q_1}{C_2 + C_3} = \frac{8\times 36}{4+8} = 24 \ \mu\text{C}$$

$$V_1 = \frac{q_1}{C_1} = \frac{36}{4} = 9 \text{ V}$$

$$V_2 = V_3 = \frac{q_2}{C_2} = \frac{q_3}{C_3} = \frac{36}{4} = 3 \text{ V}$$



Alternatively, potentials can also be calculated by dividing 12 V in the inverse ratio of C_1 and C'.

$$V_1 = \frac{C'V}{C_1 + C'} = \frac{12\times 12}{4+12} = 9 \text{ V}$$

$$V_2 = V_3 = \frac{C_1 V}{C_1 + C'} = \frac{4\times 12}{4+12} = 3 \text{ V}$$

(iii) $U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 4 \times 9^2 = 162 \ \mu\text{J}$

$$U_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2} \times 4 \times 3^2 = 18 \ \mu\text{J}$$

$$U_3 = \frac{1}{2} C_3 V_3^2 = \frac{1}{2} \times 8 \times 3^2 = 36 \ \mu\text{J}$$

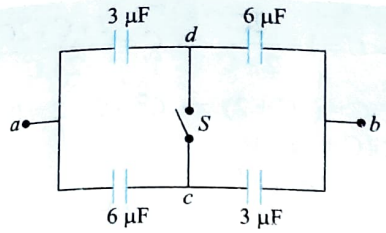
Total energy is $U_1 + U_2 + U_3 = 216 \ \mu\text{J}$

Alternatively

$$U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2} \times 3 \times (12)^2 = 216 \ \mu\text{J}$$

ILLUSTRATION 4.27

The capacitors in figure are initially uncharged and are connected as in the diagram with switch S open. The applied potential difference is $V_{ab} = +360$ V.



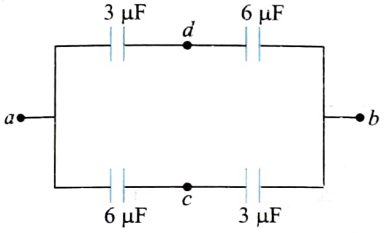
(a) What is the potential difference V_{cd}?

(b) What is the potential difference across each capacitor after switch S is closed?

(c) How much charge flowed through the switch when it was closed?

Sol.

(a) When switch is open, the potential difference across the upper and lower branch should be equal as both are connected in parallel.



The potential difference across a and d,

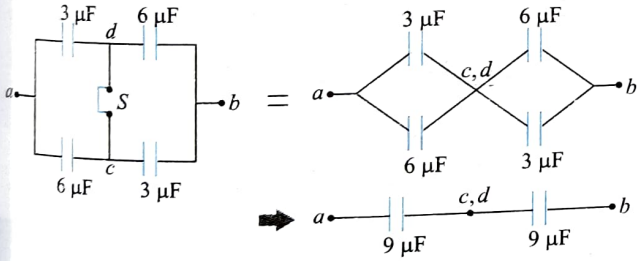
$$V_a - V_d = \frac{6}{6+3} \times 360 = 240\,\text{V} \qquad \text{...(i)}$$

and the potential difference across a and c,

$$V_a - V_c = \frac{3}{6+3} \times 360 = 120\,\text{V} \qquad \text{...(ii)}$$

From (i) and (ii), $V_c - V_d = V_{cd} = 120\,\text{V}$

(b) When switch S is closed, the circuit will be reduce as shown in figure.



From figure, $V_a - V_c = V_a - V_d = V_d - V_b = V_c - V_b$

$$= \frac{360}{2} = 180\,\text{V}$$

This is, the potential across each capacitor is 180 V

(c) Before closing the switch, the charge in each capacitor,

$$q = \frac{3\times 6}{6+3} \times 360 = 720\ \text{C}$$

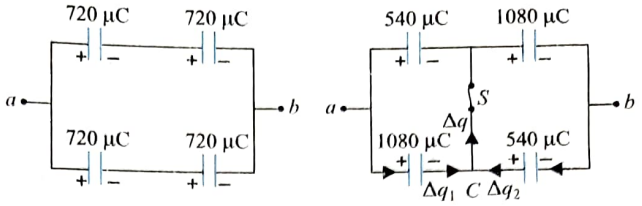
After closing the switch, the charge in the capacitors having capacitance 3μF:

$$q_{3\mu F} = 3\times 180 = 540\ \text{C}$$

The charge in the capacitors having capacitance 6μF:

$$q_{6\mu F} = 6\times 180 = 1080\ \text{C}$$

The charge appearing on each capacitor before and after closing the switch are shown in figure.



From figure we can observe, $\Delta q_1 = 1080 - 720 = 360\ \mu\text{C}$ and $\Delta q_2 = 720 - 540 = 180\,\mu\text{C}$

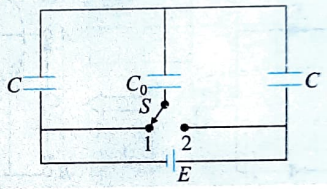
Now, at junction c incoming charge should be equal to outgoing charge.

Hence, $\Delta q = \Delta q_1 + \Delta q_2 = 360 + 180 = 540\,\mu\text{C}$

It means, charge of 540 μC flows from c to d after closing the switch.

ILLUSTRATION 4.28

In the given figure, when switch is swapped from 1 to 2, find the heat produced in the circuit.



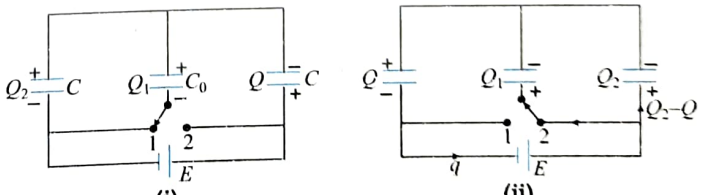
Sol. When the switch is at position 1, the circuit diagram is as shown in Fig. (i). The equivalent capacitance of the circuit,

$$C_{eq} = \frac{(C+C_0)C}{2C+C_0} \text{ and } Q = C_{eq}E$$

The charge supplied by battery, $Q = C_{eq}E = \dfrac{(C+C_0)C}{2C+C_0}E$

Hence the charge in other capacitors as connected in Fig. (i)

$$Q_1 = \frac{C_0 Q}{C+C_0} \text{ and } Q_2 = \frac{CQ}{C+C_0}$$



After shifting the switch from position 1 to position 2, the equivalent capacitance of the circuit will not change but the charge on different capacitors will be as shown in Fig. (ii). At this stage the charge flowing through the left most capacitor 'C' should be equal to the charge supplied by the battery in this process

Hence the charge flowing battery on reconnection,

$$q = Q - Q_2 = Q - \frac{CQ}{C+C_0} = \frac{C_0}{C+C_0}Q = \frac{C_0}{C+C_0}(C_{eq}E)$$

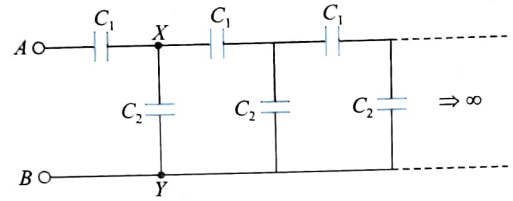
As the total energy on capacitors before and after reconnection is same. Hence whole work done by battery will go in the form of heat.

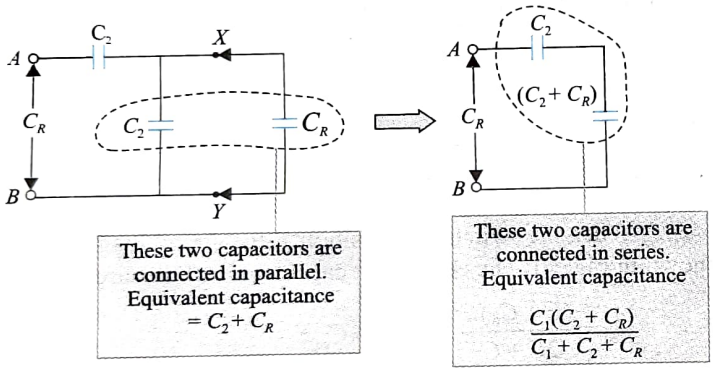
$$W_{\text{battery}} = qE = \left(\frac{C_0}{C+C_0} \times \frac{(C+C_0)C}{(2C+C_0)} E\right) E = \frac{CC_0E^2}{2C+C_0}$$

So heat generated is $H = \dfrac{CC_0E^2}{2C+C_0}$

INFINITE CHAIN OF CAPACITORS

Suppose the effective capacitance between A and B is C_R. Since the network is infinite, even if we remove one pair of capacitors from the chain, the remaining network would still have infinite pairs of capacitors, i.e., effective capacitance between X and Y would also be C_R.





These two capacitors are connected in parallel. Equivalent capacitance $= C_2 + C_R$

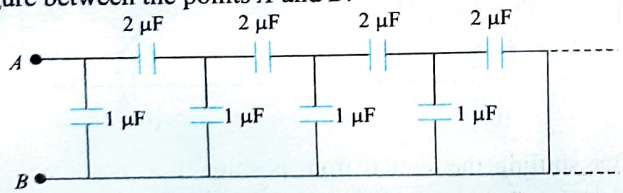
These two capacitors are connected in series. Equivalent capacitance $\dfrac{C_1(C_2+C_R)}{C_1+C_2+C_R}$

Hence, the equivalent capacitance between A and B is

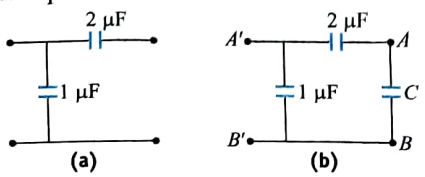
$$C_{AB} = \frac{C_1(C_2+C_R)}{C_1+C_2+C_R} = C_R \text{ or } C_{AB} = \frac{C_2}{2}\left[\sqrt{1+4\frac{C_1}{C_2}} - 1\right]$$

ILLUSTRATION 4.29

Find the equivalent capacitance of the infinite ladder shown in figure between the points A and B.



Sol. First of all we need to identify repeating unit. The repeating unit is as shown in Fig. (a). Let equivalent capacitance across A and B be C. If we add one more unit as shown in figure the equivalent capacitance across A' and B' should again be C.



$$C_{A'B'} = C = 1 + \frac{2C}{2+C} \Rightarrow C = \frac{2+C+2C}{2+C}$$

$$\Rightarrow \quad (2+C)C = (3C+2) \Rightarrow C^2 - C - 2 = 0$$

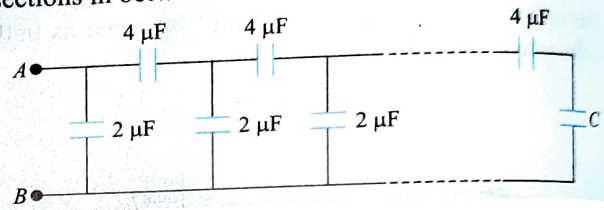
$$\Rightarrow \quad (C-2)(C+1) = 0$$

Since $C = -1$ is not acceptable

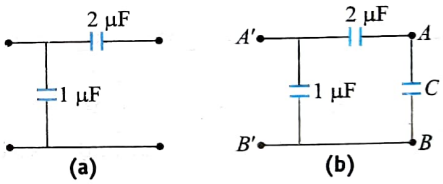
Hence equivalent capacitance, $C = 2\ \mu F$.

ILLUSTRATION 4.30

An infinite ladder is constructed by connecting several sections of 2 μF, 4 μF capacitor combinations as shown in figure. It is terminated by a capacitor of capacitance C. What value should be chosen for C such that the equivalent capacitance of the ladder between A and B becomes independent of the number of sections in between?



Sol. The equivalent capacitance of the given infinite ladder between A and B becomes independent of the number of units in between. It means, if we remove all the capacitors, other than terminal capacitor 'C', the equivalent capacitance across A and B should also be 'C'. Here the repeating unit is shown in Fig. (a). If we connect one unit across A and B, as shown in Fig. (b) the equivalent capacitance across A' and B' should remain 'C'.



$$C_{A'B'} = C = 2 + \frac{4 \times C}{4+C} \Rightarrow 4C + C^2 = 8 + 2C + 4C$$

$$\Rightarrow \quad C^2 - 2C - 8 = 0 \Rightarrow (C+2)(C-4) = 0$$

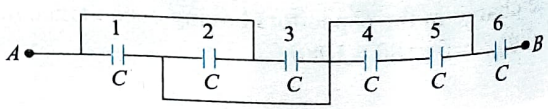
$$\Rightarrow \quad C = -2 \text{ and } 4$$

As $C = -2$ is not acceptable.

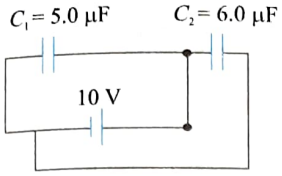
Hence equivalent capacitance, $C = 4\ \mu F$.

CONCEPT APPLICATION EXERCISE 4.3

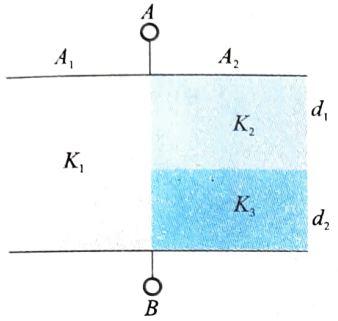
1. If you have several 2.0 μF capacitors, each capable of withstanding 200 V without breakdown, how would you assemble a combination that has an equivalent capacitance of
   (a) 0.4 μF **(b)** 1.2 μF, each withstanding 1000 V?
2. N identical capacitors are connected in parallel, and then a potential difference of V is applied to them. Find the potential difference when these capacitors are reconnected in series, their charges being left undisturbed.
3. Find the equivalent capacitance between points A and B as shown in figure.



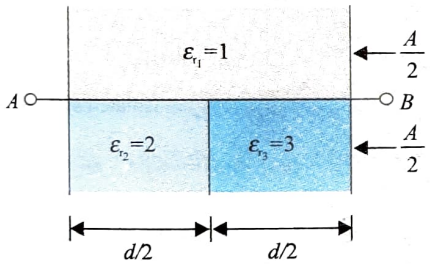
4. Find the charge supplied by the battery in the arrangement as shown in figure.



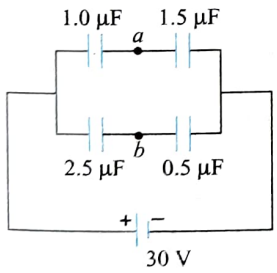
5. Find the capacitance between A and B if three dielectric slabs of dielectric constants K_1 (area A_1 and thickness d), K_2 (area A_2 and thickness d_1), and K_3 (area A_2 and thickness d_2) are inserted between the plates of a parallel plate capacitor of plate area A (figure). (Given that the distance between the two plates is $d = d_1 + d_2$.)



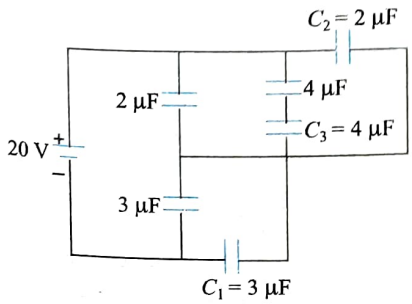
6. Three dielectrics of relative permittivities $\varepsilon_{r_1} = 1$, $\varepsilon_{r_2} = 2$, and $\varepsilon_{r_3} = 3$ are introduced in a parallel plate capacitor of plate area A and separation d. Find the effective capacitance between A and B.



7. Four capacitors are connected as shown in figure to a 30 V battery. Find the potential difference between points a and b.

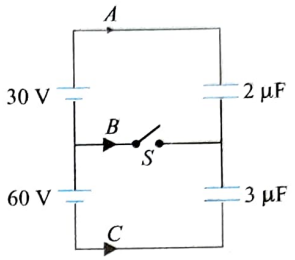


8. In figure, the battery has a potential difference of 20 V. Find

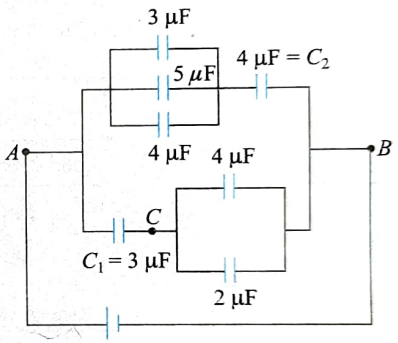


(a) the equivalent capacitance of all the capacitors across the battery and
(b) the charge stored on that equivalent capacitance. Find the charge on
(c) capacitor C_1,
(d) capacitor C_2, and
(e) capacitor C_3.

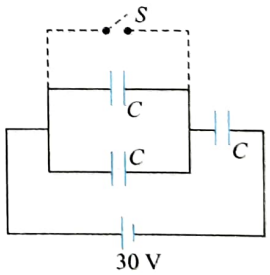
9. What charges will flow through section B of the circuit in the direction shown when switch S is closed.



10. Figure shows a network of seven capacitors. If charge on 5 μF capacitor is 10 μC, find the potential difference between points A and C



11. The capacitor each having capacitance $C = 2$ μF are connected with a battery of emf 30 V as shown in figure. When the switch S is closed. Find



(a) the amount of charge flown through the battery
(b) the energy supplied by the battery
(c) the heat generated in the circuit
(d) the amount of charge flown through the switch S

ANSWERS

2. NV **3.** $3C/4$ **4.** 110 μC

5. $\frac{A_1 k_1 \varepsilon_0}{d_1 + d_2} + \frac{A_2 k_2 k_3 \varepsilon_0}{k_2 d_2 + k_3 d_1}$ **6.** $\frac{17\,\varepsilon_0 A}{10d}$ **7.** 13 V

8. (a) 3 μF (b) 60 μC (c) 30 μC (d) 20 μC (e) 20 μC **9.** 120 μC

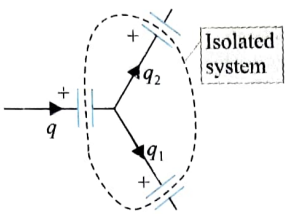
10. 5.33 V **11.** (a) 20 μC (b) 0.6 mJ (c) 0.3 mJ (d) 60 μC

KIRCHHOFF'S RULES FOR CAPACITORS

Kirchhoff's rules can be used to determine the potential difference and charge on the plates of a capacitor in any electric circuit. In a circuit with capacitors and batteries, two important rules are involved.

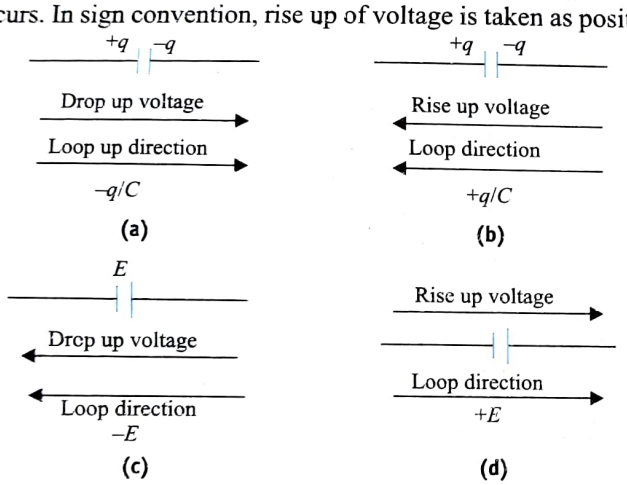
Junction rule: A capacitor circuit obeys the principle of conservation of charge. The incoming charge at any junction is equal to the outgoing charge from the junction, i.e., $q = q_1 + q_2$. In other words, in any isolated system, the net charge is conserved. So in the system shown in figure,

$$-q + q_1 + q_2 = 0 \Rightarrow q = q_1 + q_2$$



Loop rule: In a closed circuit, the algebraic sum of the rise up and drop up voltages is zero, i.e., $\Sigma V = 0$.

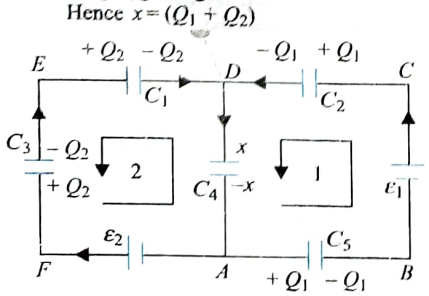
The direction of the loop is not specified and is chosen in a comfortable manner. When we go from a point of higher potential to a point of lower potential in the loop direction, drop up voltage occurs [Fig. (a)]. In sign convention, drop up voltage is taken as negative. If we go from a lower potential point to a higher potential point, rise up voltage occurs. In sign convention, rise up of voltage is taken as positive.



APPLICATION OF KIRCHHOFF'S LOOP RULE IN A CIRCUIT

If you travel from the positively charged plate of a capacitor or the terminal of a cell to the negatively charged plate of a capacitor or the terminal of a cell, then the sign of V or ε is negative and vice versa.

At junction D
in coming charge $= (Q_1 + Q_2)$
Outgoing charge $= x$
Hence $x = (Q_1 + Q_2)$



Applying loop rule in closed loop $ABCDA$

$$-\frac{Q_1}{C_5} + \varepsilon_1 - \frac{Q_1}{C_2} - \frac{Q_1 + Q_2}{C_4} = 0$$

or $$Q_1\left(\frac{1}{C_2} + \frac{1}{C_4} + \frac{1}{C_5}\right) + \frac{Q_2}{C_4} = \varepsilon_1 \quad \text{...(i)}$$

Applying loop rule in closed loop $ADEFA$

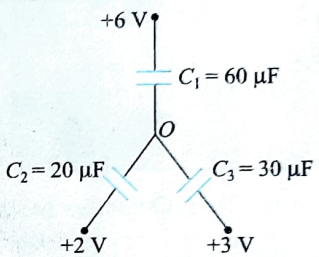
$$\frac{Q_1 + Q_2}{C_4} + \frac{Q_2}{C_1} + \frac{Q_2}{C_3} - \varepsilon_2 = 0$$

or $$\frac{Q_1}{C_4} + Q_2\left[\frac{1}{C_1} + \frac{1}{C_3} + \frac{1}{C_4}\right] = \varepsilon_2 \quad \text{...(ii)}$$

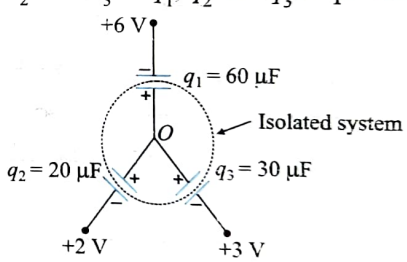
Now you have two equations and two unknowns (Q_1 and Q_2), which can be found easily by solving the equations.

ILLUSTRATION 4.31

Three uncharged capacitors of capacitance C_1, C_2 and C_3 are connected to one another as shown in figure. Find the potential at O.



Sol. Let at junction O, the potential be V and the charges in capacitors C_1, C_2 and C_3 be q_1, q_2 and q_3 respectively



At O, $q_1 + q_2 + q_3 = 0$ ...(i)

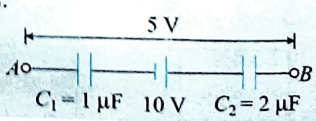
The potential difference across the capacitors C_1, C_2 and C_3 will be $(V-6), (V-2)$ and $(V-3)$ respectively. Hence equation (i) can be written as

$$60(V-6) + 20(V-2) + 30(V-3) = 0$$

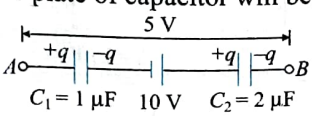
Solving this equation, we get, $V = \frac{49}{11}$ V

ILLUSTRATION 4.32

A circuit has a section AB shown in figure. The emf of the cell is 10 V and the capacitors have capacitances $C_1 = 1\ \mu F$ and $C_2 = 2\ \mu F$. The potential difference $V_{AB} = 5$V. Find the charges on the capacitors.



Sol. Let the charge on the left plate of the capacitor is q, hence the charge on right plate of capacitor C_2 will be $-q$ as charge supplied by negative plate of capacitor will be $-q$.



Moving from point A in the section of circuit AB

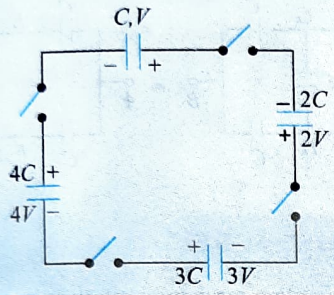
$$V_A - \frac{q}{1} + 10 - \frac{q}{2} = V_B$$

$$V_A - V_B = q + \frac{q}{2} - 10 \Rightarrow 5 = \frac{3}{2}q - 10$$

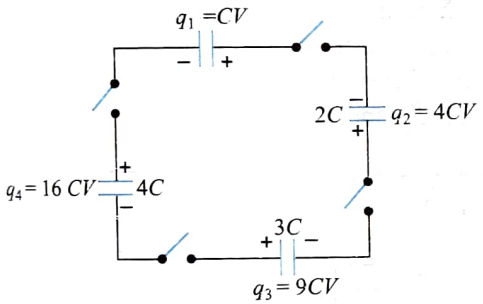
Hence the charges on the capacitors $q = 10\ \mu C$

ILLUSTRATION 4.33

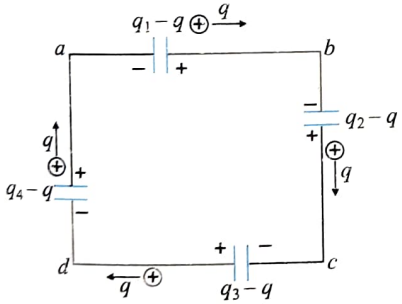
Four capacitors having capacitance C, $2C$, $3C$ and $4C$ are charged to the voltage V, $2V$, $3V$ and $4V$ correspondingly. The circuit is closed. Find the voltage on all condensers in the equilibrium.



Sol. Let a charge q flow in the clockwise direction after closing the switch.



Just before closing the switch



Just after closing the switch

Applying loop law in $abcda$, we get

$$\frac{q_1 - q}{C} + \frac{q_2 - q}{2C} + \frac{q_3 - q}{3C} + \frac{q_4 - q}{4C} = 0$$

Substituting values of q_1, q_2, q_3, and q_4, we get, $q = \frac{24}{5}CV$

The potential difference across the capacitor 'C',

$$|\Delta V|_C = \left|\frac{q_1 - q}{C}\right| = \frac{19}{5}\text{ V}$$

The potential difference across the capacitor '$2C$',

$$|\Delta V|_{2C} = \left|\frac{q_2 - q}{2C}\right| = \frac{2}{5}\text{ V}$$

The potential difference across the capacitor '$3C$',

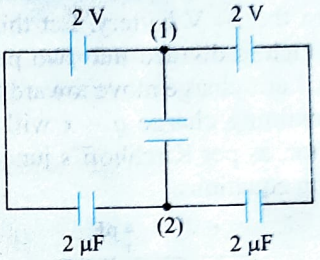
$$|\Delta V|_{3C} = \left|\frac{q_3 - q}{3C}\right| = \frac{7}{5}\text{ V}$$

and the potential difference across the capacitor '$4C$',

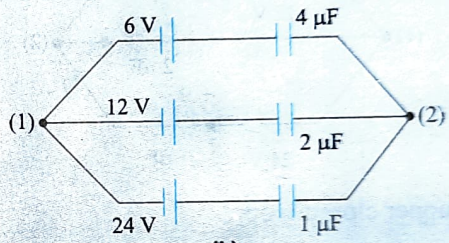
$$|\Delta V|_{4C} = \left|\frac{q_4 - q}{4C}\right| = \frac{14}{5}\text{ V}$$

ILLUSTRATION 4.34

Find the potential difference $V_a - V_b$ between the points (1) and (2) shown in each part of figure.



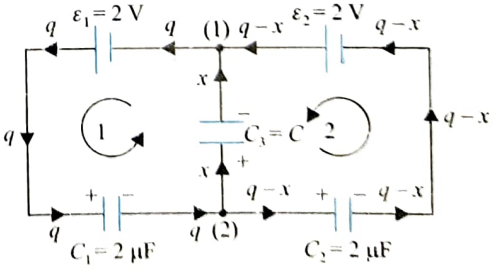
(a)



(b)

Sol.

(a) We first distribute the charges on different capacitors and branches keeping in mind Kirchhoff's junction rule. We can start from any battery. In this case, we start from battery ε_1. Let battery ε_1 supply a charge q. The charge on the left plate of capacitor C_1 will be q as this plate is receiving the charge. Now charge q reaches junction b where it is divided into two paths. Let charge x go to capacitor C_3, then the remaining charge $q - x$ will go to capacitor C_2. In junction a, we can verify Kirchhoff's junction rule, i.e., the incoming charge is equal to the outgoing charge. Now we write the loop equations.



For loop 1

$$2 - \frac{q}{2} - \frac{x}{C} = 0 \qquad \text{...(i)}$$

For loop 2

$$2 + \frac{x}{C} - \frac{(q - x)}{2} = 0$$

or $$2 + \frac{x}{C} - \frac{q}{2} + \frac{x}{2} = 0 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii)

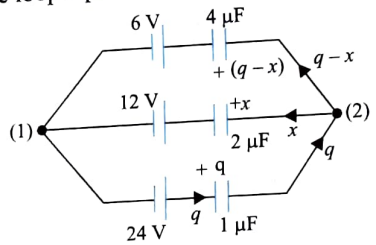
$$\frac{2x}{C}+\frac{x}{2}=0$$

or $$x\left(\frac{2}{C}+\frac{1}{2}\right)=0$$

or $$x=0\left(\text{since }\frac{2}{C}+\frac{1}{2}\neq 0\right)$$

No charge will go to the capacitor connected across (1) and (2). As there is no charge in the capacitor connected across (1) and (2), the potential difference across (1) and (2) should be zero.

(b) In this case also, we will first distribute the charges. Let us start from the 24 V battery. Let this battery supply a charge q, which is divided into two paths after reaching junction (2). Let x charge move toward the 2 μF capacitor, then the remaining charge $q - x$ will move toward the 4 μF capacitor, as per Kirchhoff's junction rule. Now we write the loop equations.



In the upper closed loop

$$+6+\frac{(q-x)}{4}-\frac{x}{2}-12=0 \text{ or } \frac{q}{4}-x=6 \quad \text{...(i)}$$

In the lower closed loop

$$12+\frac{x}{2}+\frac{q}{1}-24=0 \quad \text{or} \quad q+\frac{x}{2}=12 \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$x=\frac{-8}{3}\mu C$$

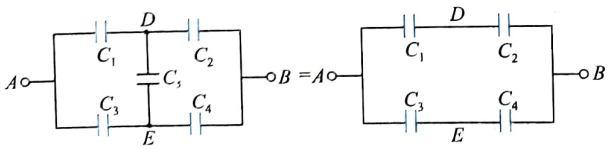
Moving from (1) to (2) through middle path

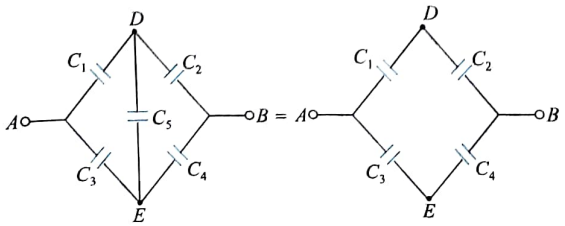
$$V_a+12-\frac{8/3}{2}=V_b$$

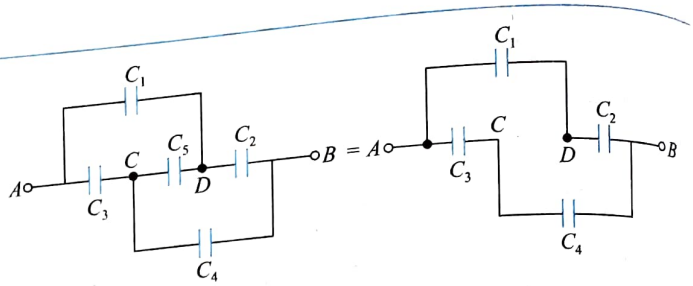
or $$V_a-V_b=\frac{4}{3}-12=\frac{-32}{3}\text{ V}$$

CIRCUITS BASED ON WHEATSTONE BRIDGE

In following circuits if $(C_1/C_2)=(C_3/C_4)$, the potential difference across D and E will be zero.







In all above cases, the equivalent capacity between A and B is

$$C_{AB}=\frac{C_1C_2}{C_1+C_2}+\frac{C_3C_4}{C_3+C_4}$$

EXTENDED WHEATSTONE BRIDGE

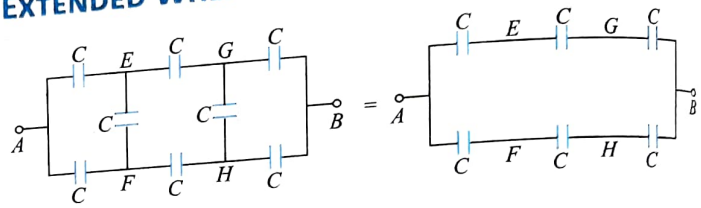
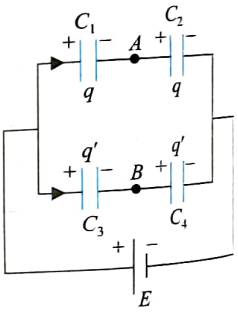


ILLUSTRATION 4.35

What is $V_A - V_B$ in the arrangement shown in figure. What is the condition such that $V_A - V_B = 0$.

Sol. Let the charges be as shown in figure. We know that capacitors in series have the same charge. Considering the loop containing C_1, C_2, and E, we get



$$\frac{q}{C_1}+\frac{q}{C_2}-E=0$$

or $$q=E\left[\frac{C_1C_2}{C_1+C_2}\right]$$

From the loop containing C_3, C_4, and E, we get

$$\frac{q'}{C_3}+\frac{q'}{C_4}-E=0 \quad \text{or} \quad q'=E\left[\frac{C_3C_4}{C_3+C_4}\right]$$

Now,

$$V_A-V_B=\frac{q}{C_2}-\frac{q'}{C_4}=E\left[\frac{C_1}{C_1+C_2}-\frac{C_3}{C_3+C_4}\right]$$

$$=E\left[\frac{C_1C_4-C_3C_2}{(C_1+C_2)(C_3+C_4)}\right]$$

For $V_A - V_B = 0$, $C_1C_4 = C_2C_3 = 0$. If $(C_1/C_2) = (C_3/C_4)$, the potential difference across A and B will be zero. This statement is very important in capacitor circuit solving. This type of the situation is called balanced Wheatstone bridge.

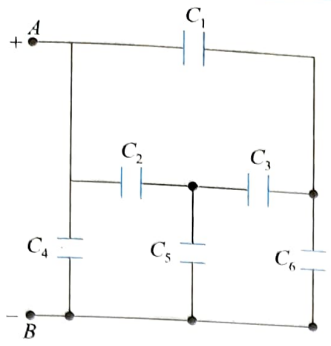
ILLUSTRATION 4.36

Six capacitors $C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C$ are arranged as shown in figure. Determine the equivalent capacitance between A and B.

Sol. Let the potential difference across the battery terminals be V and the charge supplied be Q. We have to find the capacitance

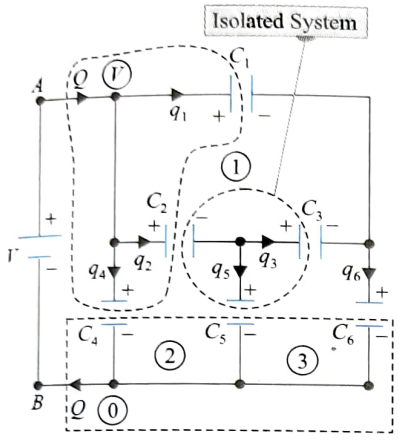
of the system and the capacitance of a capacitor that would have the same charge Q on its plates as the battery at voltage V. Hence,

$$C_{equivalent} = \frac{Q}{V_A - V_B} = \frac{Q}{V}$$



The incoming charge Q is equal to the charge received by the plates of the capacitors C_1, C_2, and C_4. Thus,

$$Q = q_1 + q_2 + q_4$$



Similarly, outgoing charge Q is equal, to charge coming out from the plates of the capacitors C_4, C_5, and C_6.

$$Q = q_4 + q_5 + q_6$$

The above equations give

$$q_1 + q_2 = q_5 + q_6 \qquad ...(i)$$

In a closed loop, the net potential drop must be zero, according to Kirchhoff's voltage law. Therefore, for loop 1, loop 2, and loop 3, we have

$$\text{1} \quad \frac{q_1}{C} - \frac{q_2}{C} - \frac{q_3}{C} = 0 \Rightarrow \begin{matrix} q_1 = q_2 + q_3 \\ q_1 + q_2 = 2q_2 + q_3 \end{matrix} \qquad ...(ii)$$

$$\text{2} \quad \frac{q_2}{C} - \frac{q_4}{C} + \frac{q_5}{C} = 0 \Rightarrow q_4 = q_2 + q_5 \qquad ...(iii)$$

$$\text{3} \quad \frac{q_3}{C} - \frac{q_5}{C} + \frac{q_6}{C} = 0 \Rightarrow \begin{matrix} q_5 = q_3 + q_6 \\ q_5 + q_6 = 2q_6 + q_5 \end{matrix} \qquad ...(iv)$$

From the isolated system shown in figure, we can conclude

$$q_3 + q_5 - q_2 = 0 \qquad ...(v)$$

From Eqs. (i), (ii), and (iv), we conclude $q_2 = q_6$ and $q_1 = q_5$. If $q_1 = q_5$, then Eq. (v) becomes

$$q_2 - q_1 = q_3 \qquad ...(vi)$$

Solving Eqs. (ii) and (vi), we get

$$q_3 = 0,\ q_1 = q_2 = q_5 = q_6 = \frac{q_4}{2},\ \text{and}\ Q = 2q_4$$

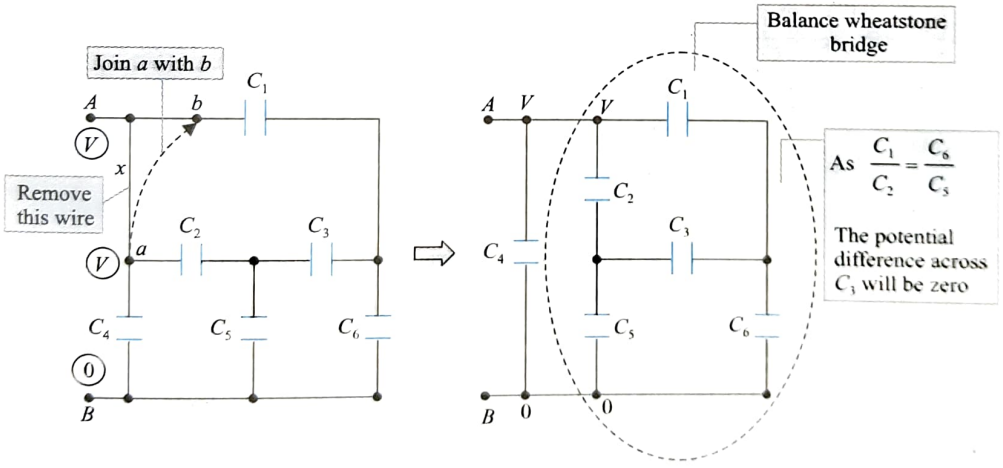
We can write

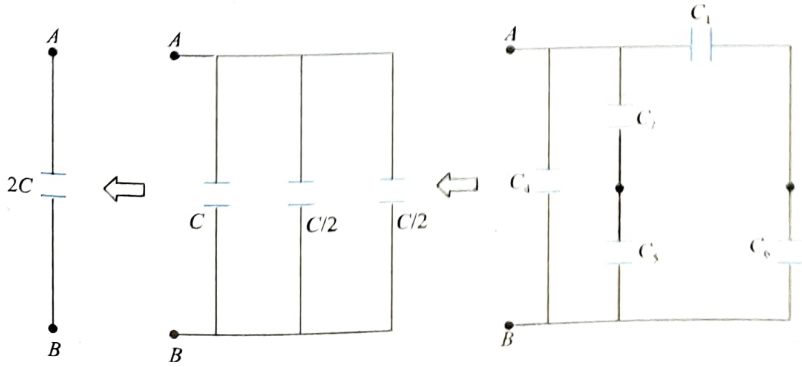
$$V = \frac{q_4}{C} = \frac{Q}{2C} \text{ or } \frac{Q}{V} = 2C$$

Hence,

$$C_{equivalent} = \frac{Q}{V_A - V_B} = \frac{Q}{V} = 2C$$

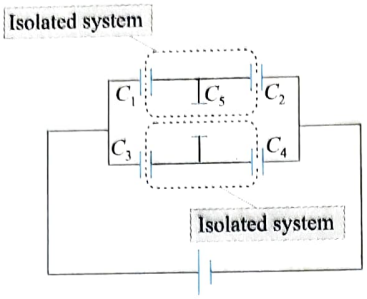
Alternative method: Connect point a to point b and note the indicated Wheatstone bridge (figure). Now simplify the circuit to obtain the desired result.





NODAL ANALYSIS OF CAPACITIVE CIRCUITS

Nodal analysis is based on conservation of charge. At any isolated part of a circuit, charge is conserved. $\Sigma q_i = 0$ if the capacitors are initially uncharged, and Σq_i is equal to the sum of the initial charges of the plates of the capacitors connected to the node if the capacitors have initial charge.



The following steps should be followed in problem solving:

(i) Mark different nodes in the circuit and assign their potentials.
(ii) Identify the isolated systems in the circuit and write the equation of conservation of charges.
(iii) Solve the equations to get the values of potentials. Let us learn this matter through some illustrations.

Step 1: Assign a node as a reference and assume its potential To be zero.

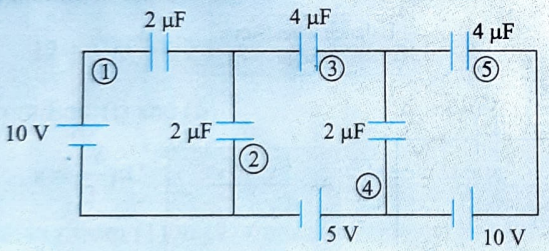
Step 2: Assign the potential of every junction of the circuit as an unknown S.

Step 3: Apply Kirchhoff's current law at the junction corresponding to the unknown potential.

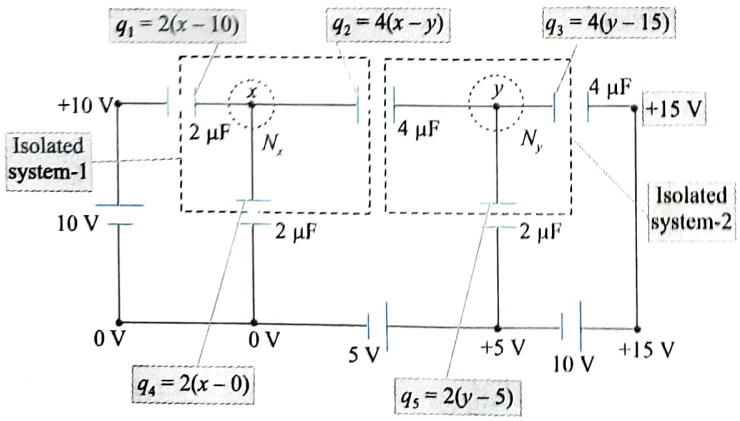
Step 4: Solve the equations to get the desired unknown potentials. Let us learn this matter through some illustrations.

ILLUSTRATION 4.37

In figure, what are the charges on all the four capacitors?



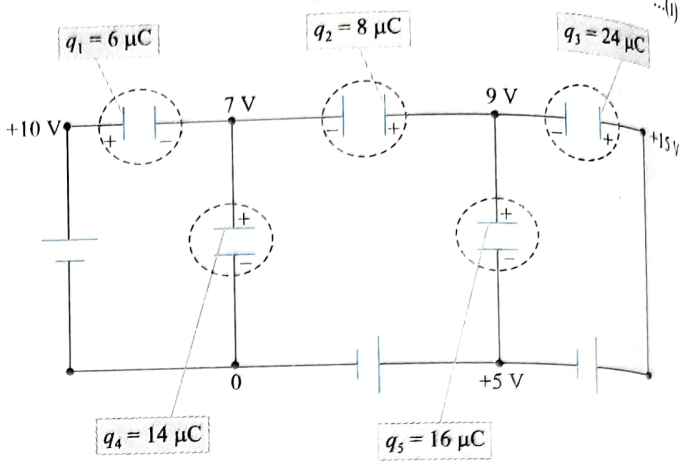
Sol. Let us assign the potentials at nodes N_1 and N_2 as x and y, respectively. We have identified two isolated systems in the circuits as marked in figure. The potentials at different junctions are marked according to the emf of the batteries as shown in the figure. The charge appearing in each capacitors is marked as per the formula $q = C\Delta V$



In the isolated systems (1) and (2), the net charge should be conserved. For the isolated system (1),

$$2(x - 10) + 4(x - y) + 2x = 0$$

or $$8x - 4y - 20 = 0 \text{ or } 2x - y = 5 \quad \ldots(i)$$



For the isolated system (2),

$$-4(x - y) + 4(y - 15) + 4(y - 5) = 0$$

or $$-4x + 12y - 80 \text{ or } -2x + 6y = 40 \quad \ldots(ii)$$

Adding Eqs. (i) and (ii), we get

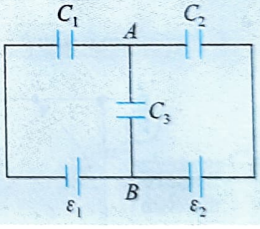
$$5y = 45$$

or $$y = 9 \text{ V and } x = 7 \text{ V}$$

Hence, the charges on different capacitors are shown in figure.

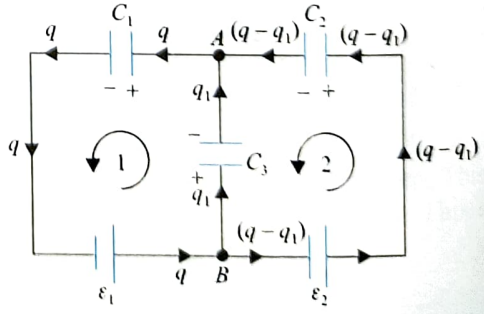
ILLUSTRATION 4.38

Find the potential difference $V_A - V_B$ between points A and B of the circuit shown in figure.



Sol.

Method 1: The distribution of electric charge is shown in figure.



In loop (1),

$$\varepsilon_1 - \frac{q_1}{C_3} - \frac{q}{C_1} = 0 \quad \ldots(i)$$

In loop (2),

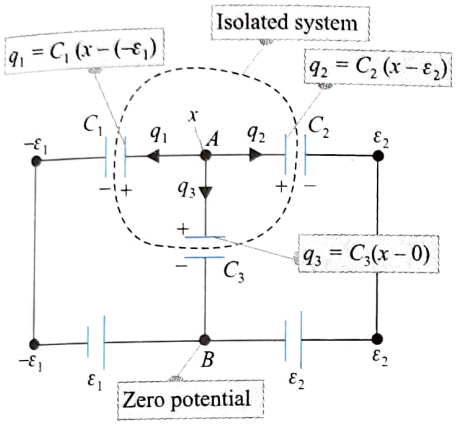
$$-\left(\frac{q - q_1}{C_2}\right) + \frac{q_1}{C_3} + \varepsilon_2 = 0 \quad \ldots(ii)$$

Solving Eqs. (i) and (ii), we get

$$q_1 = \frac{-C_3(C_2\varepsilon_2 - C_1\varepsilon_1)}{(C_1 + C_2 + C_3)}$$

$$\therefore \quad V_A - V_B = -\left\{\frac{q_1}{C_3}\right\} = \left(\frac{C_2\varepsilon_2 - C_1\varepsilon_1}{C_1 + C_2 + C_3}\right)$$

Method 2: The algebraic sum of the charge at junction A should be zero. Suppose the charges q_1, q_2, and q_3 are moving away from the junction. Let us assume that the potential of junction B is zero. Now we will write the potentials of various junctions as shown in figure. After knowing the potential difference across the capacitors, we can write the relations of q_1, q_2, and q_3 in terms of their capacitance and potential difference.



At junction A,

$$\Sigma q = 0$$

or $\quad q_1 + q_2 + q_3 = 0$

or $\quad C_1(x + \varepsilon_1) + C_2(x - \varepsilon_2) + C_3(x - 0) = 0$

or $\quad x(C_1 + C_2 + C_3) = \varepsilon_2 C_2 - \varepsilon_1 C$

$$\therefore \quad x = \frac{\varepsilon_2 C_2 - \varepsilon_1 C_1}{C_1 + C_2 + C_3}$$

$$\therefore \quad V_A - V_B = x - 0 = x = \frac{\varepsilon_2 C_2 - \varepsilon_1 C_1}{C_1 + C_2 + C_3}$$

Method 3: The electric charge enclosed by the isolated system shown in figure should be zero. Therefore,

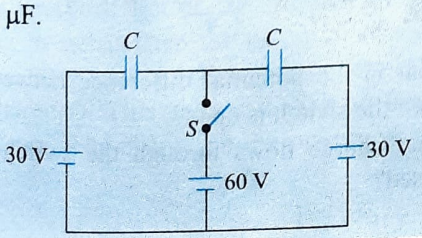
$$q_1 + q_2 + q_3 = 0$$

or $\quad \frac{x + \varepsilon_1}{C_1} + \frac{x - \varepsilon_2}{C_2} + \frac{x - 0}{C_3} = 0 \quad$ or $\quad x = \frac{C_2\varepsilon_2 - C_1\varepsilon_1}{C_1 + C_2 + C_3}$

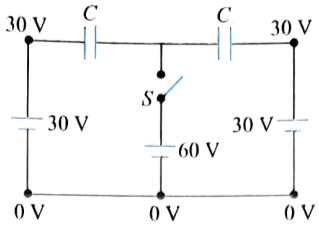
$$\therefore \quad V_A - V_B = x - 0 = \frac{C_2\varepsilon_2 - C_1\varepsilon_1}{C_1 + C_2 + C_3}$$

ILLUSTRATION 4.39

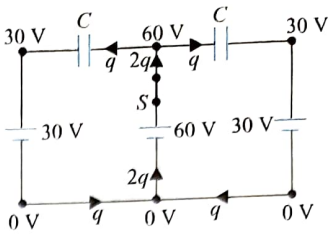
In the given circuit diagram, initially the switch S is opened. Find heat dissipated in the circuit after switch S is closed. It is given $C = 2\ \mu F$.



Sol. When switch S open the potential difference across the group of capacitors is zero. Hence the initial charge in both the capacitors will be zero.



Before closing the switch



After closing the switch

When the switch S is closed, potential difference across both of them is 30 V. Charge on each of them is

$$q = (2\ \mu F)(30\ V) = 60\ \mu C$$

The heat developed in the circuit

$$H = \sum \frac{(\Delta Q)^2}{2C} = \frac{(60-0)^2}{2\times 2} = 180\ \mu J = 1.8\ mJ$$

Approach 2: The charge flow in various path has also been shown in figure. The charge supplied by 60 V cell $= 2q = 2 \times 60 = 120\ \mu C$. Energy supplied by 60 V cell, $W_{60\,V} = 60 \times 120 = 7200\ \mu J$. Energy absorbed by each of 30 V cell,

$$W_{30\,V} = -30 \times 60 = 1800\ \mu J$$

Energy stored in each capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 2 \times 30^2 = 900\ \mu J$$

Applying conservation of energy principle,

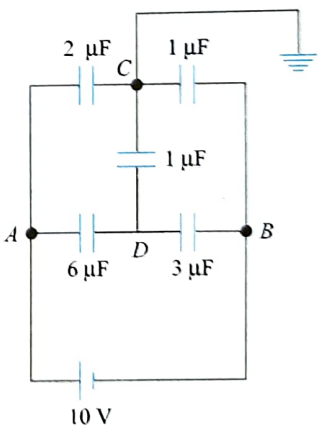
$$W_{battery} = \Delta U + \text{Heat} \Rightarrow \text{Heat} = W_{battery} - \Delta U$$

$$\therefore \text{ Heat dissipated, } H = (7200 - 2\times 1800) - 2\times 900$$

$$= 180\ \mu J = 1.8\ mJ$$

CONCEPT APPLICATION EXERCISE 4.4

1. For the network of capacitors as shown in figure.



(a) Find the potential of junction B,

(b) Find the potential of junction D,

(c) Find the charge on 2 μF capacitor.

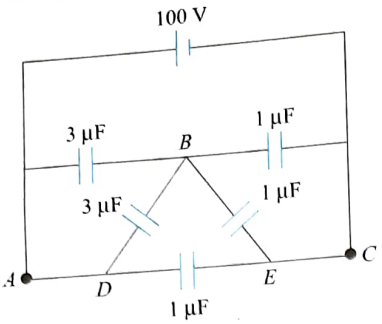
2. In figure, the system is in steady state. Then

(a) $V_A - V_B =$ _____

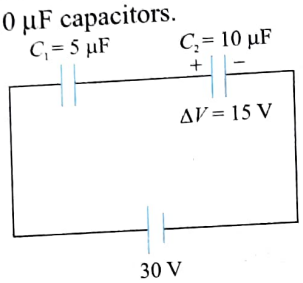
(b) $V_B - V_C =$ _____

(c) $V_D - V_E =$ _____

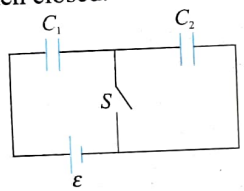
(d) The energy stored in the circuit is _____



3. A 10 μF capacitor is charged to 15 V. It is next connected in series with an uncharged 5 μF capacitor. The series combination is finally connected across a 30 V battery, as shown in figure. Find the new potential difference across the 5 μF and 10 μF capacitors.

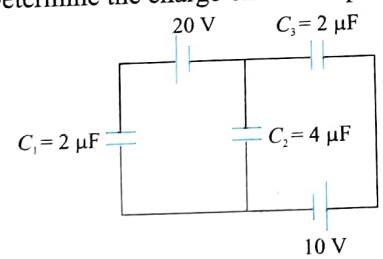


4. Two capacitors $C_1 = C_2 = C$ are connected with a battery of emf ε as shown in figure. The switch S is open for a long time and then closed.

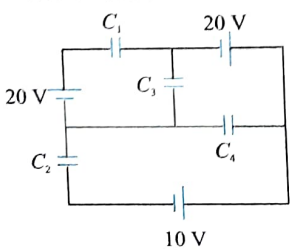


(a) Find the charge flowing through the battery when the switch S is closed.
(b) Find the work done by the battery.
(c) Find the change in energy stored in the capacitance.
(d) Find the heat developed in the system.

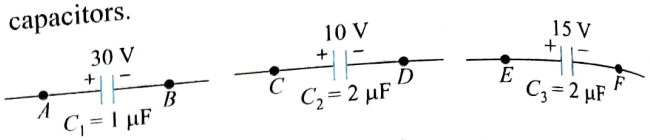
5. Three capacitors C_1, C_2, and C_3 are arranged as shown in figure. Determine the charge on each capacitor.



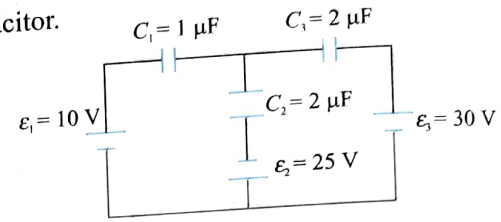
6. Four capacitors $C_1 = 8\ \mu F$, $C_2 = 2\ \mu F$, $C_3 = 6\ \mu F$, and $C_4 = 6\ \mu F$ are arranged as shown in figure. Find the charge on all the capacitors in the circuit.



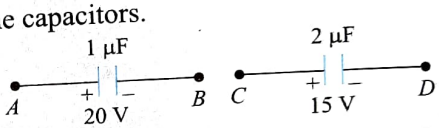
7. Three capacitors of capacitances 1 μF, 2 μF, and 2 μF are charged up to the potential difference 30 V, 10 V, and 15 V, respectively. If terminal A is connected with D, C is connected with E, and F is connected with B, then find the charge flow in the circuit and the final charges on the capacitors.



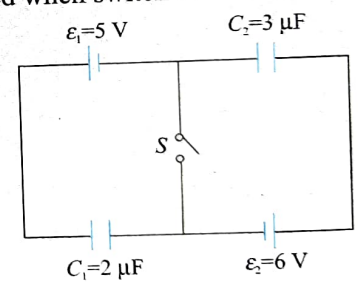
8. Three initially uncharged capacitors are arranged with batteries as shown in figure. Find the charge on each capacitor.



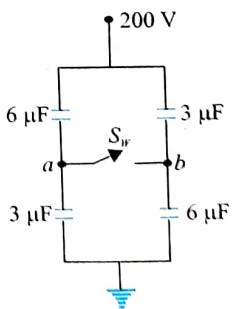
9. Two capacitors of capacitances 1 μF and 2 μF are charged to potential differences 20 V and 15 V as shown in figure. If terminals B and C are connected together and terminals A and D are connected with the positive and negative ends of the battery, respectively, then find the final charges on both the capacitors.



10. Two capacitors C_1 and C_2 are connected with two batteries of emf ε_1 and ε_2. The circuit components are connected with a switch S as shown in figure. Initially the switch is open and the capacitors are charged. Find the heat produced when switch S is closed.



11. Figure shows a capacitive circuit, with a switch S_w.



(a) What is the potential difference between a and b when the switch is open?
(b) What charge flows through the switch when it is closed?

ANSWERS

1. (a) $-\frac{20}{3}$ V (b) 0 (c) $\frac{20}{3}\mu C$

2. (a) 25 V (b) 75 V (c) 100 V (d) 125×10^{-4} J

3. 10 V, 20 V **4.** (a) $\frac{C\varepsilon}{2}$ (b) $\frac{C\varepsilon^2}{2}$ (c) $\frac{1}{4}C\varepsilon^2$ (d) $\frac{1}{4}C\varepsilon^2$

5. 35 μC, 10 μC and 25 μC

6. 100 μC, 10 μC, 30 μC and 60 μC

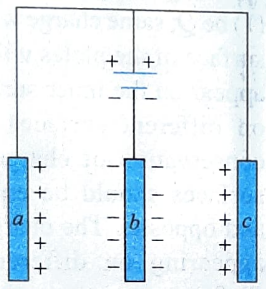
7. 12.5 μ, 17.5 μC, 42.5 μC, 7.5 μC **8.** 52 μC, 58 μC, 6 μC

9. $\frac{50}{3}\mu C, \frac{80}{3}\mu C$ **10.** 40 μJ **11.** (a) $\frac{200\,V}{3}$ (b) $300\,\mu C$

PROBLEMS INVOLVING PLATES

ILLUSTRATION 4.40

Each of the three plates shown in figure has 2.0×10^{-2} m^2 area on one side, and the gap between the adjacent plates is 0.2 mm. The emf of the battery is $\varepsilon = 20$ V. Find the distribution of charge on various surfaces of the plates. What is the equivalent capacitance of the system between the terminal points?



Sol. As the potentials of a and c are equal, the capacitors C_{ab} and C_{bc} are in parallel. Therefore,

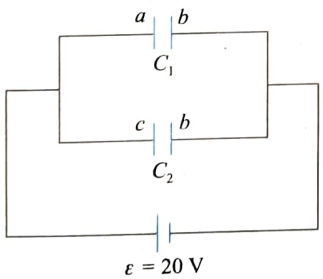
$$C_{ab} = C_{bc} = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \times 2 \times 10^{-2}}{0.2 \times 10^{-3}} = 100\,\varepsilon_0 (F)$$

Equivalent capacitance is

$$C = C_{ab} + C_{bc} = \frac{2\varepsilon_0 A}{d}$$

$$= \frac{2 \times \varepsilon_0 \times 2 \times 10^{-2}}{0.2 \times 10^{-3}} = 200\,\varepsilon_0\,(C)$$

The charge on plat b is negative on both faces. Thus, the charge on faces a and c is



$$q_a = q_c = \frac{\varepsilon_0 A}{d}\varepsilon$$

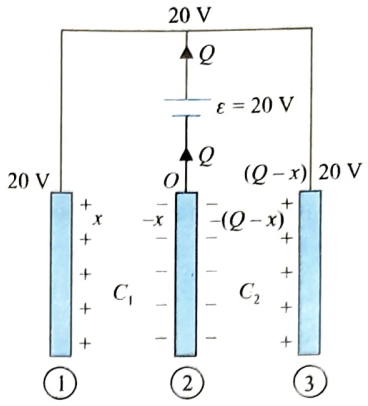
$$= 100\,\varepsilon_0 \times 20$$

$$= 2000\,\varepsilon_0\,(C)$$

$$\therefore \quad Q = 2 \times 2000\,\varepsilon_0\,(C)$$

$$= 4000\,\varepsilon_0\,(C)$$

Alternative method: Let us assume that the potential of the middle plate be zero. Then the potential of the left and right plates will be 20 V each. Let the positive terminal of the battery supply a charge Q. This charge is divided into two plates as shown in figure. The charge on the outermost surfaces will be zero.



Equal and opposite charges will appear on both surfaces of the mid plate (plate 2). The sum of the charges on both the surfaces should be $-Q$, which will flow toward the negative terminal of the battery. We can observe two capacitors C_1 and C_2. For capacitor C_1

$$x = C(20 - 0) \qquad ...(i)$$

For capacitor C_2

$$(Q - x) = C(20 - 0) \qquad ...(ii)$$

From (i)

$$x = \frac{\varepsilon_0 A}{d} \times 20 = \frac{\varepsilon_0 (2 \times 10^{-2})}{0.2 \times 10^{-3}} \times 20 = 2000\,\varepsilon_0 (C)$$

Also from (ii),

$$(Q - x) = 2000\,\varepsilon_0\,(C)$$

The net charge on the mid plate is

$$-x + (Q - x) = -2 \times 2000\varepsilon_0 = -4000\varepsilon_0\,(C)$$

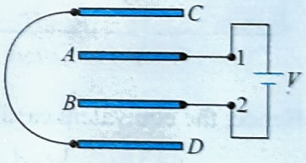
Hence, the equivalent capacity is

$$C_{eq} = \frac{Q}{\varepsilon} = \frac{4000\,\varepsilon_0}{20}$$

or $\quad C_{eq} = 200\varepsilon_0 (F)$

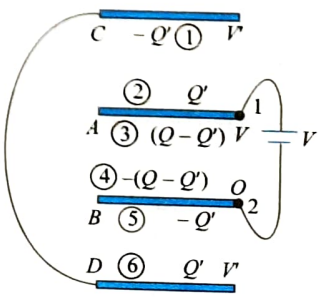
ILLUSTRATION 4.41

Four identical metal plates are arranged in air at equal distance d from each other. The area of each plate is A. A battery of emf V is connected across plates 1 and 2. Discuss the charge distribution and find the capacitance of the system between points 1 and 2 if the other two plates are connected by a conducting wire as shown in figure.



Sol. As the battery is connected to A and B, there is a charge $+Q$ on plate A and $-Q$ on plate B. The charges on A and B will induce a charge $-Q'$ on C and a charge $+Q'$ on D, so that the net charge on plates C and D remains zero.

Consequently, charge Q on A is divided into two parts: $+Q'$ on the upper side of plate A and $(Q - Q')$ on the lower side. Similarly, charge $-Q$ on B is also divided: $-Q'$ on the lower side and $-(Q - Q')$ on the upper side on plate B.



Let us assume that the potential of plate A be V, then the potential

of plate B is zero. As plates C and D are connected, they will be at a common potential, say V'. So the capacity of the system can be written as follows

$$C = \frac{Q}{V_+ - V_-} = \frac{Q}{V} \quad ...(i)$$

We can write the equation for facing surfaces (1) and (2) as

$$Q' = C_0(V - V') \quad ...(ii)$$

For facing surfaces (3) and (4)

$$(Q - Q') = C_0(V - 0) \quad ...(iii)$$

For facing surfaces (5) and (6)

$$Q' = C_0(V' - 0) \quad ...(iv)$$

From Eqs. (ii), (iii), and (iv)

$$\frac{Q}{V} = \frac{3}{2}C_0 = \frac{3}{2}\frac{\varepsilon_0 A}{d}$$

and $$Q' = \frac{C_0 V}{2} = \frac{\varepsilon_0 AV}{2d}$$

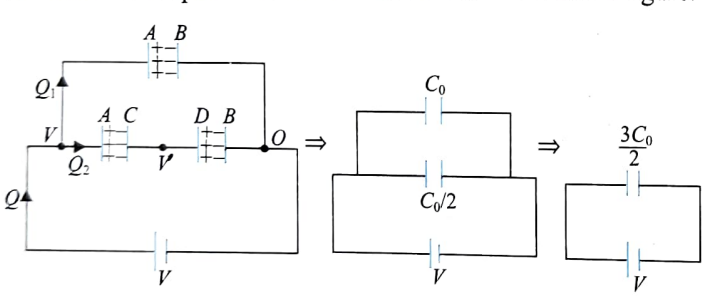
Hence, the equivalent capacitance is

$$C_{eq} = \frac{3}{2}\frac{\varepsilon_0 A}{d}$$

Charges on different surfaces are given in the following table:

Surfaces	(1), (5)	(2), (6)	(3)	(4)
Charges	$-\frac{\varepsilon_0 AV}{2d}$	$+\frac{\varepsilon_0 AV}{2d}$	$\frac{\varepsilon_0 AV}{d}$	$-\frac{\varepsilon_0 AV}{d}$

Method 2: An equivalent circuit can be drawn as shown in figure.



Hence, the equivalent capacitance is

$$C_{eq} = \frac{3}{2}C_0 = \frac{3}{2}\frac{\varepsilon_0 A}{d}$$

The charge supplied by the battery is

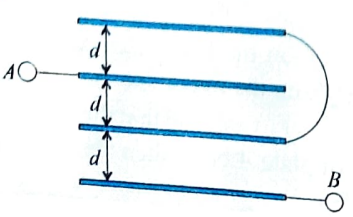
$$Q = \frac{3}{2}\frac{\varepsilon_0 A}{d} \cdot V$$

$$\therefore \quad Q_1 = Q\left(\frac{C_0}{C_0 + C_0/2}\right) = \frac{2}{3}Q \text{ and } Q_2 = \frac{Q}{3}$$

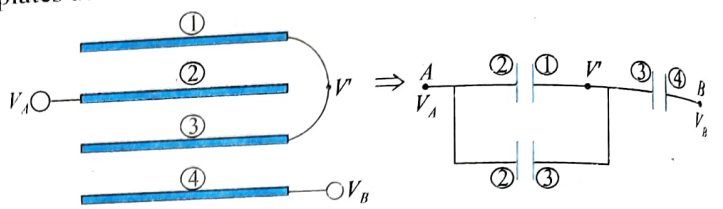
From these results, the charge on different surfaces can be calculated.

ILLUSTRATION 4.42

Four identical plates, each having area A, are arranged as shown in figure. Find the equivalent capacity of the structure between A and B.



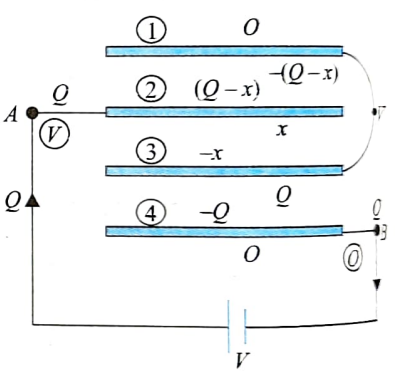
Sol. **Method 1:** Let the potentials of plates A and B be V_A and V_B, respectively. As the plates (1) and (3) (figure) are connected together, they will have a common potential, say V'. Now we will make an equivalent circuit diagram by connecting the different plates across the assumed potential difference.



The equivalent capacitance between A and B can easily be calculated as follows:

$$C_{eq} = \frac{2}{3}C = \frac{2}{3}\frac{\varepsilon_0 A}{d}$$

Method 2: Let us assume that a battery is connected between A and B. Let the potentials of A and B be V and 0. Plates (1) and (2) are connected together and both have the same potential V'. Let the charge supplied by the positive terminal of the battery to plate (1) be Q; same charge will come out from plate (4). The outermost surface of the plates will carry no charge. Hence, $-Q$ charge will appear on the inner surface of plate (4). The charge distribution on different surfaces of the plates can be done by using conservation of charge. The charge appearing on the facing surfaces should be equal and opposite. The charges appearing on difference surfaces are shown in figure. The capacitance of the system of plates can be given by



$$C = \frac{Q}{(V - 0)} \quad ...(i)$$

Between plates (1) and (2),

$$(Q - x) = C(V - V') \quad ...(ii)$$

Between plates (2) and (3),

$$x = C(V - V') \quad ...(iii)$$

Between plates (3) and (4),

$$Q = C(V' - 0) \quad ...(iv)$$

From (ii) and (iii),

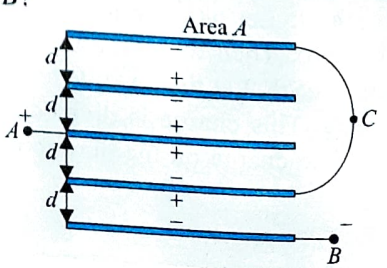
$$(Q - x) = x \text{ or } x = \frac{Q}{2} \quad ...(v)$$

Adding Eqs. (iii) and (iv), we get

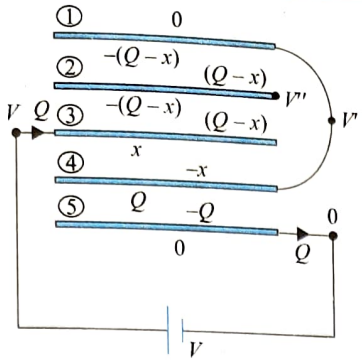
$$V = \frac{x}{C} + \frac{Q}{C} = \frac{3Q}{2C} \quad \text{or} \quad \frac{Q}{C} = \frac{2}{3}C = \frac{2}{3}\frac{\varepsilon_0 A}{d} = C_{eq}$$

ILLUSTRATION 4.43

Five identical plates, each having area A, are arranged as shown in figure. Find the equivalent capacity of the structure between A and B?



Sol. Method 1: The charge distribution on difference surfaces can be done by following conservation of charge and noting that facing surfaces have equal and opposite charges. Thus, the equivalent capacity can be given by



$$C_{eq} = \frac{Q}{V_A - V_B} \quad \ldots(i)$$

Between plates (1) and (2),

$$(Q - x) = C(V'' - V') \quad \ldots(ii)$$

Between plates (2) and (3),

$$(Q - x) = C(V - V'') \quad \ldots(iii)$$

Between plates (3) and (4),

$$x = C(V - V') \quad \ldots(iv)$$

Between plates (4) and (5),

$$Q = C(V' - 0) \quad \ldots(v)$$

Adding Eqs. (ii) and (iii), we get

$$2(Q - x) = C(V - V') \quad \ldots(vi)$$

From (iv) and (vi),

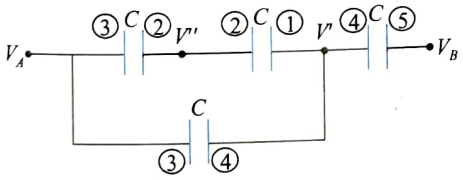
$$x = 2(Q - x) \text{ or } x = \frac{2Q}{3}$$

Adding (v) and (vi), we get

$$(V - 0) = \frac{1}{C}(Q + x) = \frac{5Q}{3C}$$

or $$\frac{Q}{(V_A - V_B)} = \frac{Q}{(V - 0)} = \frac{3}{5}C = \frac{3\varepsilon_0 A}{5d}$$

Method 2: Let the potentials of A and B be V_A and V_B, respectively. Plates (1) and (2) are connected together and have a common potential, say V'. Plate (2) is isolated and has a potential, say V''. Now we draw an equivalent circuit diagram by connecting the different plates across the assumed potential difference.

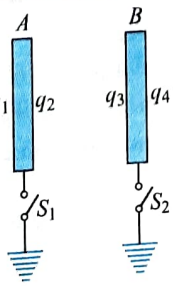


As the equivalent circuit is a simple circuit of a combination of capacitors, the equivalent capacitance can be easily calculated as follows:

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{2}{3C} = \frac{5}{3C} \quad \text{or} \quad C_{eq} = \frac{3C}{5} = \frac{3A\varepsilon_0}{5d}$$

CONCEPT APPLICATION EXERCISE 4.5

1. In figure, plate A has 100 μC charge, while plate B has 60 μC charge.



(a) When both switches are open, then

$q_1 =$ _____

$q_2 =$ _____

$q_3 =$ _____

$q_4 =$ _____

(b) When only switch S_1 is closed, then

$q_1 =$ _____ $q_2 =$ _____

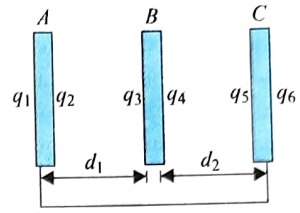
$q_3 =$ _____ $q_4 =$ _____

(c) When switch S_2 is also closed, then

$q_1 =$ _____ $q_2 =$ _____

$q_3 =$ _____ $q_4 =$ _____

2. In the arrangement shown in figure, plate B is given a charge equal to 60 μC. The ratio d_1/d_2 is 2. Then

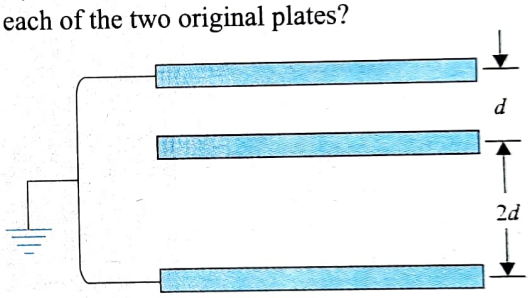


$q_1 =$ _____ $q_2 =$ _____

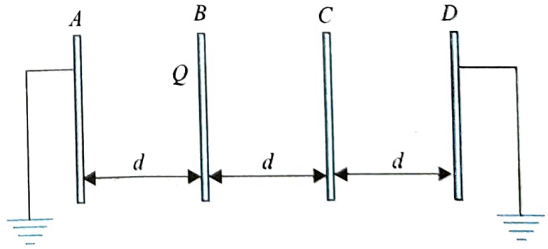
$q_3 =$ _____ $q_4 =$ _____

$q_5 =$ _____ $q_6 =$ _____

3. Two large parallel metal plates, each having area A, are oriented horizontally and separated by a distance $3d$. A grounded conducting wire joins them, and initially each plate carries no charge. Now a third identical plate carrying charge Q is inserted between the two plates, parallel to them and located a distance d from the upper plate, as shown in figure. What induced charge appears on each of the two original plates?



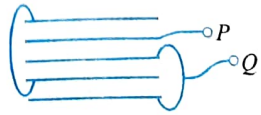
4. Four parallel large plates separated by equal distance d are arranged as shown in figure. The area of the plates is S. Find the potential difference between plats B and C if plate B is given a charge Q.



5. Identical metal plates are located in air at equal distance d from one another. The area of each plate is equal to A. Evaluate the capacitance of the system between P and Q if the plates are interconnected as shown in figure.

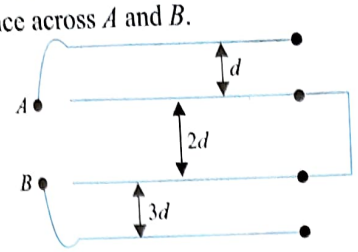
(a)

(b)

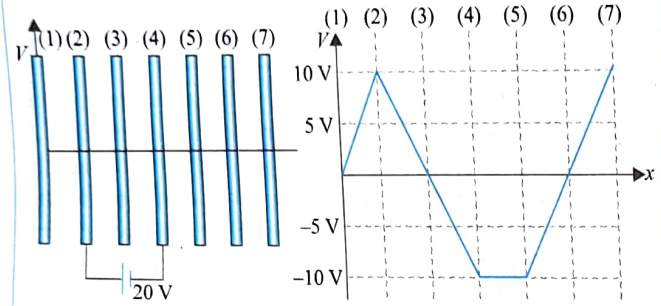
(c) 

(d)

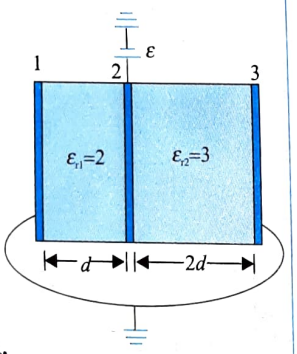
6. If the area of each plate is A (figure) and the successive separations are d, $2d$, and $3d$, then find the equivalent capacitance across A and B.



7. Figure shows an arrangement of identical metal plates placed parallel to each other. The diagram also shows the variation of potential between the plates. Using the details given in the diagram, find the equivalent capacitance connected across the battery (separation between the consecutive plates is equal to l and the cross-sectional area of each plate is A).

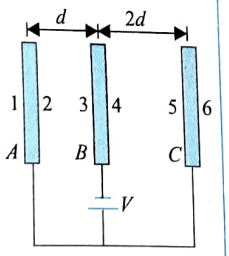


8. Two dielectric slabs of relative permittivities ε_{r_1} = 2 and ε_{r_2} = 3, thickness d and $2d$ are filled in between the grounded parallel plates each of area A. If a battery of emf ε is connected to the middle conducting plate, find the



(a) capacitance of the system,
(b) charge flowing through the battery,
(c) induced charges on the plates,
(d) energy stored in the system.

9. In the given figure the Area of each plate is A. The conducting plates are connected to a battery of emf V volts. Find charges q_1 to q_6.

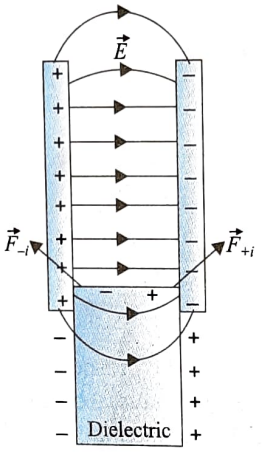


ANSWERS

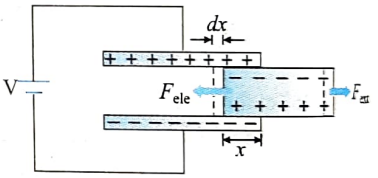
1. (a) 80 μC, 20 μC, –20 μC, 80 μC
(b) 0, –60 μC, 60 μC, 0 (c) 0, 0, 0, 0
2. $q_1 = q_6 = 30$ μC, $q_3 = 20$ μC, $q_4 = 40$ μC $q_2 = -20$ μC, $q_5 = -40$ μC
3. Induced charge on upper plate = $-2Q/3$
Induced charge on lower plate = $-Q/3$
4. $\dfrac{Qd}{3\varepsilon_0 S}$ 5. (a) $\dfrac{2\varepsilon_0 A}{3d}$ (b) $\dfrac{3\varepsilon_0 A}{2d}$ (c) $\dfrac{5}{3}\dfrac{\varepsilon_0 A}{d}$ (d) $\dfrac{3\varepsilon_0 A}{d}$
6. $\dfrac{\varepsilon_0 A}{4d}$ 7. $\dfrac{6\varepsilon_0 A}{5l}$
8. (a) $\dfrac{7\varepsilon_0 A}{2d}$ (b) $\dfrac{7\varepsilon_0 A\varepsilon}{2d}$ (c) $-\dfrac{2\varepsilon_0 A\varepsilon}{d}, \dfrac{7\varepsilon_0 A\varepsilon}{2d}, -\dfrac{3\varepsilon_0 A\varepsilon}{2d}$
(d) $\dfrac{7\varepsilon_0 A\varepsilon^2}{4d}$
9. $q_1 = q_6 = 0, q_2 = +\dfrac{\varepsilon_0 AV}{d}, q_3 = -\dfrac{\varepsilon_0 AV}{d}, q_5 = +\dfrac{\varepsilon_0 AV}{2d}$
$q_4 = -\dfrac{\varepsilon_0 AV}{2d}$

FORCE ON DIELECTRIC SLAB

When a dielectric slab is placed near a charged capacitor, the fringing field at the edges of the capacitor exerts force on the negative and positive induced surface charges of the dielectric due and the slab experiences the net force towards inside the capacitor [Fig. (a)]. Let us consider a situation a capacitor of capacity C is connected to a battery of emf V. Let a dielectric slab is inserted up to distance x as shown in [Fig. (b)].



(a)



(b)

If you pull the dielectric slab by a small distance dx, the capacitance of the system charges by dC. Hence, an excess charge $dq = VdC$ flows from (or to) the supply if the system is kept connected to the supply. In this process the battery (or supply or source) does a work dW_b in sending a charge dq against the voltage V (or emf ε of the battery). You can call it energy supplied by the battery (sources). According to the conservation of energy, part of the energy supplied by the source is spent in changing (increasing or decreasing) the electrostatic potential energy of the system by dU, say and rest of the supplied source energy is utilised in doing mechanical work dW_{mech}, say on the system; the dielectric on which you are in tending to find the work and energy loss dH in the form of heat, light and sound, etc. Then, we can write

$$dW_b = dW_{mech.} + dU + dH \quad ...(i)$$

where $dW_{mech.} = Fdx$; F = force on the dielectric. We can ignore the value of dH in the battery and conductors. Now we can write Eqn. (i) as, $dW_b = F.dx + dU$.

Hence the force F acting on the conductor (or dielectric) given as,

$$F = \frac{dW_b}{dx} - \frac{dU}{dx} \quad ...(ii)$$

The above expression is valid for all processes such as time varying or constant charge and voltage. We have two practical cases.

(i) **When the battery is disconnected after charging the capacitor(or charge in capacitor remains constant):** If you plan to find the force by disconnecting the system from the battery, in this case, put q = constant. Then, the work done by the battery should be zero. By putting $dW_b = 0$, in Eq. (ii), we have

$$F = -\left.\frac{dU}{dx}\right|_{q\ =\ \text{constant}} \qquad ...(iii)$$

(ii) **When battery remains connected (Here V remains constant):** In this case, the battery does a work $dW_b (= Vdq)$ in sending a charge dq through a potential difference V. At the same time the change in potential energy of the system can be given as

$$dU = d\left(\frac{1}{2}CV^2\right) = \frac{V^2}{2}dC = \frac{Vdq}{2} \qquad (\because\ VdC = dq)$$

Then, we have, $dW_b = 2dU$

This tells us that, half of the energy supplied by the battery rises the potential energy of the system and the other half is spent in doing a mechanical work. Then, putting $dW_b = 2dU$ in Eq. (iii), we have

$$F = \frac{2dU}{dx} - \frac{dU}{dx} = \frac{dU}{dx} \text{ we have}$$

Now we can write $F = +\left.\frac{dU}{dx}\right|_{V\ =\ \text{constant}}$ ...(iv)

ILLUSTRATION 4.44

Let us consider a parallel plate of capacitor with plates of width b and length l. The distance between the plates is d. Find the force acting on the smooth dielectric slab when

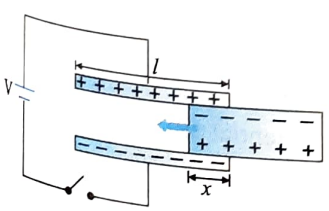
(i) When the battery is disconnected after charging the capacitor

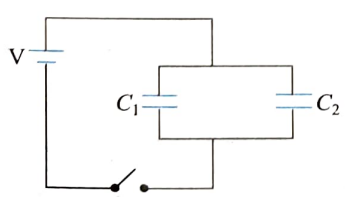
(ii) When battery remains connected

Sol.

(i) Force acting on the dielectric slab can be given as,

$$F = -\frac{dU}{dx}, \text{ where } U = \frac{Q^2}{2C}$$





or $F = +\frac{Q^2}{2C^2}\frac{dC}{dx}$ ...(i)

$$C_1 = \frac{\varepsilon_0 b(l-x)}{d}\ ;\ C_2 = \frac{K\varepsilon_0 bx}{d}$$

Hence, $C = C_1 + C_2 = \frac{\varepsilon_0 b}{d}(l + x(K-1))$ ...(ii)

$$\frac{dC}{dx} = \frac{\varepsilon_0 b}{d}(K-1) \qquad ...(iii)$$

From (i), (ii) and (iii), we get

$$F = +\frac{Q^2}{2\left[\frac{\varepsilon_0 b}{d}(l + x(K-1))\right]^2}\frac{\varepsilon_0 b}{d}(K-1)$$

$$\Rightarrow\ F = \frac{Q^2 d(K-1)}{2\varepsilon_0 b[l + x(K-1)]^2}$$

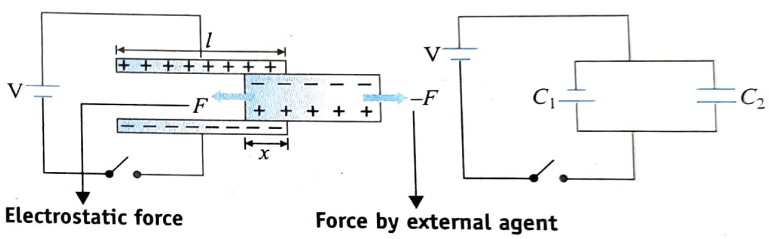
If the capacitor is connected to a constant voltage, put $\frac{Q}{C} = V$ to obtain

$$F = +(\varepsilon_r - 1)\frac{\varepsilon_0 lV^2}{2d}$$

The sign signifies that the force points to right.

The magnitude of the force on the dielectric is constant.

(ii) When battery remains connected



$$F = +\frac{dU}{dx}, \text{ where } U = \frac{1}{2}CV^2$$

or $F = V^2\frac{dC}{dx}$ ...(i)

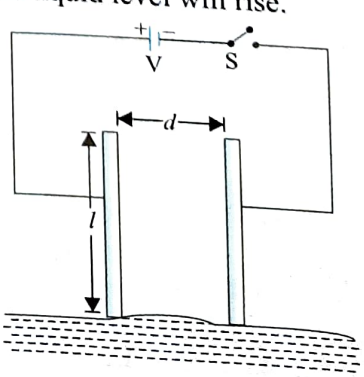
Here, $C = C_1 + C_2 = \frac{\varepsilon_0 b}{d}(l + x(K-1))$ ...(ii)

and $\frac{dC}{dx} = \frac{\varepsilon_0 b}{d}(K-1)$ ...(iii)

Hence $F = (K-1)\frac{\varepsilon_0 bV^2}{2d}$

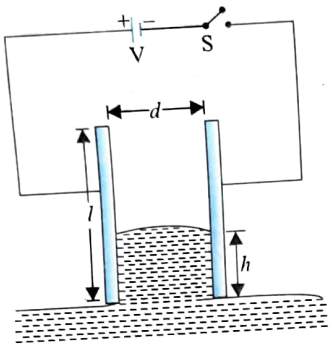
ILLUSTRATION 4.45

Figure shows a parallel plate capacitor of plate area $A = lb$ with separation d connected to a battery via a switch S. Capacitor plates are kept vertical and touched on the surface of a liquid of density ρ as shown. If S is closed find the height between plates to which the liquid level will rise.



Sol. Here the liquid acts as dielectric. At the time when the switch is closed, due to the upward force on dielectric(liquid) because of polarization it starts raising up between the plates and upto a level where the upward force on liquid balances its weight. If it is raised up to a height h as shown in figure and battery is connected to the capacitor we use result for the force on dielectric which we proved in theory. At equilibrium we use

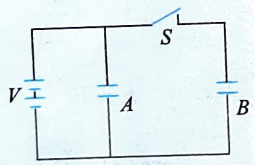
$$(K-1)\frac{\varepsilon_0 bV^2}{2d} = hbd\rho g \Rightarrow h = \frac{\varepsilon_0 V^2(K-1)}{2d^2\rho g}$$



Solved Examples

EXAMPLE 4.1

Figure shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened, and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) $K = 3$. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.



Sol. Initially, when the switch is closed, both the capacitors A and B are in parallel and, therefore, the energy stored in the capacitors is

$$U_i = 2\times\frac{1}{2}CV^2 = CV^2 \qquad ...(i)$$

When switch S is opened, B gets disconnected from the battery. The capacitor B is now isolated, and the charge on an isolated capacitor remains constant, often referred to as bound charge. On the other hand, A remains connected to the battery. Hence, potential V remains constant on it.

When the capacitors are filled with dielectric, their capacitance increases to KC. Therefore, energy stored in B changes to $Q^2/2KC$, where $Q = CV$ is the charge on B, which remains constant, and energy stored in A changes to $1/2\ KCV^2$, where V is the potential on A, which remains constant. Thus, the final total energy stored in the capacitors is

$$U_f = \frac{1}{2}\frac{(CV)^2}{KC} + \frac{1}{2}KCV^2 = \frac{1}{2}CV^2\left(K+\frac{1}{K}\right) \qquad ...(ii)$$

From Eqs. (i) and (ii), we find

$$\frac{U_i}{U_f} = \frac{2K}{K^2+1}$$

It is given that $K = 3$. Therefore, we have

$$\frac{U_i}{U_f} = \frac{3}{5}$$

EXAMPLE 4.2

Two parallel plate capacitors of capacitance C each are connected in series with a battery of emf ε. Then, one of the capacitors is filled with a dielectric of dielectric constant K.

(i) Find the change in electric field in the two capacitors, if any.

(ii) What amount of charge flows through the battery?

(iii) Find the change in the energy stored in the circuit, if any.

Sol.

(i) Two capacitors A and B initially have same charge Q and potential $V = Q/C$. The electric field between the capacitor plates is given by $E = V/d$. Since the two capacitors are connected in series with the battery, the sum of potentials across the capacitors must be equal to ε, i.e.,

$$\varepsilon = 2V = \frac{2Q}{C} \qquad ...(i)$$

(ii) When one of the capacitors, say A, if filled with a dielectric, the capacity of A increases to $C' = KC$, while that of B remains unchanged, i.e., C. Suppose, charge on the capacitors becomes Q' and potentials across A and B become V_A' and V_B', respectively, with $V_A' + V_B' = \varepsilon$. Hence we have

$$V_A' = \frac{Q'}{C'} = \frac{Q'}{KC}$$

and $$V_B' = \frac{Q'}{C'}$$

Hence,

$$\varepsilon = \frac{Q'}{C}\left(1+\frac{1}{K}\right) \qquad ...(ii)$$

From Eqs. (i) and (ii), we get

$$Q' = \frac{2K}{1+K}Q$$

Since $K > 1$, so $Q' > Q$. Also

$$V_A' = \frac{2K}{1+K}\text{ V}$$

and $$V_B' = \frac{2K}{1+K}\text{ V}$$

Thus, the electric field (or potential difference) in capacitor A increases by a factor of $2/(1+K)$ while that in B increases by a factor of $2K/(1+K)$. The amount of charge that flows into the circuit is given by

$$\Delta Q = Q' - Q = \left(\frac{2K}{1+K} - 1\right)Q = \frac{K-1}{K+1}Q$$

$$= \frac{1}{2}\frac{K-1}{K+1}C\varepsilon$$

(iii) Initially, the energy is given by

$$U_i = 2\times\frac{1}{2}CV^2 = CV^2$$

Finally, the energy is

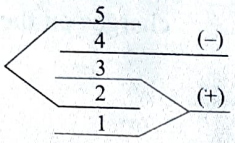
$$U_f = \frac{1}{2}C'V_A'^2 + \frac{1}{2}CV_B'^2$$

$$= \frac{1}{2}KC\left(\frac{2}{1+K}\right)^2 V^2 + \frac{1}{2}C\left(\frac{2K}{1+K}\right)^2 V^2$$

$$= \frac{4K}{1+K}\left(\frac{1}{2}CV^2\right) = \frac{2K}{1+K}U_i$$

EXAMPLE 4.3

Five identical conducting plates 1, 2, 3, 4, and 5 are fixed parallel and equidistant from each other as shown in figure. Plates 2 and 5 are connected by a conductor, while plates 1 and 3 are joined by another conductor. The junction of plates 1 and 3 and plate 4 are connected to a source of constant emf V_0.



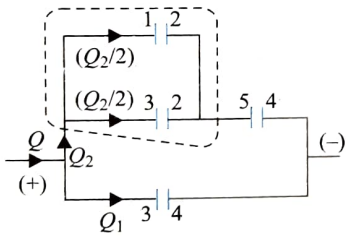
(i) Find the effective capacity of the system between the terminals of the source.

(ii) Find the charges on plates 3 and 5. Given, d is the distance between any two successive plates and A is the area of either face of each plate.

Sol.

(i) The equivalent circuit is shown in figure. The system consists of four capacitors, i.e., C_{12}, C_{32}, C_{34}, and C_{54}. The capacity of each capacitor is $(K\varepsilon_0 A/d) = C_0$.

The capacitors C_{12} and C_{32} are in parallel, and their capacity is $C_0 + C_0 = 2C_0$. The capacitor C_{54} is in series with the parallel combination of C_{12} and C_{32}. Hence, the resultant capacity will be



$$C_1 = \frac{C_0\times 2C_0}{C_0+2C_0} = \frac{2C_0}{3}$$

Further, C_{34} is again in parallel with the combination of C_{12}, C_{32}, and C_{54}. Hence, the effective capacity is

$$C_{\text{eff.}} = C_0 + \frac{C_0\times 2C_0}{C_0+2C_0} = \frac{5}{3}C_0 \frac{5}{3}K\varepsilon_0\frac{A}{d}$$

(ii) Charge on plate 5 is equal to the charge on the upper half of the parallel combination. So

$$Q_5 = V_0\left(\frac{2}{3}C_0\right) = \frac{2}{3}\frac{K\varepsilon_0 AV_0}{d}$$

Charge on plate 3 on the surface facing 4 is

$$Q_1 = V_0C_0 = \frac{K\varepsilon_0 AV_0}{d}$$

Charge on plate 3 on the surface facing 2 is equal to the potential difference across (3 – 2), i.e.,

$$C_0 = V_0\frac{C_0}{C_0+2C_0}C_0 = K\varepsilon_0\frac{AV_0}{3d}$$

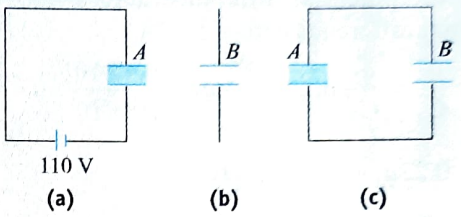
Net charge on plate 3 is

$$Q_3 = \frac{K\varepsilon_0 AV_0}{d} + K\varepsilon_0\frac{AV_0}{3d} = \frac{K\varepsilon_0 AV_0}{d}\left[1+\frac{1}{3}\right]$$

$$= \frac{4}{3}K\varepsilon_0\frac{A}{d}V_0$$

EXAMPLE 4.4

Two parallel plate capacitors A and B have the same separation $d = 8.85\times10^{-4}$ m between the plates. The plate areas of A and B are 0.04 m^2 and 0.02 m^2, respectively. A slab of dielectric constant (relative permittivity) $K = 9$ has dimensions such that it can exactly fill the space between the plates of capacitor B.



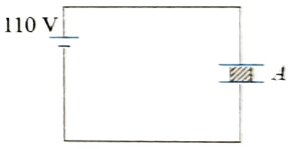
(i) The dielectric slab is placed inside A as shown in Fig. (a). A is then charged to a potential difference of 110 V. Calculate the capacitance of A and the energy stored in it.

(ii) The battery is disconnected, and then the dielectric slab is removed from A. Find the work done by the external agency in removing the slab from A.

(iii) The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in Fig. (c). Calculate the energy stored in the system.

Sol.

(i) Capacitor A with a dielectric can be regarded as two capacitors in parallel, one having a dielectric and the other having no dielectric state. Such a capacitor has an area of $A/2$. So the combined capacitance is



$$C = C_1 + C_2 = \frac{(A/2)\varepsilon_0}{d} + \frac{(A/2)\varepsilon_0 K}{d} = \frac{A}{2}\frac{\varepsilon_0}{d}(1+K)$$

$$= \frac{0.04\times8.85\times10^{-12}}{2\times8.85\times10^{-4}}(1+9) = 2\times10^{-9}\text{ F}$$

Thus, energy stored is

$$\frac{1}{2}CV^2 = \frac{1}{2}\times2\times10^{-9}\times(110)^2 = 1.21\times10^{-5}\text{ J}$$

(ii) Work done in removing the dielectric state = (Energy stored in capacitor without dielectric) – (Energy stored in capacitor with dielectric).

It may be noted that on taking out the dielectric, the charge on the capacitor plate remains the same.

$$W = \frac{q^2}{2C'} - \frac{q^2}{2C}$$

Here,

$$C = 2\times10^{-9}\text{ F},$$

$$C' = \frac{A\varepsilon_0}{d} = \frac{0.04\times8.85\times10^{-12}}{8.85\times10^{-4}} = 0.4\times10^{-9}\text{ F}$$

$$q = CV = 2\times10^{-9}\times110 = 2.2\times10^{-7}\ \text{C}$$

$$\therefore \quad W = \frac{(2.2\times10^{-7})^2}{2}\left[\frac{1}{0.4\times10^{-9}} - \frac{1}{2\times10^{-9}}\right]$$

$$= 4.84\times10^{-5}\ \text{J}$$

(iii) The capacitance of B is $\varepsilon_0 KA_B/d$. So

$$C_B = 1.8\times10^{-9}\ \text{F}$$

The charge on A, $q_A = 2.2\times10^{-7}$ C, gets distributed into two parts q_1 and q_2. So.

$$q_1 + q_2 = 2.2\times10^{-7}\ \text{C}$$

Also, the potential difference across A is equal to the potential difference across B. Thus,

$$\frac{q_1}{C_A} = \frac{q_2}{C_B} \text{ or } q_1 = \frac{C_A}{C_B}q_2 = \frac{0.4\times10^{-9}}{1.8\times10^{-9}}q_2 = 0.22q_2$$

or $\quad 0.22q_2 + q_2 = 2.2\times10^{-7}$

or $\quad q_2 = \dfrac{2.2}{1.22}\times10^{-7} = 1.8\times10^{-7}\ \text{C}$

and $\quad q_1 = 0.4\times10^{-7}\ \text{C}$

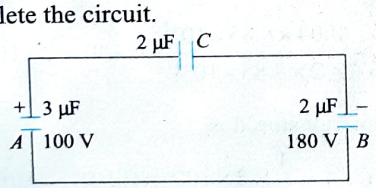
Total energy stored is

$$\frac{q_1^2}{2C_A} + \frac{q_2^2}{2C_B} = 0.2\times10^{-5} + 0.9\times10^{-5} = 1.1\times10^{-5}\ \text{J}$$

Alternatively, the combined capacitance of the two capacitors can be found. The total charge on the two capacitors is known. The energy can be found using the formula $Q^2/2C_{eq}$.

EXAMPLE 4.5

Two capacitors A and B with capacities 3 μF and 2 μF are charged to a potential difference of 100 V and 180 V, respectively. The plates of the capacitors are connected as shown in figure with one wire free from each capacitor. The upper plate of A is positive and that of B is negative. An uncharged 2 μF capacitor C with lead wires falls on the free ends to complete the circuit.



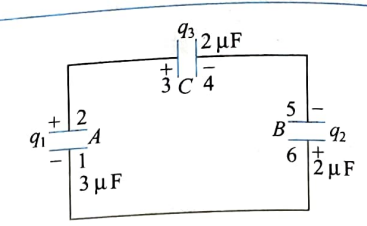
(i) Calculate the final charge on the three capacitors,

(ii) Find the amount of electrostatic energy stored in the system before and after the completion of the circuit.

Sol.

(i) Method 1: Initial charge on capacitor A is $q_A = 3\times100 = 300$ μC. Initial charge on capacitor B is $q_B = 2\times180 = 360$ μC. After completing the circuit, let the charge on capacitor A be q_1, on B be q_2, and on C be q_3 with polarities as shown in figure. Applying conservation of charge for plates 2 and 3, we get

$$q_1 + q_3 = 300 + 0 = 300 \quad \text{...(i)}$$



Applying conservation of charge for plates 4 and 5, we get

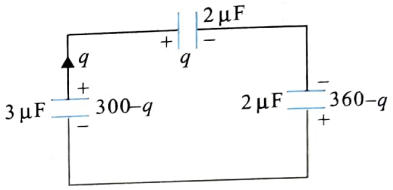
$$-q_2 - q_3 = -360 + 0 \text{ or } q_2 + q_3 = 360 \quad \text{...(ii)}$$

Applying Kirchhoff's law in the whole circuit, we get

$$\frac{q_1}{3} - \frac{q_3}{2} + \frac{q_2}{2} = 0 \quad \text{(iii)}$$

On solving, we get $q_1 = 90$ μC, $q_2 = 150$ μC, and $q_3 = 210$ μC.

Method 2: After completing the circuit, let the charge q flow in the circuit in the clockwise direction. Then the final charges on the capacitors will be as shown in figure.



Applying Kirchhoff's law, we get

$$\frac{300-q}{3} - \frac{q}{2} + \frac{360-q}{2} = 0 \text{ or } q = 210\ \mu\text{C}$$

Now charges on all three capacitors can be found.

(ii) Initially, energy stored is

$$U_i = \frac{1}{2}\times3\times10^{-6}\times(100)^2 + \frac{1}{2}\times2\times10^{-6}(180)^2$$

$$= 4.74\times10^{-2}\ \text{J}$$

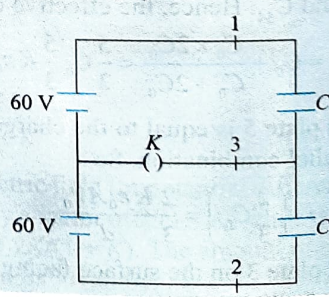
Finally, the energy stored is

$$U_f = \frac{(90\times10^{-6})^2}{2\times3\times10^{-6}} + \frac{(150\times10^{-6})^2}{2\times2\times10^{-6}} + \frac{(210\times10^{-6})^2}{2\times2\times10^{-6}}$$

$$= 1.8\times10^{-2}\ \text{J}$$

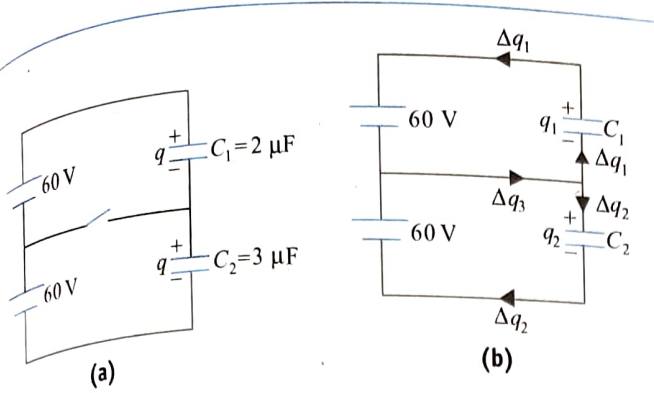
EXAMPLE 4.6

In the circuit shown (figure), the emf of each battery is 60 V and $C_1 = 2$ μF and $C_2 = 3$ μF. Find the charges that will flow through the sections 1, 2, and 3 after the key is closed.



Sol. Before closing the switch [Fig. (a)], both capacitors are in series. Their equivalent capacitance is

$$C_{eq} = \frac{2\times3}{2+3} = \frac{6}{5}\ \mu\text{F}$$



charge on each capacitor is

$$q = C_{eq} \times 120 = \frac{6}{5} \times 120 = 144\ \mu C$$

After closing the switch [Fig. (b)], the potential difference across each capacitor is 60 V. So charges on them are

$$q_1 = C_1 \times 60 = 2 \times 60 = 120\ \mu C$$

$$q_2 = C_2 \times 60 = 3 \times 60 = 180\ \mu C$$

Charge flowing through section 1 is

$$\Delta q_1 = q - q_1 = 144 - 120 = 24\ \mu C$$

Charge flowing through section 2 is

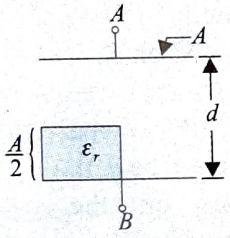
$$\Delta q_2 = q_2 - q = 180 - 144 = 36\ \mu C$$

Charge flowing through section 3 is

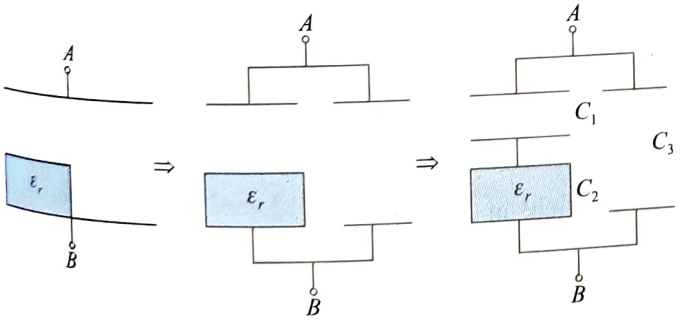
$$\Delta q_3 = \Delta q_1 + \Delta q_2 = 24 + 36 = 60\ \mu C$$

EXAMPLE 4.7

Find the capacitance between the terminals A and B if $\varepsilon_r = 2$.



Sol. The given system is broken into three capacitors C_1, C_2, and C_3 as shown in figure.



$$C_1 = \frac{\varepsilon_0 A/2}{d/2}$$

$$C_2 = \frac{\varepsilon_0 \varepsilon_r A/2}{d/2}$$

where $C_3 = \frac{\varepsilon_0 A/2}{d}$

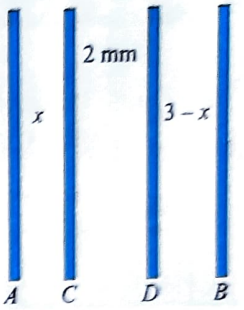
C_1 and C_2 are in series, and this series combination is in parallel with C_3. Then, the equivalent capacitance between the terminals A and B is

$$C_{AB} = \frac{C_1 C_2}{C_1 + C_2} + C_3 = \frac{\varepsilon_0 A}{d}\left(\frac{\varepsilon_r}{\varepsilon_r + 1} + \frac{1}{2}\right) \quad (\text{where } \varepsilon_r = 2)$$

$$\therefore \quad C_{AB} = \frac{7\varepsilon_0 A}{6d}$$

EXAMPLE 4.8

Two parallel plate capacitors differ only in the spacing between their (very thin) plates; AB has a spacing of 5 mm and a capacitance of 20 pF, while CD has a spacing of 2 mm. Plates A and C carry charges of +1 nC, while B and D each carry –1 nC. What are the potential differences V_{AB} and V_{CD} after the capacitor CD is slid centrally between and parallel to the plates of AB without touching them? Would it make any difference if CD was not centrally placed between A and B?

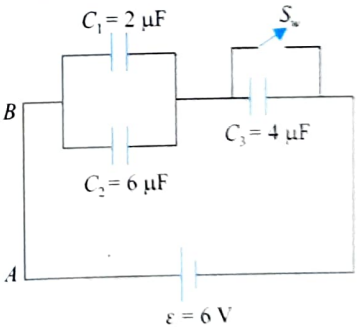


Sol. The net charges on the plates cannot change, but the charges on the plates on either side of any of the spaces must be equal and opposite. Consequently, the charges on C and D must be –1 nC and +1 nC, respectively, on their outside surfaces and +2 nC and –2 nC, respectively, on their inside surfaces. The capacitance of any pair of plates is inversely proportional to their separation, with 5 mm corresponding to 20 pF.

Thus, if AC is x mm and DB is $(3 - x)$ mm, the capacitances of the three successive capacitors are $100/x$, 50, and $100/(3 - x)$ pF. The voltage V_{CD} is, therefore, 40 V, and V_{AB} is $10x + 40 + 10(3 - x) = 70$ V, independent of the value of x.

EXAMPLE 4.9

In figure, all the capacitors are in steady state initially.



(i) What is the charge flowing through the switch when it is closed?
(ii) What is the charge flowing through section AB?
(iii) What is the work done by the battery?
(iv) What is the heat produced when S is closed?

Sol. Considering the isolated system. Before closing the switch,

$$4x + (x - 6)2 + (x - 6)2 = 0 \quad \text{or} \quad x = 4\ V$$

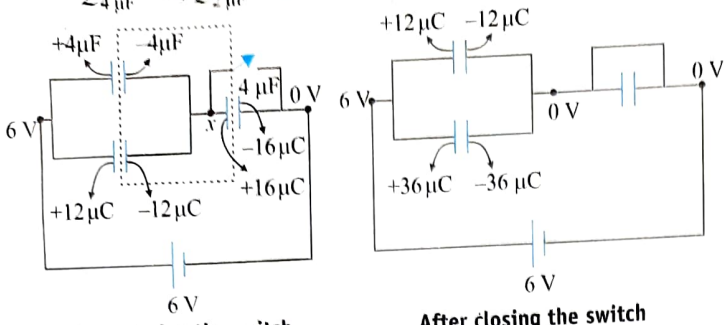
$$|Q_{4\mu F}| = 16\ \mu C;\ |Q_{2\mu F}| = 4\ \mu C$$

$$|Q_{6\mu F}| = 12\ \mu C$$

When switch S_w is closed, the 4 μF capacitor is short-circuited, and the potential difference across it becomes zero. The circuit after closing the switch is shown in figure.

The charges on each capacitor at this stage

$Q_{4\,\mu F} = 0$, $Q_{2\,\mu F} = 2 \times 6 = 12\ \mu C$ and $Q_{6\,\mu F} = 6 \times 6 = 36\ \mu C$



Before closing the switch

After closing the switch

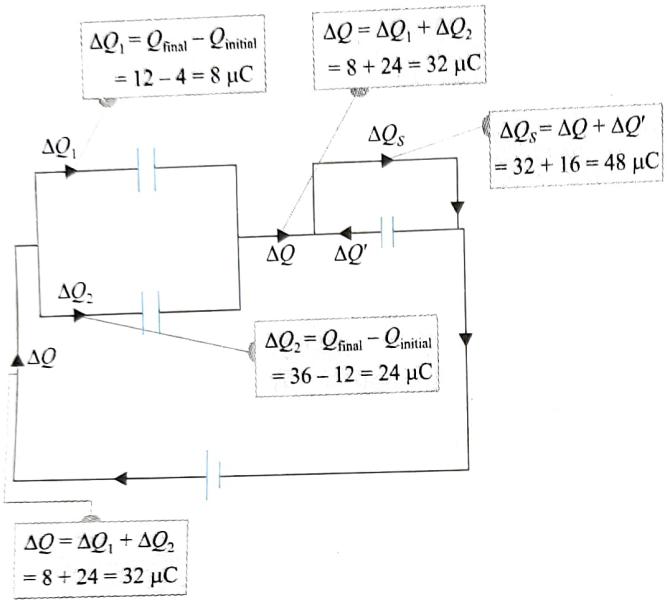


Figure shows the charges coming and going out of the capacitors. So the charge through section AB is 32 μC, and the charge through the switch is 48 μC. When the switch is closed, 16 μC charge from the left plate goes past the switch and neutralizes the −16 μC charge on the right plate. Hence, charge passing through the battery is 32 μC. So the work done by the battery is

$$W_{battery} = \Delta q V = (32\ \mu C)\ (6\ V) = 192\ \mu J$$

Heat produced is

$$H = W_{battery} - \Delta U = W_{battery} - (U_f - U_i)$$

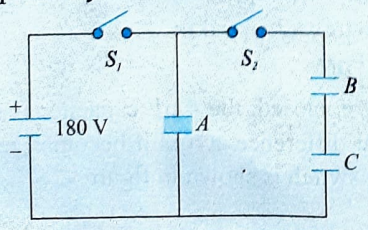
$$U_f = \frac{1}{2}(2) \times 6^2 + \frac{1}{2}(6)(6)^2 = 144\ \mu J$$

and $$U_i = \frac{1}{2}4(4)^2 + \frac{1}{2}(8)2^2 = 48\ \mu J$$

$$H = [192 - (144 - 48)]\ \mu J = 96\ \mu J$$

EXAMPLE 4.10

In the circuit shown in figure, capacitor A has capacitance $C_1 = 2\ \mu F$ when filled with a dielectric slab ($k = 2$). Capacitors B and C are air capacitors and have capacitances $C_2 = 3\ \mu F$ and $C_3 = 6\ \mu F$, respectively.



A is charged by closing the switch S_1 alone.

(i) Calculate the energy supplied by the battery during the process of charging.

Switch S_1 is now opened and S_2 is closed.

(ii) Calculate the charge on B and the energy stored in the system when an electrical equilibrium is attained.

Now switch S_2 is also opened, and the slab of A is removed. Another dielectric slab of $K = 2$, which can just fill the space in B, is inserted into it and then switch S_2 alone is closed.

(iii) Calculate by how many times the electric field in B is increased. Calculate also the loss of energy during the redistribution of charge.

Sol. When switch S_1 alone is closed, capacitor A gets directly connected across the battery. Thus, charge on A in steady state is

$$q_0 = C_1 V = 2 \times 180 = 360\ \mu C$$

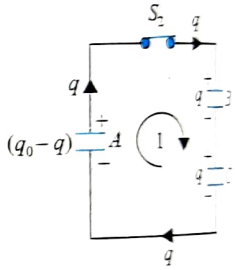
This whole charge is supplied by the battery at emf $V = 180$ V. Therefore, the energy supplied by the battery during the charging of capacitor A is

$$W_{battery} = q_0 V = 0.0648\ J$$

But the energy stored in capacitor A is

$$U_1 = \frac{1}{2} q_0 V = 0.0324\ J$$

The remaining part of the energy supplied by the battery is converted into heat during the flow of current through the connecting wires. After A is charged, switch S_1 is opened, which disconnects the battery. When S_2 is closed, some charge is transferred from capacitor A to capacitors B and C. Let the charge transferred be q. In steady state, charges on the capacitors will be as shown in figure. Applying Kirchhoff's loop law,



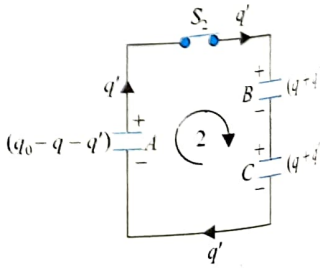
$$\circlearrowleft 1 \quad \frac{q}{C_2} + \frac{q}{C_3} - \frac{(q_0 - q)}{C} = 0 \text{ or } q = 180\ \mu C$$

Now energy stored in the system of capacitors is

$$U_2 = U_A + U_B + U_C = \frac{(q_0 - q)^2}{2C_1} + \frac{q^2}{2C_2} + \frac{q^2}{2C_3} = 0.0162\ J$$

Electric field in B is $E = q/\varepsilon_0 A$, where A is the area of plates of capacitor B. Now after the opening of switch S_2, the dielectric slab of A is removed; therefore, its capacitance becomes

$$C_1' = \frac{C_1}{K} = 1\ \mu F$$



Another plate of $K = 2$ is inserted in capacitor B. Therefore, its capacitance becomes $C_2' = KC_2 = 6\ \mu F$.

Now switch S_2 is closed and charge gets redistributed. Let a charge q' be further transferred from A to capacitors B to C. In steady state, charges will be as shown in figure. Applying Kirchhoff's loop law,

(2) $\frac{q+q'}{C_2'}+\frac{q+q'}{C_3}=\frac{q_0-q-q'}{C_1'}=0$ or $q'=90\ \mu C$

Final charge on capacitor B is $q+q'=270\ \mu C$. Hence, electric field inside it is

$$E'=\frac{q+q'}{\varepsilon_0 KA}$$

Factor, by which the electric field in capacitor B is increased is

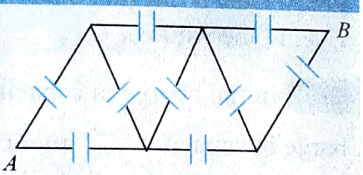
$$\frac{E'}{E}=\frac{(q+q')}{qK}=0.75$$

During the redistribution of charge, energy is lost in the form of heat produced in the connecting wires and is equal to $\Sigma(\Delta q)^2/2C$ where Δq is the increase in charge on a capacitor. Hence, energy lost is

$$\frac{(-q')^2}{2C_1'}=\frac{(q')^2}{2C_2'}+\frac{(q')^2}{2C_3}=5.4\times10^{-3}\ J$$

EXAMPLE 4.11

Some capacitors each of capacitance 30 μF are connected as shown in figure. Calculate the equivalent capacitance between terminals A and B.



Sol. Let us do the charge distribution as shown in the figure.

At junction A: $Q=Q_1+Q_2$ ...(i)

Considering closed loop $ACDA$: $-\frac{Q_2}{C}-\frac{Q_3}{C}+\frac{Q_1}{C}=0$

or $Q_1=Q_2+Q_3$ ...(ii)

Now considering closed loop $DCED$:

$$-\frac{Q_3}{C}-\frac{Q_1-Q_2+2Q_3}{C}+\frac{Q_2-Q_3}{C}=0$$

or $Q_1-2Q_2+4Q_3=0$ ...(iii)

From Eqs. (ii) and (iii); $5Q_1=6Q_2$ ...(iv)

From Eqs. (i) and (iv), we get

$6Q=11Q_1$ ...(v)

Now considering outer loop ($ADEBHA$)

$$-\frac{Q_2}{C}-\frac{(Q_2-Q_3)}{C}-\frac{Q_1}{2}+V=0$$

or $CV=3Q-3Q_1$ ...(vi)

from Eqs. (v) and (vi) we get

$$CV=3Q-3\times\frac{6Q}{11}=\frac{15}{11}Q$$

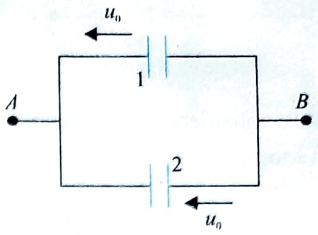
$$\Rightarrow\quad \frac{Q}{V}=\frac{11}{15}C=C_{eq}$$

Hence equivalent capacity of the system

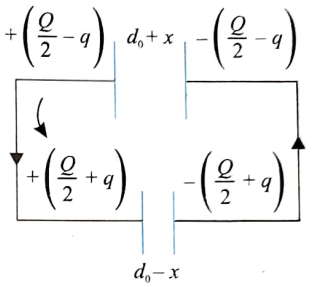
$$C_{eq}=\frac{11}{15}\times30=22\ \mu F$$

EXAMPLE 4.12

Two identical capacitors having plate separation d_0 are connected parallel to each other across the points A and B as shown in figure. A charge Q is imparted to the system by connecting a battery across A and B and the battery is removed. Now the first plate of the first capacitor and the second plate of the second capacitor start moving with constant velocity u_0 toward left. Find the magnitude of the current flowing in the loop during this process.



Sol. Let each plate move a distance 'x' from its initial position. Let q charge flow in the loop. Using KVL, we have



$$\frac{\left(\frac{Q}{2}-q\right)(d+x)}{\varepsilon_0 A}-\frac{\left(\frac{Q}{2}+q\right)(d-x)}{\varepsilon_0 A}=0$$

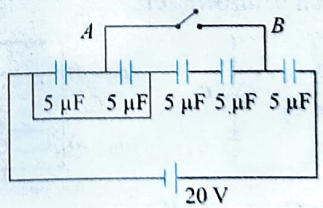
or $q=\frac{Qx}{2d_0}$ ...(i)

Differentiating Eq. (i) both sides w.r.t. time.

$$I=\frac{dq}{dt}=\frac{Q}{2d_0}\left(\frac{dx}{dt}\right)=\frac{Qu_0}{2d_0}$$

EXAMPLE 4.13

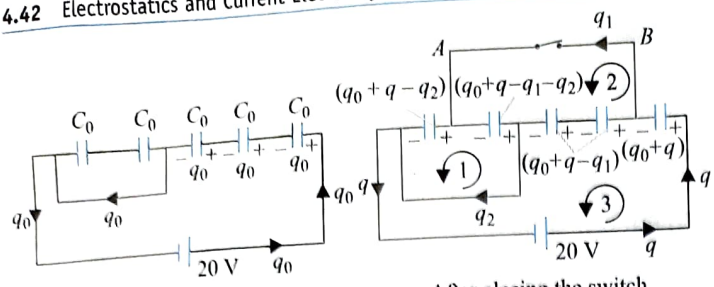
Five identical capacitors each of magnitude 5 μF are arranged with a battery of emf 20 V as shown in figure. Initially, the switch is opened. Find the charge that flows from point A to point B, when the switch is closed.



Sol. Let us consider the circuit when switch is open

$$20-\frac{q_0}{C_0}-\frac{q_0}{C_0}-\frac{q_0}{C_0}=0$$

or $20=\frac{3q_0}{C_0}\Rightarrow q_0=\frac{100}{3}\times10^{-6}\ C$



Before closing the switch After closing the switch

Now considering the situation when switch is closed.

In loop (1)

$$-\frac{(q_0+q-q_1-q_2)}{C_0}-\left(\frac{q_0+q-q_2}{C_0}\right)=0$$

or $2q_0+2q=q_1+2q_2$ ...(i)

In loop (2)

$$\frac{q_0+q-q_1-q_2}{C_0}+\frac{(q_0+q-q_1)}{C_0}+\frac{(q_0+q-q_1)}{C_0}=0$$

or $3q_0+3q-3q_1-q_2=0$ ...(ii)

In loop (3)

$$V_0-\left(\frac{q_0+q}{C_0}\right)-\left(\frac{q_0+q-q_1}{C_0}\right)=0$$

or $2q_0+2q-q_1=20\times5\times10^{-6}=100\times10^{-6}$ ...(iii)

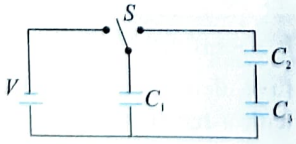
After solving Eqs. (i), (ii), and (iii), we get

$$q_1=\frac{400}{7}\mu C \text{ from } B \text{ to } A.$$

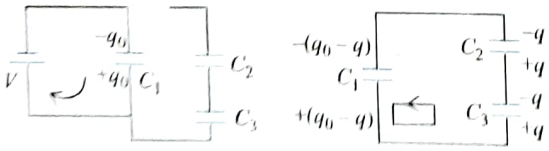
Thus, charge flowing from A to B is $-400/7\ \mu C$.

EXAMPLE 4.14

When the switch S in figure is pushed to the left, the plates of capacitor C_1 acquire a potential difference V. Initially, capacitors C_2 and C_3 are uncharged. The switch is now pushed to the right. What are the final charges q_1, q_2, and q_3 on the corresponding capacitors.



Sol. When switch is thrown left



Charge in capacitor C_1, $q_0=C_1V$

When switch is thrown right

According to loop rule,

$$\frac{q_0-q}{C_1}-\frac{q}{C_3}-\frac{q}{C_2}=0$$

or $$\frac{q_0-q}{C_1}=q\left(\frac{1}{C_2}+\frac{1}{C_3}\right)=q\left(\frac{C_2+C_3}{C_2C_3}\right)$$

$$\therefore \quad q=\left(\frac{C_2C_3q_0}{C_1C_2+C_1C_3+C_2C_3}\right)$$

$$\therefore \quad q_1=q_0-q=C_1V-\left(\frac{C_2C_3q_0}{C_1C_2+C_1C_3+C_2C_3}\right)$$

$$=\frac{C_1^2V(C_2+C_3)}{C_1C_2+C_2C_3+C_1C_3}$$

Also, $$q_2=q_3=q=\frac{C_2C_3q_0}{C_1C_2+C_2C_3+C_1C_3}$$

$$=\frac{C_2C_3C_1V}{C_1C_2+C_1C_3+C_2C_3}$$

EXAMPLE 4.15

A capacitor of capacitance C_0 is charged to a potential V_0 and then isolated. A small capacitor C is then charged from C_0, discharged and charged again, the process being repeated n times. Due to this potential of the capacitor, C_0 is decreased to V. Find the value of C.

Sol. Initial charge in capacitor C_0, $q_0=C_0V_0$

Charge in capacitor 'C_0' after connecting with capacitor 'C'.

$$q_1=q_0\left(\frac{C_0}{C_0+C}\right)$$

After discharging and again charging first time,

$$q_{0_1}=(C_0V_0)\frac{C_0}{(C_0+C)}=\frac{C_0^2V_0}{(C_0+C)}$$

i.e., $$V_{0_1}=\frac{C_0V_0}{(C_0+C)}$$

After discharging and again charging second time the charge on capacitor C_0,

$$q_{0_2}=(q_{0_1})\frac{C_0}{C_0+C}=\frac{C_0^3V_0}{(C_0+C)^2}$$

$$V_{0_2}=\left(\frac{C_0}{C_0+C}\right)^2V_0$$

After nth charging

$$V_{0_n}=V=\left(\frac{C_0}{C+C_0}\right)^nV_0$$

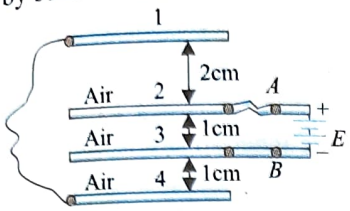
or $$\left(\frac{C_0}{C+C_0}\right)^n=\left(\frac{V}{V_0}\right)$$

$$\frac{C}{C_0}+1=\left(\frac{V_0}{V}\right)^{1/n}$$

or $$C=C_0\left(\frac{V_0}{V}\right)^{1/n}-C_0=C_0\left[\left(\frac{V_0}{V}\right)^{1/n}-1\right]$$

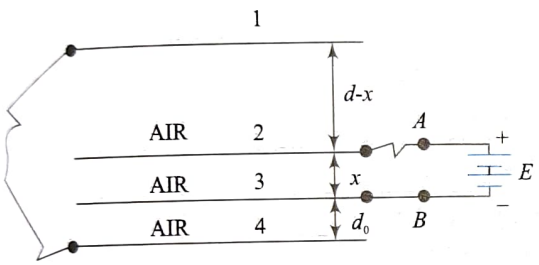
EXAMPLE 4.16

The arrangement of parallel conducting plates shown in figure are of the same surface area $A = 10\ cm^2$. A battery of emf $E =$ 10 V is connected across the ends A and B. Plate 2 is slowly moved upward by some external force.



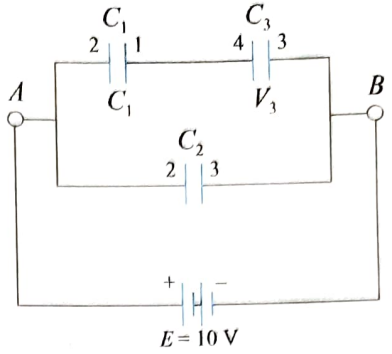
Find the position of the plate at which the energy stored in the system is minimum. Also, find this minimum energy. Take $\varepsilon_0 = 8.8 \times 10^{-12}\ Fm^{-1}$.

Sol. Let x be the instantaneous distance between the plates 2 and 3, then that between 1 and 2 will be $d - x$, where $d = 3$ cm. Also, distance between 3 and 4 is $d_0 = 1$ cm.



The system of plates can be represented as combination of three capacitors C_1, C_2, and C_3, where

$$C_1 = \frac{\varepsilon_0 A}{d-x};\ C_2 = \frac{\varepsilon_0 A}{x},\ \text{and}\ C_3 = \frac{\varepsilon_0 A}{d_0}$$



The equivalent capacitance of the system is

$$C = \frac{C_1 C_3}{C_1 + C_3} + C_2 = \frac{\varepsilon_0 A}{d_0 + d - x} + \frac{\varepsilon_0 A}{x} \qquad ...(i)$$

The potential energy of the system

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d_0 + d - x} + \frac{\varepsilon_0 A}{x}\right)V^2$$

The energy of the system is minimum when

$$\frac{dU}{dx} = 0$$

$$\therefore \quad \frac{dU}{dx} = \left(\frac{\varepsilon_0 A}{(d_0 + d - x)^2} - \frac{\varepsilon_0 A}{x^2}\right)V^2 = 0$$

or $\quad x^2 - (d + d_0 - x)^2 = 0 \quad$ or $\quad x = \dfrac{d + d_0}{2} = \dfrac{3+1}{2} = 2\ \text{cm}$

$$C_{\min} = \frac{\varepsilon_0 A}{(1+3-2)\times 10^{-2}} - \frac{\varepsilon_0 A}{2 \times 10^{-2}} = \varepsilon_0 A \times 10^2\ \text{F}$$

and the minimum energy is

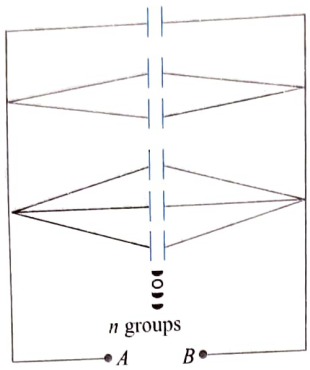
$$U_m = \frac{1}{2}C_m E^2 = \frac{1}{2}(\varepsilon_0 A \times 10^2)E^2$$

$$= \frac{1}{2}(8.8 \times 10^{-12})(10 \times 10^{-4})(10^2)(10)^2 = 4.4 \times 10^{-11}\ \text{J}$$

Exercises

Single Correct Answer Type

1. A number of capacitors, each of equal capacitance C, are arranged as shown in figure. The equivalent capacitance between A and B is



(1) n^2C

(2) $(2n+1)C$

(3) $\frac{(n-1)n}{2}C$

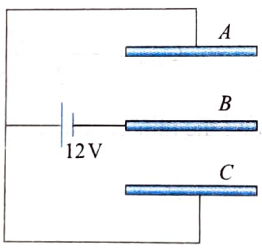
(4) $\frac{(n+1)n}{2}C$

2. The plates of a parallel plate capacitor are charged up to 100 V. Now, after removing the battery, a 2 mm thick plate is inserted between the plates. Then, to maintain the same potential difference, the distance between the capacitor plates is increased by 1.6 mm. The dielectric constant of the plate is

(1) 5 (2) 1.25

(3) 4 (4) 2.5

3. Three plates A, B, and C each of area 50 cm^2 have separation 3 mm between A and B and 3 mm between B and C. The energy stored when the plates are fully charged is



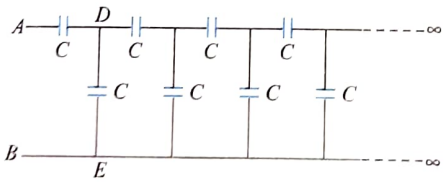
(1) 6×10^{-9} J

(2) 3.12×10^{-9} J

(3) 2.12×10^{-9} J

(4) none of these

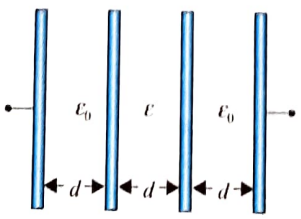
4. The capacitance of an infinite circuit formed by the repetition of the same link consisting of two identical capacitors, each with capacitance C (figure), is



(1) zero (2) $\frac{\sqrt{5}-1}{2}$ C

(3) $\frac{\sqrt{5}+1}{2}$ C (4) infinite

5. For the configuration of media of permittivities ε_0, ε, and ε_0 between parallel plates each of area A, as shown in figure, the equivalent capacitance is



(1) $\varepsilon_0 A/d$ (2) $\varepsilon\varepsilon_0 A/d$

(3) $\frac{\varepsilon\varepsilon_0 A}{d(\varepsilon+\varepsilon_0)}$ (4) $\frac{\varepsilon\varepsilon_0 A}{(2\varepsilon+\varepsilon_0)d}$

6. A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed, and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32. Determine the potential of the inner sphere.

(1) 400 V (2) 450 V

(3) 500 V (4) 300 V

7. A parallel plate capacitor has plates of area A and separation d and is charged to a potential difference V. The charging battery is then disconnected and the plates are pulled apart until their separation is $2d$. What is the work required to separate the plates?

(1) $2\varepsilon_0 AV^2/d$ (2) $\varepsilon_0 AV^2/d$

(3) $3\varepsilon_0 AV^2/2d$ (4) $\varepsilon_0 AV^2/2d$

8. A parallel plate capacitor is charged and then disconnected from the source of potential difference. If the plates of the condenser are then moved farther apart by the use of insulated handle, which of the following is true?

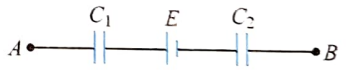
(1) The charge on the capacitor increases.

(2) The charge on the capacitor decreases.

(3) The capacitance of the capacitor increases.

(4) The potential difference across the plates increases.

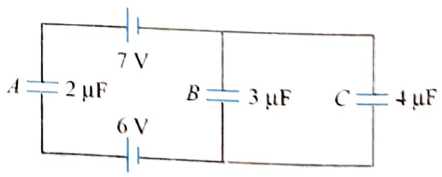
9. For section AB of a circuit shown in figure, $C_1 = 1$ μF, $C_2 = 2$ μF, $E = 10$ V, and the potential difference $V_A - V_B = -10$ V. Charge on capacitor C_1 is



(1) 0 μC (2) 20/3 μC

(3) −40/3 μC (4) none of these

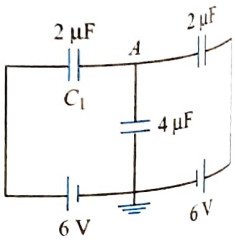
10. Three capacitors A, B, and C are connected in a circuit as shown in figure. What is the charge in μC on the capacitor B?



(1) 1/3 (2) 2/3

(3) 1 (4) 4/3

11. Three capacitors are connected as shown in figure. Then, the charge on capacitor C_1 is



(1) 6 μC

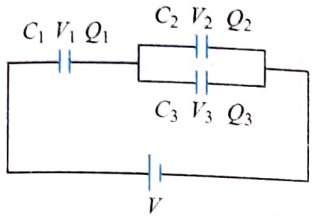
(2) 12 μC

(3) 18 μC

(4) 24 μC

12. A capacitor of capacitance $C_1 = 1\ \mu F$ charged up to a voltage $V = 110$ V is connected in parallel to the terminals of a circuit consisting of two uncharged capacitors connected in series and possessing capacitances $C_2 = 2\ \mu F$ and $C_3 = 3\ \mu F$. Then, the amount of charge that will flow through the connecting wires is
(1) 40 μC (2) 50 μC
(3) 60 μC (4) 110 μC

13. In figure, three capacitors C_1, C_2, and C_3 are joined to a battery. With symbols having their usual meaning, the correct conditions will be

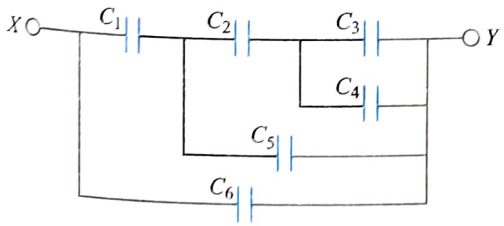


(1) $Q_1 = Q_2 = Q_3$ and $V_1 = V_2 = V_3 + V$
(2) $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_2 + V_3$
(3) $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_2$
(4) $Q_2 = Q_3$ and $V_2 = V_3$

14. An uncharged parallel plate capacitor having a dielectric of dielectric constant K is connected to a similar air cored parallel plate capacitor charged to a potential V_0. The two share the charge, and the common potential becomes V. The dielectric constant K is
(1) $\frac{V_0}{V} - 1$ (2) $\frac{V_0}{V} + 1$
(3) $\frac{V}{V_0} - 1$ (4) $\frac{V}{V_0} + 1$

15. Two identical parallel plate capacitors are connected in series and then joined in series with a battery of 100 V. A slab of dielectric constant $K = 3$ is inserted between the plates of the first capacitor. Then, the potential difference across the capacitors will be, respectively,
(1) 25 V, 75 V (2) 75 V, 25 V
(3) 20 V, 80 V (4) 50 V, 50 V

16. In the given network of capacitors as shown in figure, given that $C_1 = C_2 = C_3 = 400$ pF and $C_4 = C_5 = C_6 = 200$ pF. The effective capacitance of the circuit between X and Y is

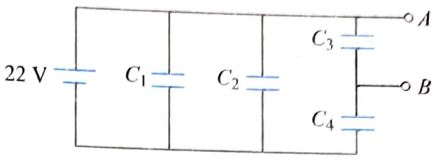


(1) 810 pF (2) 205 pF
(3) 600 pF (4) 410 pF

17. The work done in increasing the potential of a capacitor from V volt to $2V$ volt is W. Then, the work done in increasing the potential of the same capacitor from $2V$ volt to $4V$ volt will be
(1) W
(3) $4W$ (2) $2W$
(4) $8W$

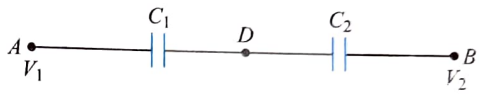
18. The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2 mm. The capacitor is charged by connecting it to a 400 V supply. Then the energy density of the energy stored (in Jm^{-3}) in the capacitor is (take $\varepsilon_0 = 8.8 \times 10^{-12}\ Fm^{-1}$)
(1) 0.113 (2) 0.117
(3) 0.152 (4) none of these

19. In figure, given $C_1 = 3\ \mu F$, $C_2 = 5\ \mu F$, $C_3 = 9\ \mu F$, and $C_4 = 13\ \mu F$. What is the potential difference between points A and B?



(1) 13 V (2) 9 V
(3) 0 V (4) 11 V

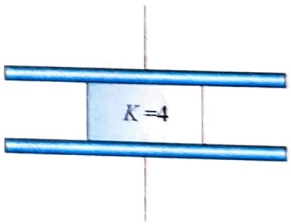
20. Two condensers C_1 and C_2 in a circuit are joined as shown in figure. The potential of point A is V_1 and that of B is V_2. The potential of point D will be



(1) $\frac{1}{2}(V_1 + V_2)$ (2) $\frac{C_1V_2 + C_2V_1}{C_1 + C_2}$
(3) $\frac{C_1V_1 + C_2V_2}{C_1 + C_2}$ (4) $\frac{C_2V_1 - C_1V_2}{C_1 + C_2}$

21. A capacitor is charged to store an energy U. The charging battery is disconnected. An identical capacitor is now connected to the first capacitor in parallel. The energy in each capacitor is now
(1) $3U/2$ (2) U
(3) $U/4$ (4) $U/2$

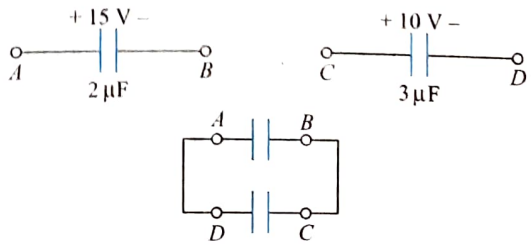
22. Consider a parallel plate capacitor of capacity 10 μF with air filled in the gap between the plates. Now, one-half of the space between the plates is filled with a dielectric of dielectric constant 4 as shown in figure. The capacity of the capacitor changes to



(1) 25 μF (2) 20 μF
(3) 40 μF (4) 5 μF

23. A 2 μF capacitor is charged to 100 V, and then its plates are connected by a conducting wire. The heat produced is
(1) 0.001 J (2) 0.01 J
(3) 0.1 J (4) 1 J

24. In figure, the initial status of capacitors and their connections is shown. Which of the following is incorrect about this circuit?



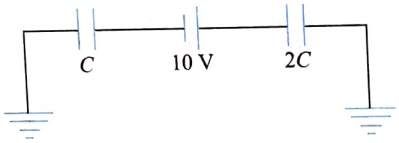
(1) Final charge on each capacitor will be zero.
(2) Final total electrical energy of the capacitor will be zero.
(3) Total charge flowing from A to D is 30 μC.
(4) Total charge flowing from A to D is –30 μC.

25. A capacitor of capacitance C_0 is charged to a potential V_0 and then isolated. A small capacitor C is then charged from C_0, discharged and charged again; the process being repeated n times. Due to this, the potential of the larger capacitor is decreased to V. The value of C is

(1) $C_0\left(\frac{V_0}{V}\right)^{1/n}$ (2) $C_0\left[\left(\frac{V_0}{V}\right)^{1/n}-1\right]$

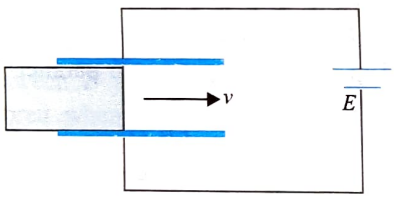
(3) $C_0\left[\left(\frac{V}{V_0}\right)-1\right]^n$ (4) $C_0\left[\left(\frac{V}{V_0}\right)^n+1\right]$

26. In the circuit shown in figure, $C = 6$ μF. The charge stored in the capacitor of capacity C is

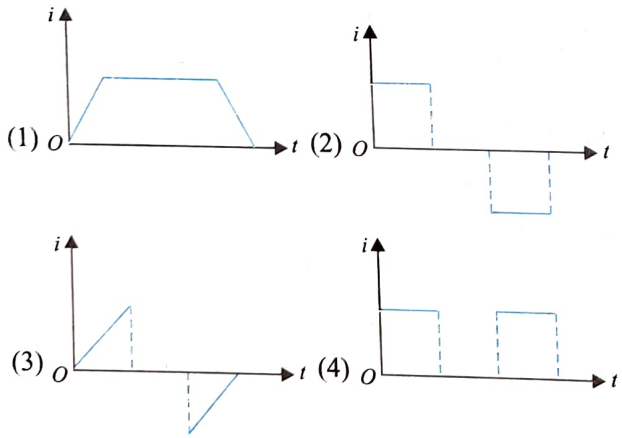


(1) zero (2) 90 μC
(3) 40 μC (4) 60 μC

27. A dielectric slab of area A and thickness d is inserted between the plates of a capacitor of area $2A$ with constant speed v as shown in figure. Distance between the plates is d.



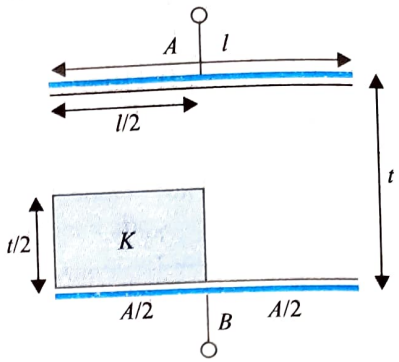
The capacitor is connected to a battery of emf E. The current in the circuit varies with time as



28. A photographic flash unit consists of a xenon-filled tube. It gives a flash of average power 2000 W for 0.04 s. The flash is due to discharge of a fully charged capacitor of 40 μF. The voltage to which it is charged before a flash is given by the unit is

(1) 1500 V (2) 2000 V
(3) 2500 V (4) 3000 V

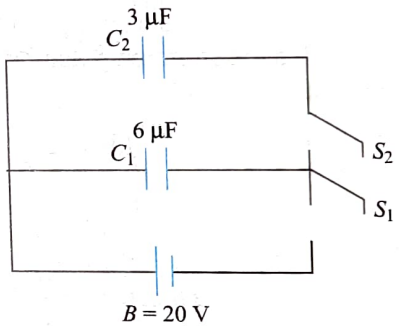
29. Two square plates ($l \times l$) and dielectric $\left(\frac{l}{2}\times\frac{t}{2}\times l\right)$ are arranged as shown in figure. Find the equivalent capacitance of the structure.



(1) $\frac{\varepsilon_0 A}{t}\left(\frac{3K+1}{2(K+1)}\right)$ (2) $\frac{2\varepsilon_0 A}{t}\left(\frac{K+1}{K+3}\right)$

(3) $\frac{\varepsilon_0 A}{t}\left(\frac{K+1}{K+3}\right)$ (4) $\frac{\varepsilon_0 A}{t}\left(\frac{2K+1}{2K+3}\right)$

30. In the circuit shown in figure, $C_1 = 6$ μF, $C_2 = 3$ μF, and battery $B = 20$ V. The switch S_1 is first closed. It is then opened, and S_2 is closed. What is the final charge on C_2?



(1) 120 μC (2) 80 μC
(3) 40 μC (4) 20 μC

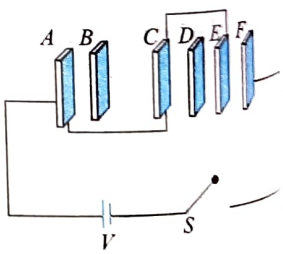
31. A, B, C, D, E, and F are conducting plates each of area A and any two consecutive plates are separated by a distance d. The net energy stored in the system after the switch S is closed is

(1) $\frac{3\varepsilon_0 A}{2d}V^2$

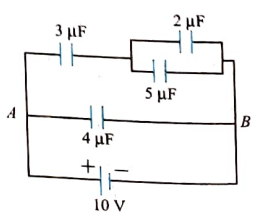
(2) $\frac{5\varepsilon_0 A}{12d}V^2$

(3) $\frac{\varepsilon_0 A}{2d}V^2$

(4) $\frac{\varepsilon_0 A}{d}V^2$

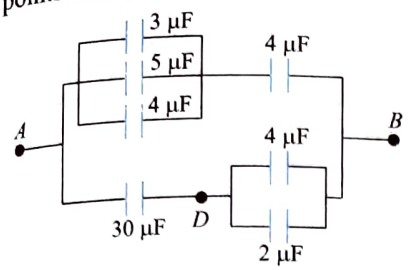


32. In the circuit shown in figure, the charge on the 5 μF capacitor will be



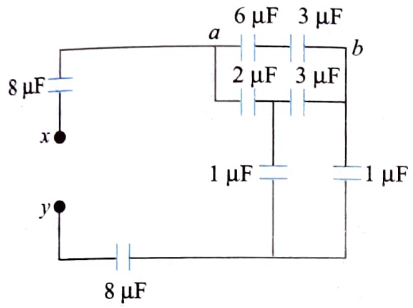
(1) 4.5 μC (2) 9 μC
(3) 15 μC (4) none of these

33. Several capacitors are connected as shown in figure. If the charge on the 5 μF capacitor is 120 μC, the potential between points A and D is



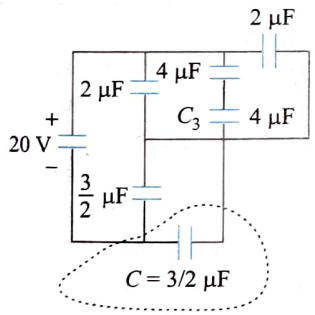
(1) 16 V (2) 32 V
(3) 64V (4) none of these

34. In the circuit shown, the effective capacitance between points X and Y is



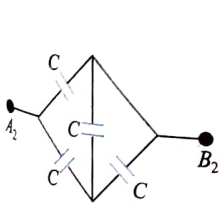
(1) 3.33 μF (2) 1 μF
(3) 0.44 μF (4) none of these

35. In figure, the battery has a potential difference of 20 V. The charge in the capacitor marked as C is

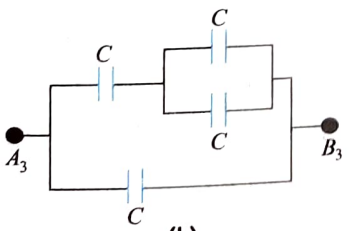


(1) 20 μC (2) 40 μC
(3) 10 μC (4) none of these

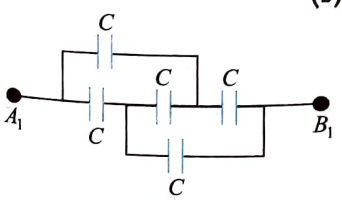
36. In figure, identical capacitors are connected in the following three configurations.



(a)



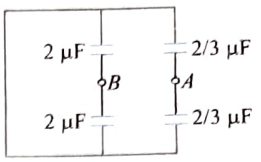
(b)



(c)

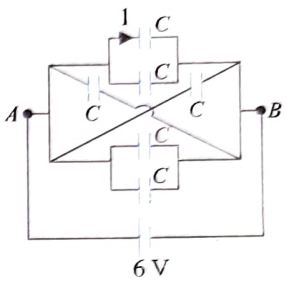
The ratio of the total capacitances in (i), (ii), and (iii), respectively, is
(1) 3 : 5 : 5 (2) 3 : 3 : 5
(3) 5 : 4 : 4 (4) 5 : 5 : 3

37. The equivalent capacitance of the circuit across the terminals A and B is equal to



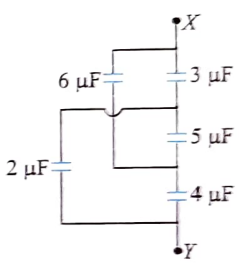
(1) 0.5 μF
(2) 2 μF
(3) 1 μF
(4) none of these

38. Six capacitors each of capacitance 1 μF are connected as shown in figure. Find the charge flowing in direction 1 as shown in the figure will be



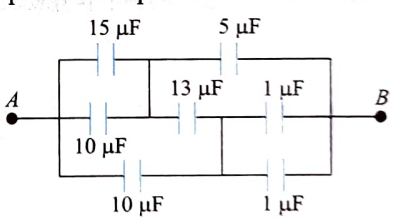
(1) 12 μC
(2) 6 μC
(3) 3 μC
(4) none of these

39. The equivalent capacitance between points X and Y in figure is



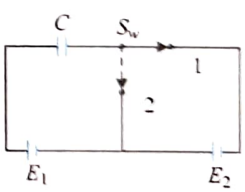
(1) $\frac{6}{5}$ μF
(2) 4 μF
(3) $\frac{18}{5}$ μF
(4) none of these

40. Find the equivalent capacitance across A and B.



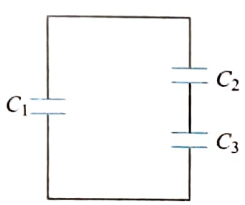
(1) $\frac{35}{6}$ μF (2) $\frac{25}{6}$ μF
(3) 15 μF (4) none of these

41. In the given circuit diagram (figure), switch S_w is shifted from position 1 to position 2. Then



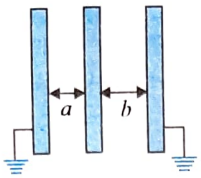
(1) a charge of amount CE_2 will be supplied to battery E_1
(2) heat generated in the circuit is $CE_2^2/2$
(3) a charge of amount CE_2 will be supplied by battery E_1
(4) heat generated in the circuit is $CE_1E_2/2$

42. Capacitor C_1 is connected to a battery and charged till the magnitude of the charge on each plate is q_0. Then, the battery is disconnected and C_1 is connected to two other uncharged capacitors C_2 and C_3 as shown (figure). Final charges on the capacitors (q_1, q_2, and q_3) are related by



(1) $q_0 = q_1 + q_2 + q_3$ (2) $q_1 + q_2 + q_3 = 0$
(3) $q_0 = q_3 + q_2, q_1 = 0$ (4) $q_0 = q_1 + q_2, q_2 = q_3$

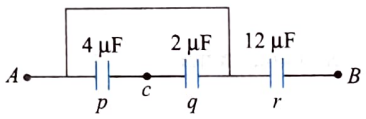
43. Three identical metallic plates are kept parallel to one another at separations a and b. The outer plates are connected to the ground and the middle plate is given charge Q (figure). Then, the charge on the right side of middle plate is



(1) $Q/2$ (2) $-\frac{Qb}{a+b}$
(3) $\frac{Qb}{a+b}$ (4) $\frac{Qa}{a+b}$

44. A 16 μF capacitor, initially charged to 5 V, is started charging at $t = 0$ by a source at the rate of $40t$ μCs^{-1}. How long will it take to raise its potential to 10 V?
(1) 1 s (2) 2 s
(3) 3 s (4) none of these

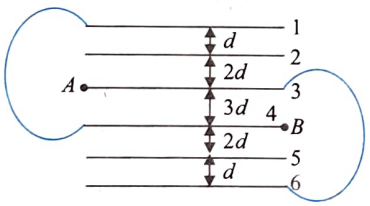
45. Find the equivalent capacitance between C and B.



(1) 6/5 μF (2) 5/6 μF
(3) 4 μF (4) none of these

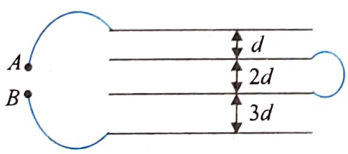
46. A parallel plate capacitor of capacitance 10 μF is connected across a battery of emf 5 mV. Now, the space between the plates of the capacitor is filled with a dielectric material of dielectric constant $K = 5$. Then, the charge that will flow through the battery till equilibrium is reached is
(1) 250 μC (2) 250 nC
(3) 200 nC (4) 200 μC

47. Six plates of equal area A and plate separation as shown (figure) are arranged. The equivalent capacitance between A and B is



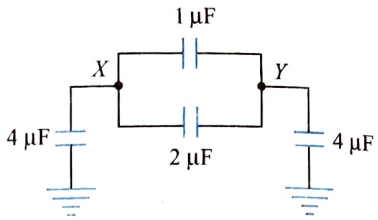
(1) $\frac{\varepsilon_0 A}{d}$ (2) $\frac{2\varepsilon_0 A}{d}$
(3) $\frac{3\varepsilon_0 A}{d}$ (4) $\frac{\varepsilon_0 A}{4d}$

48. If area of each plate is A and the successive separations are d, $2d$, and $3d$, then the equivalent capacitance across A and B is



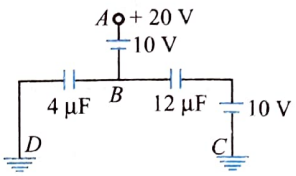
(1) $\frac{\varepsilon_0 A}{6d}$ (2) $\frac{\varepsilon_0 A}{4d}$
(3) $\frac{3\varepsilon_0 A}{4d}$ (4) $\frac{\varepsilon_0 A}{3d}$

49. In the circuit shown (figure), the equivalent capacitance between the points X and Y is



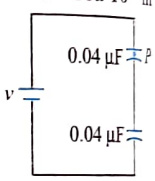
(1) 2 μF (2) 3 μF
(3) 4 μF (4) 5 μF

50. For the arrangement shown in figure, identify the correct statement.



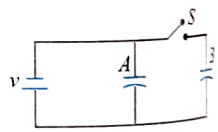
(1) The charge on the 12 μF capacitor is zero.
(2) The charge on the 12 μF capacitor is 30 μC.
(3) The charge on the 4 μF capacitor is 30 μC.
(4) None of these.

51. The particle P shown in the figure has a mass of 10 mg and a charge of –0.01 μC. Each plate has a surface area 10^{-2} m^2 on one side. What potential difference V should be applied to the combination to hold the particle P in equilibrium?



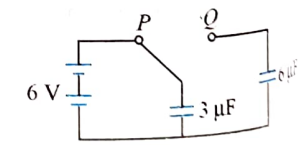
(1) 43 mV (2) 35 mV
(3) 50 mV (4) 55 mV

52. Figure shows two identical parallel plate capacitors connected to a battery through a switch S. Initially, the switch is closed so that the capacitors are completely charged. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant 3. Find the ratio of the initial total energy stored in the capacitors to the final total energy stored.



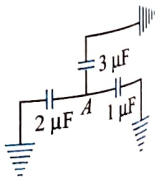
(1) 9 : 16 (2) 5 : 9
(3) 2 : 3 (4) 3 : 5

53. In the circuit shown, a capacitor of capacitance 3 μF is charged form a battery of emf 6 V with switch connected to terminal P. The switch is now connected to Q. This charges the 6 μF capacitor from the 3 μF one. What is the new potential difference across combination?



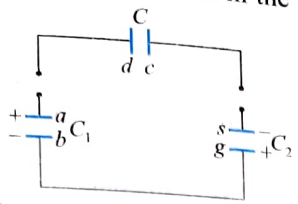
(1) 1 V (2) 2 V
(3) 4 V (4) 6 V

54. In the given arrangement of capacitors, 6 μC charge is added to point A. Find the charge on upper capacitor.



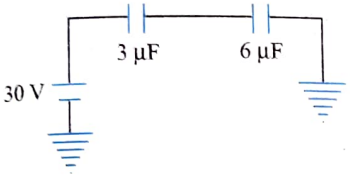
(1) 3 μC (2) 1 μC
(3) 2 μC (4) 6 μC

55. Two capacitors A and B with capacities C_1 and C_2 are charged to potential difference of V_1 and V_2, respectively. The plates of the capacitors are connected as shown in the figure. The upper plate of A is positive and that of B is negative. An uncharged capacitor of capacitance C and falls on the free ends to complete circuit, then



(1) the final charge on each capacitor are same to each other
(2) the final sum of charge on plates a and d is C_1V_1
(3) the final sum of charge on plates b and g is $C_2V_2 - C_1V_1$
(4) both (2) and (3) are correct

56. In the circuit shown in figure, the charge stored on a capacitor of capacitance 3 μF is

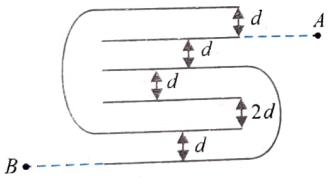


(1) zero
(2) 40 μC
(3) 60 μC
(4) 90 μC

57. A parallel plate capacitor is connected to a battery. The plates are pulled apart with uniform speed. If x is the separation between the plates, then the rate of change of electrostatic energy of the capacitor is proportional to

(1) x^2
(2) x
(3) $\frac{1}{x}$
(4) $\frac{1}{x^2}$

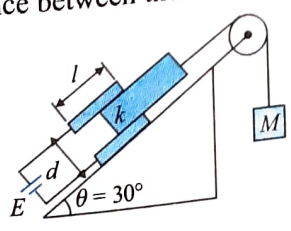
58. Find the equivalent capacitance between A and B. [Assume each conducting plate is having same dimensions and neglect the thickness of the plate, $\varepsilon_0 A/d = 7$ μF, where A is area of plates and $A >> d$].



(1) 7 μF
(2) 11 μF
(3) 12 μF
(4) 15 μF

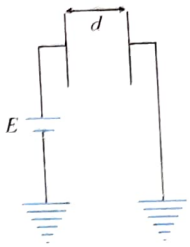
59. If the plates of a parallel plate capacitor are not equal in area, then
(1) quantity of charge on the plates will be the same, but nature of charge will differ
(2) quantity of charge as well as nature of charge on the plates will be different
(3) quantity of charge on the plates will be different, but nature of charge will be the same
(4) quantity of charge as well as nature of charge will be the same

60. The capacitor plates are fixed on an inclined plane and connected to a battery of emf E. The capacitor plates have plate area a, length l, and the distance between them is d. A dielectric slab of mass m and dielectric constant K is inserted into the capacitor and tied to mass M by a massless string as shown in the figure. Find the value of M for which the slab will stay in equilibrium. There is no friction between slab and plates.



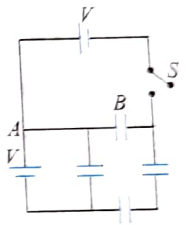
(1) $\frac{m}{2} + \frac{E^2\varepsilon_0 A(k-1)}{2lgd}$
(2) $\frac{m}{2} - \frac{E^2\varepsilon_0 A(k-1)}{2lgd}$
(3) $\frac{m}{2} + \frac{E^2\varepsilon_0 A(k-1)}{lgd}$
(4) $\frac{m}{2} - \frac{E^2\varepsilon_0 A(k-1)}{lgd}$

61. Two plates, each of area A, are placed parallel to each other at a distance d. One plate is connected to a battery of emf E and its negative is earthed. The other plate is also earthed. The charge drawn by plate is



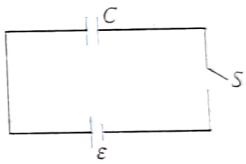
(1) $\frac{2\varepsilon_0 AE}{d}$
(2) $\frac{\varepsilon_0 AE}{d}$
(3) $\frac{\varepsilon_0 AE}{2d}$
(4) $\frac{3\varepsilon_0 AE}{d}$

62. Find the charge that will flow through the wire A to B if the switch S is closed. The capacitance of each capacitor shown in the figure is C.



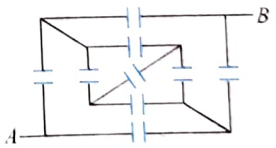
(1) CV
(2) $CV/3$
(3) zero
(4) $2CV/3$

63. The left plate of the capacitor shown in the figure carries a charge $+Q$ while the right plate is uncharged. The total final charge on the right plate after closing the switch S will be



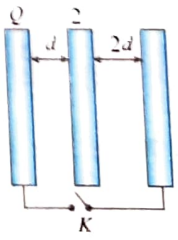
(1) $\frac{Q}{2} + C\varepsilon$
(2) $\frac{Q}{2} - C\varepsilon$
(3) $-\frac{Q}{2}$
(4) none of these

64. The effective capacitance between points A and B is (the capacitance of each of the capacitors is C)



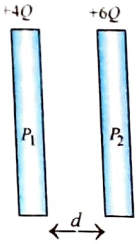
(1) C
(2) $C/2$
(3) $36C/17$
(4) $42C/17$

65. Three large plates are arranged as shown. How much charge will flow through the key k if it is closed?



(1) $5Q/6$
(2) $4Q/3$
(3) $3Q/2$
(4) none

66. Two identical conducting very large plates P_1 and P_2 having charges $+4Q$ and $+6Q$ are placed very closed to each other at separation d. The plate area of either face of the plate is A. The potential difference between plates P_1 and P_2 is



(1) $V_{P_1} - V_{P_2} = \frac{Qd}{A\varepsilon_0}$ (2) $V_{P_1} - V_{P_2} = \frac{-Qd}{A\varepsilon_0}$

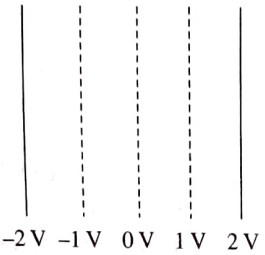
(3) $V_{P_1} - V_{P_2} = \frac{5Qd}{A\varepsilon_0}$ (4) $V_{P_1} - V_{P_2} = \frac{-5Qd}{A\varepsilon_0}$

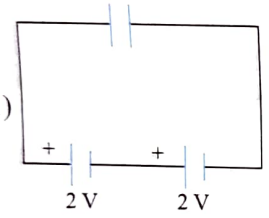
67. In the above question, if plates P_1 and P_2 are connected by a thin conducting wire, then the amount of heat produced will be

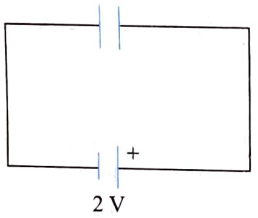
(1) $\frac{Q^2}{A\varepsilon_0}d$ (2) $\frac{5Q^2}{A\varepsilon_0}d$

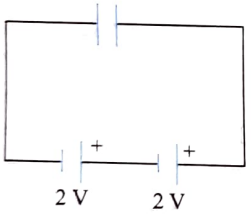
(3) $\frac{2Q^2}{A\varepsilon_0}d$ (4) none of these

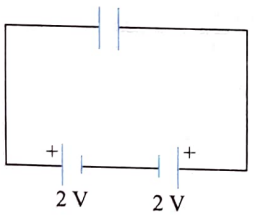
68. A battery (or batteries) connected to two parallel plates produces the equipotential lines between the plates as shown. Which of the following configurations is most likely to produce these equipotential lines?



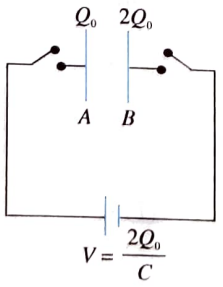
(1)


(2)


(3)


(4)


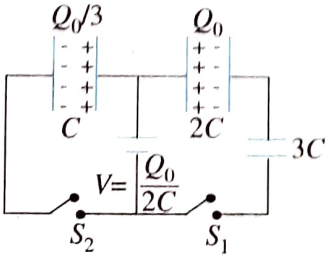
69. Charges Q_0 and $2Q_0$ are given to parallel plates A and B, respectively, and they are separated by a small distance. The capacitance of the given arrangement is C. Now, plates A and B are connected to positive and negative terminals of battery of potential difference $V = 2Q_0/C$ respectively, as shown, then the work done by the battery is



(1) $\frac{2Q_0^2}{C}$ (2) $\frac{4Q_0^2}{C}$

(3) $\frac{5Q_0^2}{C}$ (4) $\frac{6Q_0^2}{C}$

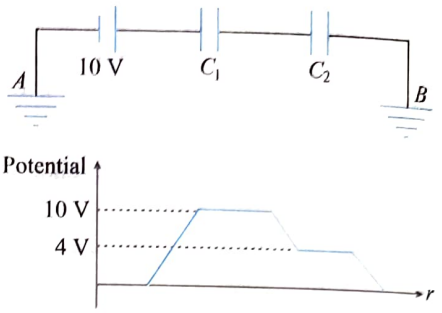
70. In the given circuit, the initial charges on the capacitors are shown in the figure. The charge flown through the switches S_1 and S_2, respectively after closing the switches are



(1) zero, $\frac{Q_0}{6}$ (2) $\frac{Q_0}{5}, \frac{Q_0}{2}$

(3) zero, $\frac{Q_0}{2}$ (4) $\frac{3}{5}Q_0, \frac{Q_0}{6}$

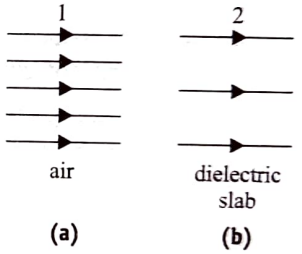
71. Figure shows two capacitors C_1 and C_2 connected with 10 V battery and terminal A and B are earthed. The graph shows the variation of potential as one moves from left to right. Then the ratio C_1/C_2 is



(1) 5/2 (2) 2/3

(3) 2/5 (4) 4/3

72. In figure, there are two patterns of electric field lines absent of dielectric [Fig. (a)] and in presence of dielectric slab [Fig. (b)]. The relative permittivity of the dielectric slab is

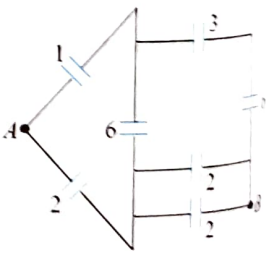


(1) $\frac{5}{3}$ (2) $\frac{5}{3} \times \varepsilon_0$

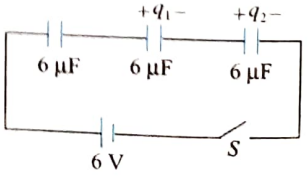
(3) 4 (4) $\frac{5}{2}$

73. All the capacitances shown in figure are in μF. Find the equivalent capacitance between A and B.

(1) 2 μF
(2) 3 μF
(3) 6 μF
(4) 9 μF



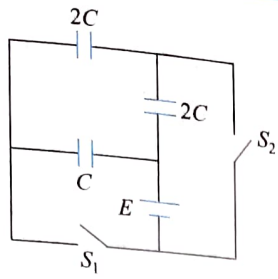
74. The left most capacitor is uncharged. The other two capacitors carry charges $q_1 = 8$ μC and $q_2 = 10$ μC. Now switch S is closed. Find the final charge on left most capacitor.



(1) 3 μC (2) 6 μC

(3) 12 μC (4) 18 μC

15. In the figure shown, all the capacitors are initially uncharged.

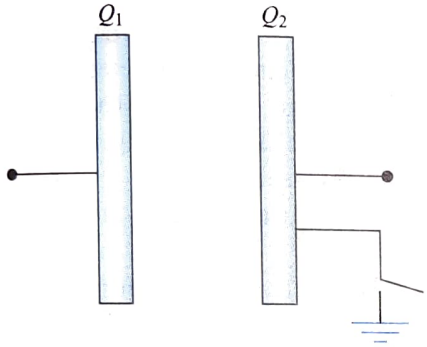


Case I: only switch S_1 is closed
Case II: only switch S_2 is closed
Case III: both switches are closed
Find the ratio of the total energy stored in the system of capacitors for the cases I, II, and III, respectively.

(1) 1 : 2 : 3 (2) 5 : 7 : 9
(3) 6 : 8 : 9 (4) 9 : 7 : 6

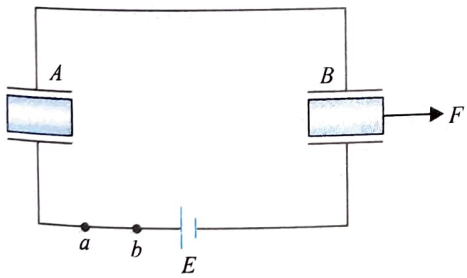
Multiple Correct Answers Type

1. Charges Q_1 and Q_2 are given to two plates of a parallel plate capacitor. The capacity of the capacitor is C. When the switch is closed, mark the correct statement(s). (Assume both Q_1 and Q_2 to be positive.)



(1) The charge flowing through the switch is zero.
(2) The charge flowing through the switch is $Q_1 + Q_2$.
(3) Potential difference across the capacitor plate is Q_1/C.
(4) The charge of the capacitor is Q_1.

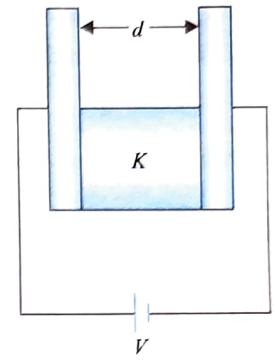
2. Two identical capacitors with identical dielectric slabs in between them are connected in series as shown in figure. Now, the slab of one capacitor is pulled out slowly with the help of an external force F at steady state as shown. Mark the correct statement(s).



(1) During the process, charge (positive) flows from b to a.
(2) During the process, the charge of capacitor B is equal to the charge on A at all instants.
(3) Work done by F is positive.
(4) During the process, the battery has been charged.

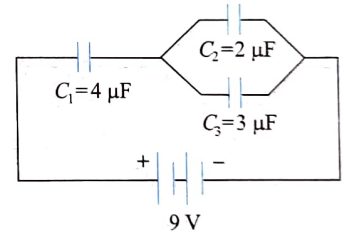
3. A parallel plate air capacitor has initial capacitance C. If plate separation is slowly increased from d_1 to d_2, then mark the correct statement(s). (Take potential of the capacitor to be constant, i.e., throughout the process it remains connected to battery.)
(1) Work done by electric force = negative of work done by external agent.
(2) Work done by external force = $-\int \vec{F} \cdot \vec{dx}$, where $\vec{F}$ is the electric force of attraction between the plates at plate separation x.
(3) Work done by electric force ≠ negative of work done by external agent.
(4) Work done by battery = two times the change in electric potential energy stored in capacitor.

4. A dielectric slab fills the lower half of a parallel plate capacitor as shown in figure. (Take plate area as A)



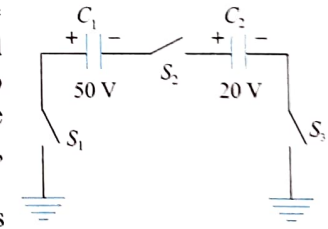
(1) Equivalent capacity of the system is $(\varepsilon_0 A/2d)(1 + K)$.
(2) The net charge of the lower half of the left hand plate is $1/K$ times the charge on the upper half of the plate.
(3) Net charges on the lower and upper halves of the left hand plate are different.
(4) Net charge on the lower half of the left hand plate is $\frac{K\varepsilon_0 A}{2d} \times V$.

5. In figure, the charges on C_1, C_2, and C_3 are Q_1, Q_2, and Q_3, respectively. Then,



(1) $Q_2 = 8$ μC (2) $Q_3 = 12$ μC
(3) $Q_1 = 20$ μC (4) $Q_2 = 12$ μC

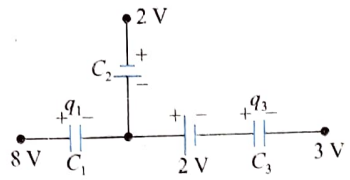
6. In the circuit shown, $C_1 = C_2 = 2$ μF. Capacitor C_1 is charged to 50 V and C_2 is charged to 20 V. After charging, they are connected as shown. When S_1, S_2, and S_3 are closed,



(1) 70 μC of charge will pass through S_1
(2) 100 μC of charge will pass through S_1
(3) 70 μC of charge will pass through S_3
(4) 40 μC of charge will pass through S_3

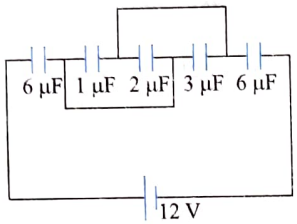
7. A capacitor of 5 μF is charged to a potential of 100 V. Now, this charged capacitor is connected to a battery of 100 V with the positive terminal of the battery connected to the negative plate of the capacitor. For the given situation, mark the correct statement(s).
(1) The charge flowing through the 100 V battery is 500 μC.
(2) The charge flowing through the 100 V battery is 1000 μC.
(3) Heat dissipated in the circuit is 0.1 J.
(4) Work done on the battery is 0.1 J.

8. Figure shows a part of a circuit. If all the capacitors have a capacitance of 2 μF, then the



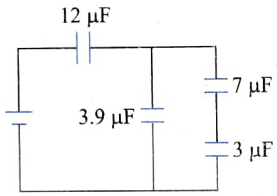
(1) charge on C_3 is zero (2) charge on C_3 is 12 μC
(3) charge on C_1 is 6 μC (4) charge on C_2 is 6 μC

9. In the circuit diagram shown in figure,



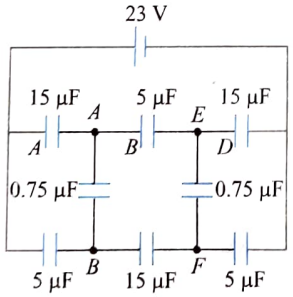
(1) the charge on 2 μF capacitor is 8 μC
(2) the charge on each 6 μF capacitor is 72 μC
(3) the potential drop across 1 μF capacitor is 4 V
(4) the potential drop across 3 μF is 4 V

10 Four capacitors and a battery are connected as shown in figure. If the potential difference across the 7 μF capacitor is 6 V, then which of the following statement(s) is/are correct?



(1) The potential drop across the 12 μF capacitor is 10 V.
(2) The charge in the 3 μF capacitor is 42 μC.
(3) The potential drop across the 3 μF capacitor is 10 V.
(4) The emf of the battery is 30 V.

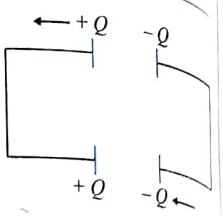
11. Find the potential difference between the points A and B and that between E and F of the circuit shown in figure.



(1) $V_{AB} = 5$ V (2) $V_{EF} = 5$ V
(3) $V_{AB} = 0$ (4) $V_{EF} = 0$

12. Two identical parallel plate capacitors are connected in one case in parallel and in the other in series. In each case the plates of one capacitor are brought closer by a distance a and the plates of the other are moved apart by the same distance a. Then
(1) total capacitance of first system increases
(2) total capacitance of first system decreases
(3) total capacitance of second system decreases
(4) total capacitance of second system remains constant

13. A charge Q is imparted to two identical capacitors in parallel. Separation of the plates in each capacitor is d_0. Suddenly, the first plate of the first capacitor and the second plate of the second capacitor start moving to the left with speed v, then



(1) charges on the two capacitors as a function of time are

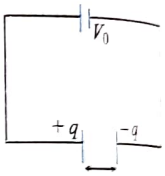
$$\frac{Q(d_0 - vt)}{2d_0}, \frac{Q(d_0 + vt)}{2d_0}$$

(2) charges on the two capacitors as a function of time are

$$\frac{Qd_0}{2(d_0 - vt)}, \frac{Qd_0}{2(d_0 + vt)}$$

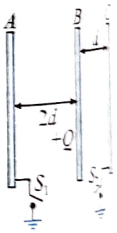
(3) current in the circuit will increase as time passes on
(4) current in the circuit will be constant

14. Two plates of a parallel plate capacitor carry charges q and $-q$ and are separated by a distance a from each other. The capacitor is connected to a constant voltage source V_0. The distance between the plates is changed to $x + dx$. Then in steady state



(1) change in electrostatic energy stored in the capacitor is $-Udx/x$
(2) change in electrostatic energy in the capacitor is $-Udx/dx$
(3) attraction force between the plates is $1/2qE$
(4) attraction force between the plates is qE (where E is electric field between the plates)

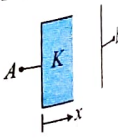
15. Three identical parallel conducting plates A, B, and C are placed as shown. Switches S_1 and S_2 are open, and can connect A and C to earth when closed. $+Q$ charge is given to B.

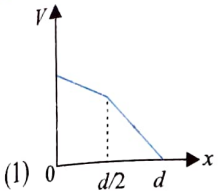


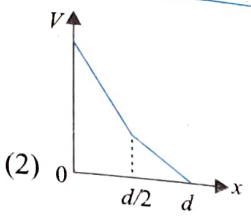
(1) If S_1 is closed with S_2 open, a charge of amount Q will pass through S_1.
(2) If instead S_2 were closed with S_1 open, a charge of amount Q will pass through S_2.
(3) If S_1 and S_2 are closed together, a charge of amount $Q/3$ will pass through S_1, and a charge of amount $2Q/3$ will pass through S_2.
(4) If S_1 and S_2 are closed together, a charge of amount $2Q/3$ will pass through S_1, and a charge of amount $Q/3$ will pass through S_2.

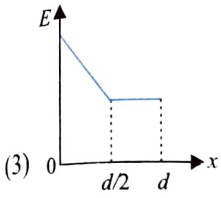
16. A parallel plate capacitor is charged from a cell and then isolated from it. The separation between the plates is now increased.
(1) The force of attraction between the plates will decrease.
(2) The field in the region between the plates will not change.
(3) The energy stored in the capacitor will increase.
(4) The potential difference between the plates will decrease.

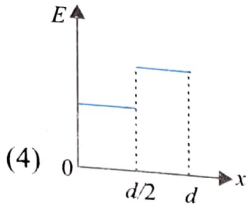
17. In figure, half of the space between the plates of a parallel plate capacitor is filled with dielectric material of constant K. Then which of the plots are possible?



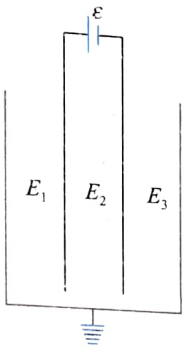
(1) 

(2) 

(3) 

(4) 

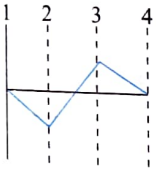
18. Four identical metallic plates (1, 2, 3 and 4) are arranged in air at same distance d from each other with their outer plates being connected together and earthed. If the plates 2 and 3 are connected with a cell of constant emf E, then ratio of electric fields between the plate is

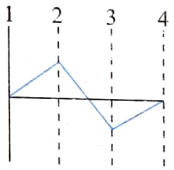


(1) $E_1 : E_2 : E_3 = \frac{2}{3} : 1 : \frac{2}{3}$

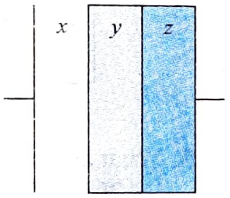
(2) $E_1 : E_2 : E_3 = \frac{1}{2} : 1 : \frac{1}{2}$

and variation of electric potential will be

(3) 

(4) 

19. Figure shows a capacitor having three layers between its plates. Layer x is vacuum, y is conductor, and z is a dielectric. Which of the following change (s) will result in increase in capacitance?

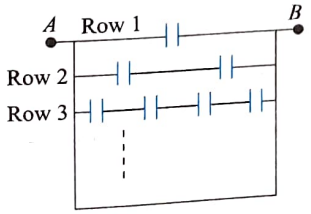


(1) replace x by conductor (2) replace y by dielectric
(3) replace z by conductor (4) replace x by dielectric

20. A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved further apart by means of insulating handles.
(1) The charge on the capacitor increases.
(2) The voltage across the plates increases.
(3) The capacitance increases.
(4) The electrostatic energy stored in the capacitor increases.

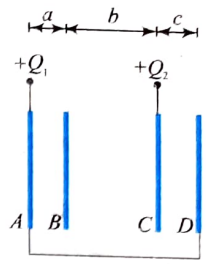
21. The plates of a parallel plate capacitor are not exactly parallel. The surface charge density is, therefore,
(1) higher at the closer end
(2) the surface charge density will not be uniform
(3) each plate will have the same potential at each point
(4) the electric field is smallest where the plates are closest

22. Rows of capacitors containing 1, 2, 4, 8, …, ∞ capacitors, each of capacitance $2F$, are connected in parallel as shown in figure. The potential difference across AB is 10 V, then



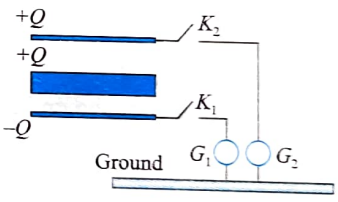
(1) total capacitance across AB is $4F$
(2) charge on each capacitor will be same
(3) charge on the capacitor in the first row is more than on any other capacitor
(4) energy of all the capacitors is 50 J

23. Figure shows an arrangement of four identical rectangular plates A, B, C, and D each of area S. Find the charges appearing on each face (from left to right) of the plates. Ignore the separation between the plates in comparison to the plate dimensions.



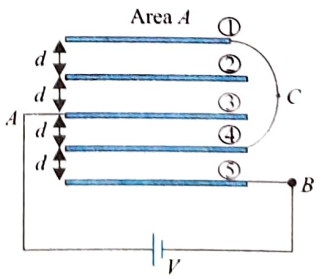
(1) Potential difference between plates A and B is independent of Q_1.
(2) Potential difference between plates C and D is independent of Q_1.
(3) Potential difference between plates A and B is independent of Q_2.
(4) Potential difference between plates C and D is independent of Q_2.

24. A parallel plate capacitor is charged as shown (Q is given). A metal slab with the total charge $+Q$ is placed inside the capacitor as shown. The thickness of the slab is d. The distance between the top plate and the top of the slab is $2d$, and the distance between the bottom plate and the bottom of the slab is d. Each plate is grounded through a galvanometer as shown. Find the charge that passes through each galvanometer after both switches are closed simultaneously.



(1) A total charge of $-Q/3$ flows (through G_1) from the bottom plate to ground.
(2) A total charge of $+4Q/3$ flows (through G_2) from the top plate to ground.
(3) A total charge of $+Q/3$ flows (through G_1) from the bottom plate to ground.
(4) A total charge of $-4Q/3$ flows (through G_2) from the top plate to ground.

25. Five plates are arranged as shown in the figure and connected across a battery of emf V. The separation between each plate is d and surface area of each plate is A.



(1) Equivalent capacity between A and B is $\varepsilon_0 A/5d$
(2) Equivalent capacity between A and B is $3\varepsilon_0 A/5d$
(3) Charge on plate 1 is $\varepsilon_0 AV/5d$
(4) Charge on plate 3 is $3\varepsilon_0 AV/5d$

Linked Comprehension Type

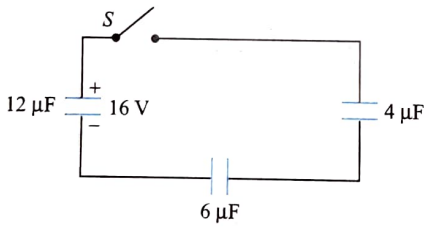
For Problems 1–2

A 1 μF and a 2 μF capacitor are connected in series across a 1200 V supply.

1. The charged capacitors are disconnected from the line and from each other and are now reconnected with the terminals of like sign together. Find the final charge on each capacitor and the voltage across each capacitor.
   (1) Charges on capacitors are 1400/3 μC and 3200/3 μC, and potential difference across each capacitor is 1600/3 V.
   (2) Charges on capacitors are 1600/3 μC and 3200/3 μC, and potential difference across each capacitor is 1600/3 V
   (3) Charge on each capacitor is 1600 μC, and potential difference across each capacitor is 800 V.
   (4) Charge and potential difference across each capacitor are zero.
2. If the charged capacitors are reconnected with the terminals of opposite signs together, find the final charge and voltage across each capacitor.
   (1) Charges on capacitors are 1400/3 μC and 3200/3 μC, and potential difference across each capacitor is 1600/3 V.
   (2) Charges on capacitors are 1600/3 μC and 3200/3 μC, and potential difference across each capacitor is 1600/3 V.
   (3) Charge on each capacitor is 1600 μC, and potential difference across each capacitor is 800 V.
   (4) Charge and potential difference across each capacitor are zero.

For Problems 3–5

In the arrangement shown in figure, when the switch S is closed, find



3. the final charge on the 6 μF capacitor
   (1) 12 μC (2) 24 μC
   (3) 32 μC (4) 48 μC
4. the final potential difference across the 4 μF capacitor
   (1) 12 V (2) 8 V
   (3) 20 V (4) 32 V
5. the final potential difference across the 12 μF capacitor
   (1) $\frac{40}{3}$ V (2) $\frac{20}{3}$ V
   (3) 12 V (4) 24 V

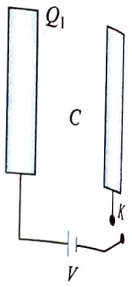
For Problems 6–7

Each plate of a parallel plate air capacitor has area $S = 5 \times 10^{-3}$ m^2 and the distance between the plates is $d = 8.80$ mm. Plate A has positive charge $q_1 = +10^{-10}$ C, and plate B has charge $q_2 = +2 \times 10^{-10}$ C. A battery of emf $E = 10$ V has its positive terminal connected to plate A and the negative terminal to plate B. (Given $\varepsilon_0 = 8.8 \times 10^{-12}$ Nm^2 C^{-2})

6. Charge supplied by the battery is
   (1) 120 pC (2) 100 pC
   (3) 60 pC (2) 50 pC
7. Energy supplied by the battery is
   (1) 10^{-9} J (2) 5×10^{-9} J
   (3) 50×10^{-9} J (4) 25×10^{-9} J

For Problems 8–9

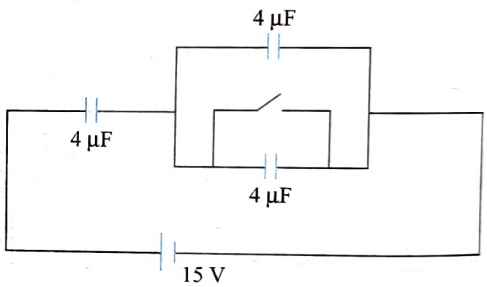
For the system shown in figure, capacitance is C. The left plate is given a charge Q_1, and the right plate is uncharged. Now, the switch is closed.



8. Find the amount of charge that will flow through the battery before the steady state is achieved.
   (1) CV (2) $CV - Q_1$
   (3) $CV + \frac{Q_1}{2}$ (4) $CV - \frac{Q_1}{2}$
9. Find the charge appearing on the inner face of the left plate.
   (1) $CV - \frac{Q_1}{2}$ (2) $CV + Q_1$
   (3) $CV + \frac{Q_1}{2}$ (4) CV

For Problems 10–11

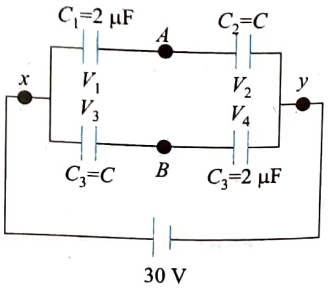
Consider the circuit shown in figure, after switch S is closed.



10. What amount of charge will flow through the battery?
    (1) 20 μC (2) 60 μC
    (3) 40 μC (4) no charge will flow
11. What amount of charge will flow through the switch?
    (1) 20 μC (2) 60 μC
    (3) 40 μC (4) no charge will flow

For Problems 12–16

The given circuit shows an arrangement of four capacitors. A potential difference 30 V is applied across the combination. It is observed that potentials at points A and B differ by 5 V. Also if a conducting wire is connected between A and B, electrons will flow from A to B. Of course, we have not actually connected any wire between A and B, we have described only an 'if' situation. Answer the following questions.



12. Potential difference across C_4 is
(1) 12.5 V (2) 15.5 V
(3) 17.5 V (4) 22.5 V

13. Equivalent capacitor between X and Y is
(1) 2.34 μF (2) 1.54 μF
(3) 1.22 μF (4) 0.77 μF

14. Charge on capacitor C_2 is
(1) 60 μC (2) 52 μC
(3) 42 μC (4) 35 μC

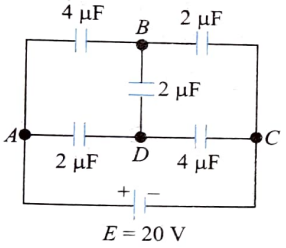
Let us now connect two more capacitors in the circuit. One of them, C_5, is connected in the part of the circuit between X and A. It could be either in series or in parallel with C_1. The other, C_6, is connected between A and B. It is observed whether we increase C_6 or reduce it, equivalent capacitance between X and Y has the same value.

15. Capacitance C_5 is
(1) 1.24 μF in series with C_1
(2) 1.92 μF in parallel with C_1
(3) 2.28 μF in series with C_1
(4) 2.56 μF in parallel with C_1

16. Charge on C_5 will be
(1) 32 μC (2) 28 μC
(3) 24 μC (4) 16 μC

For Problems 17–18

Figure shows a diagonal symmetric arrangement of capacitors and a battery.

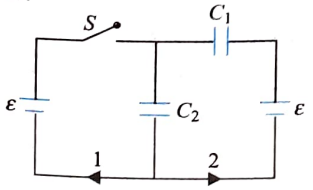


17. Identify the correct statements.
(1) Both the 4 μF capacitors carry equal charges in opposite sense.
(2) Both the 4 μF capacitors carry equal charges in the same sense.
(3) $V_B - V_D = 0$
(4) $V_D - V_B > 0$

18. If the potential of C is zero, then identify the incorrect statement.
(1) $V_A = +15$ V
(2) $4(V_A - V_B) + 2(V_D - V_B) = 2V_B$
(3) $2(V_A - V_D) + 2(V_B - V_D) = 4V_D$
(4) $V_A = V_B + V_D$

For Problems 19–20

In the circuit shown in figure, initially the switch is opened. The switch is closed now.

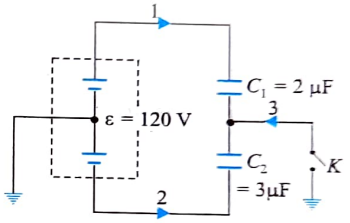


19. The charge that will flow in direction 1 is
(1) $-\frac{C_2^2\varepsilon}{C_1 + C_2}$ (2) $-\left(\frac{C_1C_2}{C_1 + C_2}\right)\varepsilon$
(3) $\frac{C_1^2\varepsilon}{C_1 + C_2}$ (4) $C_2\varepsilon$

20. The charge that will flow in direction 2 is
(1) $-\frac{C_2^2\varepsilon}{C_1 + C_2}$ (2) $\left(\frac{C_1C_2}{C_1 + C_2}\right)\varepsilon$
(3) $\frac{C_1^2\varepsilon}{C_1 + C_2}$ (4) $C_2\varepsilon$

For Problems 21–23

Capacitors $C_1 = 2$ μF and $C_2 = 3$ μF are connected in series to a battery of emf $\varepsilon = 120$ V whose midpoint is earthed. The wire connecting the capacitors can be earthed through a key K. Now, key K is closed. Determine the charge flowing through the sections 1, 2, and 3 in the directions indicated in figure.



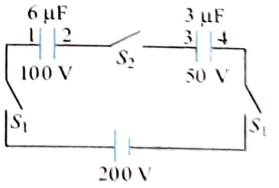
21. In section 1,
(1) −24 μC (2) −36 μC
(3) −60 μC (4) 60 μC

22. In section 2,
(1) −24 μC (2) −36 μC
(3) −60 μC (4) 60 μC

23. In section 3,
(1) −24 μC (2) −36 μC
(3) −60 μC (4) 60 μC

For Problems 24–26

Two capacitors of capacity 6 μF and 3 μF are charged to 100 V and 50 V separately and connected as shown in figure. Now all the three switches S_1, S_2, and S_3 are closed.



24. Which plate(s) form an isolated system?
(1) Plate 1 and plate 4 separately
(2) Plate 2 and plate 3 separately
(3) Plate 1 and plate 4 jointly
(4) Plate 2 and plate 3 jointly

25. Charges on 6 μF and 3 μF capacitors in steady state will be
(1) 400 μC, 400 μC (2) 700 μC, 250 μC
(3) 800 μC, 350 μC (4) 300 μC, 450 μC

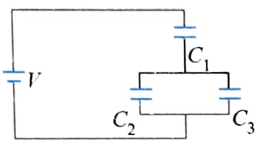
26. Let q_1, q_2, and q_3 be the magnitudes of charges flown from switches S_1, S_2, and S_3 after they are closed. Then

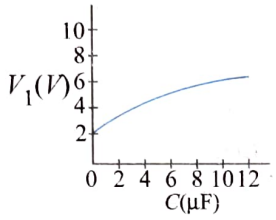
(1) $q_1 = q_3$ and $q_2 = 0$ (2) $q_1 = q_3 = q_2/2$

(3) $q_1 = q_3 = 3q_2$ (4) $q_1 = q_2 = q_3$

For Problems 27–29

Capacitor C_3 in the circuit is a variable capacitor (its capacitance can be varied). C_1 and C_2 are of fixed values. Graph is plotted between potential difference V_1 (across capacitor C_1) versus C_3. Electric potential V_1 approaches on asymptote of 10 V as $C_3 \to \infty$.





27. The electric potential V across the battery is equal to

(1) 10 V (2) 12 V

(3) 16 V (4) 20 V

28. Relation between C_1 and C_2 is

(1) $C_1 = C_2$ (2) $C_1 = 4C_2$

(3) $4C_1 = C_2$ (4) any relation

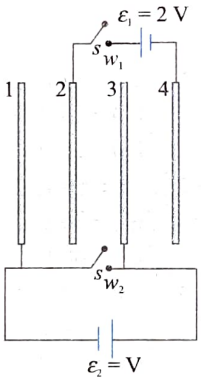
29. When $V_1 = 4$ V, then C_3 is equal to

(1) $5C_2/2$ (2) $5C_1/3$

(3) $5C_2/3$ (4) none of these

For Problems 30–32

Figure shows four plates each of plate area A and separated between plates is d. Two switches S_{w_1} and S_{w_2} can activate two batteries in the circuit.



30. Switch S_{w_1} and S_{w_2} are closed. What is the charge on plate 2?

(1) $\frac{3}{2}\frac{\varepsilon_0 AV}{D}$ (2) $\frac{3}{2}\frac{\varepsilon_0 AV}{d}$

(3) $\frac{1}{3}\frac{\varepsilon_0 AV}{d}$ (4) $\frac{4}{3}\frac{\varepsilon_0 AV}{d}$

31. What is the charge passing through battery 1?

(1) $\frac{3}{2}\frac{\varepsilon_0 AV}{D}$ (2) $\frac{3}{2}\frac{\varepsilon_0 AV}{d}$

(3) $\frac{1}{3}\frac{\varepsilon_0 AV}{d}$ (4) $\frac{4}{3}\frac{\varepsilon_0 AV}{d}$

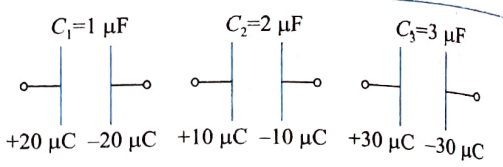
32. Now switch S_{w_2} is opened, what is the charge on plate 4?

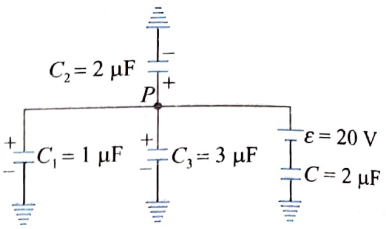
(1) $-\frac{5}{3}\frac{\varepsilon_0 AV}{d}$ (2) $+\frac{5}{3}\frac{\varepsilon_0 AV}{d}$

(3) $-\frac{1}{3}\frac{\varepsilon_0 AV}{d}$ (4) $+\frac{1}{3}\frac{\varepsilon_0 AV}{d}$

For Problems 33–34

Three capacitors C_1, C_2, and C_3 of capacitance 1 μF, 2 μF, and 3 μF, respectively, are charged separately as shown in the figure. Now these charged capacitors are connected to a battery of $\varepsilon = 20$ V and an uncharged capacitor of $C = 2$ μF as shown in figure.





33. The charge on capacitor marked C is

(1) 3.75 μC (2) 7.5 μC

(3) 15 μC (4) none of these

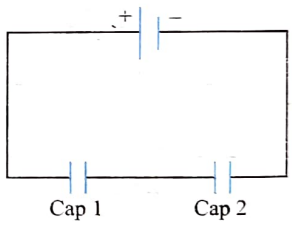
34. The potential of point P is

(1) 12.5 V (2) 25 V

(3) 6.25 V (4) none of these

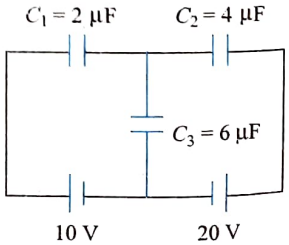
Matrix Match Type

1. Two identical capacitors are connected in series, and the combination is connected with a battery, as shown. Some changes in capacitor 1 are now made independently after the steady state is achieved as listed in column I. Some effects that may occur in the new steady state due to these changes on capacitor 2 are listed in column II. Match the changes on capacitor 1 in column I with the corresponding effect on capacitor 2 in column II.



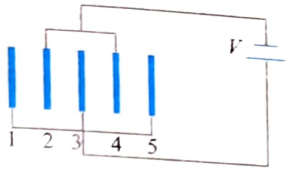
Column I	Column II
i. A dielectric slab is inserted	a. Charge on the capacitor increases
ii. Separation between plates is increased	b. Charge on the capacitor decreases
iii. A metal plate is inserted connecting both plates	c. Energy stored in the capacitor increases
iv. The left plate is grounded	d. No change occurs

2. Observe the circuit in figure and match the following (assume q_1, q_2, and q_3 be the charges on three capacitors).



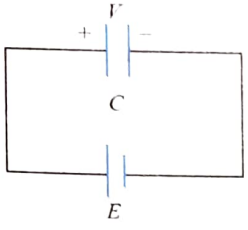
Column I	Column II
i. q_1 (in μC)	a. 50
ii. q_2 (in μC)	b. $\frac{10}{3}$
iii. q_3 (in μC)	c. $\frac{140}{3}$
iv. Potential difference across the 6 μF capacitor is (in volt)	d. $\frac{25}{3}$

3. Five identical capacitor plates, each of area A, are arranged such that the adjacent plates are at a distance d apart. The plates are connected to a source of emf V as shown in figure. Match the following.



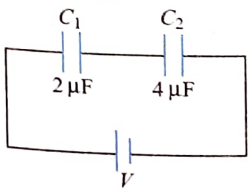
Column I	Column II
i. Charge on plate 1	a. $-2\varepsilon_0 AV/d$
ii. Charge on plate 4	b. $+\varepsilon_0 AV/d$
iii. Potential difference between plates 2 and 3	c. zero
iv. Potential difference between plates 1 and 5	d. V

4. A capacitor of capacitance C is charged to a potential V. Now it is connected to a battery of emf E as shown in figure. Based on this information, match the entries of Column I with the entries of Column II in the following table.



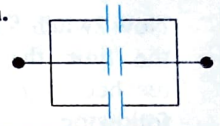
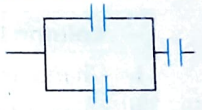
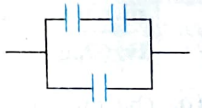
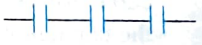
Column I	Column II
i. If $V = E$, then	a. charge flows in the circuit
ii. If $V > E$, then	b. no charge flows in the circuit
iii. If $V < E$, then	c. nonzero thermal energy will be dissipated in the circuit
iv. If the plates of capacitor are shorted, then	d. outer surfaces of the plates of capacitor have zero charge

5. In figure, the separation between the plates of C_1 is slowly increased to double of its initial value. Now match the entries in columns I and II.

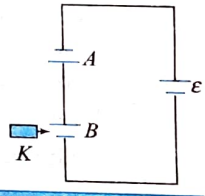


Column I	Column II
i. The potential difference across C_1	a. increases
ii. The potential difference across C_2	b. decreases
iii. The energy stored in C_1	c. increases by a factor of 6/5
iv. The energy stored in C_2	d. decreases by a factor of 18/25

6. Match the column.

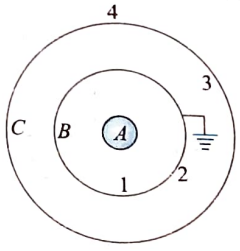
Column I	Column-II
i. If breakdown voltage of each capacitor is same, which combination can withstand largest external voltage.	a. 
ii. If same charge is supplied, which combination acquires largest potential difference.	b. 
iii. If same potential difference is applied, which combination can store maximum charge.	c. 
iv. If capacitors are identical, which combination gives maximum equivalent capacitance.	d. 

7. Two identical capacitors A and B are connected to a battery of emf ε as shown in figure. Now a dielectric slab is inserted between the plates of capacitor B while battery remains connected. Due to this insertion, some physical quantities may change, which are mentioned in Column I and the effect is mentioned in Column II. Match the Column I with Column II.



Column I	Column II
i. Charge on A	a. increases
ii. Charge on B	b. decreases
iii. Potential difference across A	c. remains constant
iv. Potential difference across B	d. will change

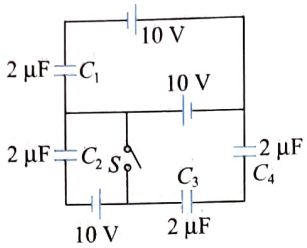
8. Figure shows three concentric thin spherical shells A, B, and C of radii R, $2R$, and $3R$. The shell B is earthed and A and C are given charges q and $2q$, respectively. If the magnitude of charge appearing on surfaces 1, 2, 3, and 4 are q_1, q_2, q_3, and q_4, respectively, then match the following columns.



Column I	Column II
i. q_1	a. q
ii. q_2	b. $\frac{4q}{3}$
iii. q_3	c. $\frac{2q}{3}$
iv. q_4	

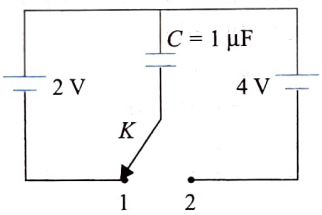
9. Before connecting the circuit shown in figure, all capacitors were uncharged. The circuit now is in steady state.

Now switch S is closed. During the time the steady state is reached again, match the following:



Column I	Column II
i. Charge on C_1	a. increases
ii. Charge on C_2	b. decreases
iii. Charge on C_3	c. remains same
iv. Charge on C_4	d. becomes zero

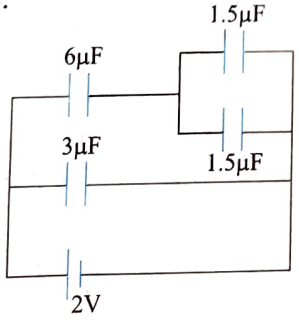
10. The circuit involves two ideal cells connected to a 1 μF capacitor via a key K. Initially the key K is in position 1 and the capacitor is charged fully by 2 V cell. The key is pushed to position 2. Column I gives physical quantities involving the circuit after the key K is pushed from position 1 to 2. Column II given corresponding results. Match the statements in Column I with the corresponding values in Column II.



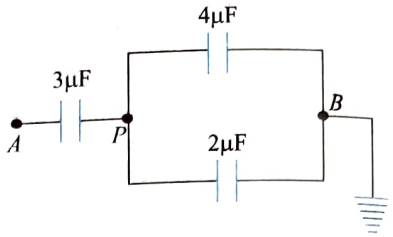
Column I	Column II
i. The net charge crossing the 4 V cell (in μC) is	a. 2
ii. The magnitude of work done by 4 V cell (in μJ) is	b. 6
iii. The gain in potential energy of capacitor (in μJ) is	c. 8
iv. The net heat produced in circuit (in μJ) is	d. 16

Numerical Value Type

1. Each capacitance shown in figure is in μF. Find the charge on 6 μF in μC.



2. In figure, a potential of +12 V is given to point A, and point B is earthed. What is the potential at the point P in V?



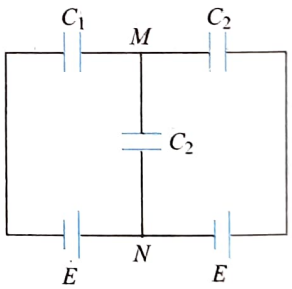
3. The capacitance of a capacitor becomes 4/3 times its original value if a dielectric slab of thickness $t = d/2$ is inserted between the plates (d is the separation between the plates). What is the dielectric constant of the slab?

4. A spherical drop of capacitance 12 μF is broken into eight drops of equal radius. What is the capacitance of each small drop in μF?

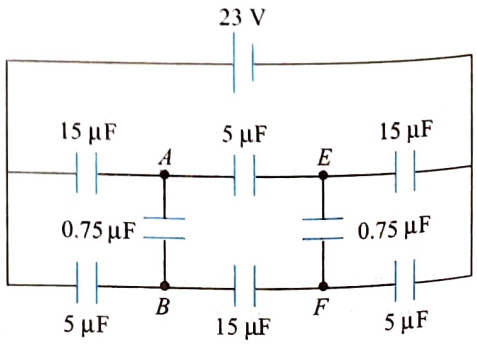
5. A parallel plate capacitor of capacity C_0 is charged to a potential V_0. E_1 is the energy stored in the capacitor when the battery is disconnected and the plate separation is doubled, and E_2 is the energy stored in the capacitor when the charging battery is kept connected and the separation between the capacitor plates is doubled. Find the ratio E_1/E_2.

6. A capacitor of capacitance C_1 = 1 μF can withstand a maximum voltage of V_1 = 6 kV, and another capacitor of capacitance C_2 = 2 μF can withstand a maximum voltage of V_2 = 4 kV. If they are connected in series, what maximum voltage will the system withstand? (in kV)

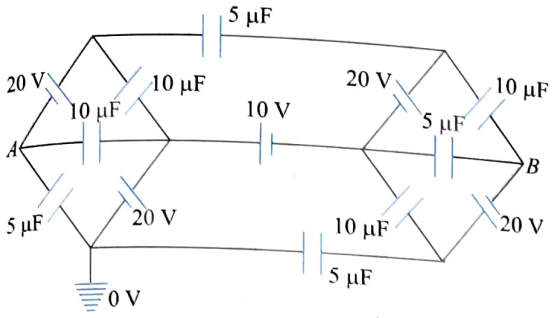
7. Find the potential difference between the points M and N (in V) of the system shown in figure if the emf is equal to E = 110 V and the capacitance ratio C_2/C_1 is 23.



8. Find the ratio of the magnitudes of the potential difference between the points A and B and that between E and F of the circuit shown in figure.



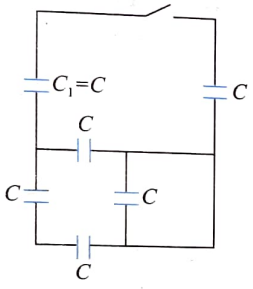
9. Find the potential difference between the points A and B (in V) in the circuit shown in figure.



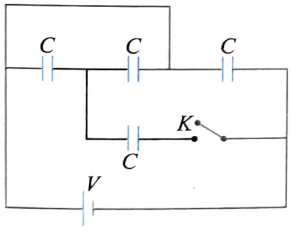
10. A charge of 20 μC is placed on the positive plate of an isolated parallel plate capacitor of capacitance 10 μF. Calculate the potential difference (in V) developed between the plates.

11. A capacitor with stored energy 4.0 J is connected with an identical capacitor with no electric field in between. Find the total energy stored (in J) in the two capacitors.

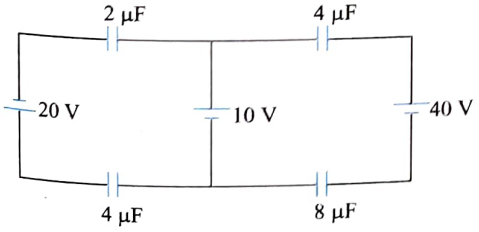
12. In the circuit shown, all capacitors are identical. Initially, the switch is open and the capacitor marked C_1 is the only one charged to a value Q_0. After the switch is closed and the equilibrium is reestablished, the charge on the capacitor marked C_1 is Q. Find the ratio of initial change to final charge in capacitor C_1.



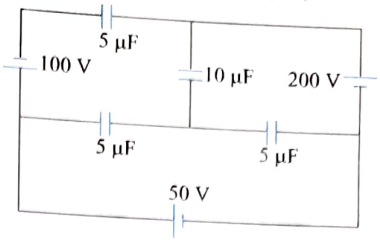
13. Find the amount by which the total energy stored in the capacitor (in μJ) will increase in the circuit shown in the figure after switch K is closed. Consider all capacitors are of capacitance 3 μF and battery voltage is 10V.



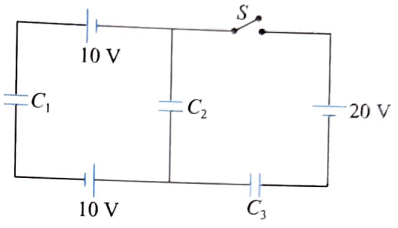
14. In the circuit shown in figure. Calculate the charge on 3 μF capacitor in steady state (in μC) .



15. In the circuit shown in figure. Calculate the charge stored in 10 μF capacitor in steady state (in μC).

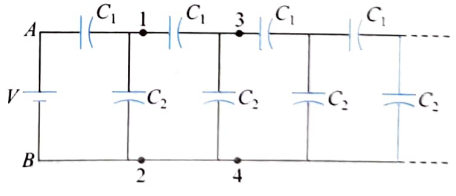


16. Find the amount of heat produced (in mJ) in the circuit shown in figure when switch is closed (μJ). Each capacitor in circuit is of capacitance 2 μF.

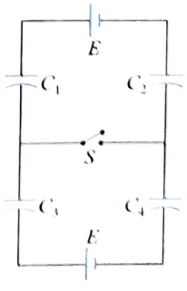


17. A parallel plate capacitor is to be constructed which can store $q = 10$ μC charge at $V = 1000$ volt. The minimum plate area of the capacitor is required to be A_1 when space between the plates has air. If a dielectric of constant $K = 3$ is used between the plates, the minimum plate area required to make such a capacitor is A_2. The breakdown field for the dielectric is 8 times that of air. Find A_1/A_2.

18. In the given figure the capacitor circuit continues to infinity. The battery connected has e.m.f equal to V. The potential difference between points 1 and 2 is $V/2$, that between points 3 and 4 is $V/4$ and so on; i.e., the potential difference becomes 1/2 after every step of the ladder. Find the ratio C_1/C_2.



19. In the given circuit diagram, $E = 12$ V, $C_1 = 4$ μF, $C_2 = 2$ μF, $C_3 = 6$ μF and $C_4 = 3$ μF. Find the heat produced in the circuit (in μJ) after switch S is shorted.



JEE Main

Single Correct Answer Type

1. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then

(1) $3C_1 = 5C_2$ (2) $3C_1 + 5C_2 = 0$

(3) $9C_1 = 4C_2$ (4) $5C_1 = 3C_2$

(JEE Main 2013)

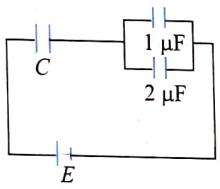
2. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to

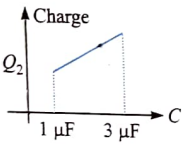
(1) 3×10^4 C/m^2 (2) 6×10^4 C/m^2

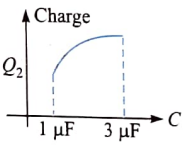
(3) 6×10^{-7} C/m^2 (4) 3×10^{-7} C/m^2

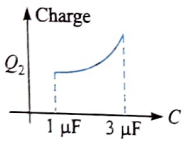
(JEE Main 2014)

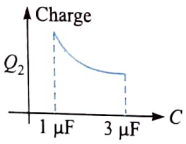
3. In the given circuit, charge Q_2 on the 2 μF capacitor changes as C is varied from 1 μF to 3 μF. Q_2 as a function of 'C' is given properly by: (figures are drawn schematically and are not to scale)



(1) 

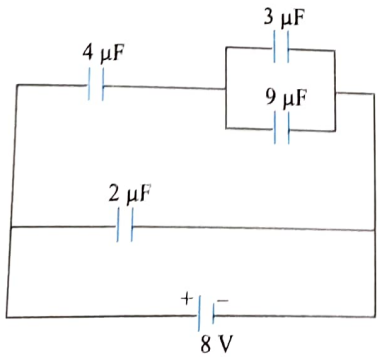
(2) 

(3) 

(4) 

(JEE Main 2015)

4. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point distant 30 m from it, would equal



(1) 240 N/C (2) 360 N/C

(3) 420 N/C (4) 480 N/C

(JEE Main 2016)

5. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is

(1) 24 (2) 32

(3) 2 (4) 16

(JEE Main 2017)

6. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant K = 5/3 is inserted between the plates, the magnitude of the induced charge will be

(1) 0.9 nC (2) 1.2 nC

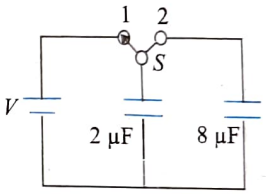
(3) 0.3 nC (4) 2.4 nC

(JEE Main 2018)

JEE Advanced

Single Correct Answer Type

1. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after switch S is turned to position 2 is

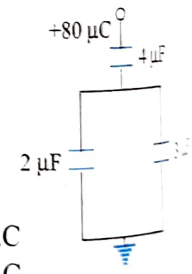


(1) 0% (2) 20%

(3) 75% (4) 80%

(IIT-JEE 2011)

2. In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is

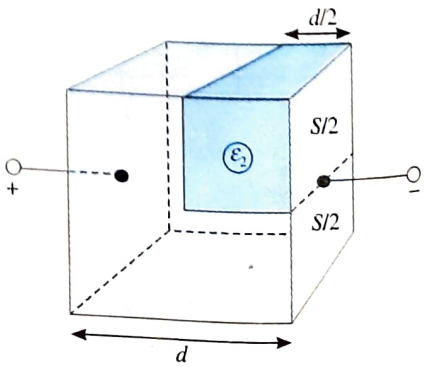


(1) +32 μC (2) +40 μC

(3) +48 μC (4) +80 μC

(IIT-JEE 2012)

3. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is

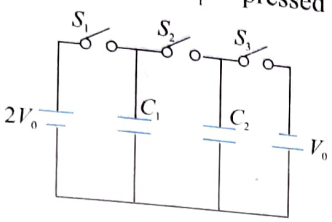


(1) 6/5 (2) 5/3
(3) 7/5 (4) 7/3

(JEE Advanced 2015)

Multiple Correct Answers Type

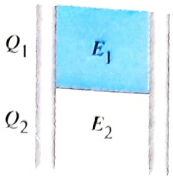
1. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S is pressed. After some time,



(1) the charge on the upper plate of C_1 is $2CV_0$
(2) the charge on the upper plate of C_1 is CV_0
(3) the charge on the upper plate of C_1 is 0
(4) the charge on the upper plate of C_2 is $-CV_0$

(IIT-JEE 2013)

2. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/ options, ignoring edge effects.



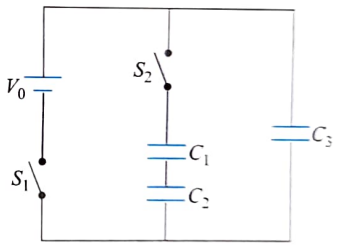
(1) $\frac{E_1}{E_2}=1$ (2) $\frac{E_1}{E_2}=\frac{1}{K}$

(3) $\frac{Q_1}{Q_2}=\frac{3}{K}$ (4) $\frac{C}{C_1}=\frac{2+K}{K}$

(JEE Advanced 2014)

Numerical Value Type

1. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity $\in_r$. The cell electromotive force (emf) $V_0 = 8V$. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5 μC. The value of $\in_r$.



(JEE Advanced 2018)

Answers Key

EXERCISES

Single Correct Answer Type

1. (4)	2. (1)	3. (3)	4. (2)	5. (4)
6. (2)	7. (4)	8. (4)	9. (3)	10. (2)
11. (1)	12. (3)	13. (3)	14. (1)	15. (1)
16. (4)	17. (3)	18. (2)	19. (1)	20. (3)
21. (3)	22. (1)	23. (2)	24. (4)	25. (2)
26. (3)	27. (2)	28. (2)	29. (1)	30. (3)
31. (3)	32. (3)	33. (1)	34. (2)	35. (1)
36. (4)	37. (3)	38. (2)	39. (3)	40. (1)
41. (2)	42. (4)	43. (4)	44. (2)	45. (3)
46. (3)	47. (1)	48. (2)	49. (4)	50. (1)
51. (1)	52. (4)	53. (2)	54. (1)	55. (4)
56. (3)	57. (4)	58. (2)	59. (1)	60. (1)
61. (2)	62. (4)	63. (2)	64. (1)	65. (1)
66. (2)	67. (4)	68. (3)	69. (3)	70. (1)
71. (2)	72. (1)	73. (1)	74. (2)	75. (3)

Multiple Correct Answers Type

1. (2),(3),(4)	2. (2),(3)(4)	3. (1),(2),(4)
4. (1),(3),(4)	5. (1),(2),(3)	6. (1),(3)
7. (2),(3)	8. (1),(3),(4)	9. (1),(3),(4)
10. (1),(2),(4)	11. (1),(2)	12. (1),(4)
13. (1),(4)	14. (1),(3)	15. (1),(2),(3)
16. (2),(3)	17. (1),(4)	18. (2),(4)
19. (1),(3),(4)	20. (2),(4)	21. (1),(2),(3)
22. (1),(3)	23. (1),(2)	24. (1),(2)
25. (2),(3),(4)		

Linked Comprehension Type

1. (2)	2. (4)	3. (3)	4. (2)	5. (1)
6. (2)	7. (1)	8. (4)	9. (4)	10. (1)
11. (2)	12. (3)	13. (1)	14. (4)	15. (2)
16. (3)	17. (2)	18. (1)	19. (2)	20. (4)
21. (1)	22. (2)	23. (4)	24. (4)	25. (2)
26. (4)	27. (2)	28. (3)	29. (4)	30. (4)
31. (4)	32. (1)	33. (3)	34. (1)	

Matrix Match Type

1. i. → a., c.; ii. → b.; iii., → a., c.; iv. → d.
2. i. → b.; ii. → c.; iii. → a.; iv. → d.
3. i. → b.; ii. → a.; iii. → d.; iv. → c.
4. i. → b., d.; ii. → a., c., d.; iii. → a., c., d.; iv. → a., c., d.
5. i. → a., c.; ii. → b.; iii. → b., d.; iv. → b.
6. i. → d.; ii. → d.; iii. → a.; iv. → a.;
7. i. → a., d.; ii. → b., d.; iii. → b., d.; iv → b., d.
8. i. → a.; ii. → b.; iii. → b.; iv. → c.
9. i. → c.; ii. → a.; iii. → b.; iv. → b.
10. i. → a.; ii. → c.; iii. → b.; iv. → a.

Numerical Value Type

1. (4)	**2.** (4)	**3.** (2)	**4.** (6)	**5.** (4)
6. (9)	**7.** (22)	**8.** (5)	**9.** (10)	**10.** (1)
11. (2)	**12.** (1.60)	**13.** (100)	**14.** (40)	**15.** (700)
16. (0.30)	**17.** (24)	**18.** (2)	**19.** (240)	

ARCHIVES

JEE Main

Single Correct Answer Type

1. (1) **2.** (3) **3.** (2) **4.** (3) **5.** (2)
6. (2)

JEE Advanced

Single Correct Answer Type

1. (4) **2.** (3) **3.** (4)

Multiple Correct Answers Type

1. (2),(4) **2.** (1),(4)

Numerical Value Type

1. (1.50)

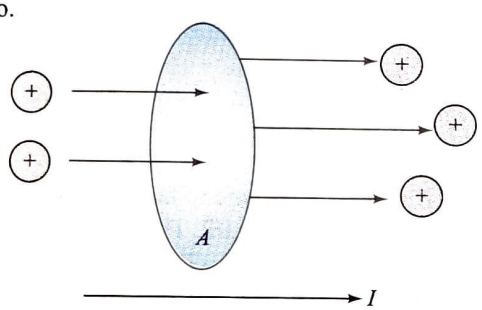
5 Electric Current and Circuits

INTRODUCTION

In electrostatics, we discussed the electric phenomena of charges at rest. In the previous chapter, we treated the concept of electric potential, which is measured in volt. Here we will find that voltage acts like an "electrical pressure" that can produce a flow of charge or current, which is measured in ampere (or simply, amp and abbreviated as A) and that the resistance that restrains this flow is measured in ohm (Ω).

ELECTRIC CURRENT

An isolated metallic conductor, say a wire, contains a few electrons moving at random with high speeds. These are called conduction electrons. The rate at which these electrons pass from left to right through an area in a wire is the same as the rate at which they pass from right to left through the same area, i.e., net rate is zero.



Charges in motion through an area A. The time rate at which a charge flows through the area is defined as the current I. The direction of the current is the direction in which positive charges flow when free to do so.

To define current mathematically, suppose that the charged particles are moving perpendicular to a surface of area A as shown in figure (this area could be the cross-sectional area of a wire, for example). The current is defined as the rate at which electric charge flows through this surface. If ΔQ is the amount of charge that passes through this area in time interval Δt, then the average current, I_{avg}, over this time interval through this area is the ratio of the charge to the time interval, i.e.,

$$I_{avg} = \frac{\Delta Q}{\Delta t} \quad \text{...(i)}$$

It is possible for the rate at which the charge flows to vary with time. We define the instantaneous current I as the limit of the preceding expression as Δt tends to zero, i.e.,

$$I \equiv \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

We can also write

$$dQ = Idt \text{ or } Q = \int Idt \quad \text{...(ii)}$$

From Eq. (ii), total charge flowing in a time interval can be found.

Important Points:

- The particles flowing through a surface can be charged positively or negatively, or we can have two or more types of particles moving, with charges of both the signs in the flow. Conventionally, we define the direction of the current as the direction of flow of positive charges.
- In a common conductor such as copper, the current is in physical state due to the motion of the negatively charged electrons. Therefore, when we speak of current in such a conductor, the direction of the current is opposite to the direction of flow of electrons.
- On the other hand, if one considers a beam of positively charged protons in a particle accelerator, then the current is in the direction of the motion of the protons.
- In some cases, gases and electrolytes, for example, the current is the result of the flow of both positive and negative charges.
- It is common to refer to a moving charged particle (whether it is positive or negative) as a mobile charge carrier. For example, the charge carriers in a metal are electrons.

When a wire is connected to a battery, an electric field is set up at every point within the wire. This field exerts a force on each conduction electron. Although the electrons are continuously accelerated by the field, due to their frequent collisions with the atoms of the wire, they simply drift at a small constant speed in the direction opposite to the field. Thus, there is a net flow of charge in the wire at a small rate. The total charge passing through any cross section per second is the electric current in the wire.

In the steady state, current through each section of the conducting loop would be same, no matter what is the location or orientation of the area of that section.

Unit of Electric Current

In SI system, the unit of current is ampere (A).

"The current is said to be 1 A when 1 C of charge flows past any cross section of a conductor every second."

STATEMENT OF OHM'S LAW

"The electric current in any conductor is proportional to the potential difference between its ends, other factors remaining constant."

The ratio of the potential difference to current is termed as the resistance of the conductor. Accordingly,

$$R = \frac{V}{I} \text{ or } V = IR$$

where I is the current, V is the potential difference, and R is the resistance. The resistance of the conductor is the opposition offered by the conductor to the flow of electric current passing through it. The resistance R not only depends on the material of the conductor but also on the dimensions of the conductor.

The resistance of an ohmic conducting wire is found to be proportional to its length l and inversely proportional to its cross-sectional area A, i.e.,

$$R = \rho \frac{l}{A} \quad \text{...(iii)}$$

where the constant of proportionality ρ is called the resistivity of the material, which has the unit ohm meter (Ωm). To understand the relationship between resistance and resistivity, we should know that ρ depends on the properties of the material and on temperature. On the other hand, the resistance R of a particular conductor depends on its size and shape as well as on the resistivity of the material.

The inverse of resistivity is defined as conductivity σ. Hence, the resistance of an ohmic conductor can be expressed in terms of its conductivity as

$$R = \frac{l}{\sigma A}, \quad \text{where } \sigma = \frac{1}{\rho}$$

Resistance and resistivity: Resistivity is a property of a substance, whereas resistance is a property of an object. We have seen similar pairs of variables before. For example, density is a property of a substance, whereas mass is a property of an object. Equation (iii) relates resistance to resistivity.

SI Unit of Resistance

The unit of resistance is ohm (Ω).

"The resistance of a conductor is said to be 1 Ω if a current of 1 A flows through it when the potential difference. across its ends is 1 V."

DEPENDENCE OF RESISTANCE ON VARIOUS FACTORS

We have

$$R = \rho \frac{l}{A}$$

Therefore, the relation of resistance with length and area of cross section will be as follows:

$R \propto l$ and $R \propto 1/A$, which gives $R \propto l/A$.

CHANGE IN RESISTANCE

On stretching a wire keeping volume (V) constant

If the length of wire is changed,

$$R \propto \frac{l}{A}\frac{l}{l} = \frac{l^2}{V}$$

or $R \propto l^2$

Hence,

$$\frac{R_1}{R_2} = \frac{l_1^2}{l_2^2}$$

If the radius of cross section is changed

$$R \propto \frac{l}{A}\cdot\frac{A}{A} = \frac{V}{A^2} \text{ or } R \propto \frac{1}{A^2}$$

If the radius of the wire is r, then $A \propto r^2$ and $R \propto 1/r^4$.

Hence,

$$\frac{R_1}{R_2} = \frac{r_2^4}{r_1^4}$$

where R_1 and R_2 are initial and final resistances, l_1 and l_2, are initial and final lengths, and r_1 and r_2 are initial and final radii, respectively.

Effect of percentage change in length of wire

$$\frac{R_2}{R_1} = \frac{l^2\left[1+\frac{x}{100}\right]^2}{l^2}$$

where l is the original length and x is the percentage increment in length. If x is quite small (say < 5%), then percentage change in resistance is

$$\frac{\Delta R}{R}\% = \frac{R_2 - R_1}{R_1} \times 100 = \frac{\left(1+\frac{x}{100}\right)^2 - 1}{1} \cong 2x\%$$

ILLUSTRATION 5.1

If a wire is stretched to double its length, find the new resistance if the original resistance of the wire was R.

Sol. We know that

$$R = \frac{\rho l}{A}$$

Let new resistance be

$$R' = \frac{\rho l'}{A'}$$

By conservation principle, volume of the wire remains constant. So $A'l' = Al$; if $l' = 2l$, then $A' = A/2$. Thus

$$R' = \frac{\rho \times 2l}{A/2} = 4 \quad \text{or} \quad \frac{\rho l}{A} = 4R$$

ILLUSTRATION 5.2

The wire is stretched to increase the length by 1%. Find the percentage change in the resistance.

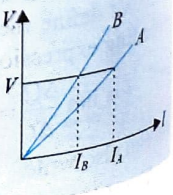
Sol. As we known that, for constant volume $R \propto l^2$, so

$$\frac{\Delta R}{R}\% = \frac{2\Delta l}{l}\% = 2 \times 1\% = 2\%$$

As percentage change is positive, hence resistance will increase by 2%.

ILLUSTRATION 5.3

The voltage-current graphs for two resistors of the same material and the same radius with lengths, L_1 and L_2 are shown in figure. If $L_1 > L_2$, state with reason which of these graphs represents voltage-current change for L_1.



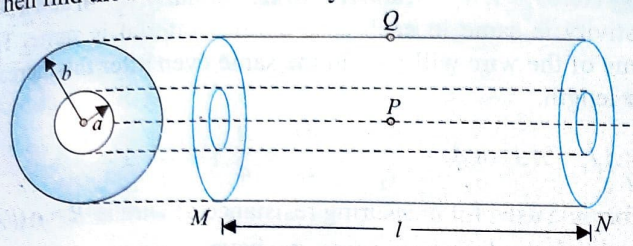
Sol. As $R_A = V/I_A$ and $R_B = V/I_B$,

$$\frac{I_A}{I_B} = \frac{R_B}{R_A}$$

From figure, as $I_A > I_B$, $R_B > R_A$. Since, $R \propto L$, graph B represents the voltage-current change for L_1.

ILLUSTRATION 5.4

A hollow cylinder of length l, has inner and outer radius a and b respectively. If the resistivity of material of the cylinder is ρ. Then find the resistance of the cylinder.



(i) across its ends (across M and N)

(ii) across its inner and outer surface (across P and Q)

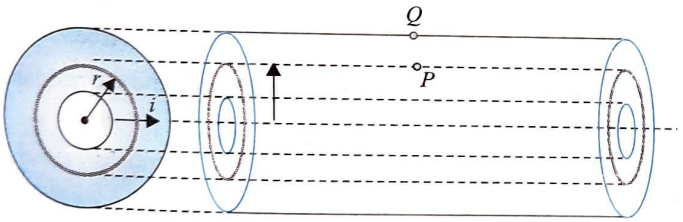
Sol.

(i) Resistance across the ends of the cylinder (across M and N)

Resistance $R = \dfrac{\rho l}{A}$; here $A = \pi(b^2 - a^2)$

Hence $R_{MN} = \dfrac{\rho l}{\pi(b^2 - a^2)}$

(ii) If we calculate the resistance across P and Q, we need to apply potential difference across inner and outer surface. It means the current will flow in radial direction. Here we can consider the cylinder is made of a number of elemental cylinders placed coaxially. Let us consider an elemental cylinder of radius r and thickness dr.



The resistance of this elemental cylinder,

$$dR = \frac{\rho \cdot dr}{A} \quad \text{...(i)}$$

Here $A = 2\pi rl$ ...(ii)

From (i) and (ii) $dR = \dfrac{\rho\, dr}{2\pi rl}$

Hence $R = \int dR = \int_a^b \dfrac{\rho dr}{2\pi rl}$

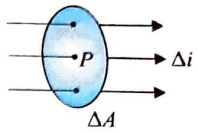
or $R = \dfrac{\rho}{2\pi l}\int_a^b \dfrac{dr}{r} = \dfrac{\rho}{2\pi l}\ln\dfrac{b}{a}$

CURRENT DENSITY

Current density at a point is defined as the amount of current flowing per unit area around that point, provided the area is normal to the direction of current. It is denoted by J. So we can write $J = I/A$, where I is current and A is area. The unit of current density is Am^{-2}.

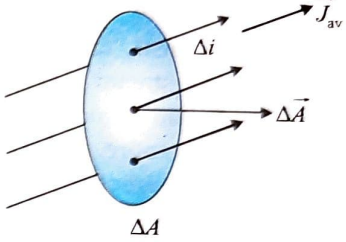
Current density is a vector quantity. So it must have direction also. Its direction at a point is the direction of motion of the positive charge or direction of current at that point.

Average current density: Let through an area ΔA around a point P, a current Δi is passing normal to the area. Then the average current density over the area is given by $J_{av} = \Delta i/\Delta A$.



Current density at a point: Now suppose we want to find current density at point P in the preceding figure. Take an infinitesimally small area dA around point P. Let the current through this small area be di. Then the current density at point P is given by $J = di/dA$.

Current not perpendicular to area: Here we divide the current by the component of area in the direction of the current. Hence, the average current density is given by



$$J_{av} = \frac{\Delta i}{\Delta A \cos\theta}$$

We can also write

$$\Delta i = J_{av}\Delta A\cos\theta = \vec{J}_{av}\cdot\Delta\vec{A}$$

Current density at *P*: Take an infinitesimally small area dA around point P. Let the current through this small area be di. Then the current density at point P is given by $J = di/(dA\cos\theta)$.
We can also write,

$$di = JdA\cos\theta = \vec{J}\cdot\vec{dA} \text{ or } i = \int\vec{J}\cdot\vec{dA}$$

Note: While electric current is a scalar quantity, electric current density is a vector quantity.

In case of conductors,

$$J = \frac{I}{A} = \frac{V}{RA} = \frac{EL}{(\rho L/A)A} = \frac{E}{\rho} = \sigma E$$

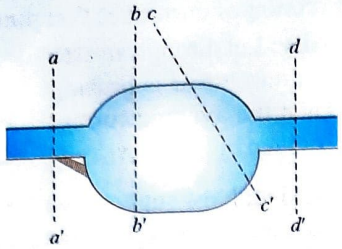
or $\vec{J} = \sigma\vec{E}$

- Current density is proportional to electric field, and direction of current density is the same as that of electric field.
- If electric field is uniform (i.e., constant), current density will be constant.
- If electric field is zero (as in electrostatics inside a conductor), current density and hence current will be zero.

Important Points:

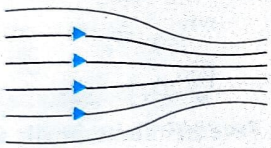
- If the current has not reached a steady state, i.e., the flow of charge is not constant, then the current through different cross sections at a particular instant may have different values.
- Electric current may be distributed nonuniformly over the surface through which it passes. Hence, to characterize the current in greater detail, current density vector $\vec{J}$ is introduced.

- Current density, $\vec{j}$, tells us how charge flows at a certain point and its direction tells us about the direction of the flow of charge at that point. Current describes how charge flows through an extended object.



Current through all sections would be same in steady state

- The direction of current density is the same as that of the velocity of positive charge or opposite to the direction of the velocity of negative charge.
- Current density can be represented by a similar set of lines known as stream lines. The spacing of the stream lines suggests the value of current density. Narrower stream lines mean more current density, and spaced stream lines mean less current density (figure).



A conductor having nonuniform cross section

ILLUSTRATION 5.5

In a hydrogen discharge tube, the number of protons drifting across a cross section per second is 1.0×10^{18}, while the number of electrons drifting in the opposite direction across the same cross section is 2.7×10^{18} per second. Find the current flowing in the tube.

Sol. As electrons and protons are moving in the opposite directions, they will effectively produce current in the same direction and the total current in the tube is

$$I = (n_p + n_e)e/t$$
$$= (1.0 \times 10^{18} + 2.7 \times 10^{18}) \times 1.6 \times 10^{-19}/1$$
$$= 3.7 \times 1.6 \times 10^{-1}\ \text{A} = 0.592\ \text{A}$$

ILLUSTRATION 5.6

You need to produce a set of cylindrical copper wires 2.5 m long that will have a resistance of 0.125 Ω each. What will be the mass of each of these wires? (Density of copper is $8.9 \times 10^3\ \text{kgm}^{-3}$ and resistivity of copper is $1.72 \times 10^{-8}\ \Omega\text{m}$).

Sol. Given that $L = 2.5$ m and $R = 0.125\ \Omega$. To find the mass of each wire, we need to calculate the volume of one of the wires.

Mass = Density × Volume

Volume = Area × Length $\quad \left[\because R = \frac{\rho L}{A} \text{ or } A = \frac{\rho L}{R}\right]$

$$\text{Volume} = AL = \frac{\rho L^2}{R} = \frac{1.72 \times 10^{-8} \times (2.5)^2}{0.125} = 8.6 \times 10^{-7}\ \text{m}^3$$

$$m = d \times V = 8.9 \times 10^3 \times 8.6 \times 10^{-7}$$
$$= 7.654 \times 10^{-3}\ \text{kg} = 7.654\ \text{g}$$

ILLUSTRATION 5.7

Consider a wire of length l, area of cross section A, and resistivity ρ with resistance 10 Ω. Its length is increased by applying a force, and it becomes four times of its original value. Find the changed resistance of the wire.

Sol. Here $l_1 = l$, $A_1 = A$, and $R = 10\ \Omega$. Similarly, $l_2 = 4l$ and $R_2 = ?$ Resistivity is same in each case as the material is same. The volume of the wire will remain the same even after the increase in the length.

$$A_1 l_1 = A_2 l_2 \text{ or } A_2 = \frac{A_1 l_1}{l_2} = \frac{Al}{4l} = \frac{A}{4}$$

The formula used for measuring resistance of wire is $R = \rho(l/A)$. Using this formula in both cases, we have

$$R_1 = \rho \frac{l_1}{A_1} = \frac{\rho l}{A} \quad \text{...(i)}$$

and $$R_2 = \rho \frac{l_2}{A_2} = \rho \frac{4l}{A/4} = 16\,\rho \frac{l}{A} \quad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i),

$$\frac{R_2}{R_1} = 16 \text{ or } R_2 = 160\ \Omega$$

ILLUSTRATION 5.8

A wire of mass m, length l, density d, and area of cross section A is stretched in such a way that its length increases by 10% of its original value. Express the changed resistance in percentage.

Sol. Given mass m, length $l_1 = l$, density d, and area of cross section $A_1 = A$. Let ρ be the resistivity and R_1 be the resistance of the wire. Mass of wire m = volume × density $= Al \times d = Ald$.

Therefore, area of cross section is $A_1 = m/ld$, and the resistance of the wire is

$$R_1 = \rho \frac{l}{A_1} = \rho \frac{l^2 d}{m} = \left(\frac{\rho d}{m}\right) l^2 = k l^2 \quad \text{...(i)}$$

Let l_2 be the new length, then

$$l_2 = l + \frac{10}{100} l = l + 0.1l = 1.1l$$

Let R_2 be the resistance of the wire after stretching, then

$$R_2 = k l_2^2 \quad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i), we have $\frac{R_2}{R_1} = \frac{l_2^2}{l_1^2} = \frac{(1.1)^2 l_1^2}{l_1^2} = 1.21$

or $\quad R_2 = 1.21\,R_1 = R_1 + 0.21\,R_1$

or $\quad R_2 - R_1 = 0.21\,R_1$

Hence, the percentage change in the resistance is

$$\frac{R_2 - R_1}{R_1} \times 100 = 21\%$$

ILLUSTRATION 5.9

A uniform copper wire of mass 2.23×10^{-3} kg carries a current of 1 A when 1.7 V is applied across it. Calculate the length and the area of cross section. If the wire is uniformly stretched to double its length, calculate the new resistance. Density of copper is 8.92×10^3 kgm^{-3} and resistivity is 1.7×10^{-8} Ωm.

Sol. As m = volume × density = $(L \times S) \times d$,

(as volume = $L \times S$)

$$L \times S = \frac{m}{d} = \frac{2.23 \times 10^{-3}}{8.92 \times 10^3} = \frac{1}{4} \times 10^{-6} \quad \text{...(i)}$$

And as $V = IR$, i.e.,

$$R = \frac{V}{I} = \frac{1.7}{1} = 1.7\ \Omega \quad \text{...(ii)}$$

But by definition,

$R = \rho\,(L/S)$

or $$\frac{L}{S} = \frac{R}{\rho} = \frac{1.7}{1.7 \times 10^{-8}} = 10^8 \quad \text{...(iii)}$$

Solving Eqs. (i) and (iii) for L and S, we get L = 5 m and $S = 5 \times 10^{-8}$ m^2. When the wire is stretched uniformly to double its length, the volume will remain unchanged, i.e.,

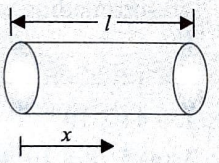
$SL = S'(2L)$ or $S' = S/2$

Hence, the new resistance will be

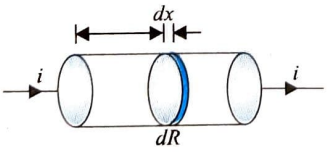
$$R' = \rho \frac{(2L)}{(S/2)} = 4\rho \frac{L}{S} = 4R = 4 \times 1.7 = 6.8\ \Omega$$

ILLUSTRATION 5.10

A cylindrical wire has length l and area of cross section A. The conductivity of wire material changes with distance from left end as $\sigma = \sigma_0 \frac{l}{x}$, where σ_0 is a constant. Calculate electric field inside the conductor as a function of x, when a battery of emf V is connected across its ends.



Sol. Let us calculate the resistance of the cylindrical conductor. Here, we can consider the cylinder to be made of a number o felemental discs. Resistance of one such elemental disc (shown in figure) will be



$$dR = \frac{dx}{\sigma A} = \frac{x\,dx}{A\sigma_0 l}$$

Hence, total resistance of the cylindrical wire

$$R = \int dR = \frac{1}{Al\sigma_0} \int_0^l x\,dx = \frac{1}{Al\sigma_0} \left[\frac{x^2}{2}\right]_0^l = \frac{l}{2A\sigma_0}$$

Current through the wire, $I = \frac{V}{R} = \frac{2VA\sigma_0}{l}$

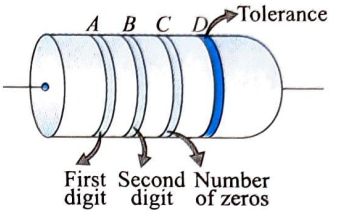
Current density, $j = \frac{I}{A} = \frac{2V\sigma_0}{l}$

Using ohm's law in microscopic form,

$$E = \frac{j}{\sigma} = \frac{2V\sigma_0}{l} \frac{x}{\sigma_0 l} = \frac{2V}{l^2} x$$

COLOUR CODING AND TOLERANCES FOR RESISTANCES

In electrical and electronic circuits, carbon resistors with a wide range of values are extensively used. To indicate the resistance value and its percentage reliability, a colour code is used. The resistor has a set of coloured concentric rings A, B, C and D on it with their significant indicated in table.



'A' denotes the first digit (i.e., the first significant figure), 'B' denotes the second digit (i.e., the second significant figure), 'C' denotes the number of zeros (or power of ten) by which the above two significant figures are to be multiplied. 'D' denotes the tolerance limits (i.e., the error in the value of the resistance).

	Color	Strip A	Strip B	Strip C Multiplier	Strip D Tolerance (%)
B	black	0	0	× 1	
B	white	1	1	× 10	1
R	red	2	2	× 10^2	2
O	orange	3	3	×10^3	
Y	yellow	4	4	×10^4	
G	green	5	5	×10^5	0.5
B	blue	6	6	×10^6	0.25
V	violet	7	7	×10^7	0.1
G	grey	8	8	×10^8	0.05
W	white	9	9	×10^9	
G	gold			×0.1	±5
S	silver			×0.01	±10
	none				±20

Example: If A is green, B is violet, C is orange and D is silver, resistance is (57000 ± 10%) Ω.

Example: If A is yellow, B is red, C is orange and D is gold, resistance is (42000 ± 5%) Ω.

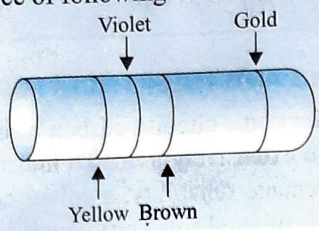
ILLUSTRATION 5.11

Draw a colour code for 42 kΩ ± 10% carbon resistance.

Sol. According to colour code colour for digit 4 is yellow, for digit 2 it is red, for 3 colour is orange and 10% tolerance is represented by silver colour. So colour code should be yellow, red, orange and silver.

ILLUSTRATION 5.12

What is resistance of following resistor.



Sol. Number for yellow is 4, Number of violet is 7

Brown colour gives multiplier 10^1, Gold gives a tolerance of $\pm 5\%$ ·

So resistance of resistor is

$$47\times10^{1}\,\Omega\pm5\%=470\pm5\%\,\Omega.$$

ILLUSTRATION 5.13

A voltage of 30 V is applied across a colour coded carbon resistor with first, second and third rings of blue, black and yellow colours. What is the current flowing through the resistor?

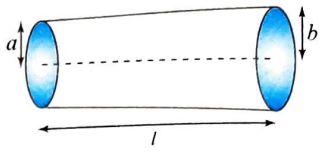
Sol. First significant figure = 6, second significant figure = 0, number of zeros to be attached = 4

Thus, $R=60{,}000=60\times10^{4}\,\Omega$ (blue : 6, black : 0, yellow : 4)

As $V=30$ V, $I=\dfrac{V}{R}=\dfrac{30V}{60\times10^{4}\,\Omega}=0.5\times10^{-4}$ A

CONCEPT APPLICATION EXERCISE 5.1

1. How many electrons per second pass through a section of wire carrying a current of 0.7 A?
2. A current of 7.5 A is maintained in a wire for 45 s. In this time **(a)** how much charge and **(b)** how many electrons flow through the wire?
3. If 0.6 mol of electrons flows through a wire in 45 min, what are **(a)** the total charge that passes through the wire and **(b)** the magnitude of the current?
4. The current in a wire varies with time according to the equation $i = 4 + 2t$, where i is in ampere and t is in second. Calculate the quantity of charge that passes through a cross section of the wire during the time $t = 2$ s to $t = 6$ s.
5. A wire has a resistance R. What will be its resistance if **(a)** it is double on itself and **(b)** it is stretched so that (i) its length is doubled and (ii) its radius is halved.
6. If a copper wire is stretched to make it 0.1% longer, what is the percentage change in its resistance?
7. The current density across a cylindrical conductor of radius R varies in magnitude according to the equation $J=J_0\left(1-\dfrac{r}{R}\right)$ where r is the distance from the central axis. Thus, the current density is a maximum J_0 at that axis ($r=0$) and decreases linearly to zero at the surface ($r=R$). Calculate the current in terms of J_0 and the conductor's cross-sectional area $A=\pi R^2$.
8. Figure shows a conductor of length l having a circular cross section.



The radius of cross section varies linearly from a to b. The resistivity of the material is ρ. Assuming that $b-a \ll l$, find the resistance of the conductor.

9. The space between the plates of a parallel plate capacitor is completely filled with a material of resistivity 2×10^{11} Ωm and dielectric constant 6. Capacity of the capacitor with the given dielectric medium between the plates is 20 nF. Find the leakage current if a potential difference 2500 V is applied across the capacitor.
10. A uniform copper wire of mass 2.23×10^{-3} kg carries a current of 1 A when 1.7 V is applied across it. Calculate its length and area of cross section. If the wire is uniformly stretched to double its length, calculate the new resistance. Density of copper is 8.92×10^{3} kgm^{-3} and resistivity is 1.7×10^{-8} Ωm.
11. A hollow metallic sphere has inner radius a, outer radius b and resistivity ρ. If a potential difference V is applied between the inner and outer surface. Find the
    (i) resistance between inner and outer surface
    (ii) total current
    (iii) current density in function of radial distance $J=f(r)$
    (iv) electric field in function of radial distance $E=f(r)$
12. A conducting sphere of radius r is surrounded by a poorly conducting material of inner radius a and outer radius b whose resistivity ρ varies with $\rho=\rho_0 r^3$. Find the resistance between the sphere and the outer surface of surrounding conducting material.

ANSWERS

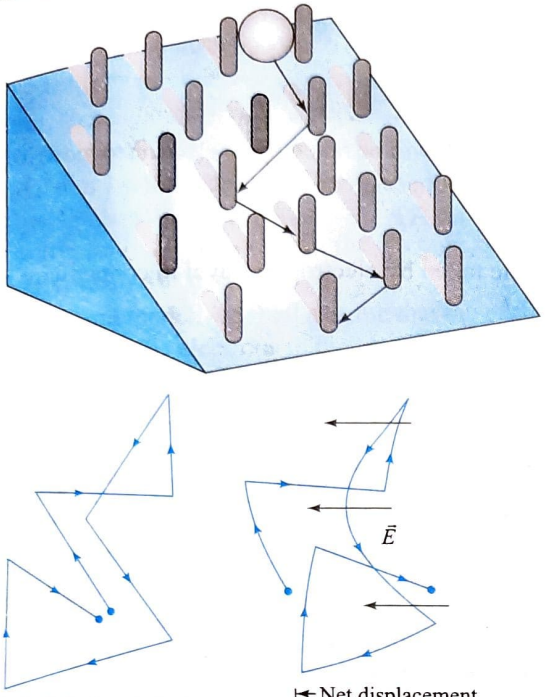
1. 0.44×10^{19} **2.** (a) 337.5 C (b) 2.1×10^{21}
3. (a) 5.78×10^{4} C (b) 21.4 A **4.** 48 C
5. (a) $\dfrac{1}{4}R$ (b) (i) $4R$ (ii) $16R$ **6.** 0.2% **7.** $\dfrac{J_0A}{3}$
8. $\dfrac{\rho l}{\pi ab}$ **9.** 4.7 μA **10.** 5 m, 5×10^{-8} m^2, 6.8 Ω
11. (i) $\dfrac{\rho}{4\pi}\left(\dfrac{b-a}{ab}\right)$ (ii) $\dfrac{4\pi Vab}{\rho(b-a)}$ (iii) $\dfrac{Vab}{\rho(b-a)r^2}$ (iv) $\dfrac{Vab}{(b-a)r^2}$
12. $\dfrac{\rho_0(b^2-a^2)}{8\pi}$

DRIFT VELOCITY

Under the normal conditions of temperature and pressure and without the influence of any external electrostatic field, the motion of free electrons in a conductor is due to the thermal energy.

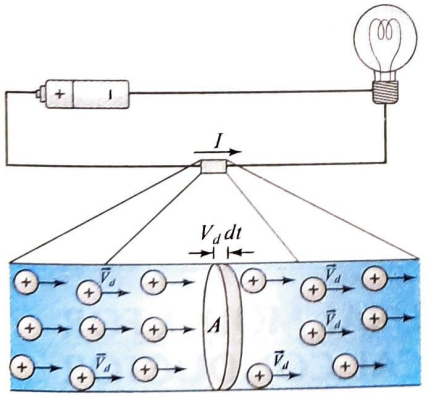
Now, when this conductor is placed in an external field, these free electrons start experiencing electric force and start moving under the influence of this force (see figure). However, although

they are free to move, they are not able to move in a straight line because they encounter other electrons, ions, atoms, or molecules in their way (see figure). Hence, they experience collisions after collisions but are able to drift in a particular direction because of this external field. The drifting of these free electrons over some period of time is called drift velocity.

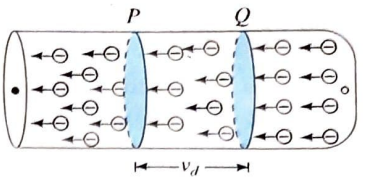


Relation Between Drift Velocity and Current

Let A be the area of cross section of the conductor, e be the charge on each electron, v_d be the drift velocity, n be the number of free electrons per unit volume, and I be the current, then the total number of electrons between cross sections P and Q, which are v_d distance apart is (volume between P and Q) $\times n = Av_d n = nAv_d$. Therefore, the total charge in this volume is $nAv_d e = neAv_d$.



Now, the electron which is present at cross section Q will reach cross section P after 1/sec because P and Q are so selected that the distance between them is v_d, which is the drift speed of the electron. Therefore, $neAv_d$ is the charge that will pass through cross section at P (where P can be any point on the conductor). Hence, this is the electric current that flows through the conductor (see figure).



A is the cross section of conductor. Therefore, $I = neAv_d$. Accordingly current density is $J = I/A = nev_d$.

The equation $J = nev_d$ can be written in vector form as follows: $\vec{J} = nq\vec{v}_d$, where q is the charge of the charge carrier and v_d is the average drift velocity. This equation is correct for both the signs of q. If $q > 0$, $\vec{v}_d$ is in the direction of electric field $\vec{E}$, and $\vec{J}$ is in the direction of $\vec{E}$. If $q < 0$ ($q = -e$ for electrons), as it is in metallic conductor, $\vec{v}_d$ is opposite to $\vec{E}$, and $\vec{J} = -ne\vec{v}_d$ continues to be in the direction of $\vec{E}$.

Illustration 5.14

A copper wire has a square cross section of 6 mm on a side. The wire is 10 m long and carries a current of 3.6 A. The density of free electrons is $8.5 \times 10^{28}/\text{m}^3$. Find the magnitude of **(a)** the current density in the wire; **(b)** the electric field in the wire. **(c)** How much time is required for an electron to travel the length of the wire? (ρ, electrical resistivity, is $1.72 \times 10^{-8}\ \Omega\text{m}$.)

Sol. Given $r = 6$ mm, $l = 10$ m, $I = 3.6$ A, $n = 8.5 \times 10^{28}/\text{m}^3$.

(a) To find the current density, formula used should be

$$J = I/A = \frac{3.6}{(6\times10^{-3})^2} = \frac{3.6}{36\times10^{-6}} = 10^5\ \text{Am}^{-2}$$

(b) Electric field is

$$E = \rho J = 1.72 \times 10^{-8} \times 10^5 = 1.72 \times 10^{-3}\ \text{Vm}^{-1}$$

(c) Time taken is

$$t = \frac{l}{v_d} = \frac{l\,ne\,A}{I}$$

$$= \frac{10 \times 8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times (6 \times 10^{-3})^2}{3.6}$$

$$= 1.36 \times 10^6\ \text{s}$$

Illustration 5.15

Consider a wire of length 0.1 m with an area of cross section 1 mm^2 connected to 5 V. Find the current flowing through the metallic wire where $\mu = 5 \times 10^{-6}\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$, $e = 1.6 \times 10^{-19}$ C, and $n = 8 \times 10^{28}\ \text{m}^{-3}$.

Sol. Given $A = 1\ \text{mm}^2 = 10^{-6}\ \text{m}^2$, $l = 0.1$ m, $V = 5$ V.

Due to the applied potential difference across the wire, an electric field is set up in the conductor. So

$$E = \frac{V}{l} = \frac{5}{0.1} = 50\ \text{V m}^{-1}$$

The current flowing through the wire is given by

$$I = nAev_d = nAe\mu E \qquad [\text{where, } \mu = v_d/E]$$

$$= 8 \times 10^{28} \times 10^{-6} \times 1.6 \times 10^{-19} \times 5 \times 10^{-6} \times 50 = 3.2\ \text{A}$$

ILLUSTRATION 5.16

(a) Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area 1.0×10^{-7} m^2 carrying a current of 1.5 A. Assume that each copper atom contributes roughly one conduction electron. The density of copper is 9.0×10^3 kgm^{-3}, and its atomic mass is 63.5 u.

(b) Compare the drift speed obtained with the speed of propagation of electric field along the conductor, which causes the drift motion.

Sol.

(a) The direction of drift velocity of conduction electrons is opposite to the electric field direction, i.e., the electrons drift in the direction of increasing potential. The drift speed v_d is given by $v_d = (I/neA)$.

Now, $e = 1.6 \times 10^{-10}$ C, $A = 1.0 \times 10^{-7}$ m^2, $I = 1.5$A, the density of n conduction electrons is equal to the number of atoms per cubic meter (assuming one conduction electron per Cu atom is reasonable from its valence electron count of one). A cubic meter of copper has a mass of 9.0×10^3 kg. Since 6.0×10^{23} copper atoms have a mass of 63.5 g,

$$n = \frac{6.0\times10^{23}}{63.5}\times 9.0\times10^{6} = 8.5\times10^{28}$$

which gives

$$v_d = \frac{1.5}{8.5\times10^{28}\times1.6\times10^{-19}\times1.0\times10^{-7}}$$

$$= 1.1 \times 10^{-3}\,\text{ms}^{-1} = 1.1\ \text{mm s}^{-1}$$

(b) An electric field traveling along the conductor has a speed of an electromagnetic wave, i.e., equal to 3.0×10^8 ms^{-1}. The drift speed is, in comparison, extremely small, smaller by a factor of 10^{-11}.

ILLUSTRATION 5.17

Find the approximate total distance traveled by an electron in the time interval in which its displacement is one meter along the wire.

Sol. Time taken to travel length of the wire is

$$t = \frac{\text{displacement}}{\text{drift velocity}} = \frac{S}{V_d}$$

So $V_d = 1\ \text{mm s}^{-1} = 10^{-3}\ \text{ms}^{-1}$ (normally the value of drift velocity is 1 mm s^{-1})

and $S = 1$ m

$$\Rightarrow \quad t = \frac{1}{10^{-3}} = 10^3\ \text{s}$$

As distance travelled = speed × time, so speed = 10^6 ms^{-1} (The speed of an electron between two successive collision is around 10^6 ms^{-1}). So required distance is $10^6 \times 10^3$ m = 10^9 m.

ILLUSTRATION 5.18

Find the total linear momentum of the electrons in a conductor of length $l = 1000$ m carrying a current $i = 70$ A.

Sol. Linear momentum of electrons is

$$P = (nmAl)v_d \qquad \ldots(i)$$

where n is the density of electrons, and v_d is the drift velocity. We know,

$$v_d = \frac{i}{neA} \qquad \ldots(ii)$$

Then linear momentum is

$$p = nAlm\left(\frac{i}{neA}\right) = \frac{mil}{e} = 0.4\times10^{-6}\ \text{Ns}$$

ILLUSTRATION 5.19

How much time will be taken by an electron to move a distance $l = 1$ km in a copper wire of cross section $A = 1$ mm^2 if it carries a current $I = 4.5$ A?

Sol. Time taken by electron to travel in copper wire is

$$t = \frac{l}{v_d}$$

where $v_d = \frac{J}{ne}$

or $t = \frac{l}{J/ne}$ where $J = i/A$

Hence,

$$t = \frac{neAl}{i}$$

$$= \frac{10^{28}\times1.6\times10^{-19}\times10^{3}\times10^{-6}}{4.5} = 3\times10^{6}\,\text{s}$$

ILLUSTRATION 5.20

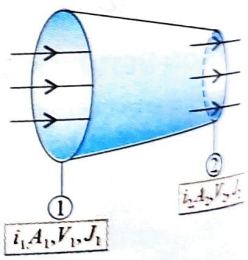
Current is flowing from a conductor of nonuniform cross-sectional area. If $A_1 > A_2$, then find relation between

(a) i_1 and i_2

(b) j_1 and j_2

(c) $(v_d)_1$ and $(v_d)_2$ (drift velocity)

where i is current, j is current density, and V is drift velocity.



Sol.

(a) The charge flowing through a cross section per unit time is the current. So, $i_1 = i_2$.

(b) Current density is $J = i/A$. As $A_1 > A_2$, then $j_1 < j_2$.

(c) We know $j = nev_d$. So drift velocity is $v_d = j/ne$. As $j_1 < j_2$ then $(v_d)_1 < (v_d)_2$.

STRUCTURAL MODEL FOR ELECTRICAL CONDUCTOR

Consider a conductor as a regular array of atoms containing free electrons (sometimes called conduction electrons). Such electrons are free to move throughout the conductor. In the absence of an electric field, the free electrons move in random directions with average speeds of the order of 10^6 ms^{-1}. The situation is similar to the motion of the gas molecules confined in a vessel that we studied in kinetic theory of gases. In fact, the conduction electrons in a metal are often called electron gas.

Conduction electrons are not totally free because they are confined to the interior of the conductor and undergo frequent collisions with the array of atoms. The collisions are the predominant mechanism contributing to the resistivity of a metal at normal temperatures. Note that there is no current in a conductor in the absence of an electric field because the average velocity of the free electrons is zero. On an average, just as many electrons move in one direction as in the opposite direction, so there is no net flow of charge.

However, the situation is modified, when an electric field is applied to the metal. In addition to the random thermal motion, the free electrons drift slowly in a direction opposite to that of the electric field, with an average drift speed of v_d, which is much less (typically 10^{-4} ms^{-1}) than the average speed between collisions (typically 10^6 ms^{-1}).

In our structural model, we shall assume that the excess kinetic energy acquired by the electrons in the electric field is lost to the conductor in the collision process. The energy given up to the atoms in the collisions increases the total vibrational energy of the atoms, causing the conductor to warm up. The model also assumes that an electron's motion after a collision is independent of its motion before the collision.

On the basis of our model, we now take the first step toward obtaining an expression of the drift speed. Let a mobile charged particle of mass m and charge q is subjected to an electric field $\vec{E}$. For electrons in a metal, $\vec{F}_e = e\vec{E}$. The motion of the electron can be determined from Newton's second law, $\sum \vec{F} = m_e \vec{a}$. The acceleration of the electron is

$$\vec{a} = \frac{\sum \vec{F}}{m_e} = \frac{\vec{F}_e}{m_e} = \frac{-e\vec{E}}{m_e} \qquad \text{...(i)}$$

The acceleration, which occurs only for a short time interval between collisions, changes the velocity of the electron. Because the force is constant, the acceleration is constant, and we can model the electron as a particle under constant acceleration. If $\vec{v}_0$ is the velocity of the electron just after a collision, at which we define the time as $t = 0$, the velocity of the electron at time t is

$$\vec{v} = \vec{v}_0 + \vec{a}t = \vec{v}_0 - \frac{e\vec{E}}{m_e}t \qquad \text{...(ii)}$$

The motion of the electron through the metal is characterized by a very large number of collisions per second. Consequently, we consider the average value of $\vec{v}$ over a time interval compared with the time interval between collisions, which gives us the drift velocity $\vec{v}_d$. Because the velocity of the electron after a collision is assumed to be independent of its velocity before the collision, the initial velocities are randomly distributed in direction, so that the average value of $\vec{v}_0$ is zero. In the second term on the right of Eq. (ii), the charge, electric field, and mass are all constant. Therefore, the only factor affected by the averaging process is the time t. The average value of this term is $(-e\vec{E}/m_e)\tau$, where τ is the average time interval between collisions. Therefore, after the averaging process, Eq. (ii) becomes

$$\vec{v}_d = \frac{-e\vec{E}}{m_e}\tau \qquad \text{...(iii)}$$

Substituting the magnitude of this drift velocity (the drift speed) in equation $I_{avg} = \Delta Q/\Delta t = nev_d A$, we have

$$I = nev_d A = ne\left(\frac{eE}{m_e}\tau\right)A = \frac{ne^2E}{m_e}\tau A \qquad \text{...(iv)}$$

According to Ohm's law, the current is related to the macroscopic variables of potential difference and resistance, i.e.,

$$I = \frac{\Delta V}{R}$$

Substituting $R = \rho(l/A)$, we can write this expression as

$$I = \frac{\Delta V}{\left(\rho\frac{l}{A}\right)} = \frac{\Delta V}{\rho l}A$$

In the conductor, the electric field is uniform; therefore, we use $\Delta V = El$, for the magnitude of the potential difference across the conductor, i.e.,

$$I = \frac{El}{\rho l}A = \frac{E}{\rho}A \qquad \text{...(v)}$$

Equating Eqs. (iv) and (v) for the current, we solve for the resistivity, i.e.,

$$I = \frac{ne^2E}{m_e}\tau A = \frac{E}{\rho}A$$

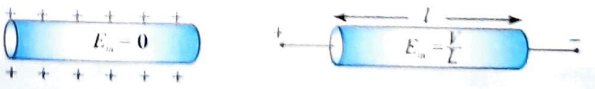
or $$\rho = \frac{m_e}{ne^2\tau} \qquad \text{...(vi)}$$

According to this structural model, resistivity does not depend on the electric field or, equivalently, on the potential difference, but depends only on fixed parameters associated with the material and the electron. This feature is characteristic of a conductor obeying Ohm's law. The model shows that resistivity can be calculated from the knowledge of the density of the electrons, their charge and mass, and the average time interval τ between collisions. We can also write current density as

$$J = nev_d = (ne)\left(\frac{eE}{m}\tau\right) = \frac{ne^2}{m}\tau E$$

Note:

It is worth noting that electric field inside a charged conductor is zero, but it is nonzero inside a current-carrying conductor and is given by $E = V/l$ where V is the potential difference across the conductor and l is the length of the conductor. Electric field outside the current carrying is zero.



The small value of drift velocity produces a large amount of electric current, due to the presence of extremely large number of free electrons in a conductor. The propagation of current is almost at the speed of light and involves electromagnetic process. It is due to this reason that the electric bulb glows immediately when switch is on.

ILLUSTRATION 5.21

A copper wire of cross-sectional area 3.00×10^{-6} m^2 carries a current 10.0 A.

(a) Find the drift speed of the electrons in the wire. Assume that each copper atom contributes one free electron to the body of material.

(b) Find the average time between collisions for electrons in the copper at 20°C. The density of copper is 8.95 gcm^{-3}, molar mass of copper is 63.5 gmol^{-1}, Avogadro number is 6.02×10^{23} electrons per mol and resistivity of copper is 1.7×10^{-8} Ωm.

Sol.

(a) The volume occupied by 63.5 g of copper is

$$V = \frac{M}{\rho} = \frac{63.5}{8.95} = 7.09 \text{ cm}^3 \text{ mol}^{-1}$$

As each copper atom contributes one free electron to the body of the material, the density of free electrons is

$$n = \frac{6.02 \times 10^{23}}{7.09 \times 10^{-6}} = 8.48 \times 10^{28} \text{ electrons per cubic meter}$$

The drift speed is

$$v_d = \frac{I}{neA}$$

$$= \frac{10.0}{8.48 \times 10^{28} \times 1.60 \times 10^{-19} \times 3 \times 10^{-6}} = 2.46 \times 10^{-4} \text{ ms}^{-1}$$

(b) Average time between collision of electrons is

$$\tau = \frac{m_e}{ne^2\rho} = \frac{9.10 \times 10^{-31}}{8.48 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 1.7 \times 10^{-8}}$$

$$= 2.5 \times 10^{-14} \text{ s}$$

MOBILITY

We have seen that conductivity arises from mobile charge carriers. In metals, these mobile charge carriers are electrons; in an ionized gas, they are electron and positive charged ions; in an electrolyte, there can be both positive and negative ions. In a semiconductor material such as germanium or silicon, conduction is partly due to electrons and partly due to electron vacancies called holes. Holes are sites of missing electrons that act like positive charges. An important quantity is the mobility μ defined as the magnitude of the drift velocity per unit electric field, i.e.,

$$\mu = \frac{v_d}{E}$$

ILLUSTRATION 5.22

Consider a conductor of length 40 cm where a potential difference of 10 V is maintained between the ends of the conductor. Find the mobility of the electrons provided the drift velocity of the electrons is 5×10^{-6} ms^{-1}.

Sol. Given $L = 40$ cm, $V = 10$ V, $v_d = 5 \times 10^{-6}$ ms^{-1}.

To find the electron mobility, we need the value of the electric field, which can be obtained using the following formula:

$$E = \frac{V}{l} \text{ or } E = \frac{10}{0.4} = 25 \text{ Vm}^{-1}$$

Also the formula used for electron mobility is

$$\mu = \frac{v_d}{E} = \frac{5 \times 10^{-6}}{25} = \frac{1}{5} \times 10^{-6}$$

$$= 0.2 \times 10^{-6} = 2 \times 10^{-7} \text{ m}^2 \text{ V}^{-1} \text{s}^{-1}$$

EFFECT OF TEMPERATURE ON RESISTIVITY AND RESISTANCE

As long as the temperature of the material is constant, the resistivity of material also remains constant. As the temperature of the material increases, the relaxation time (t) decreases, the resistivity increases, and hence the resistance also increases.

As the number of electrons per unit volume goes up, the resistance decreases. This can be easily visualized, since if the density of electrons increases, more electrons can flow in response to the potential difference and hence the current will increase. Therefore, the resistance will decrease.

TEMPERATURE COEFFICIENTS OF RESISTIVITY

Let us study the equation

$$R_T = R_{T_0}[1 + \alpha(T - T_0) + \beta(T - T_0)^2]$$

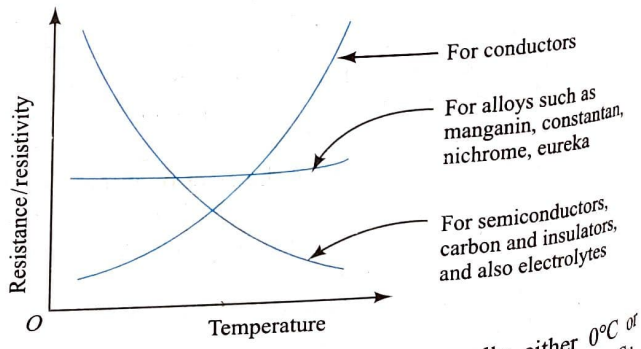
where both α and β are called temperature coefficients of resistance, though different in magnitude.

TEMPERATURE COEFFICIENTS OF RESISTANCE

In case of pure metals, β is negligibly small, so the resistance varies linearly with the rise of temperature (see figure).

$$R_T = R_{T_0}[1 + \alpha(T - T_0)]$$

where R_T is the resistance at temperature T, R_{T_0} is the resistance at temperature T_0 (called reference temperature), and α is a constant for a given metal and for a given reference temperature and is called temperature coefficient of resistance. Its unit is per degree temperature (°C^{-1}).



T_0 is some reference temperature, generally either 0°C or 20°C. α is positive for metals; it is 0.004 per degree for Cu, but very small (almost zero) for alloys such as manganin ($\alpha = 0.00001$ per degree), nichrome, constantan, and eureka. It is negative for semiconductors and insulators (see figure). Let

$$\rho_T = \rho_0[1 + \alpha(T - T_0)]$$

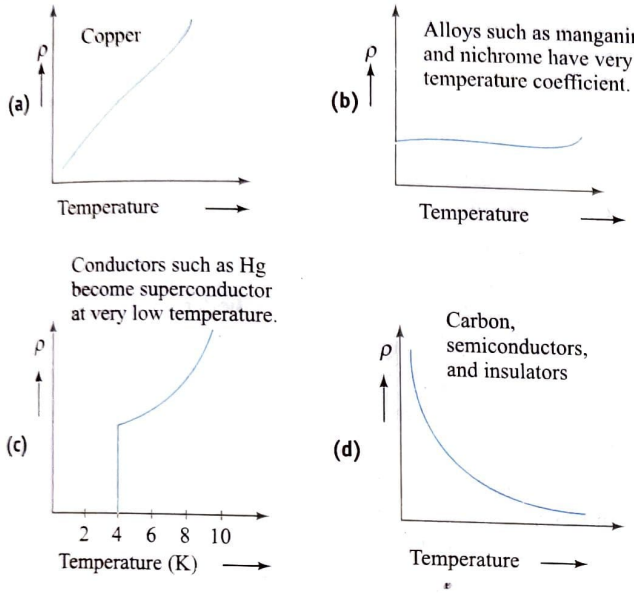
or $$\rho_T - \rho_0 = \alpha\rho_0 (T - T_0)$$

or $\alpha = \frac{1}{\rho_0} \times \frac{(\rho_T - \rho_0)}{(T - T_0)}$

or $\rho = \frac{1}{\rho} \times \frac{d\rho}{dT}$

These observations can be understood qualitatively using the relation for resistivity, i.e., $\rho = m/ne^2\tau$.

For metals, the number of free electrons is fixed. As temperature increases, the amplitude of vibration of atoms/ions increases and collisions of electrons with them become more effective and frequent, resulting in the decrease in τ and, hence, increase in ρ. Thus, for metals, ρ increases with temperature.



FINDING COEFFICIENT OF RESISTANCE

- $\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$

  where R_2 is the resistance at t_2°C and R_1 is the resistance at t_1°C.

  Derivation:

  $R_1 = R_0(1 + \alpha t_1)$ ...(i)

  and $R_2 = R_0(1 + \alpha t_2)$ ...(ii)

  Dividing Eq. (ii) by Eq. (i), we have

  $$\frac{R_2}{R_1} = \frac{1 + \alpha t_2}{1 + \alpha t_1} = (1 + \alpha t_2)(1 + \alpha t_1)^{-1}$$
  $$= (1 + \alpha t_2)(1 + \alpha t_1)$$

  (Using binomial theorem and thus neglecting higher terms)

  $$\frac{R_2}{R_1} = (1 + \alpha)(t_2 - t_1)$$

  (since α^2 is very small and hence neglected)

  or $R_2 = R_1 + R_1\alpha(t_2 - t_1)$

  or $\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$

- $\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$

  where R_2 is the resistance at t_2°C.

  Derivation:

  Dividing Eq. (ii) by Eq. (i), we have

  $$\frac{R_2}{R_1} = \frac{1 + \alpha t_2}{1 + \alpha t_1}$$

  or $R_2 + R_2\alpha t_1 = R_1 + R_1\alpha t_2$

  or $R_2 - R_1 = \alpha(R_1 t_2 - R_2 t_1)$

  or $\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$

- $\alpha = \frac{R_t - R_0}{R_0 t}$

  where R_t is the resistance at t °C and R_0 is the resistance at 0°C.

  Derivation:

  $R_t = R_0(1 + \alpha t)$ or $R_t - R_0 = R_0\alpha t$

  or $\alpha = \frac{R_t - R_0}{R_0 t}$

ILLUSTRATION 5.23

A copper coil has a resistance of 20.0 Ω at 0°C and a resistance of 26.4 Ω at 80°C. Find the temperature coefficient of resistance of copper.

Sol. $R_{80°C} = R_{0°C}[1 + \alpha\Delta T]$

or $26.4 = 20.0[1 + \alpha \times (80 - 0)]$ or $\frac{26.4}{20} = 1 + 80\,\alpha$

On solving, we get $\alpha = 4 \times 10^{-3}$°C^{-1}.

ILLUSTRATION 5.24

A metallic wire has a resistance of 120 Ω at 20° C. Find the temperature at which the resistance of same metallic wire rises to 240 Ω where the temperature coefficient of the wire is 2×10^{-4} °C^{-1}.

Sol. Given $R_1 = R_{20} = 120\,\Omega$, $R_2 = R_T = 240\,\Omega$, $t = ?$, $\alpha = 2 \times 10^{-4}$°C^{-1}, $t_1 = 20$°C, $t_2 = T = ?$

Formula used to find the temperature coefficient of resistance is

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} = \frac{R_T - R_{20}}{R_{20}(T - 20)}$$

or $2 \times 10^{-4} = \frac{240 - 120}{120(T - 20)}$

or $T - 20 = \frac{120}{120 \times 2 \times 10^{-4}}$

or $T = \frac{1}{2 \times 10^{-4}} + 20 = 0.5 \times 10^4 + 20 = 5020$°C

ILLUSTRATION 5.25

A resistance R of thermal coefficient of resistivity α is connected in parallel with a resistance $3R$, having thermal coefficient of resistivity 2α. Find the value of α_{eff}.

Sol. The equivalent resistance at 0°C is

$$R_0 = \frac{R_{10}R_{20}}{R_{10} + R_{20}} \quad ...(i)$$

The equivalent resistance at t°C is

$$R = \frac{R_1 R_2}{R_1 + R_2} \quad ...(ii)$$

But $R_1 = R_{10}(1 + \alpha t)$ ...(iii)

$R_2 = R_{20}(1 + 2\alpha t)$ ...(iv)

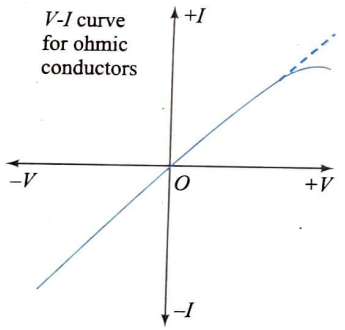
and $R = R_0(1 + \alpha_{eff} t)$ ...(v)

Putting the value of (i), (iii), (iv), and (v) in Eq. (ii), we have

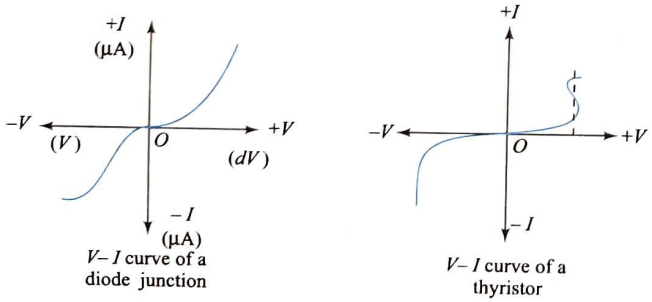
$$\alpha_{eff} = \frac{5}{4}\alpha$$

VALIDITY AND FAILURE OF OHM'S LAW

Ohm's law is not a law of nature, i.e., it is not a universal law that applies everywhere under all conditions. Ohm's law is obeyed by metallic conductors, which accordingly are called ohmic conductors, that too at about normal working temperatures. At very high currents/voltages, even ohmic conductors do not follow this law as shown in Figs. (a) and (b). Semi-conductors also do not follow Ohm's law as shown in the figures.



(a)



(b)

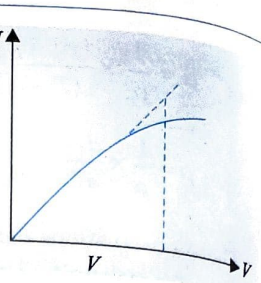
Thus, Ohm's law is not followed in the following cases:

Materials: (i) Vacuum tubes; (ii) crystal rectifiers; (iii) transistors; (iv) thermistors, thyristors; (v) superconductors.

Conditions: (i) At very high temperatures; (ii) at very low temperatures (superconductivity); (iii) at very high potential differences.

ILLUSTRATION 5.26

The I-V characteristics of a resistor is observed to deviate from a straight line for higher value of current as shown in figure. Why?



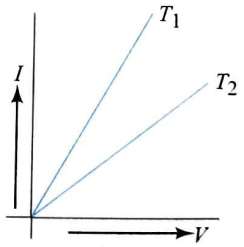
Sol. For higher value of current, the resistor gets heated and consequently its resistance increases. The resistor becomes non-ohmic due to which the I-V characteristic deviates from straight line thereby showing lesser current for the same voltage.

CONCEPT APPLICATION EXERCISE 5.2

1. **(a)** A steady current flows in a metallic conductor of non-uniform cross section. State which of the quantities i.e., current, current density, electric field, and drift velocity remain constant?
   (b) A steady current passes through a cylindrical conductor. Is there an electric field inside the conductor?
2. A potential difference V is applied to copper wire of diameter d and length L. What will be the effect on the electron drift speed by doubling **(a)** voltage V, **(b)** length L, and **(c)** diameter d?
3. The following table gives the length of three copper rods, their diameters, and the potential differences between their ends. Rank the rods according to **(a)** the magnitude of the electric field within them, **(b)** the current density within them, and **(c)** the drift speed of electrons through them, greatest first.

Rod	Length	Diameter	P.D.
1	L	$3d$	V
2	$2L$	d	$2V$
3	$3L$	$2d$	$2V$

4. The V-I graph for a metallic wire at two different temperatures T_1 and T_2 is shown in figure. Which of the two temperatures T_1 and T_2 is higher and why?



5. A beam contains 2.0×10^8 doubly charged positive ions per cubic centimeter, all of which are moving toward north with a speed of 1.0×10^5 ms^{-1}. **(a)** What are the magnitude and direction of the current density $\vec{J}$? **(b)** Can you calculate the total current i in this ion beam? If not, what additional information is needed.
6. A typical copper wire might have 2×10^{21} free electrons in 1 cm of its length. Suppose that the drift speed of the electrons along the wire is 0.05 cms^{-1}. How many electrons would pass through a given cross section of the

wire each second. How large would a current be flowing in the wire?

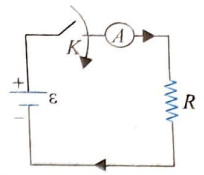
7. A coil of wire has a resistance of 25.00 Ω at 20°C and a resistance of 25.17 Ω at 35°C. What is its temperature coefficient of resistance?
8. A metal wire of diameter 2 mm and of length 300 m has a resistance of 1.6424 Ω at 20°C and 2.415 Ω at 150°C. Find the values of α, R_0 and ρ_0, where the zero subscript refers to 0°C, and $\rho_{20°C}$. Identify the metal.
9. It is desired to make a 20 Ω coil of wire, which has a zero thermal coefficient of resistance. To do this, a carbon resistor of resistance R_1 is placed in series with an iron resistor of resistance R_2. The proportions of iron and carbon are so chosen that $R_1 + R_2 = 20$ Ω for all temperatures near 20°C. Find the values of R_1 and R_2?
$\alpha_{carbon} = -0.5 \times 10^{-3}\,°C^{-1}$, $\alpha_{iron} = 5 \times 10^{-3}\,°C^{-1}$
10. A resistance thermometer measures temperature with the increase in resistance of a wire of high temperature. If the wire is platinum and has a resistance of 10 Ω at 20°C and a resistance of 35 Ω in a hot furnace, what is the temperature of the furnace? ($\alpha_{platinum} = 0.0036°C^{-1}$)
11. A conductive wire has resistance of 10 ohm at 0°C and α is 1/273°C, then determine its resistance at 273°C.
12. What is the drift speed of the conduction electrons in a copper wire with radius $r = 900$ μm when it has a uniform current $i = 17$ mA? Assume that each copper atom contributes one conduction electron to the current and the current density is uniform across the wire's cross section.
13. **(a)** At what temperature would the resistance of a copper conductor be double of its value of 0°C? **(b)** Does this same temperature hold for all copper conductors, regardless of shape and size? $[\alpha_C = 4.0 \times 10^{-3}\,°C^{-1}]$
14. A potential difference is applied across the filament of a bulb at $t = 0$, and it is maintained at a constant value while the filament gets heated to its equilibrium temperature. We find that the final current in the filament is one-sixth of the current drawn at $t = 0$. If the temperature of the filament at $t = 0$ is 20°C and the temperature coefficient of resistivity at 20°C is 0.0043 $°C^{-1}$, find the final temperature of the filament.

ANSWERS

1. (a) Current (b) Yes
2. (a) Drift velocity will be doubled
   (b) Drift velocity will be halved (c) It will not change
3. (a) $E_1 = E_2 > E_3$ (b) $V_{d_1} = V_{d_2} > V_{d_3}$ (c) $I_1 > I_3 > I_2$
4. T_2 is greater than T_1
5. (a) 6.4 Am^{-2}; towards North
   (b) No, we need area of cross section of the wire
6. 1×10^{20} electrons/sec, 16 A
7. $4.5 \times 10^{-4}\,C^{-1}$
8. $\alpha = 3.9 \times 10^3\,°C^{-1}$, $R_0 = 1.5236$ Ω, $\rho_0 = 1.596 \times 10^{-8}$ Wm, $\rho_{20°C} = 1.720 \times 10^{-8}$ Ωm, material is copper
9. $R_1 = 18.18$ Ω, $R_2 = 1.82$ Ω
10. 714°C
11. $10e$
12. $4.9 \times 10^{-7}\,ms^{-1}$
13. (a) 250°C (b) Yes
14. 1182.8°C

ELECTROMOTIVE FORCE AND POTENTIAL DIFFERENCE

When we close the key, a battery sets up a direct current in the conductor. Each charge carrier moves from positive terminal of the battery by the electric field set up by the battery. The electric field inside the conductor does positive work in driving a positive charge carrier and does equal negative work, while the charge carrier moves back to the positive pole (electrode) from negative pole (electrode) of the battery against the electric field inside the battery because $\oint \vec{E} \cdot \vec{dl} = 0$.



The seat of emf ε does a positive work *dW* circulating an elementary charge *dq*; *dW/dq*

Hence, the electric field cannot take the credit of circulating a charge carrier along the closed loop. Then, it must be the battery that does a positive work in pushing the charges from negative to positive terminal of the battery to set up a permanent potential difference across the terminals.

The work done by the battery in pushing the positive charges from its negative terminal to its positive terminal through a distance l with a force F_b is given by

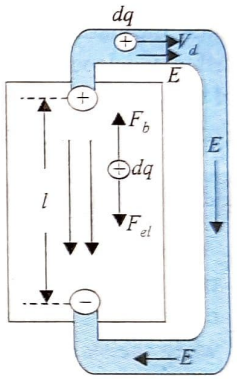
$$W = F_b l = F_{el} l = qE_{el} l = qV_b$$

(F_b is the force acting on the test charge $+q$ inside the battery due to the electrochemical action of the battery.)

Then, the work done per unit charge is

$$\frac{W}{q} = V_b$$

which is called **electromotive force** (emf) of the battery given by $\varepsilon = W_b/q$.

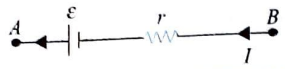
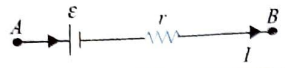
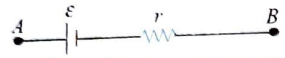
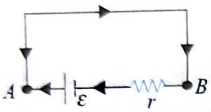


The elementary charges (positive charge carries) move from the negative to the positive electrode of the battery due to emf against the static electric field inside the battery.

Emf is numerically equal to the work done by the battery in circulating a unit positive charge. A cell (seat of emf) generates a potential difference between its terminals across the circuit, which is numerically equal to its emf when the circuit is open.

INTERNAL RESISTANCE OF A CELL

Real sources in a circuit do not behave in exactly the way we have described; the potential difference across a real source in a circuit is not equal to the emf. The reason is that the charge moving through the material of any real source encounters resistance. We call this the internal resistance of the source, denoted by r. If this resistance behaves according to Ohm's law, r is constant and independent of the current I. As the current moves through r, it experiences an associated drop in potential equal to Ir. Thus, when a current is drawn through a source, the potential difference between the terminal of the source is $V = \varepsilon - ir$.

Situation	Potential difference (V) across the terminal of cell
Discharging of a battery 	$V_A - \varepsilon + ir = V_B$ or $V_A - V_B = \varepsilon - ir$ $V_{AB} = \varepsilon - ir$ or $V_{AB} < \varepsilon$
Charging of a battery 	$V_A - \varepsilon - ir = V_B$ or $V_A - V_B = \varepsilon + ir$ $V_{AB} = \varepsilon + ir$ or $V_{AB} > \varepsilon$
Battery is open circuited 	$V_{AB} = \varepsilon$ as $i = 0$
Battery is short circuited 	$i = \frac{\varepsilon}{r}$ or $\varepsilon = ir$ $\therefore \quad \varepsilon - ir = 0$ or $V = 0$

Note:

- Emf is independent of the resistance of the circuit and depends upon the nature of electrolyte of the cell, while potential difference depends upon the resistance between the two points of the circuit and current flowing through the circuit.
- Emf is the cause, and potential difference is the effect.

ILLUSTRATION 5.27

(a) A car has a fresh storage battery of emf 12 V and internal resistance 5.0×10^{-2} Ω. If the starter motor draws a current of 90 A, what is the terminal voltage of the battery when the starter is on?

(b) After long use, the internal resistance of the storage battery increases to 500 Ω. What maximum current can be drawn from the battery? Assume the emf of the battery to remain unchanged.

(c) If the discharged battery is charged by an external emf source, is the terminal voltage of the battery during charging greater of less than its emf 12V?

Sol. We have emf of the storage battery as ε =12 V, internal resistance of the battery as $r = 0.5 \times 10^2$ Ω, and current flowing through the circuit as I = 90 A.

(a) If V is the terminal voltage,

$$V = \varepsilon - Ir = 12 - 90(5.0 \times 10^{-2}) = 7.5 \text{ V}$$

(b) We know that $I = \varepsilon/(R + r)$, where R is the external resistance. For I to be maximum (i.e., I_{max}), $R = 0$ (i.e., the battery is to be shorted). Thus,

$$I_{max} = \frac{\varepsilon}{r} = \frac{12V}{500\ \Omega} = 24 \text{ mA}$$

(as after long use, r =500 Ω)

Clearly, the battery can now no longer be used for starting the car.

(c) During charging, the current inside the battery flows in a direction opposite to that when it is discharging.

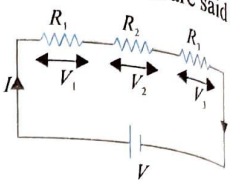
Clearly, replacing I by $-I$, we get $V = \varepsilon - (-I)r = \varepsilon + Ir$.

Hence, V should be greater than ε (= 12 V) during charging.

COMBINATION OF RESISTANCES

RESISTANCES IN SERIES

If resistances are connected as shown in figure such that the current flowing through them is the same, the resistances are said to be in series.



If I is the current flowing through the resistances, then potential drop across each is

$$V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3$$

Adding, $V_1 + V_2 + V_3 = I(R_1 + R_2 + R_3)$ ...(i)

But $V_1 + V_2 + V_3 = V$, so we get from Eq. (i)

$$V = I(R_1 + R_2 + R_3)$$

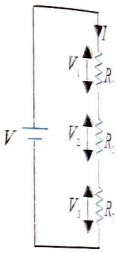
where V is the potential difference across the combinations. Also from Ohm's law

$$V = IR \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get $R = R_1 + R_2 + R_3$ where R is known as equivalent resistance.

VOLTAGE DIVIDER

In a series circuit, current through each resistor is the same (see figure).



$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

$$I = \frac{V}{R_1 + R_2 + R_3}$$

$$V_1 = \frac{VR_1}{R_1 + R_2 + R_3}$$

$$V_2 = \frac{VR_2}{R_1 + R_2 + R_3}$$

$$V_3 = \frac{VR_3}{R_1 + R_2 + R_3}$$

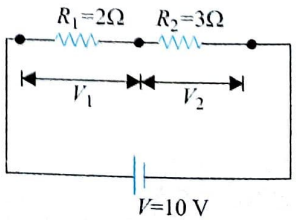
Note:

- If n identical resistances are connected in series, $R_{eq} = nR$ and potential difference across each resistance is $V' = V/n$.
- The potential difference (V_i) across any resistance R_i

$$V_{R_i} = V_{\text{total}}\left[\frac{R_i}{\text{Total resistance}}\right]$$

ILLUSTRATION 5.28

In the circuit given, find the potential difference across the resistances



Sol. The potential difference across R_1,

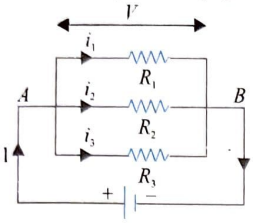
$$V_1 = V\left(\frac{R_1}{R_1 + R_2}\right) = 10\left(\frac{2}{2+3}\right) = 4 \text{ V}$$

The potential difference across R_2,

$$V_2 = V\left(\frac{R_2}{R_1+R_2}\right) = 10\left(\frac{3}{2+3}\right) = 6\text{ V}$$

RESISTANCES IN PARALLEL

If the resistances are connected between the same two points such that the potential drop across each resistance is same, then the resistances are said to be in parallel (figure).



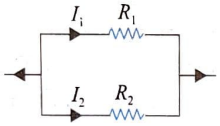
In this case, if V is the potential difference between the points A and B, then

$$V = i_1R_1,\ V = i_2R_2, \text{ and } V = i_3R_3$$

$$\Rightarrow\ i_1 = \frac{V}{R_1},\ i_2 = \frac{V}{R_2} \text{ and } i_3 = \frac{V}{R_3}$$

Therefore, the total current flowing through the battery is

$$i = i_1 + i_2 + i_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$



i.e., $$i = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Also by Ohm's law, $i = V/R$, where R is the equivalent resistance between A and B. So we get

$$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

CURRENT DIVIDER FOR TWO RESISTANCES

From figure, we have

$$I = I_1 + I_2 \qquad ...(i)$$

$$I_1R_1 = I_2R_2 \qquad ...(ii)$$

On solving Eqs. (i) and (ii), we get

$$I_1 = \left(\frac{R_2}{R_1+R_2}\right)I;\quad I_2 = \left(\frac{R_1}{R_1+R_2}\right)I$$

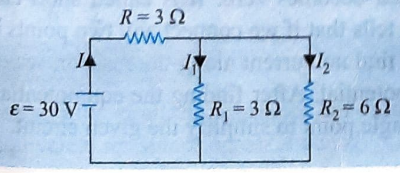
The division of current in the branches of a parallel circuit is inversely proportional to their resistances.

Note: Current through any resistance is

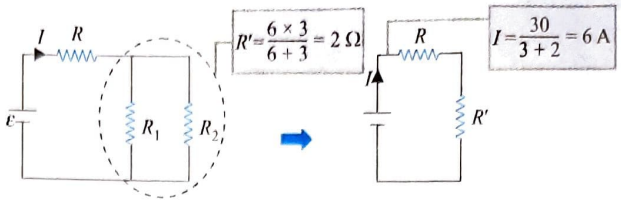
$$i = I_{\text{total}} \times \left[\frac{\text{Resistance of opposite branch}}{\text{Total resistance}}\right]$$

ILLUSTRATION 5.29

In the circuit given, find the currents I, I_1, and I_2 in the circuit.



Sol. The resistances R_1 and R_2 are connected in parallel.



$$\therefore\quad I_1 = I\left[\frac{R_2}{R_1+R_2}\right] = 6\left[\frac{6}{3+6}\right] = 4\text{A}$$

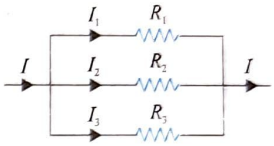
and $$I_2 = I\left[\frac{R_1}{R_1+R_2}\right] = 6\left[\frac{3}{3+6}\right] = 2\text{A}$$

CURRENT DIVIDER FOR THREE RESISTANCES

The division of current in the branches of a parallel circuit is inversely proportional to their resistances (see figure).

$$I = I_1 + I_2 + I_3$$

$$I_1R_1 = I_2R_2 = I_3R_3$$



and $$R_{\text{eq}} = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

$$I_1 = I\left[\frac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

$$I_2 = I\left[\frac{R_1R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

$$I_3 = I\left[\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

It is easy to remember the expressions for I_1, I_2, and I_3. Notice which resistance is missing in the numerator.

In all parts of figure, the resistances R_1, R_2, and R_3 are connected in parallel between the points M and N.

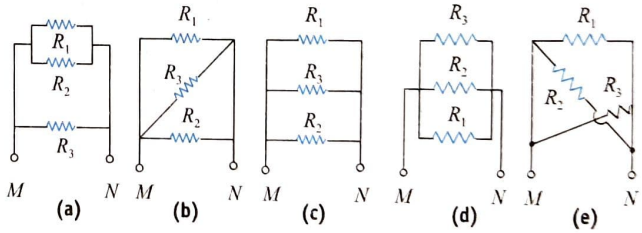
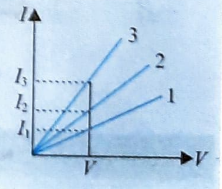


ILLUSTRATION 5.30

The V-I graphs for two resistors and their series combination are shown in figure. Which one of these graphs represents the series combination of the two resistors? Give reason for your answer.



Sol. From figure,

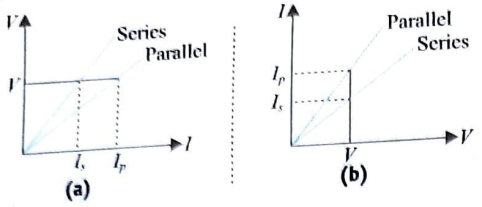
$$R_1 = \frac{V}{I_1},\ R_2 = \frac{V}{I_2},\ R_3 = \frac{V}{I_3}$$

Since $I_1 < I_2 < I_3$, so $R_1 > R_2 > R_3$

The resistance of the series combination, being the sum of the two resistances, is greater than each of the resistances. Since $R_1 > R_2$, $R_1 > R_3$, graph 1 represents the series combination of the two resistors.

ILLUSTRATION 5.31

Two students perform experiments on series and parallel combinations of two given resistors R_1 and R_2 and plot the following V-I graphs (figure). Which of the graphs is/are correctly labeled in terms of the words 'series' and 'parallel'? Justify your answer.



Sol. We know that the resistance (R_s) in series combination of two resistors is more than the resistance (R_p) in parallel combination of the same two resistors. As $I = V/R$, for a given potential (V), $I_s < I_p$. In both the graphs, for a given potential difference (V) across each combination, $I_S < I_P$. Hence, both the graphs are properly labeled.

CALCULATION OF EFFECTIVE RESISTANCE

Method of Successive Reduction

It is the most common method to determine the equivalent resistance. This method is applicable only when we are able to identify resistances in series or in parallel. The method is based on the simplification of the circuit by successive reduction of the series and parallel combinations.

To calculate the equivalent resistance between the points M and N, the network shown in figure, may be successively reduced as described step by step.

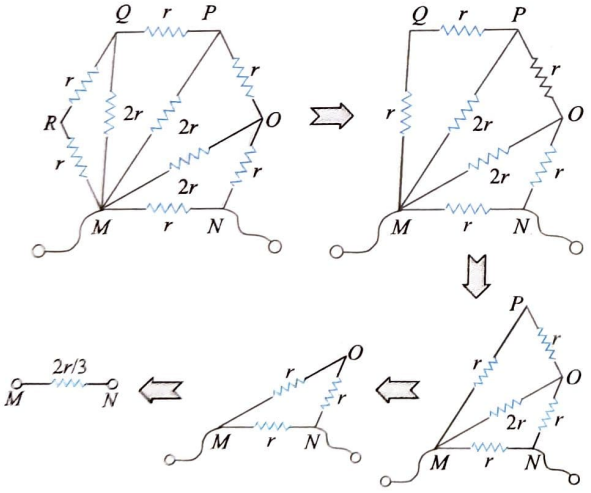
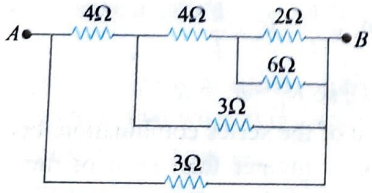
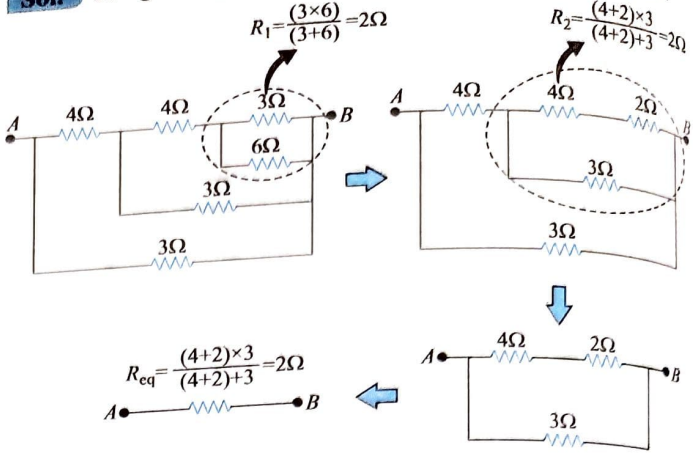


ILLUSTRATION 5.32

In the circuit given, find the effective resistance across A and B.



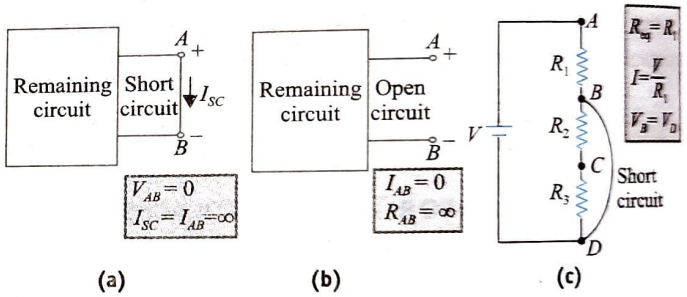
Sol. The given system can be analysed as shown in figure.



SHORT AND OPEN CIRCUITS

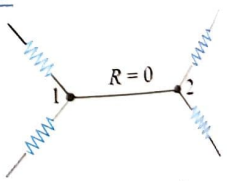
When two points of a circuit are commenced together by a conducting wire, they are said to be **short-circuited**. The connecting wire is assumed to have zero resistance. No voltage can exist across a 'short', and the current through it is very large (theoretically infinity).

Two points are said to be **open-circuited** when there is no direct connection between them; a break in the continuity of the circuit exists. Due to this break, the resistance between the two points is infinite and there is no of current between the two points.

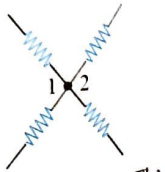


Whole of the applied voltage is felt across the 'open', i.e., across terminals A and B.

EQUIPOTENTIAL POINTS



In a current-carrying electrical network, two points are said to be equipotential if they are at the same potential. Between the points 1 and 2, $V_1 = V_2$, if $\Delta V = iR = 0$.



Then we have two cases: if $R = 0$, $\Delta V = 0$ ($i \neq 0$) and if $i = 0$ (R is finite), $\Delta V = 0$. The first case tells that when we connect any two points by an ideal conductor, the potential difference between them becomes zero. It is called short-circuiting. The second case tells that if we connect any two points by a nonzero resistor and find no current along the resistor, we can call these points equipotential. After finding the equipotential points, join them to a single point to simplify the given circuit.

ELECTRICAL SYMMETRY

If the branches ab and ac have same resistances and same current, same potential will be dropped along them. Hence, the branches ab and ac are electrically symmetrical. In this case, the points b and c are equipotential points. Then you can join these points.

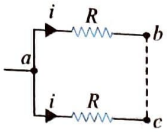
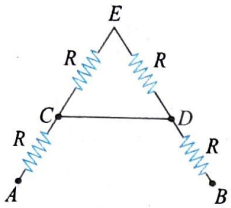


ILLUSTRATION 5.33

Four identical resistances each having value R are arranged as shown in figure. Find the equivalent resistance between A and B.



Sol. Since C and D are connected with a zero resistor, they are equipotential. Then superimpose C and D to obtain the simplified circuit as shown. Since no current flows in the branches CE and ED, cut and then throw them to have

$$R_{AB} = R + R = 2R$$

If we get a closed loop of resistors without any battery, it carries no current. Then remove the total loop to get a simpler circuit or if the current in any branch is zero, remove it.

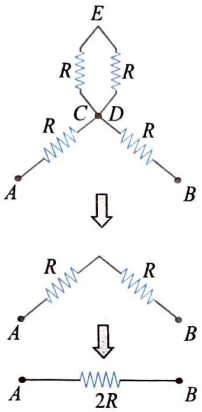
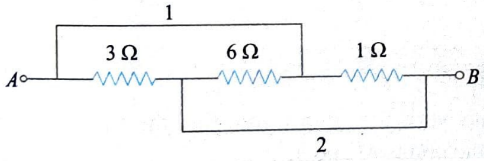
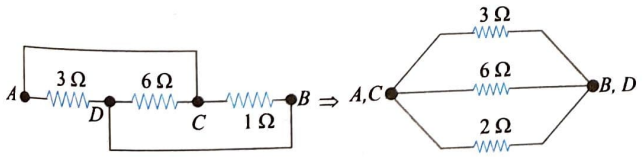


ILLUSTRATION 5.34

Find the R_{AB} in the given network.



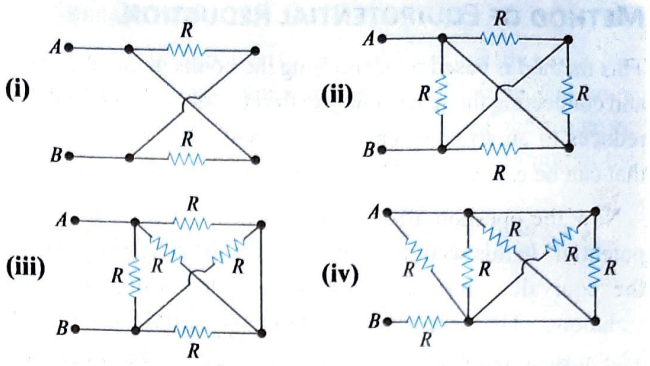
Sol. As A, C and B, D are short-circuited, A and C are at the same potential; B and D are at the same potential. Bringing A and C to one point and B and D to one point, we have redrawn the circuit. You can see that all resistors are in parallel. Then,



$$\frac{1}{R} = \frac{1}{3} + \frac{1}{6} + \frac{1}{2} \quad \text{or} \quad R = 1\ \Omega$$

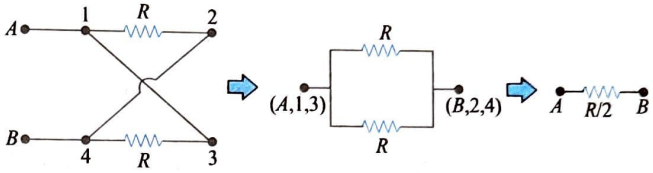
ILLUSTRATION 5.35

In the given circuits, if all the resistance has value equal to R. Find the equivalent resistance across A and B.

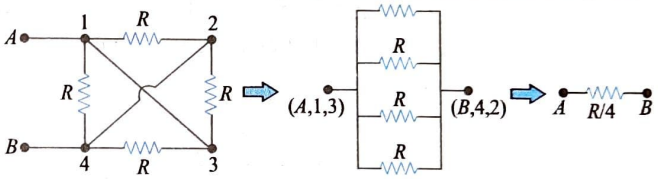


Sol.

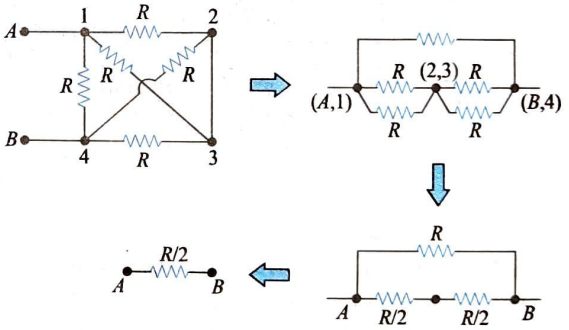
(i) The potentials of the points (A, 1 and 3) and (B, 2 and 4) should be same. We can redraw the circuit to reduce it into a simple circuit.



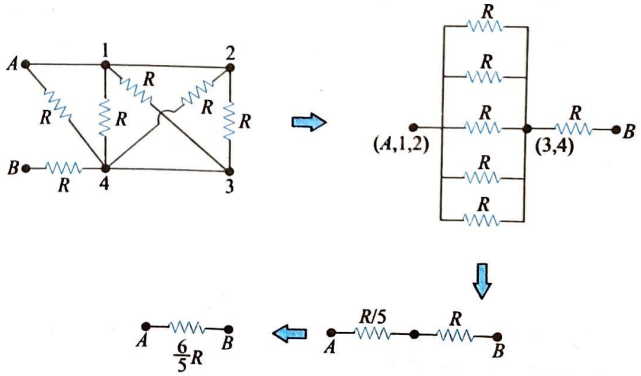
(ii) The potentials of points (A, 1 and 3) and (B, 4 and 2) should be same. We can simplify the circuit as shown in figure.



(iii) The potentials of points (A and 1), (B and 4) and (2 and 3) should be same. Now we can simplify the circuit as



(iv) Here the potentials of points (A, 1 and 2) and (4 and 3) are same. The circuit can be simplify as shown in figure.



METHOD OF EQUIPOTENTIAL REDUCTION

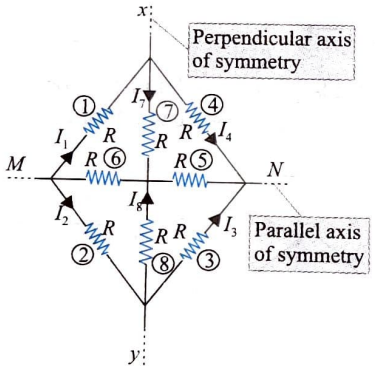
This method is based on identifying the points of equal potential and connecting them. By doing so the electric resistance network reduces to an arrangement of series and parallel combinations that can be easily solved by the successive reduction method.

Now the question arises how to identify the points of same potential? In this section, we will discuss the method to calculate the equivalent resistance and capacitance using symmetry techniques. There are various kinds of symmetry considerations. The most common are

- parallel axis of symmetry
- perpendicular axis of symmetry
- shifted symmetry or shifted asymmetry
- path symmetry

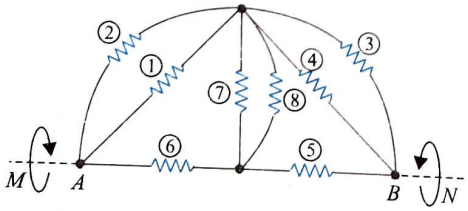
We will discuss each one of these in the following sections:

Parallel axis of symmetry: It is along the direction of current flow. Let us discuss this concept by an example. In the circuit shown in figure, even though the resistors 1 and 2 do not appear to be connected in parallel, but they can be treated as parallel, why? For explaining this, we have to use the concept of symmetry. Note that the circuit is symmetric about the line *MN*. Therefore, all characteristics such as potential and current should also be symmetrical.



From this, it means that current in 1 (I_1) = current in 2 (I_2), current in 3 (I_3) = current in 4 (I_4), current in 7 (I_7) = current in 8 (I_8), hence the potential difference across 1 is equal to the potential difference across 2.

It means if we fold the circuit about *MN*, the resistances (1 and 2), (7 and 8) and (3 and 4) becomes in parallel combination. Now the circuit can be simplify for calculation of effective resistance across *A* and *B*.

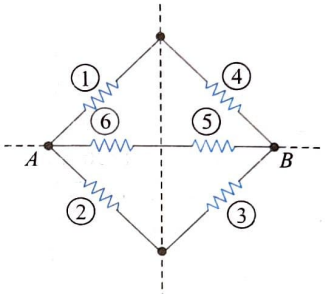


Note that the conditions for 1 and 2 to be in parallel are satisfied. Hence, we can consider them to be in parallel; however, if one of the resistors had a different value, we cannot use this method.

Perpendicular axis of symmetry: It is perpendicular to the direction of flow of current. Consider the circuit diagram given in figure. The circuit has perpendicular axis of symmetry about *XY*. The perpendicular axis of symmetry means that the circuit diagram is symmetric except for the fact that the input and output are reversed. That is only the flow of current will not be a mirror image about this particular axis. For example, in the above case, elements 1 and 4 are symmetric about *XY*, but the current flow condition is not a mirror image. The current flow condition is in the same direction.

This implies that current into 6 = current out of 5, current into 1 = current out of 4, etc. Perpendicular axis of symmetry is a very powerful principle. In fact just by looking at the circuit, we can easily say that since the circuit has perpendicular axis of symmetry about *XY*, no current will flow in elements 7 and 8.

It means if we remove the resistance (7) and (8), there will not be any change in the nature of the circuit. Now the final circuit reduces to a simple circuit for calculation of resistance across *A* and *B*.

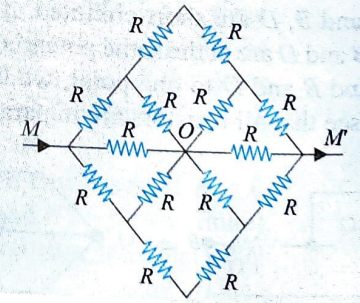


Therefore, we can ignore these two elements completely. In some cases, we may not be able to use symmetry to simplify the circuit, but we can find out some of the characteristics of current/potential based on symmetry. We should always look out for these characteristics and use them as much as possible.

Note: All the points lying on perpendicular axis of symmetry can be treated as equipotential.

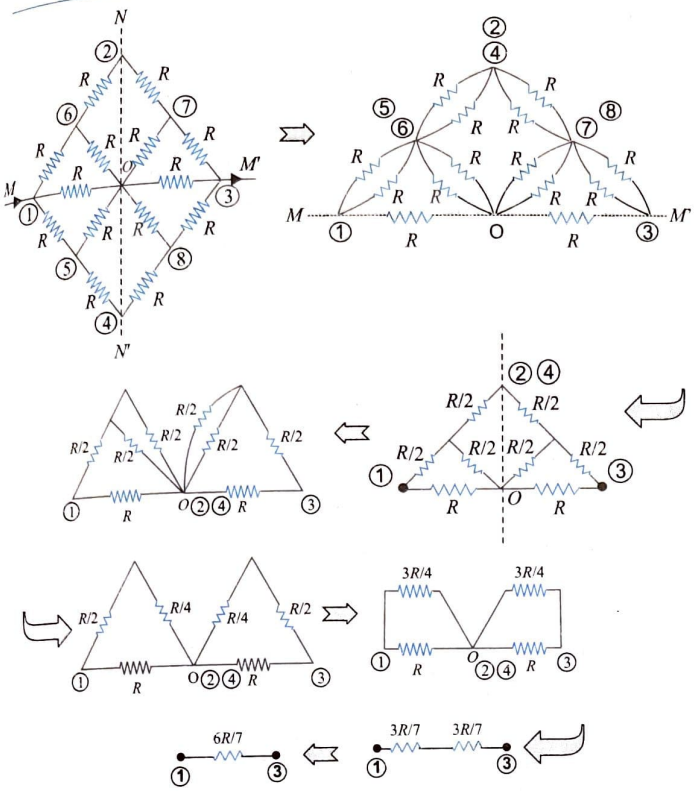
ILLUSTRATION 5.36

In the network shown in figure find the equivalent resistance across the points *M* and *M'*.



Sol.

(i) The axis *MM'* is the parallel axis of symmetry, and the axis *NN'* is the perpendicular axis of symmetry.



(ii) Points lying on the perpendicular axis of symmetry may have same potential. In the given network, points 2, 0, and 4 are at the same potential.

(iii) Points lying on the parallel axis of symmetry can never have the same potential.

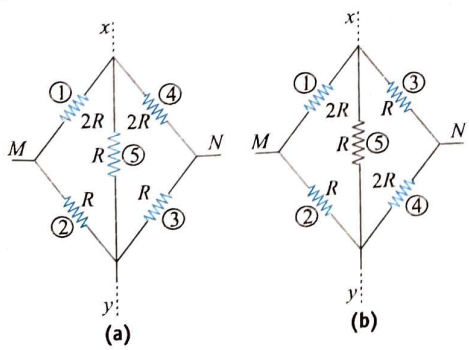
(iv) The network can be folded about the parallel axis of symmetry, and the overlapping nodes have same potential.

Thus, as shown in figure, the following points have same potential: (5 and 6), (2, 0, 4), and (7 and 8). After folding the network about the axis MM', the circuit may be simplified by using the method of successive reduction.

SHIFTED SYMMETRY

Shifted symmetry is the same as the parallel axis of symmetry and the perpendicular axis of symmetry principles, except that the symmetry is shifted (figure).

In Fig. (a), the system has a perpendicular axis of symmetry about xy, so we can say that the current in 1 must be equal to the current out of 4. In Fig. (b), if we interchange the positions of 3 and 4, then the current in 1 is equal to the current in 4 and current in (2) is equal to the current in (3).



(a) (b)

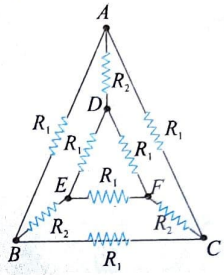
PATH SYMMETRY

Path symmetry is also a very powerful method that one can use. According to path symmetry, if all paths from one point to another point have the same configuration of resistance or capacitance, then the charge or current into the beginning of the path must be the same.

Let us learn the application of this concept through some selected illustrations.

ILLUSTRATION 5.37

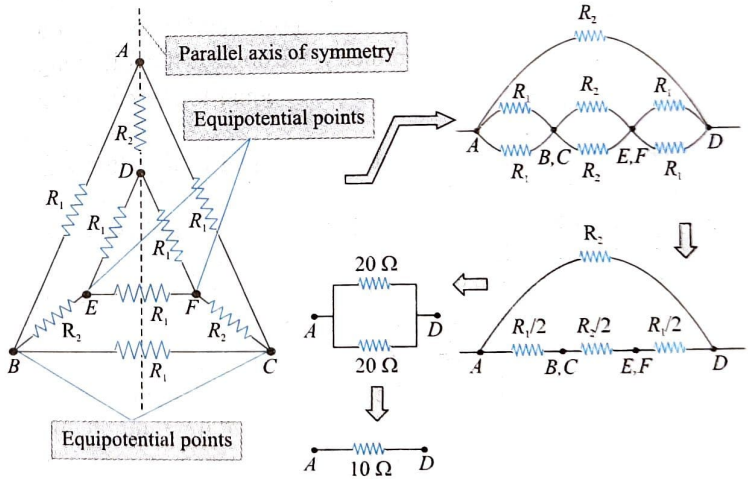
In figure, the resistances are connected as shown. Given $R_1 = 10\ \Omega$ and $R_2 = 20\ \Omega$. Determine the equivalent resistance between points A and D.



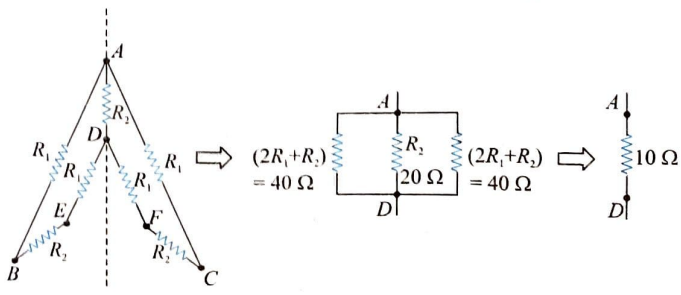
Sol. **Calculation of equivalent resistance between A and D:**

There exists parallel axis of symmetry. The points across the parallel axis of symmetry can be treated as equipotential points.

Method 1: Points B and C, and E and F are at the same potential, so the circuit can be redrawn as shown in figure. Thus, the equivalent resistance is 10 Ω.



Method 2: The points with charge symmetrical to parallel axis are equipotential points. The resistances connected between these points can be removed. A step-by-step procedure for the calculation of equivalent resistance is shown in figure.



Method 3: We can fold the circuit about the parallel axis of symmetry. The step-by-step procedure is shown in figure.

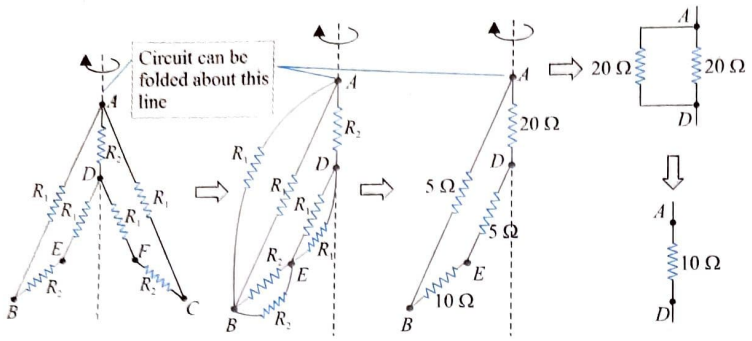
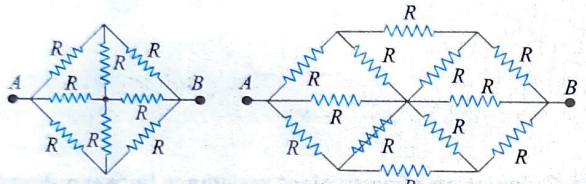


ILLUSTRATION 5.38

In the given circuits [Figs. (a) and (b)], calculate the resistance between points A and B.



Sol.

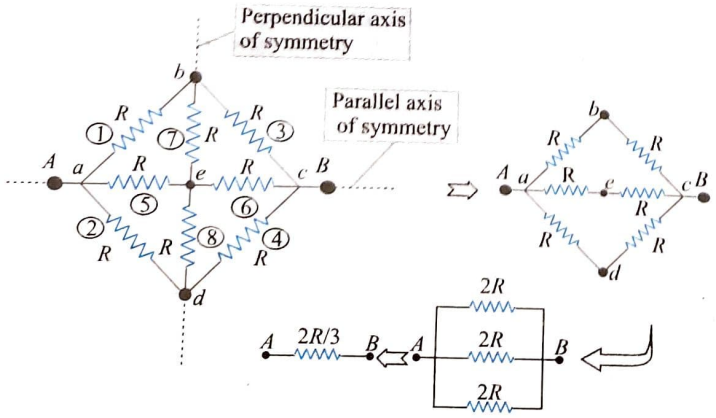
(a) The circuit has both parallel axis and perpendicular axis of symmetry (by symmetry we mean that the two parts are mirror images of each other).

Method 1: Let us solve this problem first using the perpendicular axis of symmetry. We observe the following:

- The current in resistance (1) is equal to the current in resistance (3).
- The current in resistance (5) is equal to the current in resistance (6).
- The current in resistance (2) is equal to the current in resistance (4).

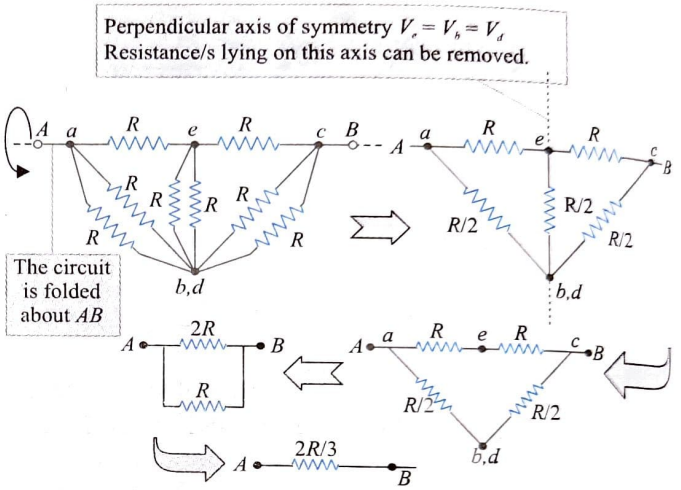
From these observations it is clear that there is no mingling of current from upper and lower branches into the middle branch. Hence, resistors (7) and (8) are ineffective.

We also observe that all the points on the symmetry line are equipotential. Hence, points b, e, and d are equipotential points, and so resistors (7) and (8) can be removed. The calculation of effective resistance is shown in figure.



The current flow is not a mirror image in branches ab and bc because the flow is in the same direction. This is called asymmetric condition. The special thing about this asymmetry is that current incoming at b is equal to the outgoing current. A similar situation exists at b and d also. Thus, resistors in branches be and de are ineffective.

Method 2: The circuit has parallel axis symmetry about the line ac, so the potential and current must also be symmetrical. Therefore, currents in ab and ad are same. Currents in dc and bc are also same. Potentials of the points b, e, and d are same. The circuit can be folded about the parallel axis of symmetry. The calculations of equivalent resistance are shown in figure.

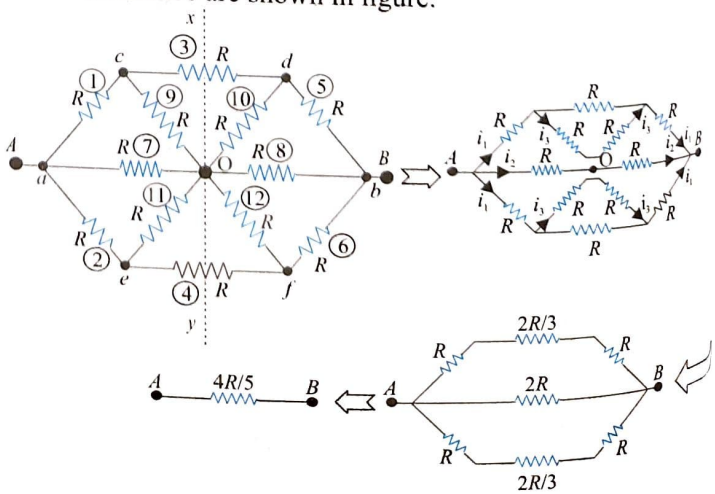


(b) The current has parallel axis of symmetry about the line passing through a and b and perpendicular axis of symmetry about the line x-y.

Method 1: From perpendicular axis of symmetry, it is clear that

- The current in resistance (7) is equal to the current in resistance (8).
- The current in resistance (9) is equal to the current in resistance (10).
- The current in resistance (11) is equal to the current in resistance (12).

From these observations, it is clear that there is no mingling of currents from upper and lower branch into the middle branch. The upper and lower branches can be separated from the middle branch. The calculations of equivalent resistance are shown in figure.



Method 2: We can observe the parallel axis symmetry. The potentials of c and e and that of d and f should be equal. Hence, the circuit can be folded about the line passing through A and B. The calculations of equivalent resistance are shown in figure. As shown in the figure, finally $2R$ and $4R/3$ are in parallel. Hence, the equivalent resistance will be

$$R_{eq} = \frac{2R \times \frac{4R}{3}}{\frac{10R}{3}} = \frac{4R}{5}$$

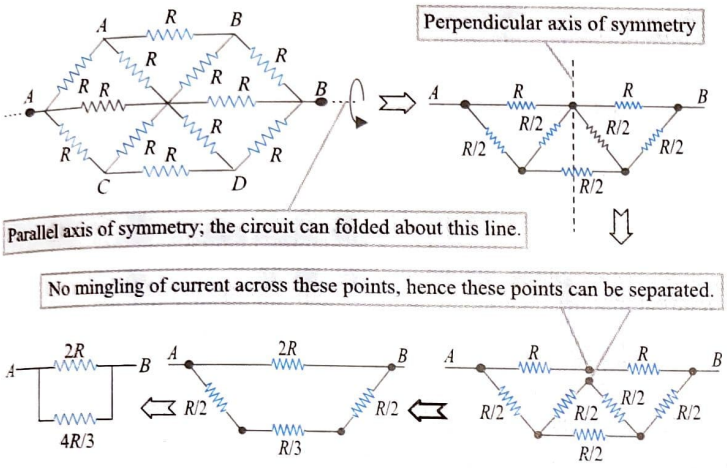
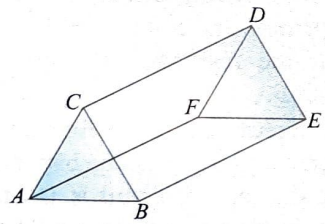


ILLUSTRATION 5.39

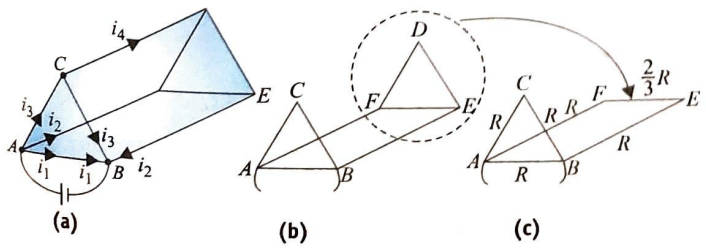
Nine resistances each of magnitude R are connected to form prism shaped network of resistors as shown in the figure. Find the equivalent resistance of the network between

(a) A and B **(b)** C and D.



Sol.

(a) The resistance across A and B: If we connect a battery across A and B, it can be observed that the points A and B are identical. The distribution of the current entering at point A should be same way as the current coming out from point B as shown in Fig. (a). We can say the points A and B has end point symmetry. It means at junction C, there should not be any distribution current to the branch CD. It permits us to remove resistance CD without changing the nature of the circuit, as shown in Fig. (b). Now we get a simplify circuit which can be analyze easily.



(a) (b) (c)

Net resistance across EF, $R_{FE} = \frac{R \cdot 2R}{R+2R} = \frac{2R}{3}$

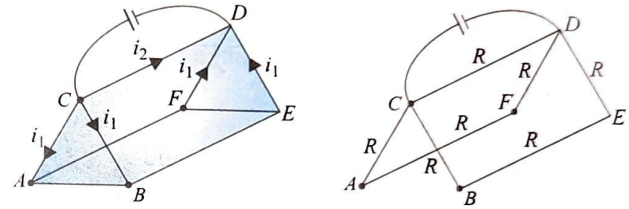
Net resistance of the section $AFEB$,

$$R_{AFEB} = R + \frac{2R}{3} + R = \frac{8R}{3}$$

Now the resistance AB, ACB and $AFEB$ are connected in parallel, hence required equivalent resistance across AB

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{8R/3} = \frac{15}{8R} \Rightarrow R_{eq} = \frac{8R}{15}$$

(b) The resistance across C and D: If we connect a battery across C and D, here the points C and D has end point symmetry. Also, the currents in the resistances CA and CB will be equal. It means the points A and B should be equipotential. Similarly, E and F are equipotential. So here we can remove the resistances AB and EF without changing the nature of circuit. Now, we left with a series parallel circuit



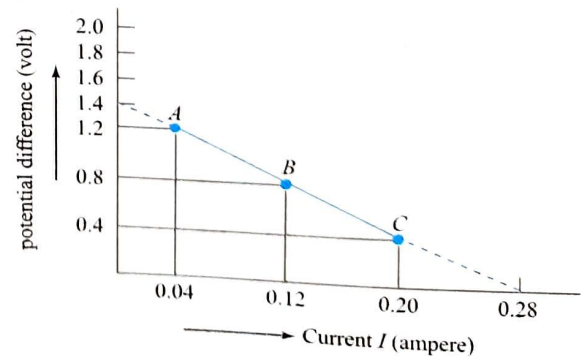
(a) (b)

Now the resistance CD, $CAFD$ and $CBED$ are connected in parallel, hence required equivalent resistance across CD

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{3R} + \frac{1}{3R} = \frac{5}{3R} \Rightarrow R_{eq} = \frac{3R}{5}$$

CONCEPT APPLICATION EXERCISE 5.3

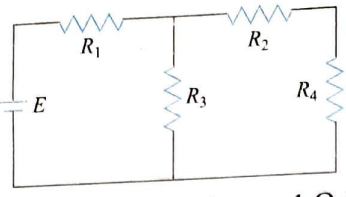
1. A battery of emf 2 V and internal resistance 0.1 Ω is being charged with a current of 5 A. In what direction will the current flow inside the battery? What is the potential difference between the two terminals of the battery?
2. Potential difference across the terminals of a cell was measured (in volts) against different currents (in ampere) flowing through the cell. A graph was drawn, which was a straight line ABC. Using the data given in the graph (see figure), determine



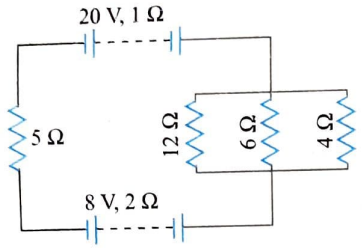
(a) the emf, and

(b) the internal resistance of the cell.

3. Determine the voltage drop across the resistor R_1 in the circuit given below with $E = 65$ V, $R_1 = 50\ \Omega$, $R_2 = 100\ \Omega$, $R_3 = 100\ \Omega$, and $R_4 = 300\ \Omega$.



4. A 20 V battery of internal resistance 1 Ω is connected to three coils of 12 Ω, 6 Ω, and 4 Ω in parallel, a resistor of 5 Ω and a reversed battery (emf 8 V and internal resistance 2 Ω) as shown in figure.



Calculate

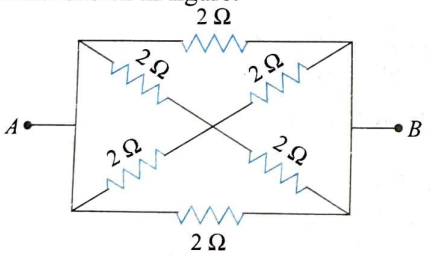
(a) the current in the circuit,

(b) current in resistor of 12 Ω coil, and

(c) potential difference across each battery.

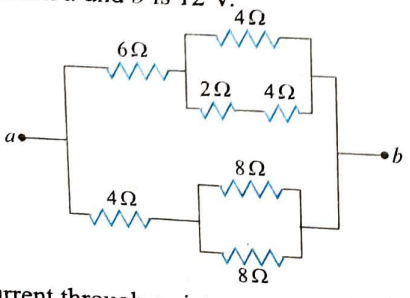
5. The current in a simple series circuit is 5 A. When an additional resistance of 2 Ω is introduced, the current is reduced to 4 A. Calculate the resistance of the original circuit. Assume that the applied potential difference is the same in both the cases.

6. Calculate the resistance between the terminals A and B of the network shown in figure.



7. A parallel combination of an 8 Ω resistor and an unknown resistor R is connected in series with a 16 Ω resistor and a battery. This circuit is then disassembled, and the three resistors are then connected in series with each other and the same battery. In both arrangements, the current through the 8 Ω resistor is the same. What is the unknown resistance R?

8. For the resistor network shown in figure, the potential drop between a and b is 12 V.

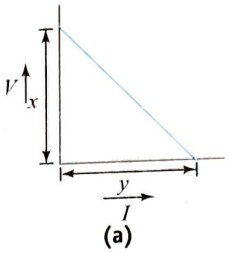


(a) Current through resistance of 6 Ω is _____.

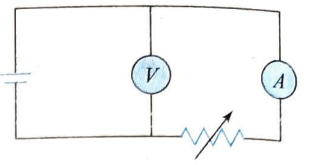
(b) Current through resistance of 2 Ω is _____.

(c) Current through resistance of 8 Ω is _____.

9. In an experiment, a graph (figure) was plotted of the potential difference V between the terminals of a cell against circuit current I by varying load rheostat. Find the internal conductance of the cell.

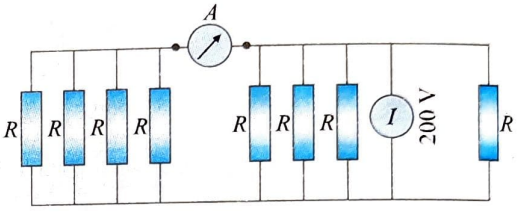


(a)



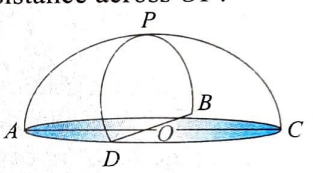
(b)

10. Each resistance in the circuit (Figure) is of 2000 Ω.

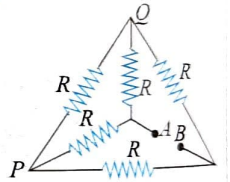


The combination is put across a supply of 200 V. Find the reading of ammeter.

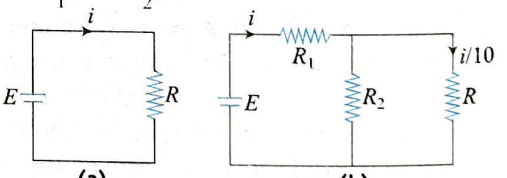
11. A hemispherical network of radius a is made by using a conducting wire of resistance per unit length r. Find the equivalent resistance across OP.



12. If each of the resistances in the shown network is R, then what is the resistance between A and B.

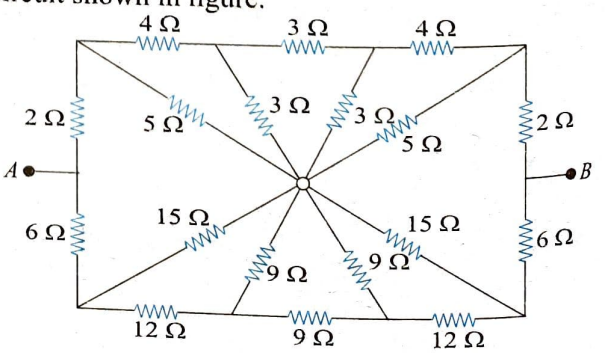


13. Consider the circuits shown in figure. Both the circuits are taking same current from the battery, but the current through R in the second circuit is 1/10th of the current through R in the first circuit. If R is 11 Ω, then find the value of R_1 and R_2.

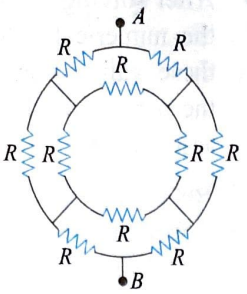


(a) (b)

14. Calculate equivalent resistance between A and B of the circuit shown in figure.



15. In the given network of resistance, if all the resistances have value equal to R. Find the equivalent resistance across points A and B.

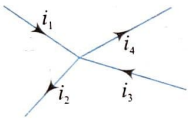


ANSWERS

1. From positive terminal to the negative terminal, 2.5 V
2. (a) 1.4 V (b) 5 Ω **3.** 25 V
4. (a) 1.2 A (b) 0.2 A (c) 18.8 V, 10.4 V **5.** 8 Ω
6. $\frac{2}{3}\Omega$ **7.** $\sqrt{128}\,\Omega$ **8.** (a) $\frac{10}{7}$A (b) $\frac{4}{7}$A (c) $\frac{3}{4}$A
9. $\frac{x}{y}$ **10.** 0.4 A **11.** $\frac{ar}{8}(2+\pi)$ **12.** R
13. $R_1 = 9.9\,\Omega$, $R_2 = 11/9\,\Omega$ **14.** 6.75 Ω **15.** $\frac{5R}{4}$

KIRCHHOFF'S LAW FOR ELECTRICAL NETWORKS

Kirchhoff's First Law: This law is also known as junction rule or current law (*KCL*). According to it, the algebraic sum of currents meeting at a junction is zero, i.e., $\Sigma i = 0$.



In a circuit, at any junction the sum of the currents entering the junction must equal the sum of the currents leaving the junction, i.e.,

$$I_1 + i_3 = i_2 + i_4$$

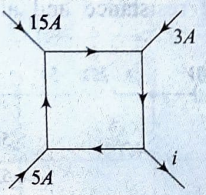
Here it is worthy to note that

(i) If a current comes out to be negative, the actual direction of current at the junction is opposite to that assumed. $I + i_1 + i_2 = 0$ can be satisfied only if at least one current is negative, i.e., leaving the junction.

(ii) This law is simply a statement of conservation of charge as if current reaching a junction is not equal to the current leaving the junction, charge will not be conserved.

ILLUSTRATION 5.40

Figure shows a network of currents. The magnitude of the current is shown here. Find the current i.

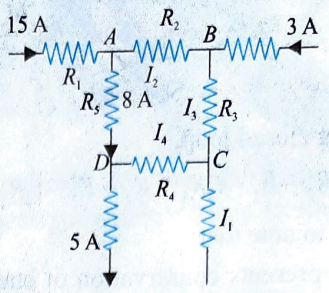


Sol. The net incoming current in circuit is 15 + 3 + 5 = 23 A.

As incoming current in circuit = Outgoing current from circuit, hence i = 23 A.

ILLUSTRATION 5.41

Calculate the values of currents I_1, I_2, I_3, and I_4 in the section of networks shown in figure.



Sol. The distribution of currents in different resistances can be calculated using Kirchhoff's junction rule. We will apply Kirchhoff's junction rule at junctions A, B, C, and D.

At junction '*A*': The current through resistance R_2 is unknown. Let the current I_2 is leaving the junction.

It means: $I_2 + 8 = 15 \Rightarrow I_2 = 15 - 8 = 7$ A

At junction '*B*': The unknown current at this junction is I_3. Let this current is leaving the junction.

It means: $I_3 + 3 = I_3 \Rightarrow I_3 = 7 + 3 = 10$ A

At junction '*D*': The unknown current at this junction is I_4. Let this current is leaving the junction.

It means: $8 = 5 + I_4 \Rightarrow I_4 = 3$ A

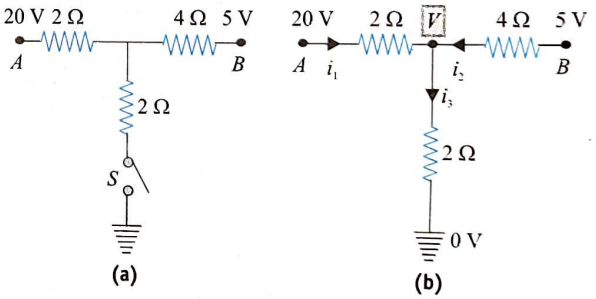
At junction '*C*': Let the unknown current I_1 is leaving the junction.

Hence $I_4 + I_3 = I_1 \Rightarrow 3 + 10 = I_1$ or $I_1 = 13$ A

ILLUSTRATION 5.42

Figure (a) shows three resistances are connected with switch S initially the switch is open. When the switch S is closed find the current passed through it.

Sol. Let V be the potential of the junction as shown in Fig. (b).



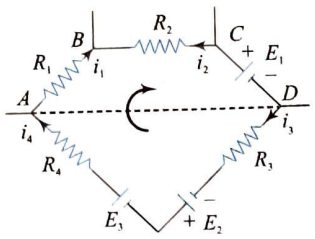
Applying junction law, we have

$$\frac{20 - V}{2} + \frac{5 - V}{4} = \frac{V - 0}{2}$$

or $40 - 2V + 5 - V = 2V$ or $5V = 45$ or $V = 9V$

$$\therefore \quad i_3 = \frac{V}{2} = 4.5\,\text{A}$$

Kirchhoff's second law: This law is also known as loop rule or voltage law (*KVL*) and according to it "the algebraic sum of the changes in potential in complete traversal of a mesh (closed loop) is zero", i.e., $\Sigma V = 0$.



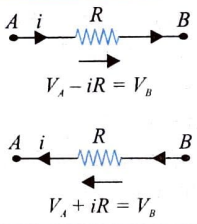
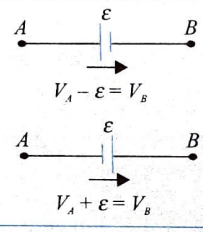
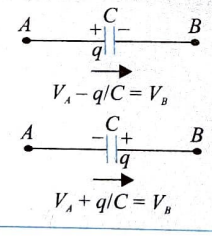
In the following closed loop,

$$-i_1R_1 + i_2R_2 - E_1 - i_3R_3 + E_2 + E_3 - i_4R_4 = 0$$

Here it is worthy to note that

- This law represents conservation of energy as if the sum of potential changes around a closed loop is not zero, unlimited energy could be gained by repeatedly carrying a charge around a loop.
- If there are n meshes in a circuit, the number of independent equations in accordance with loop rule will be $(n-1)$.

Sign convention for the application of Kirchhoff's law: For the application of Kirchhoff's laws, the following sign conventions are to be considered.

The change in potential in traversing a resistance in the direction of the current is $-iR$, while in the opposite direction it is $+iR$.	
The change in potential in traversing an emf source from negative to positive terminal is $+\varepsilon$, while in the opposite direction it is $-\varepsilon$ irrespective of the direction of current in the circuit.	
The change in potential in traversing a capacitor from the negative terminal to the positive terminal is $+q/C$, while in the opposite direction it is $-q/C$.	

GUIDELINES FOR APPLYING KIRCHHOFF'S LAW

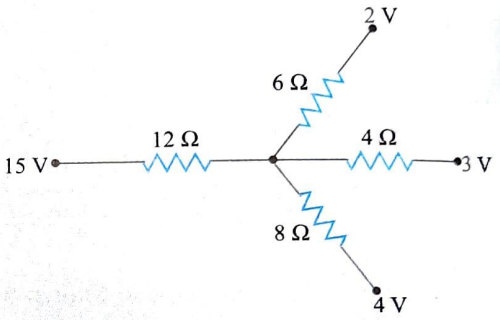
- Starting from the positive terminal of the battery having highest emf, distribute current at various junctions in the circuit in accordance with *junction rule*. It is not always easy to correctly guess the direction of current, so there is no problem if one assumes the wrong direction.
- After assuming the current in each branch, we pick a point and begin to walk (mentally) around a closed loop. As we traverse each resistor, capacitor, inductor, or battery, we must write down the voltage change for that element according to the above sign convention.
- By applying *KVL*, we get one equation, but in order to solve the circuit, we require as many equations as there are unknowns. So we select the required number of loops and apply Kirchhoff's voltage law across each such loop.
- After solving the set of simultaneous equations, we obtain the numerical values of the assumed currents. If any of these values comes out to be negative, it indicates that the particular current is in the opposite direction from the assumed one.

Note:

- The number of loops must be selected so that every element of the circuit must be included in at least one of the loops.
- While traversing through a capacitor or battery, we do not consider the direction of current.
- While considering the voltage drop or gain across an inductor, we always assume current to be in increasing function.

ILLUSTRATION 5.43

Find the current through 12 Ω resistor in figure.



Sol. Let V be the potential at P; then applying *KCL* at junction P, we get

$$I = I_1 + I_2 + I_3 \Rightarrow \frac{15-V}{12} = \frac{V-2}{6} + \frac{V-3}{4} + \frac{V-4}{8}$$

or $$V = \frac{68}{15}\text{ V}$$

and $$I = \frac{15-(68/15)}{12} = \frac{157}{180}\text{ A}$$

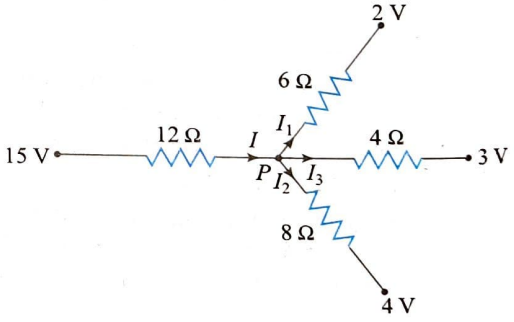
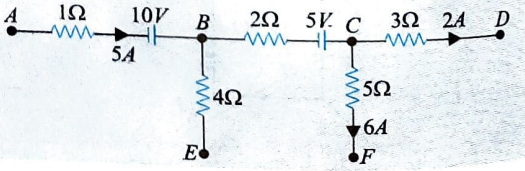
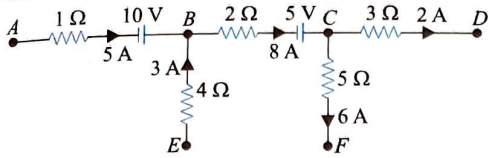


ILLUSTRATION 5.44

The figure given below is a part of a circuit. Calculate the current through 4Ω resistance and also find the potential difference $V_A - V_D$?



Sol. If we Apply KCL at junction C, current in section BC should be 8 A from B to C. Again, using KCL at terminal B the current in 4Ω resistance should be 3 A towards B.



The potential difference across points A and D can be given by writing equation of potential drop from terminal A to D which can be written as

$$V_A - 5\times1 + 10 - 8\times2 + 5 - 3\times2 = V_D$$

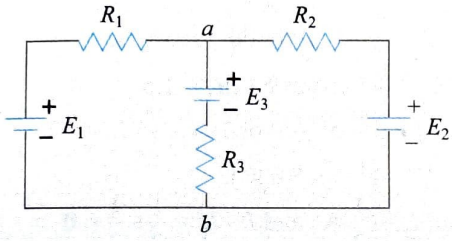
$$\Rightarrow \quad V_A - V_D = 5 - 10 + 16 - 5 + 6 \Rightarrow V_A - V_D = 11\text{ V}$$

ILLUSTRATION 5.45

Calculate the current through each resistance in the given circuit (see figure). Also calculate the potential difference between the points a and b.

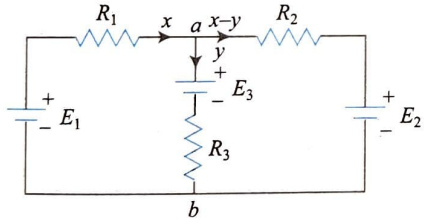
$E_1 = 6$ V, $E_2 = 8$ V, $E_3 = 10$ V,
$R_1 = 5\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 4\ \Omega$

Assume that all the cells have no internal resistance.



Sol. The process of solving a circuit involves three steps:

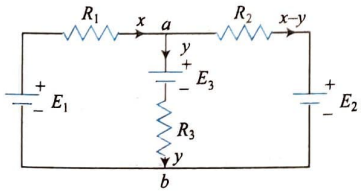
(i) Assume unknowns (x, y, ...) for currents in different branches of the circuit. Use Kirchhoff's current law at the junctions so that the number of unknowns introduced is minimum. Let x be the current through R_1 and y be the current through R_3 as shown in figure. Kirchhoff's current law at the junction a gives a current $(x - y)$ through R_2.



(ii) Select as many loops as the number of unknowns introduced for currents. Apply Kirchhoff's voltage law through every loop. Going anticlockwise through the loop containing R_1 and R_3 (starting from junction a), we get

$$+x\,R_1 - E_1 + y\,R_3 + E_3 = 0$$

or $\quad 5x + 4y = -4 \qquad \ldots(i)$



Going clockwise through the loop containing R_2 and R_3 (starting from junction a), we get

$$-R_2(x - y) - E_2 + yR_3 + E_3 = 0$$

or $\quad 5x - 7y = 1 \qquad \ldots(ii)$

Some currents may come out to be negative. This simply means that their directions were incorrectly assumed. So the signs of the currents will give us the correct direction of each current. Solving Eqs. (i) and (ii), we get

$$x = \frac{-24}{55}\text{ A} \quad \text{and} \quad y = \frac{-5}{11}\text{ A}$$

$$\therefore \quad x - y = \frac{1}{55}\text{ A}$$

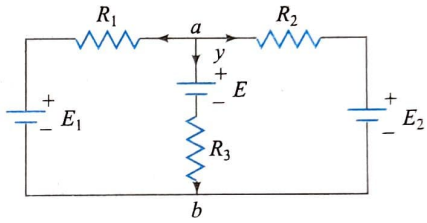
The signs indicate that the directions of x and y were assumed incorrectly, while the direction of $(x - y)$ was correct. So

current i_1 (through R_1) $= \dfrac{24}{55}$ A toward left

current i_2 (through R_2) $= \dfrac{1}{55}$ A toward right

current i_3 (through R_3) $= \dfrac{5}{11}$ A upward

The current directions are shown in the circuit diagram figure.



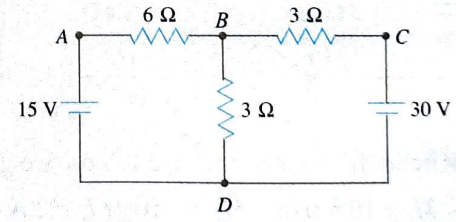
(iii) Potential difference between a and b

The potential difference between any two points in a circuit is calculated by adding the changes in potential while going through any path from one point to the other point. Hence, let us go from b to a through R_3.

$$V_a - V_b = +yR_3 + E_3 = \left(\frac{-5}{11}\right)\times4 + 10 = \frac{90}{11}\text{ V}$$

ILLUSTRATION 5.46

In the circuit shown in figure, find the current through the branch BD.



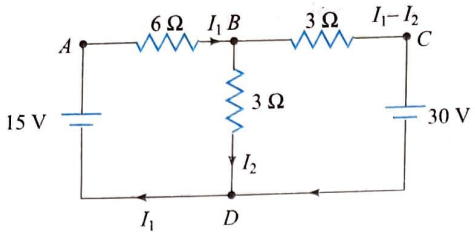
Sol. To find the current in a particular branch in a circuit where resistances and voltages are given, the lower loop rule is applicable. In order to find out the solution, the given circuit is directed into parts or closed loops, which are analyzed using Kirchhoff's second law and corresponding sign conventions.

Assume the currents in the circuit as shown in figure. Applying KVL along the loop $ABDA$ and moving in the clockwise direction, we get

$$-6I_1 - 3I_2 + 15 = 0$$

or $\quad 2I_1 + I_2 = 5 \qquad \ldots(i)$

While moving in the direction of the current corresponding IR, products are taken as negative and if the negative terminal of the battery comes first, then emf is taken as positive (figure).



Applying KVL along the loop $BCDB$, we get

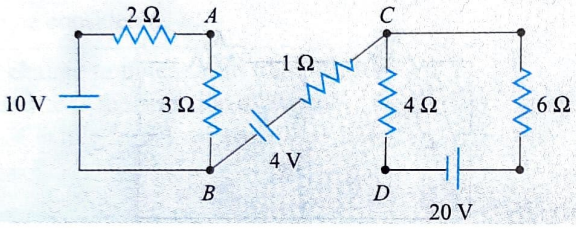
$$-3(I_1 - I_2) - 30 + 3I_2 = 0$$

or $$-I_1 + 2I_2 = 10 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii) for I_2, we get $I_2 = 5\,\text{A}$.

ILLUSTRATION 5.47

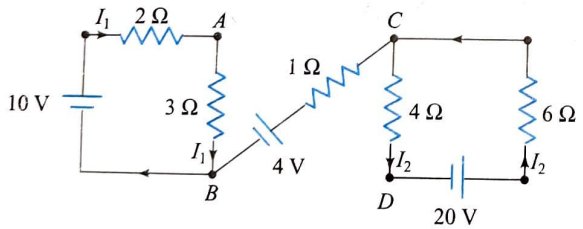
In the circuit shown in figure, determine the voltage drop between A and D.



Sol. We need to divide this circuit into three parts. We have left loop, right loop, and the central part. To find out voltage drop between points A and D, we have to apply Kirchhoff's second law to these circuits one by one.

Also let us assume that current I_1 flows in the left circuit in the clockwise direction and current I_2 flows in the right circuit in the anticlockwise direction.

Direction of both the currents is decided by the battery present in the circuit as current flows from positive to negative.



Applying Kirchhoff's second law to left loop, we get

$$-2I_1 - 3I_1 + 10 = 0 \text{ or } -5I_1 = -10 \text{ or } I_1 = 2\,\text{A}$$

Now applying Kirchhoff's second law to right loop, we get

$$4I_2 + 6I_2 - 20 = 0 \text{ or } 10I_2 = 20 \text{ or } I_2 = 2\,\text{A}$$

Applying the law between points A and D, we get

$$V_A - 3I_1 + 4 - 4I_2 = V_D \text{ or } V_A - V_D = 3I_1 + 4I_2 - 4$$

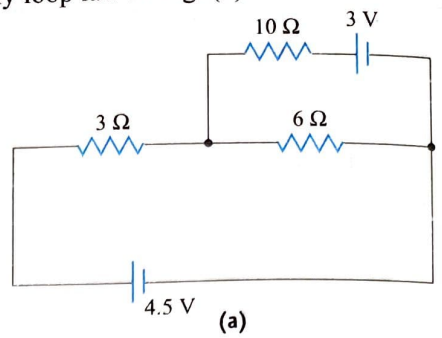
Putting values of I_1 and I_2 in the above expression, we get

$$V_A - V_D = 3 \times 2 + 4 \times 2 - 4 = 6 + 8 - 4 = 10\,\text{V}$$

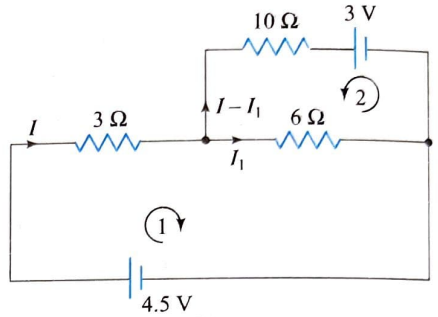
ILLUSTRATION 5.48

Find the current in each part of the circuit given in Fig. (a).

Sol. Apply loop law in Fig. (b).



(a)



(b)

(1) $-3I - 6I_1 + 4.5 = 0$ or $I + 2I_1 = 1.5$ ...(i)

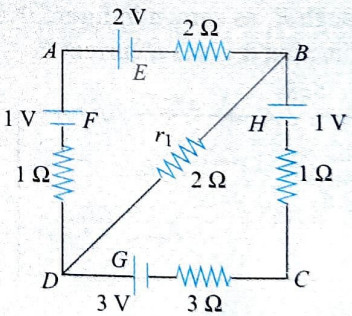
(2) $10(I - I_1) + 3 - 6I_1 = 0$ or $10I - 16I_1 = -3$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$I = \frac{1}{2}\,\text{A},\ I_1 = \frac{1}{2}\,\text{A},\ \text{and } I - I_1 = \frac{1}{2} - \frac{1}{2} = 0$$

ILLUSTRATION 5.49

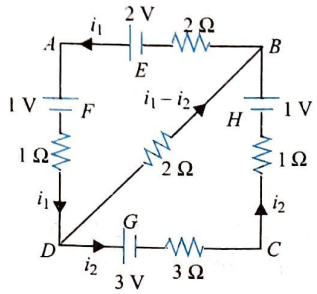
In the circuit shown in figure, E, F, G, and H are cells of emf 2, 1, 3, and 1 V, respectively. The resistances 2 Ω, 1 Ω, 3 Ω, and 1 Ω are their respective internal resistances.



(i) Find the potential difference between B and D.

(ii) Calculate the potential differences across the terminals of each of the cells G and H.

Sol. The circuit with the currents shown is redrawn in figure. Applying the loop law to $BADB$, we get



$$-2i_1 + 2 - 1 - 1i_1 - 2(i_1 - i_2) = 0 \text{ or } -5i_1 + 2i_2 = -1 \quad \text{...(i)}$$

Applying the same law to loop $DCBD$, we get

$$-3-3i_2-1i_2+1+2(i_1-i_2)=0 \text{ or } 2i_1-6i_2=2 \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$i_1=\frac{1}{13}\text{ A}, i_2=-\frac{4}{13}\text{ A} \quad \therefore \quad i_1-i_2=\frac{5}{13}\text{ A}$$

(i) $V_D-V_B=(2\ \Omega)\left(\frac{5}{13}\text{ A}\right)=\frac{10}{13}\text{ V}$

(ii) Potential differences across the cell G is

$$V_D-V_C=3+3i_2=3+3(-4/13)=27/13\text{ V}$$

Potential difference across the cell H is

$$V_B-V_C=1-i_2=1+4/13=17/13\text{ V} \quad (\because V=E+ir)$$

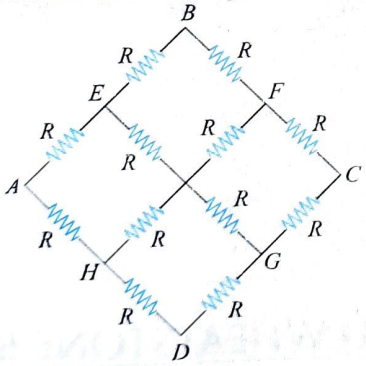
DETERMINATION OF EQUIVALENT RESISTANCE BY KIRCHHOFF'S METHOD

This method is useful when we are not able to identify any two resistances in series or in parallel. It is based on Kirchhoff's laws. The method may be described in the following steps:

- Assume an imaginary battery of emf E connected between the two terminals across which we have to calculate the equivalent resistance.
- Assume some value of current, say i, coming out of the battery and distribute it among each branch by applying Kirchhoff's current law.
- Apply Kirchhoff's voltage law to formulate as many equations as there are unknowns. It should be noted that at least one of the equations must include the assumed battery.
- Solve the equations to determine the E/i ratio, which is the equivalent resistance of the network.

ILLUSTRATION 5.50

In the following network of 12 identical resistances, calculate the equivalent resistance between points A and C.



Sol. **Step 1:** An imaginary battery of emf ε is assumed across the terminals A and C.

Step 2: The current in each branch is distributed by assuming $4i$ current coming out of the battery.

Step 3: We apply KVL along the outer loop including the nodes A, B, C, and the battery E. Voltage equation is

$$-2iR-iR-iR-2iR+E=0$$

Step 4: After solving the above equation, we get $6iR = E$. The equivalent resistance between A and C is

$$R=\frac{E}{4i}=\frac{6iR}{4i}=\frac{3}{2}R$$

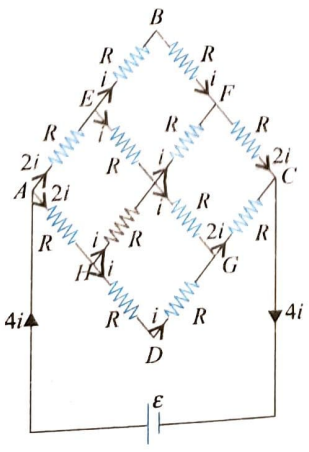
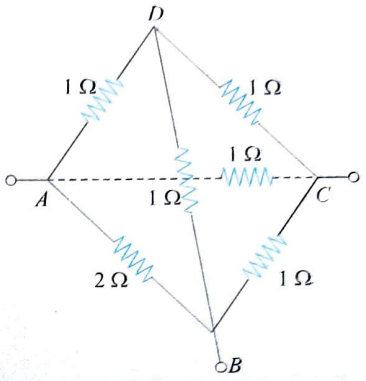
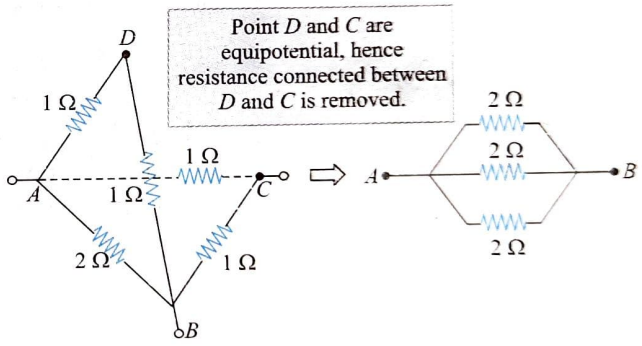


ILLUSTRATION 5.51

Six resistors are arranged along the edges of a pyramid as shown in figure. The values of resistances are mentioned with resistances in the figure. Find the effective resistance between A and B.

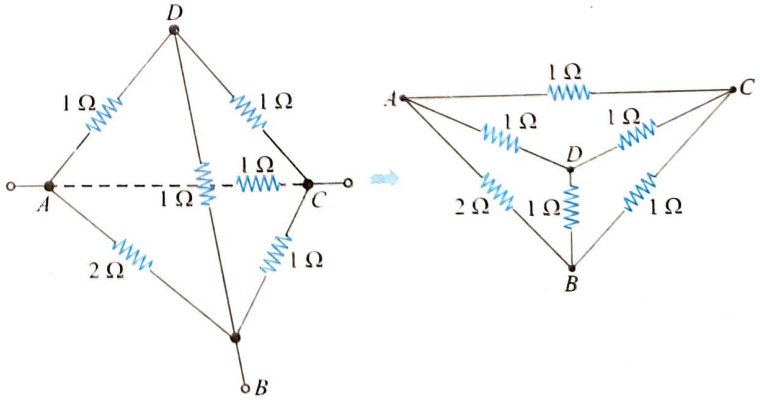


Sol. The branches ADB and ACB are symmetrical relative to the terminals A and B. Hence, the points D and C are equipotential. Since $R_{DC}\neq 0$, $i_{DC}=0$. Then remove the branch DC and then the circuit is reduced to a simpler one as shown in the figure. Then,

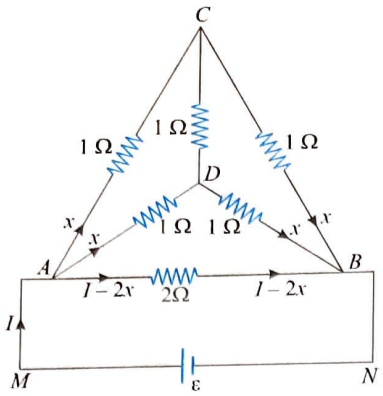


$$\frac{1}{R_{AB}}=\frac{1}{2}+\frac{1}{2}+\frac{1}{2} \qquad R_{AB}=\frac{2}{3}\Omega$$

Method 2: We can redraw the circuit as shown in the figure.



For calculating resistance across A and B, let us connect an imaginary battery of emf ε across the terminals A and B. We can distribute the currents in different branches using symmetry properties.



Applying Kirchhoff's loop law in loop $ADBA$,

$$-x.1-x.1+(I-2x)=0$$

$$\Rightarrow \quad x=\frac{I}{3} \qquad \text{...(i)}$$

Now considering the loop $ABNMA$

$$-(I-2x)\cdot 2+\varepsilon=0$$

or $$\varepsilon=2(I-2x)=0 \qquad \text{...(ii)}$$

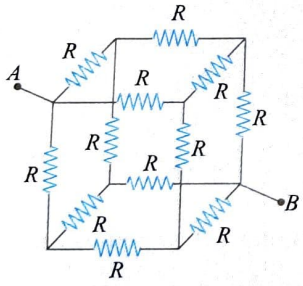
From (i) and (ii), $\varepsilon=2\left(I-2\cdot\frac{I}{3}\right)=\frac{2I}{3}$

or $$\frac{\varepsilon}{I}=\frac{2}{3}\Omega=R_{eq}$$

Hence equivalent capacity across A and B is $\frac{2}{3}\Omega$.

ILLUSTRATION 5.52

Twelve identical resistances are arranged on all edges of a cube. The resistors are all the same. Then find the equivalent resistance between the edges A and B as shown in figure.

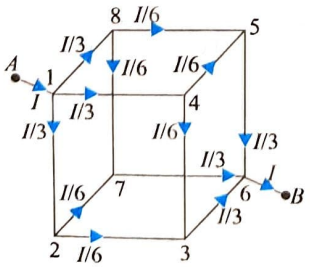


Sol. **Method 1:** Let us number the corners as 1, 2, 3, ... as shown in figure. Let us identify various paths from A to B.

$a\,(1-8-7-6)$, $b(1-2-7-6)$, $c(1-8-5-6)$,
$d(1-2-3-6)$, $e(1-4-5-6)$, $f\,(1-4-3-6)$

Now for each of these paths we have identical resistances. Let the current entering at point A be I. The current will be distributed equally among all the three paths. Therefore, we can say that current is

$$I_{12}=I_{14}=I_{18}=I/3.$$



Now look at all paths from 2 to 6. We have the following paths $g(2\to 7\to 6)$ and $h(2\to 3\to 6)$.

Since they also have the same resistances, we can assume that current in 2→3 and current in 2→7 must be same. Therefore,

$$I_{27}=I_{23}=\frac{1}{2}\left(\frac{I}{3}\right)=\frac{I}{6}$$

Similarly, we can say that $I_{43}=I_{45}=I_{85}=I_{87}=I/6$. Therefore, to calculate the potential difference across the path $(1\to 2\to 7\to 6)$, we have

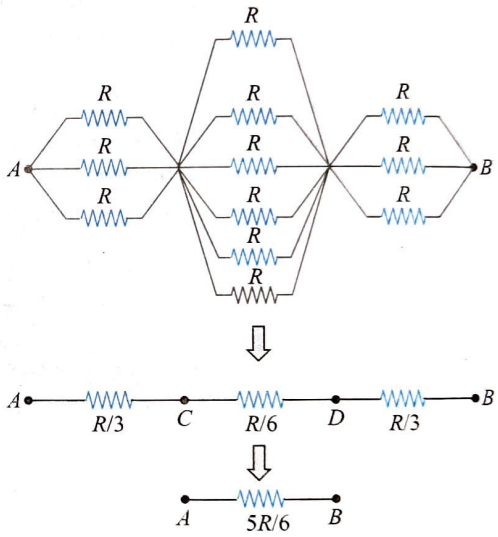
$$V_1-\frac{I}{3}R-\frac{I}{6}R-\frac{I}{3}R=V_6$$

or $$V_1-V_6=I\left[\frac{R}{3}+\frac{R}{6}+\frac{R}{3}\right]=\frac{5}{6}IR$$

or $$\frac{(V_1-V_6)}{I}=\frac{5}{6}R$$

Therefore, the equivalent resistance is $5R/6$.

Method 2: The network is symmetrical about the body diagonal AB. Since equal currents flow in the branches between A and (1, 2, and 3), the points 1, 2, and 3 are equipotential. Similarly, the points 4, 5, and 6 are equipotential. Let us now superimpose the points 1, 2, and 3 at C and 4, 5, and 6 at D. You can now see that there are three resistors between A and C, six resistors between C and D, and three resistors between D and B. Then,



$$R_{AB}=R_{AC}+R_{AD}+R_{DB}$$

$$=\frac{R}{3}+\frac{R}{6}+\frac{R}{3}=\frac{5R}{6}$$

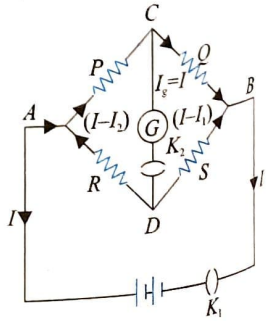
BALANCED WHEATSTONE BRIDGE

Wheatstone bridge is also known as a meter bridge or slide wire bridge. If four resistances P, Q, R, and S are joined as shown in figure, both the keys (K_1 and K_2) are on and no current flows through the galvanometer (i.e., $I_g=0$). Then the combination of resistances is called a balanced Wheatstone bridge. So

$$\frac{P}{Q}=\frac{R}{S}$$

If S is an unknown resistance, then

$$S=R\times\frac{Q}{P}$$



Now if we are given R, P, and Q or R and ratio Q/P, then we can calculate the value of S easily.

Proof: Applying loop rule to loop $ABDA$ (moving in clockwise direction), we get

$$-I_1P + (I - I_1)R = 0 \qquad \{\because I_g = 0\}$$

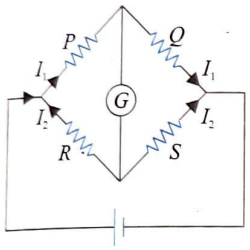
or $$I_1P = (I - I_1)R \qquad ...(i)$$

Applying loop rule to loop $BCDB$, we get

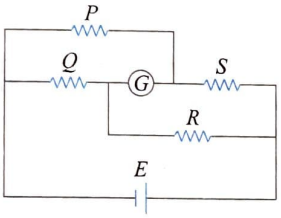
$$-I_1Q + (I - I_1)S = 0 \quad \text{or} \quad I_1Q = (I - I_1)S \qquad ...(ii)$$

Dividing Eq. (i) by Eq. (ii), we get $\dfrac{P}{Q} = \dfrac{R}{S}$

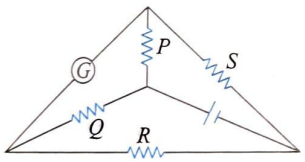
The balanced Wheatstone bridge method is an accurate method because we do not have to read out deflection, but we only have to see that the needle remains at dead zero. It is not affected by internal resistance of cells, resistances of galvanometers, etc. This is the principle used in the meter bridge or in the slide-wire bridge. Other circuits that can form Wheatstone bridge are shown in figure.



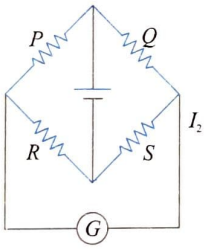
(a)



(b)



(c)

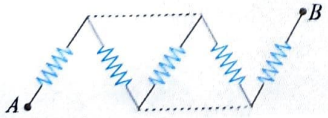


(d)

All the four circuits (a), (b), (c), and (d) represent a Wheatstone bridge network.

ILLUSTRATION 5.53

What will be the change in the resistance of a circuit between A and B, consisting of five identical resistances, if two similar resistances are added as shown by the dashed line in figure.



Sol. When two identical resistances are added in place of dashed lines.

$$R_2 = R_{eq} = 3r$$

But before adding the resistances, the equivalent resistance is

$$R_1 = R'_{eq} = 5r$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{3}{5}$$

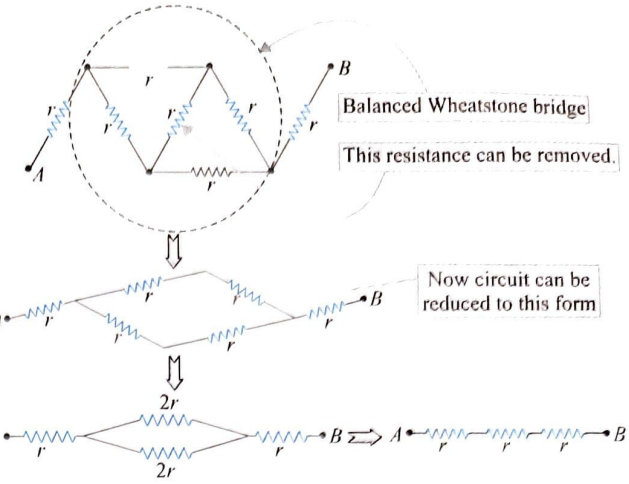
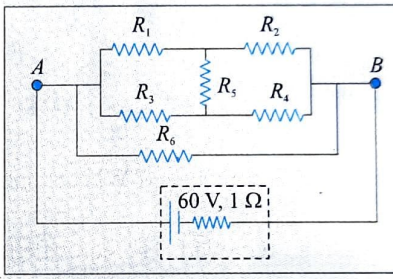


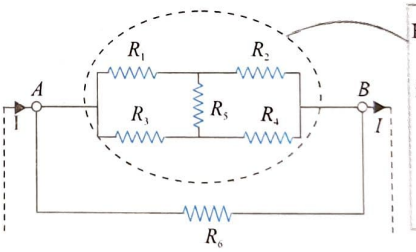
ILLUSTRATION 5.54

Resistances R_1, R_2, R_3, R_4, R_5, and R_6 are connected with a 6 V battery with internal resistance 1 Ω as shown in figure. Find (a) equivalent resistance and (b) current in each resistance.



$R_1 = 10\ \Omega$
$R_2 = 6\ \Omega$
$R_3 = 5\ \Omega$
$R_4 = 3\ \Omega$
$R_5 = 20\ \Omega$
$R_6 = 16\ \Omega$

Sol.

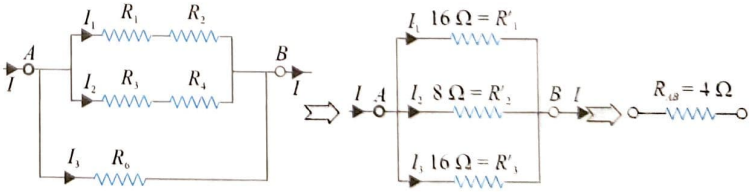


Here $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$

i.e., $\dfrac{10}{6} = \dfrac{5}{3}$

It is balanced Wheatstone bridge. The potential difference across R_5 will be zero and R_5 can be removed from circuit.

The circuit reduces to



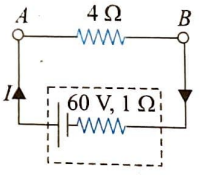
Hence, the equivalent resistance between A and B is

$$\frac{1}{R_{AB}} = \frac{1}{16} + \frac{1}{8} + \frac{1}{16} = \frac{4}{16}$$

or $$R_{AB} = 4\ \Omega$$

Now the circuit reduces to the one shown in figure. The current supplied by the battery is

$$I = \frac{60}{4+1} = 12\ \text{A}$$



As potential difference across R_5 is zero, no current will be in R_5. Now using current distribution law, we can find the current in each branch.

The currents in R_1 and R_2 will be equal.

$$I_1 = I\left(\frac{R_2'R_3'}{R_1'R_2' + R_2'R_3' + R_3'R_1'}\right)$$

$$= 12\left[\frac{8\times 16}{16\times 8 + 8\times 16 + 16\times 16}\right] = 3\text{ A}$$

The currents in R_3 and R_4 is

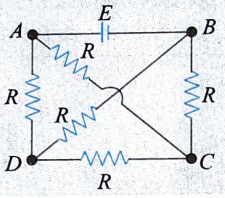
$$I_2 = I\left(\frac{R_1'R_3'}{R_1'R_2' + R_2'R_3' + R_3'R_1'}\right)$$

$$= 12\left[\frac{16\times 16}{16\times 8 + 8\times 16 + 16\times 16}\right] = 6\text{ A}$$

Hence, current in R_6 is $I_3 = 12 - (3 + 6) = 3$ A.

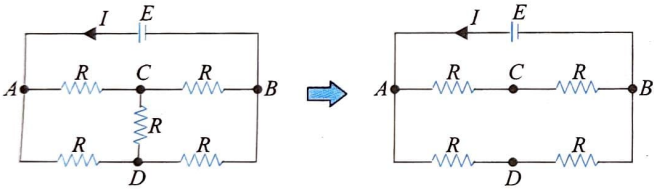
ILLUSTRATION 5.55

Four resistance each of magnitude R are connected with an ideal battery as shown in figure. Find current through the battery and potential difference between A and D.



Sol. We can redraw the circuit, it is a balanced Wheatstone bridge.

Resistance between C and D can be removed. The equivalent resistance between A and B,



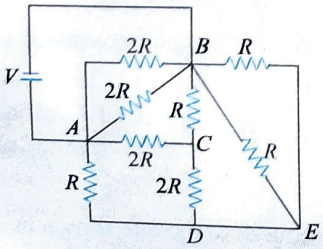
$$\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R} \Rightarrow R_{eq} = R$$

The current through the battery, $I = \dfrac{E}{R}$

As potential difference across AB is E, hence potential difference across AD should be $E/2$.

ILLUSTRATION 5.56

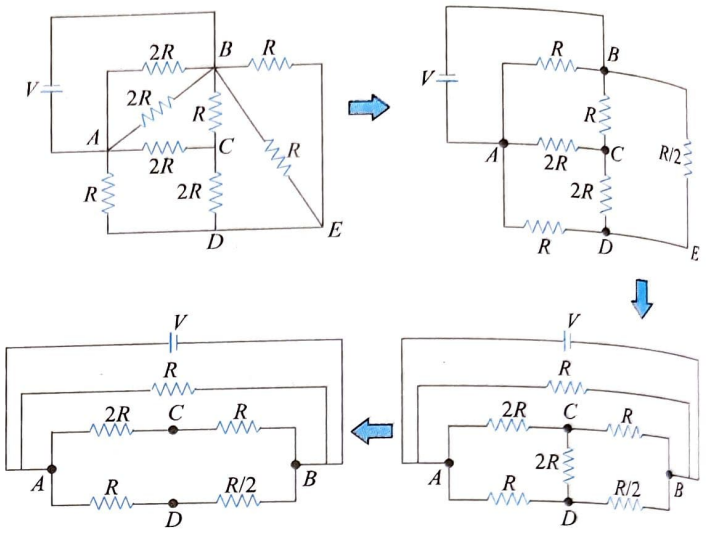
In the given circuit given in figure,find the current



(i) supplied by battery.

(ii) in the resistance connected across C and D.

Sol. We can redraw the circuit, it is a balanced Wheatstone bridge. Hence the potential difference in the resistance connected between C and D should be zero. It means the current in this resistance is zero.



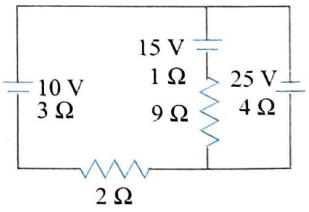
If we remove the resistance connected across C and D, the nature of the circuit will not change. The equivalent resistance across the battery

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{3R} + \frac{1}{3R/2} \Rightarrow R_{eq} = \frac{R}{2}$$

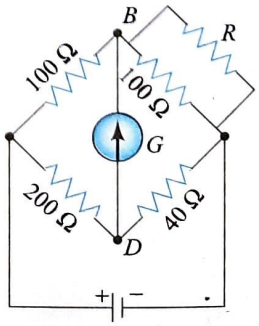
Hence current supplied by battery $I = \dfrac{V}{R/2} = \dfrac{2V}{R}$

CONCEPT APPLICATION EXERCISE 5.4

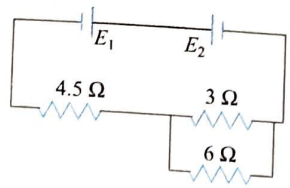
1. Find the current in each resistor in the circuit as shown in figure.



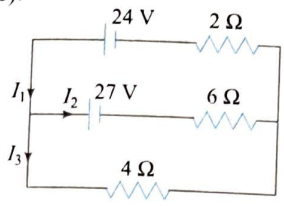
2. The given Wheatstone bridge is showing no deflection in the galvanometer joined between the points B and D in figure. Calculate the value of R.



3. In the circuit (figure), the cells E_1 and E_2 have emfs of 4 V and 8 V and internal resistances 0.5 Ω and 1.0 Ω, respectively. Calculate the current through 6 Ω resistance.

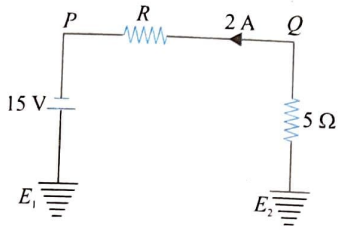


4. Determine the currents I_1, I_2, and I_3 for the network shown below (figure).

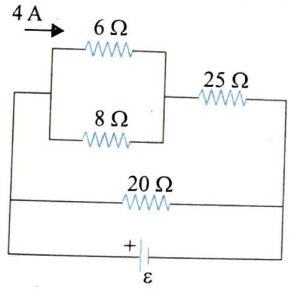


(a) I_1 = ___ (b) I_2 = ___ (c) I_3 = ___

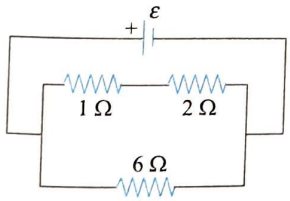
5. In the following circuit, the potential difference between P and Q is



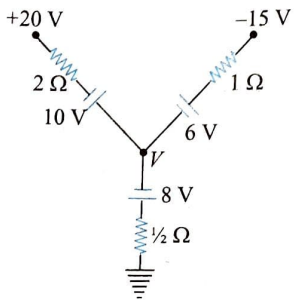
6. Consider the circuit shown in figure. The current through the 6 Ω resistor is 4A, in the direction shown. What are the currents through the 25 Ω and 20 Ω resistors?



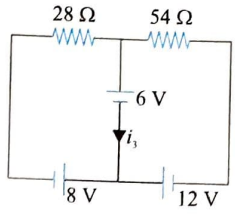
7. In the circuit shown in figure, the voltage across the 2 Ω resistor is 12 V. What is the emf of the battery and the current through the 6 Ω resistor?



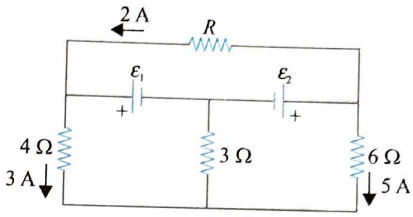
8. In the network of three cells, find the potential V of their junction.



9. Consider the circuit shown in figure. The current i_3 is equal to

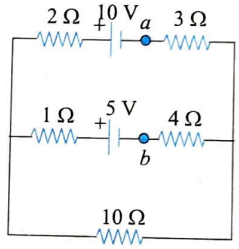


10. In the circuit shown in figure,



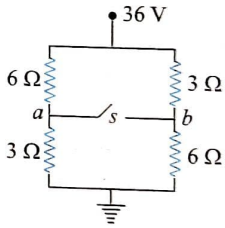
(a) find the current in the 3 Ω resistor
(b) find the unknown emfs ε_1 and ε_2
(c) find the resistance R

11. In the circuit shown in figure,

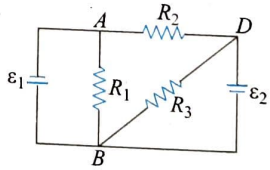


(a) find the current in each branch
(b) find the potential difference V_{ab} of point a relative to point b

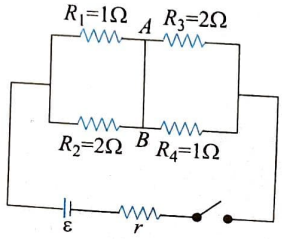
12. In the circuit shown in figure, calculate the following:
(a) Potential difference between points a and b when switch S is open.
(b) Current through S in the circuit when S is closed.



13. In this circuit if $\varepsilon_1 = 6$ V, $\varepsilon_2 = 3$ V, $R_1 = 1$ Ω and $R_2 = R_3 = 3$ Ω, find the current through each cell and resistors.



14. In the given circuit, $R_1 = 1$ Ω, $R_2 = 2$ Ω, $R_3 = 2$ Ω, $R_4 = 1$ Ω, $r = \frac{2}{3}$ Ω and $\varepsilon = 10$ V. Find the:



(a) total current drawn from the battery
(b) current in the wire AB.

ANSWERS

1. $i_{2\Omega} = 3$ A, $i_{9\Omega} = 2$ A **2.** 25 Ω **3.** 1/6 A
4. $I_1 = 3$ A, $I_2 = -1.5$ A, $I_3 = 4.5$ A **5.** 5 V
6. $i_{25\Omega} = 7$ A, $i_{20\Omega} = 9.95$ A **7.** 18 V, 3 A **8.** –44/7 V

9. $-\frac{5}{6}$ A **10.** (a) 8A (b) $\varepsilon_1 = 36$ V, $\varepsilon_2 = 54$ V (c) 9 Ω

11. (a) $i_{2\Omega} = \frac{4}{5}$ A, $i_{1\Omega} = \frac{1}{5}$ A and $i_{10\Omega} = \frac{3}{5}$ A (b) $V_{ab} = -\frac{16}{5}$ V

12. (a) –12 V (b) 3 A

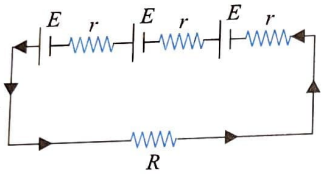
13. The current through $R_1 = 6$ A (From B to A)
The current through $R_2 = 3$ A (From D to A)
The current through $R_3 = 1$ A (From D to B)

14. (a) 5 A (b) $\frac{5}{3}$ A

COMBINATION OF CELLS

SERIES GROUPING

Suppose n cells each of emf E and internal resistance r are connected in series as shown in figure. Then net emf is nE and total resistance is $nr + R$.



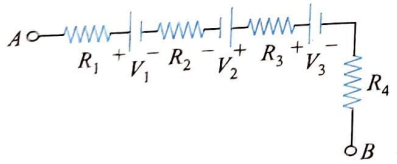
Therefore the current in the circuit is

$$i = \frac{\text{Net emf}}{\text{Total resistance}} = \frac{nE}{nr + R}$$

Note: If polarity of m cells is reversed, then equivalent emf is $(n - 2m)E$. While total resistance is still $nr + R$.

$$i = \frac{(n-2m)E}{nr + R}$$

If the same current passes through every resistor in a given branch, irrespective of the presence of sources in that branch, the resistors are in series even though they are not directly connected to each other. Same is true about capacitors.



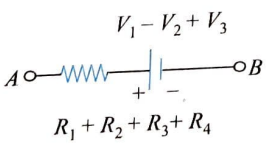
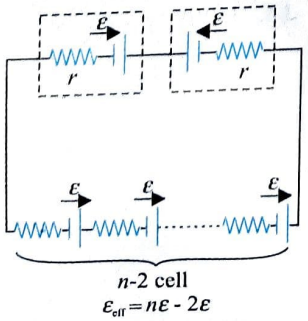


ILLUSTRATION 5.57

n identical cells each of emf 6 V, connected in series with an external resistor of 5 Ω, carry a current of 10 A. If two cells are connected wrongly in series with the same external resistor, the current flowing through the cells will be 6.45 A. Find the value of n and the internal resistance of each cell.



Sol. Let r be the internal resistance of each cell. For series connection, $\varepsilon_{eff} = n\varepsilon = 6n$.

$$r_{eff} = nr$$

or $$i = \frac{n\varepsilon}{nr + R} = \frac{6n}{nr + 5} = 10 \quad \text{...(i)}$$

For wrong connection of two cells, the effective emf decreases by 2ε because the emfs of two identical cells counteract with each other. Then,

$$\varepsilon'_{eff} = n\varepsilon - \varepsilon = (n-2)\varepsilon = (n-2)6$$

and $$\varepsilon'_{eff} = nr$$

or $$i' = \frac{(n-2)\varepsilon}{nr + R} = \frac{(n-2)6}{nr + 5} = 6.45 \quad \text{...(ii)}$$

By solving Eqs. (i) and (ii), we have $n = 10$ and $r = 0.1$ Ω.

ILLUSTRATION 5.58

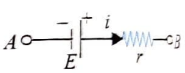
n identical cells, each of emf E and internal resistance r, are joined in series to form a closed circuit. Find the potential difference across any one cell.

Sol. Current in the circuit is

$$i = \frac{nE}{nr} = \frac{E}{r}$$

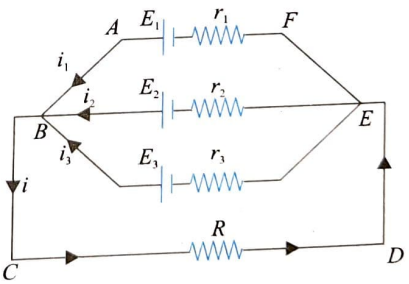
The equivalent circuit of one cell is shown in figure. Potential difference across the cell is

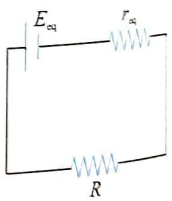
$$V_A - V_B = -E + ir = -E + \frac{E}{r} \cdot r = 0$$



PARALLEL GROUPING

Case 1: E and r of each cell are different, but still the positive terminals of all cells are connected at one junction while the negative terminals at the other.





Applying Kirchhoff's second law in loop $ABCDEFA$, we get

$$E_1 - iR - i_1 r_1 = 0 \text{ or } i_1 = -\frac{iR}{r_1} + \frac{E_1}{r_1} \quad \text{...(i)}$$

Similarly, we can write,

$$i_2 = -i\frac{R}{r_2} + \frac{E_2}{r_2} \quad \text{...(ii)}$$

Adding all above equations, we have

$$(i_1 + i_2 + \ldots + i_n) = -iR\sum\left(\frac{1}{r}\right) + \sum\left(\frac{E}{r}\right)$$

But $i_1 + i_2 + \ldots + i_n = i$, so

$$i = -iR\sum\left(\frac{1}{r}\right) + \sum\left(\frac{E}{r}\right)$$

$$\therefore \quad i = \frac{\Sigma\left(\frac{E}{r}\right)}{1 + R\Sigma\left(\frac{1}{r}\right)} = \frac{E_{eq}}{R_{eq}}$$

$$\text{where } E_{eq} = \frac{\Sigma\left(\frac{E}{r}\right)}{\Sigma\left(\frac{1}{r}\right)} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$$

$$r_{eq} = \frac{1}{\Sigma\left(\frac{1}{r}\right)} = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$$

$$\text{and} \quad R_{eq} = R + \frac{1}{\Sigma\left(\frac{1}{r}\right)}$$

From the above expression, we can see that $i = E/(R + r/n)$ if n cells of same emf E and internal resistance r are connected in parallel. This is because

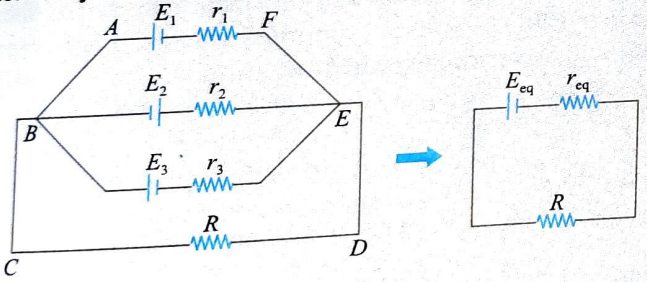
$$\Sigma\left(\frac{E}{r}\right) = \frac{nE}{r} \text{ and } \Sigma\left(\frac{1}{r}\right) = \frac{n}{r}$$

$$\therefore \quad i = \frac{nE/r}{1 + nR/r} = \frac{E}{R + r/n}$$

We can also write

$$i = \frac{(E_1/r_1) + (E_2/r_2) + (E_3/r_3)}{1 + R\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)}$$

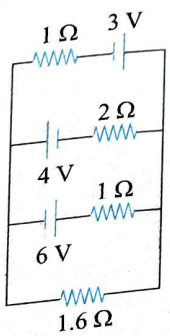
Note: If any cell is connected with reversed polarity



Then, $E_{eq} = \dfrac{\frac{E_1}{r_1} - \frac{E_2}{r_2} + \frac{E_3}{r_3}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$ and $r_{eq} = \dfrac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$

ILLUSTRATION 5.59

Three cells of emf 3 V, 4 V, and 6 V are connected in parallel. If their internal resistances are 1 Ω, 2 Ω, and 1 Ω, find the ε_{eff}, r_{eff}, and the current in the external load $R = 1.6$ Ω.



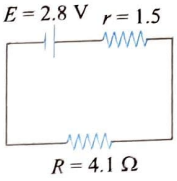
Sol. The cells are connected in parallel. The equivalent cell emf is

$$\varepsilon_{eff} = \frac{\Sigma\varepsilon_i/r_i}{\Sigma 1/r_i} = \frac{\frac{3}{1} + \left(-\frac{4}{2}\right) + \frac{6}{1}}{\frac{1}{1} + \frac{1}{2} + \frac{1}{1}} = 2.8 \text{ V}$$

The internal resistance of the cell is

$$\frac{1}{r_{eff}} = \Sigma\frac{1}{r_i} = \frac{1}{1} + \frac{1}{2} + \frac{1}{1} = \frac{5}{2}$$

or $\quad r_{eff} = \dfrac{2}{5}\Omega$



The current in the circuit is

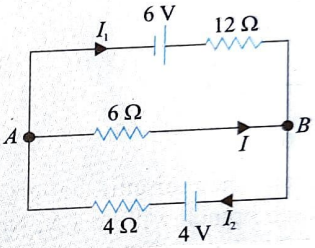
$$i = \frac{\varepsilon}{R + r} = \frac{2.8}{\frac{8}{5} + \frac{2}{5}} = 1.4 \text{ A}$$

ILLUSTRATION 5.60

In the circuit shown,

(i) $V_A - V_B =$ ..... (ii) $I_1 =$ .....

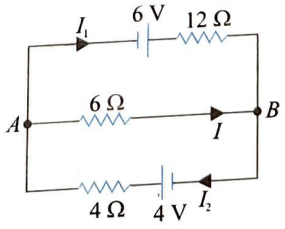
(iii) $I_2 =$ ..... (iv) $I =$ .....



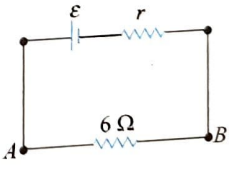
Sol. **Method 1:** The given circuit may be replaced by an equivalent battery of emf ε and resistance r as shown in figure.

$$\varepsilon = \frac{\frac{4}{4} + \left(-\frac{6}{12}\right)}{\frac{1}{4} + \frac{1}{12}} = \frac{3}{2} V$$

and $\quad r = \dfrac{1}{\frac{1}{4} + \frac{1}{12}} = 3\ \Omega$



(a)



(b)

Thus, current through the external resistor is (from A to B)

$$I = \frac{E}{r + R} = \frac{3/2}{3 + 6} = \frac{1}{6} A$$

For potential difference $V_A - V_B$:

$$V_A - 6 \times \frac{1}{6} = V_B$$

or $\quad V_A - V_B = 6 \times \dfrac{1}{6} = 1 \text{ V}$

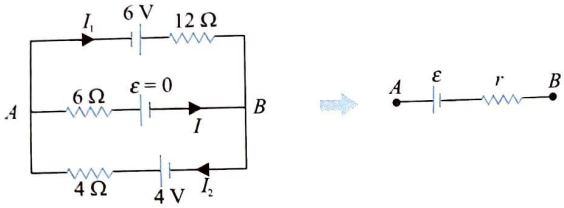
For upper branch: $V_A - I_1 \times 12 + 6 = V_B$

or $(V_A - V_B) + 6 = I_1 \times 12$ or $1 + 6 = I_1 \times 12$ or $I_1 = \frac{7}{12} A$

For lower branch: $V_A + I_2 \times 4 - 4 = V_B$

or $(V_A - V_B) - 4 = -I_2 \times 4$ or $I_2 = \frac{3}{4} A$

Method 2: Instead of making equivalent battery of two branches, we can make equivalent battery of all three branches. In the middle branch, we can assume a battery of zero emf connected with resistance 6 Ω. The given circuit may be replaced by an equivalent battery of emf ε and resistance r as shown in figure.



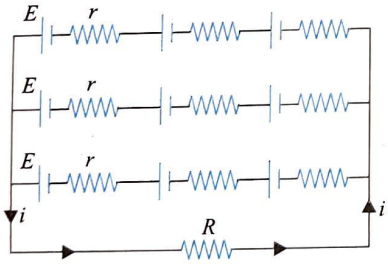
$$\varepsilon = \frac{\frac{4}{4} + \left(-\frac{6}{12}\right) + \frac{0}{6}}{\frac{1}{4} + \frac{1}{12} + \frac{1}{6}} = 1 \text{ V}$$

and $$r = \frac{1}{\frac{1}{4} + \frac{1}{12} + \frac{1}{6}} = 2\ \Omega$$

This battery is an open circuit battery, hence potential difference across each branch will be 1 V. Now we will get the same results as we have in previous method.

MIXED GROUPING

The situation is shown in figure. There are n identical cells in a row, and number of rows is m. The emf of each cell is E, and internal resistance is r. Treating each row as a single cell of emf nE and internal resistance nr, we have



Net emf = nE

Total internal resistance = $\frac{nr}{m}$

Total external resistance = R

Therefore, the current through the external resistance R is

$$i = \frac{nE}{R + \frac{nr}{m}}$$

This expression after some rearrangement can also be written as

$$i = \frac{mnE}{(\sqrt{mR} - \sqrt{nr})^2 + 2\sqrt{mnrR}}$$

From this expression we see that i is maximum when,

or $\sqrt{mR} = \sqrt{nR}$ or $R = \frac{nr}{m}$

or total external resistance = total internal resistance.

Thus, we can say that the current and hence the power transferred to the load are maximum when the load resistance is equal to the internal resistance. This is known as **maximum power transfer theorem**.

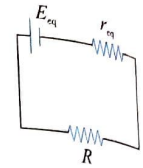


ILLUSTRATION 5.61

In a mixed grouping of identical cells, five rows are connected in parallel and each row contains 10 cell. This combination sends a current i through an external resistance of 20 Ω. If the emf and internal resistance of each cell is 1.5 V and 1 Ω, respectively, then find the value of i.

Sol. The number of cells in a row is $n = 10$, and the number of such rows is $m = 5$.

$$i = \frac{nE}{\left(R + \frac{nr}{m}\right)} = \frac{10 \times 1.5}{\left(20 + \frac{10 \times 1}{5}\right)} = \frac{15}{22} = 0.68$$

ILLUSTRATION 5.62

100 cells each of emf 5 V and internal resistance 1 Ω are to be arranged to produce maximum current in a 25 Ω resistance. Each row contains equal number of cells. Find the number of rows.

Sol. Total number of cells is

$mn = 100$. ...(i)

Current will be maximum when

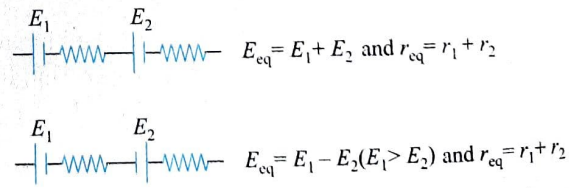
$$R = \frac{nr}{m} \text{ or } 25 = \frac{n \times 1}{m}$$

$n = 25\ m$ ...(ii)

From Eqs. (i) and (ii), we get $n = 50$ and $m = 2$.

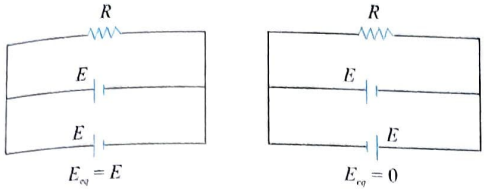
Important Points:

- In series grouping of cells, their emfs are additive or subtractive, while their internal resistances are always additive. If dissimilar plates of cells are connected together, their emfs are added to each other, while if their similar plates are connected together, their emfs are subtractive.

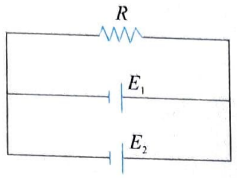


- In series grouping of identical cells, if one cell is wrongly connected, then it will cancel out the effect of two cells. If in the combination of n identical cells (each having emf E and internal resistance r) x cells are wrongly connected, then equivalent emf is $E_{eq} = (n - 2x)E$ and equivalent internal resistance is $r_{eq} = nr$.

- In parallel grouping of two identical cells having no internal resistance, we have the situation in figure.



- When two cells of different emf and no internal resistance are connected in parallel, then equivalent emf is indeterminate. Note that connecting a wire with a cell but with no resistance is equivalent to short-circuiting. Therefore, the total current that will be flowing will be infinite.



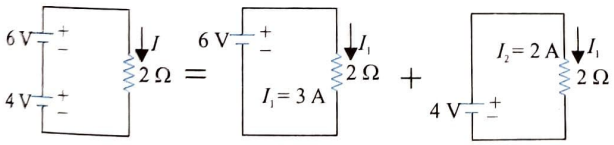
SUPERPOSITION PRINCIPLE

CONCEPTS

Whenever a circuit has more than one cell or battery, the superposition principle may be used to find current and voltages. This principle is based on the fact that every cell or battery acts independently of the presence of others.

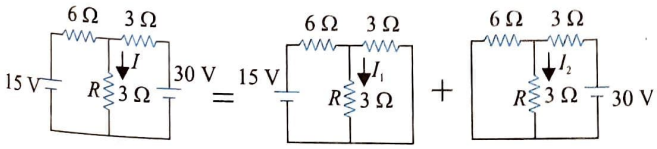
According to this principle, the total current I in the circuit equals the algebraic sum of currents (I_1, I_2, ..., I_n) produced by each source (cell or battery), taken one at a time. Mathematically, $I = I_1 + I_2 + \cdots + I_n$.

The superposition splits the original two-source problem into two one-source problems. Instead of solving a difficult two-source problem, here we learn this concept through two simple situations. In first situation two batteries are connected with a resistance and it is required to calculate current in the resistance. In this case, first we consider the current supplied by 6 V battery, ignoring the presence of the 4 V battery, then we calculate the current supplied by 4 V battery, ignoring 6 V battery. Combining these two currents, we get net current in the resistance under consideration.



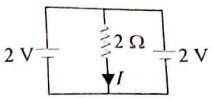
$$I = I_1 + I_2 = \frac{6}{2} + \frac{4}{2} = 3 + 2 = 5 \text{ A}$$

Now consider one more situation as shown in figure, here we need to calculate current in resistance R. Here also, first we consider current supplied by 15 V battery (I_1), ignoring the presence of the 30 V battery. In next step we consider current supplied by 30 V battery (I_2), ignoring the presence of the 15 V battery. Then by adding these two currents given net current in resistance (R).



$I_1 = 1$ A and $I_2 = 4$ A, so $I = I_1 + I_2 = 1 + 4 = 5$ A.

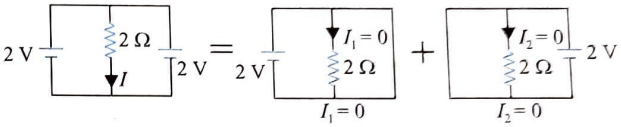
Situation where superposition principle is not valid: In the circuit shown in figure, find the current I flowing through the 2 Ω resistor.



Many students come out with an answer $I = 2$ A with a wrong reason. Each battery contributes a current of 1 A; as there are two batteries, so the total current in the 2 Ω resistor is 2 A.

The circuit may be split up into two parts as shown in figure. With each battery, the current in the 2 Ω resistor is zero. It happens so because when we remove one battery from a branch, that particular branch gets short-circuited, as there is no other resistance present in that branch.

Note that in such a situation the principle of superposition is not applicable.



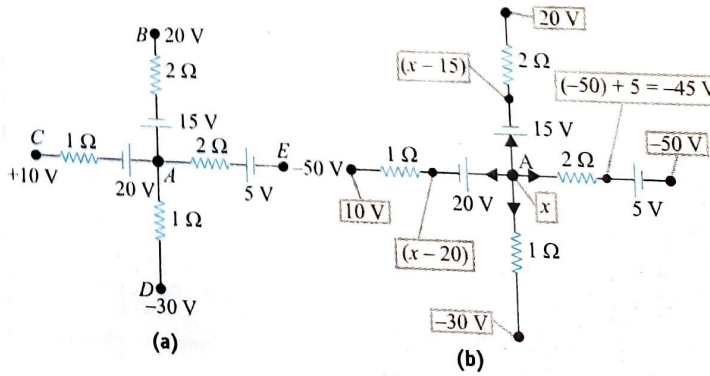
Conditions for the applicability of principle of superposition: Whenever a cell or a battery is present in a branch, there must be some resistance (internal or external or both) present in that branch in order to apply the superposition principle. In practical situations it always happens because we can never have an ideal cell or a battery with zero resistance. In this case, the current flowing through the 2 Ω resistor is 1A, since voltage drop across it is 2 V (correct).

NODAL METHOD OF CIRCUIT ANALYSIS

It is based on Kirchhoff's junction law. At any node (or junction) in electrical circuit, $\Sigma I = 0$ (it will be referred as nodal equation). Assign potential of every junction of circuit taking potential of any one of the junctions of the circuit zero (this will be called reference node or reference junction). Apply nodal equation to solve for unknown potential introduced in the circuits. Current in nodal equation will be written using the resistor equation, i.e., $I = V/R$.

ILLUSTRATION 5.63

The resistances and batteries are connected as shown in figure. The potentials at points B, C, D, and E are assigned as shown. Find the potential at point A.



Sol. Let potential at A be x. Applying Kirchhoff's current law at junction A, we get

$$\frac{(x-20)-10}{1}+\frac{(x-15)-20}{2}+\frac{x-(-45)}{2}+\frac{x-(-30)}{1}=0$$

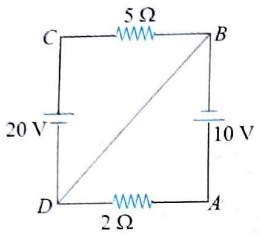
or $$\frac{(2x-60)+(x-35)+(x+45)+(2x+60)}{2}=0$$

$$6x+10=0 \quad \text{or} \quad x=-5/3$$

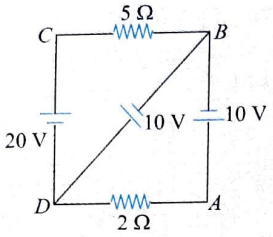
Hence potential at A is $-5/3$ V.

ILLUSTRATION 5.64

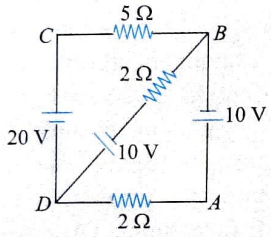
In figure, given below find the current in the wire BD.



(a)



(b)



(c)

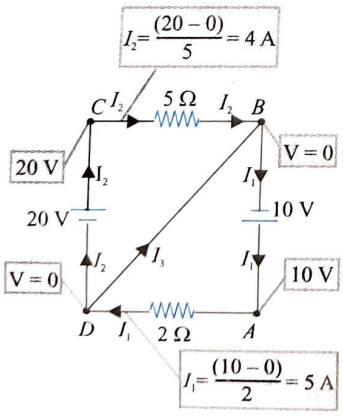
Sol.

(i) Let us assign B as reference node and take its potential = 0. As nodes B and D are directly connected with a conducting wire, hence potential of node D will also be zero. Then write the potential of other nodes; corresponding currents are shown in figure. At node D,

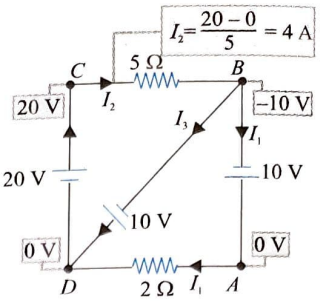
$$I_1 = I_2 + I_3$$

or $$I_3 = 5 - 4 = 1 \text{ A}$$

Therefore, current in wire BD is 1 A, from D to B.

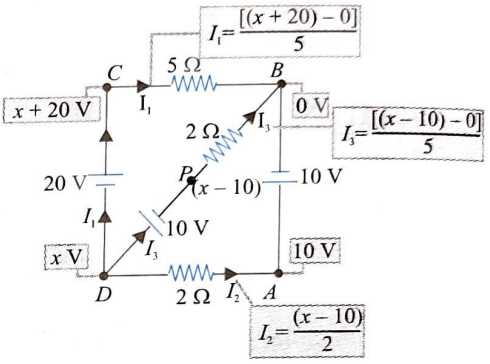


(ii) Let us assume the potential of point D to be zero. The potentials at different points can be assigned as shown in figure. From circuit diagram, it is clear that current in 2 Ω resistance will be zero as potential difference across it will be zero. Hence $I_1 = 0$.



The current through 5 Ω resistance is $I_2 = 4$ A. Hence, at junction B, $I_2 = I_1 + I_3$. As $I_1 = 0$, so $I_3 = I_2 = 4$ A. Hence, current in wire BD will be 4 A.

(iii) Let us assume the potential of point B and D be 0 V and xV, respectively. Then the potential at different points can be assigned as shown in figure.



At node D, $I_1 + I_2 + I_3 = 0$

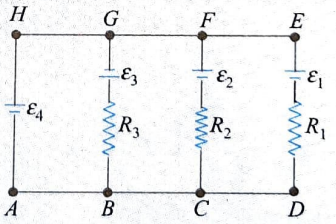
$$\frac{(x+20)}{5}+\frac{(x-10)}{2}+\frac{(x-10)}{2}=0 \quad \text{or} \quad x=5 \text{ V}$$

$$i_3=\left(\frac{x-10}{2}\right)=\frac{5-10}{2}=-\frac{5}{2} \text{ A} \qquad [\text{from } D \text{ to } B]$$

or 5/2 A current will flow from B to D.

ILLUSTRATION 5.65

Find the current in each wire



Given.

$\varepsilon_1 = 50$ V, $\varepsilon_2 = 40$ V
$\varepsilon_3 = 30$ V, $\varepsilon_4 = 10$ V
$R_1 = 2\ \Omega$, $R_2 = 2\ \Omega$,
$R_3 = 1\ \Omega$

Sol. Let potential at point A be 0 volt, then potential of other points is shown in figure. Analysis of current in different branches are shown in the figure.

$i_4 + (i_1 + i_2) = i_3$
$i_4 = i_3 - (i_1 + i_2)$
$i_4 = 40 - (20 + 15)$
$= 5$ A

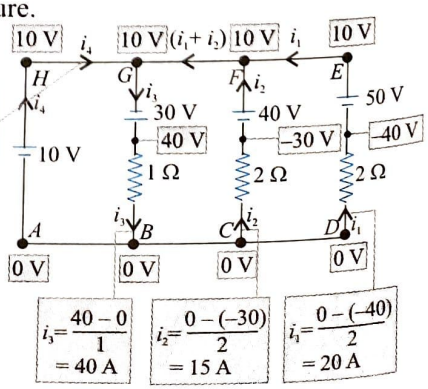
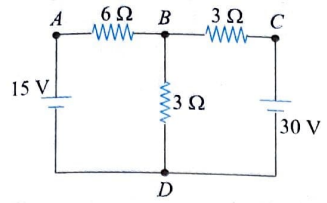


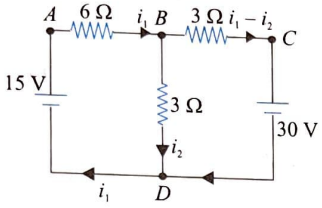
ILLUSTRATION 5.66

In the circuit shown in figure, find the current through the branch BD.



Sol. **Method 1: Using Kirchhoff's law**

The currents in the circuit are assumed as shown in figure. Applying *KVL* along the loop *ABDA*, we get



$$-6i_1 - 3i_2 + 15 = 0$$

or $2i_1 + i_2 = 5$ ...(i)

Applying *KVL* along the loop *BCDB*, we get

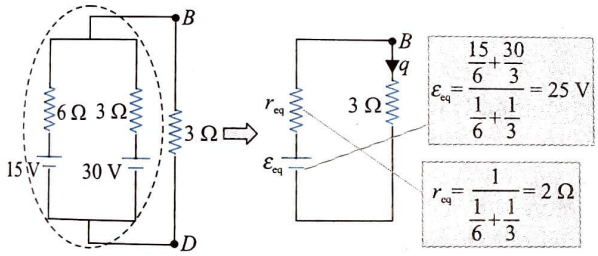
$$-3(i_1 - i_2) - 30 + 3i_2 = 0$$

or $-i_1 + 2i_2 = 10$ ...(ii)

Solving Eqs. (i) and (ii) for i_2, we get $i_2 = 5$ A.

Method 2: Using equivalent battery method (i)

We can treat 6 Ω and 3 Ω resistances connected with 15 V and 30 V batteries, respectively, as internal resistances of the batteries. Now we can assume these batteries are connected in parallel, and equivalent battery of these two is connected with resistance connected across B and D.

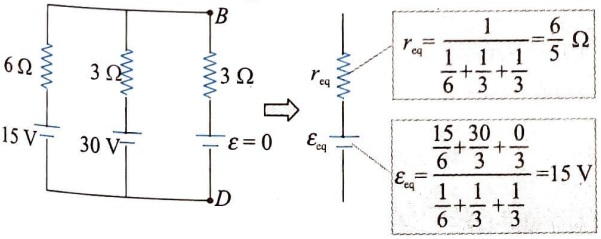


Hence, current through resistance connected across B and D is

$$i = \frac{20}{2+3} = 5 \text{ A}$$

Method 3: Using equivalent battery method (ii)

We can assume a battery of zero emf is connected in series with resistance connected across BD. This resistance can be treated as internal resistance of zero emf battery. Now we have three batteries in parallel. The equivalent of these batteries will be a single battery which is the open circuit.

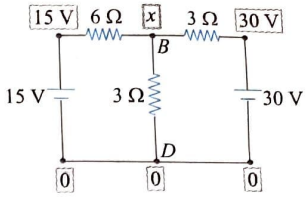


As equivalent battery is an open circuit, the potential difference across each of the battery branch will be 15 V. Hence, current through the resistance connected across BD is

$$i = \frac{15}{3} = 5 \text{ A}$$

Method 4: Using nodal method

Let us assume the potential of node D zero and the potential of node B x. Then we can assign the potentials at different nodes as shown in figure. At node B,



$$\frac{(x-15)}{6} + \frac{(x-30)}{3} + \frac{(x-0)}{3} = 0$$

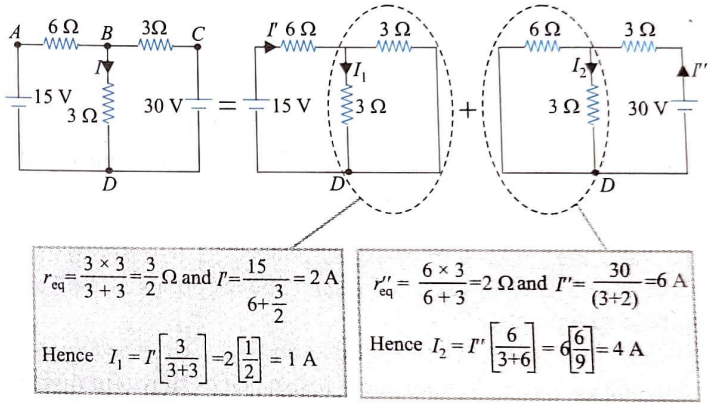
or $5x = 75$ or $x = 15$ V

Hence, current through resistance connected across BD is

$$i = \frac{15-0}{3} = 5 \text{ A (from } B \text{ to } D)$$

Method 5: Using superposition principle

In superposition method, we can take the effect of one battery at a time. From figure, it is clear that the effect of two batteries can be taken by adding the effect of individual batteries.



$r_{eq} = \frac{3\times3}{3+3} = \frac{3}{2}\ \Omega$ and $I' = \frac{15}{6+\frac{3}{2}} = 2$ A

Hence $I_1 = I'\left[\frac{3}{3+3}\right] = 2\left[\frac{1}{2}\right] = 1$ A

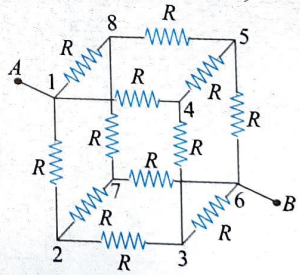
$r'_{eq} = \frac{6\times3}{6+3} = 2\ \Omega$ and $I' = \frac{30}{(3+2)} = 6$ A

Hence $I_2 = I'\left[\frac{6}{3+6}\right] = 6\left[\frac{6}{9}\right] = 4$ A

Hence, current through resistance connected across B and D is $i_1 + i_2 = 1 + 4 = 5$ A (from B to D).

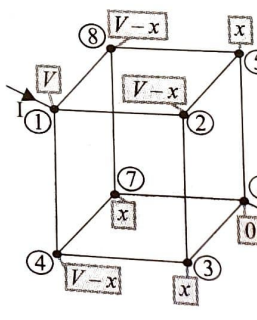
ILLUSTRATION 5.67

Twelve identical resistors are arranged on all edges of a cube. Then find the equivalent resistance between the corners 1 and 6 as shown in figure (use nodal method).



Sol. Let us assume we have connected a battery of emf V across points 1 and 6. If potential of point 1 is V, then the potential of point 6 is 0.

From symmetry, it is clear that the current entering at point 1 will equally divide the resistance. Hence, the potential differences across resistances, connected across (1, 8), (1, 2), and (1, 4), will be equal.



Similarly, the current coming out at point 6 will be equal to current entering at point 1. By symmetry, we can say the current in the resistances connected across points (7, 6), (3, 6), and (6, 5) will be equal. Hence, potential difference across these resistances should be equal. Let us assume the potentials at points 3, 5, and 7 be x, then the potentials at points 2, 4, and 8 should be $(V - x)$. At point 7,

$$\frac{[x-(V-x)]}{R}+\frac{(x-0)}{R}+\frac{[x-(V-x)]}{R}=0$$

or $$x=\frac{2}{5}V \qquad ...(i)$$

At point 1,

$$I=\frac{3[V-(V-x)]}{R}=\frac{3x}{R} \qquad ...(ii)$$

From (i) and (ii),

$$\frac{V}{I}=\frac{5R}{6}$$

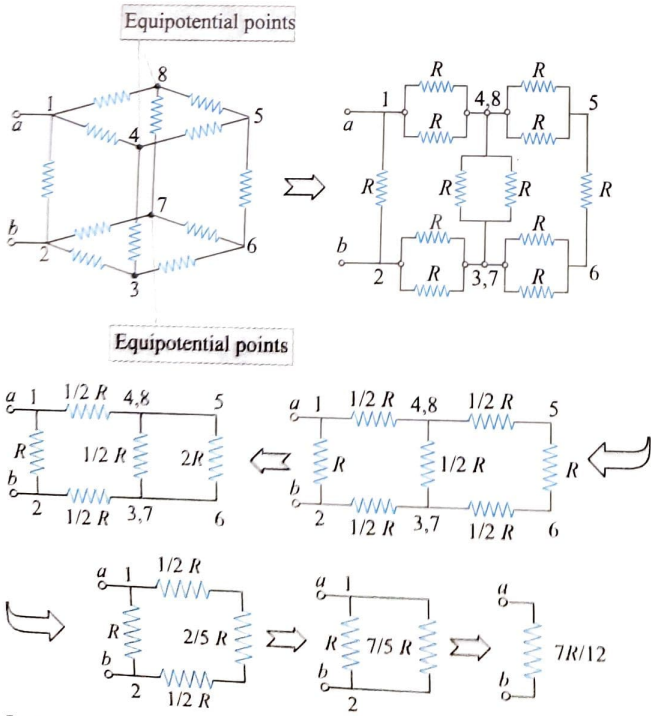
Hence, effective resistance will be $5R/6$.

ILLUSTRATION 5.68

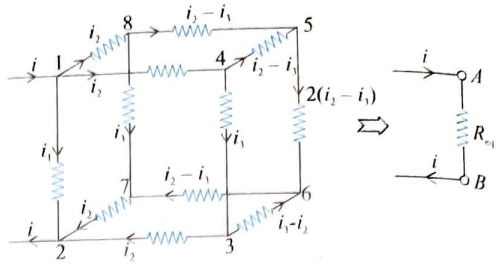
In previous illustration, find the equivalent resistance of the network between 1 and 2.

Sol. **Method 1:** From symmetry, it is clear the current in resistors connected between points (1 and 8) and (1 and 4) should be same, hence points 4 and 8 are equipotential points. Similarly, the current in resistors connected between points (2 and 7) and (2 and 3) should be same, hence points 3 and 7 are equipotential points.

We can join points (4 and 8) and (3 and 7). We can redraw the circuit and calculate the equivalent resistance as shown in figure.



Method 2: Let a current i enter the point a and leave the point b. From symmetry we can distribute currents in different resistors as shown in figure.



From *KCL*, we have

$$i = i_1 + 2i_2 \qquad ...(i)$$

Applying *KVL* for the loop $(1 \to 4 \to 3 \to 2 \to 1)$, we have

$$-i_2R - i_3R - i_2R + i_1R = 0$$

or $$i_1 = 2i_2 + i_3 \qquad ...(ii)$$

Applying *KVL* for the loop $(4 \to 3 \to 6 \to 5 \to 4)$, we have

$$-i_3R + (i_2 - i_3)R + 2(i_2 - i_3)R + (i_2 - i_3)R = 0$$

or $$4i_2 - 5i_3 = 0 \qquad ...(iii)$$

On solving Eqs. (i), (ii), and (iii), we get

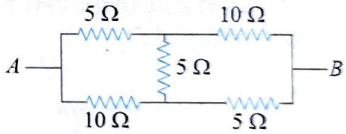
$$i_1=\frac{7}{12}i;\ i_2=\frac{5}{24}i;\ i_3=\frac{1}{6}i$$

The potential difference across $(1 \to 2)$ is $i_1R = 7/12\ iR$. The potential difference across equivalent resistance is iR_{eq}. Thus

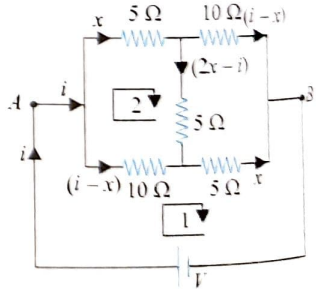
$$iR_{eq}=\frac{7}{12}iR \text{ or } R_{eq}=\frac{7}{12}R$$

ILLUSTRATION 5.69

Find the equivalent resistance between A and B.



Sol. **Method 1:** It is a Wheatstone bridge, but it is not balanced. There are no series parallel connections. But there are similar values on input and output sides. Here we see that even after using symmetry, the circuit does not reduce to series parallel combination as in previous examples. Therefore, we apply Kirchhoff's voltage law.



In loop 1:

$$V - 10(i - x) - 5x = 0$$

or $$V - 10i + 5x = 0 \qquad ...(i)$$

In loop 2:

$$10(i - x) - 5x - 5(2x - i) = 0$$

or $$10i - 10x - 10x + 5i - 5x = 0$$

or $$15i - 25x = 0$$

or $$x=\frac{15}{25}i \quad 5x = 3i \qquad ...(ii)$$

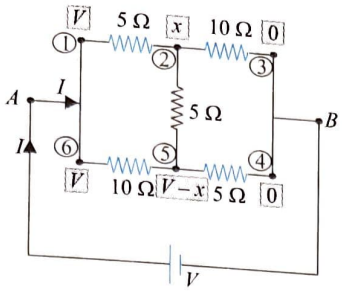
Using (ii) and (i);

$$V - 10i + 3i = 0$$

or $$\frac{V}{I} = 7\ \Omega \qquad \text{or} \quad R_{eq} = 7\ \Omega$$

Method 2: From shifted symmetry, it is clear that the current in the resistances connected across nodes 1 and 2 will be same as the resistance connected across nodes 5 and 4. Hence, potential differences across these resistances will be equal.

We can state the same for the resistances connected across the nodes 6 and 5 and nodes 2 and 3. Let us assign the potentials of points A and B to be V and 0, respectively, then we can assign potentials of different nodes as shown in the figure. At node 2



$$\frac{(x-V)}{5}+\frac{[x-(V-x)]}{5}+\frac{(x-0)}{10}=0$$

or $x=\frac{4}{7}V$ ...(i)

At point A, using Kirchhoff's junction rule

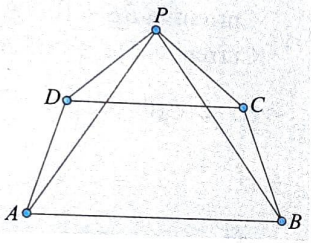
$$I=\frac{(V-x)}{5}+\frac{[V-(V-x)]}{10}=\frac{2V-x}{10} \quad ...(ii)$$

From (i) and (ii),

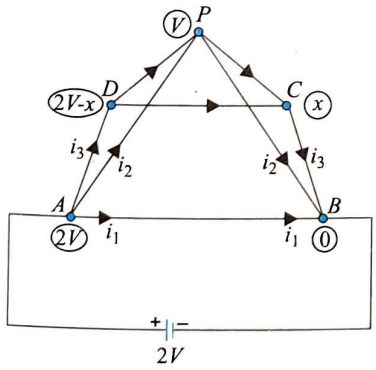
$$\frac{V}{I}=7\,\Omega=R_{eq}$$

ILLUSTRATION 5.70

Eight identical resistances R each are connected as shown in figure. Find equivalent resistance between A and B.



Sol. To find the equivalent resistance across A and B, let us connect a battery of emf '$2V$'. Due to symmetry we can assign the potential of point P as 'V'. As the circuit has end point symmetry, the magnitude of the currents in resistances AD and CB should be equal. Hence the potential difference across these resistances should be same. If 'V' be the potential of point C, then the potential of point D should be $(2V\text{-}x)$.



To calculate unknown potential, let us write KCL at junction C,

$$\frac{V-x}{R}+\frac{(2V-x)-x}{R}=\frac{x-0}{R}$$

$$\Rightarrow \quad 4x=3V \Rightarrow x=\frac{3V}{4}$$

Current through battery,

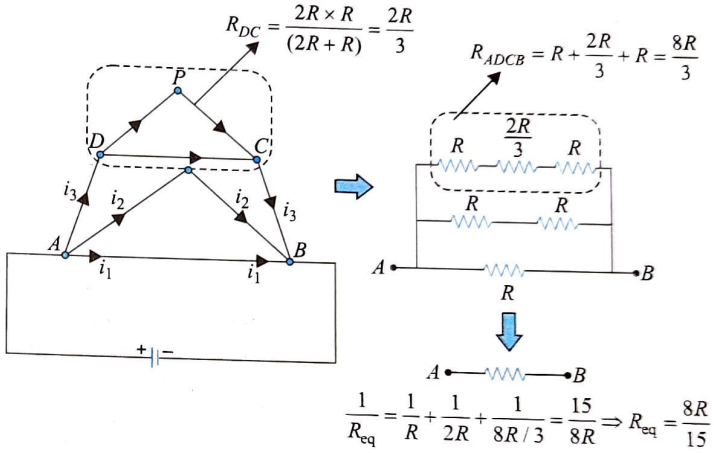
$$I_{battery}=i_1+i_2+i_3=\frac{x}{R}+\frac{2V}{R}+\frac{V}{R}=\frac{x}{R}+\frac{3V}{R}$$

$$=\frac{3V}{4R}+\frac{3V}{R}=\frac{15V}{4R}$$

Equivalent resistance across terminals A and B is given as

$$R_{eq}=\frac{V_{battery}}{I_{battery}}=\frac{2V\times 4}{15V}=\frac{8R}{15}$$

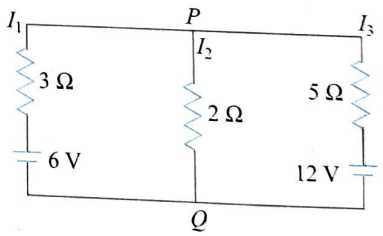
2nd Approach: The circuit has end point symmetry. Now we can distribute the currents in different branches as shown in the figure. There is no distribution of currents in the resistances AP and PB at junction P, hence we can separate these resistances from this junction. This will make no change in the calculation of effective resistance of the circuit. Now we can calculate the effective resistance as shown in figure.



CONCEPT APPLICATION EXERCISE 5.5

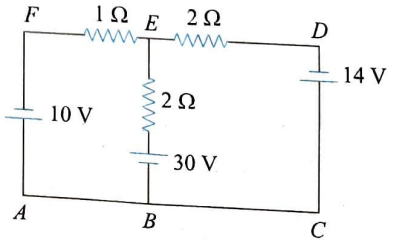
1. Calculate the value of the electric currents I_1, I_2, and I_3 in the given electrical network.

   (a) I_1 = ___ **(b)** I_2 = ___ **(c)** I_3 = ___

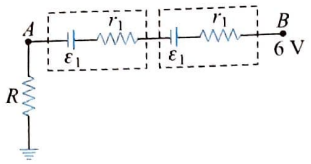


2. Find the minimum number of cells required to produce a current of 1.5 A through a resistance of 30 Ω. Given that the emf of each cell is 1.5 V and the internal resistance is 1 Ω.

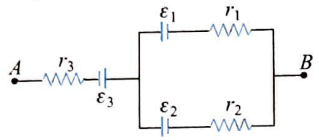
3. Find the current in each wire.



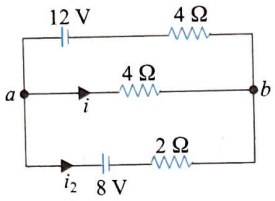
4. Find the (i) effective emf across AB, (ii) current, assuming $R = 10\ \Omega$, $\varepsilon_1 = 6$ V, $r_1 = 1.5\ \Omega$, $\varepsilon_2 = 4$ V and $r_2 = \frac{1}{2}\ \Omega$.



5. Find the emf and internal resistance of the equivalent cell between A and B. Put $\varepsilon_1 = 6$ V, $r_1 = 2\ \Omega$, $\varepsilon_2 = 4$ V, $r_2 = 2\ \Omega$, and $\varepsilon_3 = 3$ V, $r_3 = 3\ \Omega$.



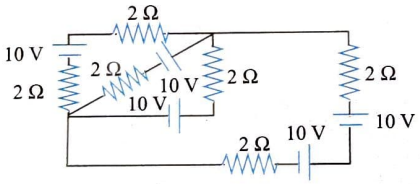
6. In the given network, find the:



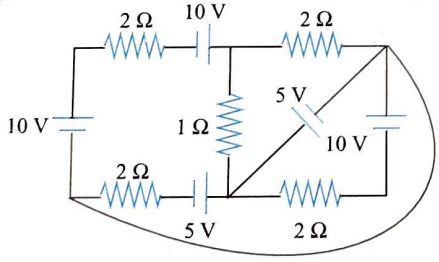
(a) potential difference between a and b.

(b) currents in each branch.

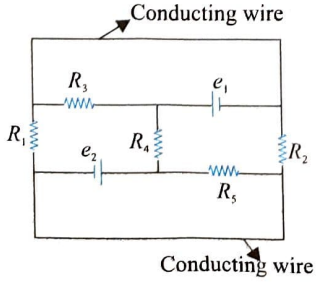
7. In the given circuit in figure, all batteries have emf 10 V and internal resistance negligible. All resistors are in ohm. Calculate the current in the rightmost 2 Ω resistor.



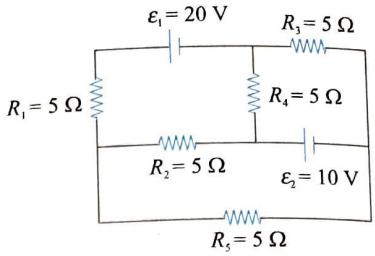
8. In the circuit diagram shown in figure, find the current through the 1 Ω resistor.



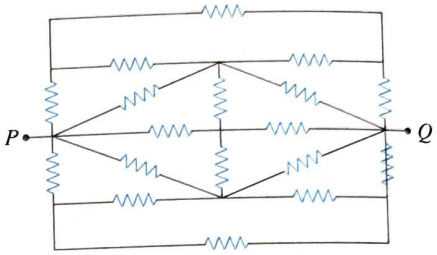
9. In the given circuit, determine current through branch having indicated resistor R_4. Given $R_1 = R_2 = 4\ \Omega$, $R_3 = R_4 = R_5 = 2\ \Omega$, $\varepsilon_1 = 5$ V, $\varepsilon_2 = 10$ V



10. Determine current through batteries ε_1 and ε_2.



11. Find the equivalent resistance across terminals A and B as shown in figure. Each resistance in circuit is R.



ANSWERS

1. (a) $\frac{18}{31}$ A (b) $\frac{66}{31}$ A (c) $\frac{48}{31}$ A **2.** 120

3. Current in $EF = 1$ A (from F to E)
Current in $BE = 10.5$ A (from B to E)
Current in $DE = 11.5$ A (from E to D)

4. (i) –2 V (directed to left) (ii) $\frac{2}{3}$ A **5.** 2 V, 4 Ω

6. (a) $\frac{4}{3}$ V, $i_1 = \frac{11}{4}$ A, $i_2 = 2.5$ A (b) $\frac{11}{4}$ A, $\frac{1}{4}$ A and 2.5 A

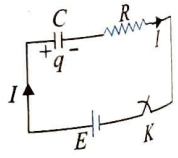
7. $\frac{25}{12}$ A **8.** $\frac{5}{2}$ A **9.** 3.75 A

10. Current through each battery will be 2 A **11.** $4R/9$

CHARGING AND DISCHARGING OF A CAPACITOR THROUGH A RESISTANCE

CHARGING

Consider an uncharged capacitor C connected to a resistor R through a battery of emf E. Let switch K be closed at $t = 0$. Let at any time t, charge on the capacitor be q and current in the circuit be I. We want to find the values of q and I.



Initial conditions: At $t = 0$, $q = 0$ because capacitor is uncharged at $t = 0$. At any time, $I = dq/dt$. Applying Kirchhoff's law at any time, we get

$$E = \frac{q}{C} + IR \qquad \text{...(i)}$$

or $$E = \frac{q}{C} + \frac{dq}{dt}R \text{ or } \frac{dq}{dt}R = -\frac{q}{C} + E$$

or $$\int_0^q \frac{dq}{EC - q} = \int_0^t \frac{dt}{RC} \text{ or } \left(\frac{\ln(EC - q)}{-1}\right)_0^q = \frac{t}{RC}$$

or $\ln(EC-q)-\ln EC=-\dfrac{t}{RC}$

or $\ln\left(\dfrac{EC-q}{EC}\right)=-\dfrac{t}{RC}$ or $\dfrac{EC-q}{EC}=e^{-t/RC}$

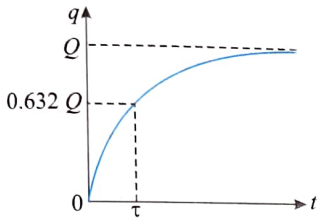
or $q=EC[1-e^{-t/RC}]=Q(1-e^{-t/\tau})$ ...(ii)

where $Q=EC$ is the maximum charge on capacitor and $\tau=RC$ is the time constant of the circuit. Let us find the charge at $t=\tau$. Putting $t=\tau$ in Eq. (ii), we get

$$q=Q(1-e^{-\tau/\tau})=Q(1-e^{-1})$$

$$=Q\left(1-\frac{1}{e}\right)=\frac{Q(e-1)}{e}=0.632Q$$

So time constant (τ) is the time in which charge on the capacitor becomes 63.2% of maximum charge. The variation of charge q with time t is as shown in the given in figure.



Current or rate of charging: Differentiating Eq. (ii), we get

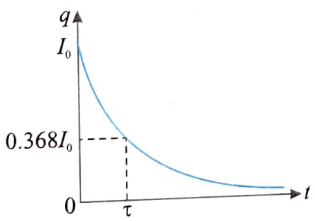
$$I=\frac{dq}{dt}=Q\left(0-e^{-t/\tau}\left(-\frac{1}{\tau}\right)\right)=\frac{Q}{\tau}e^{-t/\tau}=\frac{EC}{RC}e^{-t/\tau}$$

$$=\frac{E}{R}e^{-t/\tau}=I_0e^{-t/\tau} \quad ...(iii)$$

where $I_0=E/R$ is the maximum current in the circuit. Let us find the current at $t=\tau$, putting $t=\tau$ in Eq. (iii), we get

$$I=I_0e^{-\tau/\tau}=\frac{I_0}{e}=0.368I_0$$

So time constant (τ) may also be defined as the time in which current decreases to 36.8% of its initial maximum value. The variation of current I with time t is as shown in figure.



Notes:

- Putting $t=0$ in Eqs. (ii) and (iii), we get $q=0$ and $I=I_0$. Initially, the capacitor is uncharged and the current is maximum. It means when the capacitor is uncharged, it offers no resistance to the flow of current or capacitor behaves like a simple connecting wire (or short-circuited).
- Putting $t=\infty$ in Eqs. (ii) and (iii), we get $q=Q$ and $I=0$. Now the capacitor is fully charged and no current flows through the circuit. The capacitor here offers infinite resistance (or behaves like an open circuit). This is known as steady state. In steady state, no current flows through the branch containing capacitor.
- Initial rate of charging, i.e., initial current (at $t=0$) is $I=I_0=E/R$. Let us see how much time it will take to charge the capacitor if it were to charge with a constant rate of initial rate of charging. Here total time taken is

$$t=\frac{Q}{I_0}=\frac{EC}{E/R}=RC=\tau$$

This comes out to be equal to one time constant. So time constant may also be defined as the time during which the charging would have been completed, had the growth rate been as it began initially.

- Remember this result of integration:

  If $\int_0^x\frac{dx}{a-bx}=\int_0^t cdt$, then $x=\frac{a}{b}\left(1-e^{-bct}\right)$

- From Eqs. (ii) and (iii), we can see that at $t=0$, $q=0$, then $I=E/R$ and at $t=\infty$, $q=Q=EC$, then $I=0$
- Dimensional formula of $\tau=RC$ is $[T]$.

CHARGING OF THE CAPACITOR—OTHER APPROACH

Differentiating Eq. (i) with respect to time on both the sides, we get

$$0=\frac{1}{C}\frac{dq}{dt}+R\frac{dI}{dt}$$

or $$\frac{1}{C}I+R\frac{dI}{dt}=0$$

or $$\frac{dI}{I}=-\frac{1}{RC}dt$$

Integrating both sides, we get

or $$\int_{I_0}^{I}\frac{dI}{I}=-\frac{1}{RC}\int_0^t dt \quad \text{or} \quad [\ln I]_{I_0}^{I}=-\frac{1}{RC}[t]_0^t$$

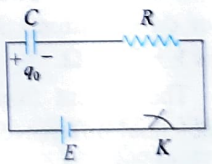
$$\ln\frac{I}{I_0}=-\frac{t}{RC}$$

or $$\frac{I}{I_0}=e^{-\frac{t}{RC}} \quad \text{or} \quad I=I_0e^{-\frac{t}{RC}}$$

This is same as Eq. (iii) obtained earlier.

ILLUSTRATION 5.71

A resistor with resistance 10 MΩ is connected in series with a capacitor of capacitance 1.0 μF and a battery with emf 12.0 V. Before the switch is closed at time $t=0$, the capacitor is uncharged.



(a) What is the time constant?

(b) What fraction of the final charge is on the plates at time $t=20$ s?

(c) What fraction of the initial current remains at $t=20$ s?

Sol.

(a) The time constant is $\tau=RC=10\times10^6\times1\times10^{-6}=10$ s

(b) The fraction f of the final charge on capacitor is q/Q

$$\frac{q}{Q}=1-e^{-t/RC}=1-e^{-20/10}=1-\frac{1}{e^2}$$

(c) $$\frac{I}{I_0}=e^{-20/10}=\frac{1}{e^2}$$

ILLUSTRATION 5.72

An uncharged capacitor is connected to a 15 V battery through a resistance of 10 Ω. It is found that in a time of 2 μs, potential difference across the capacitor becomes 5 V. Find the capacitance of the capacitor. Take ln(1.5) = 0.4.

Sol. We know that charge on the capacitor at any time is given by $q = Q(1 - e^{-t/\tau})$ where $Q = EC = 15C$. Here charge q at any time is given by $q = VC$ where V is potential difference across capacitor at that time. Here $V = 5$ V, so $q = 5C$. Putting the values, we get

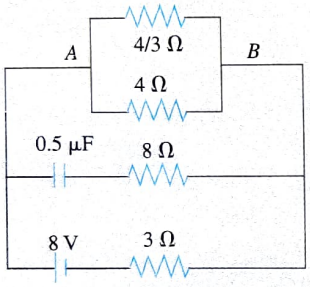
$$5C = 15C(1 - e^{-t/\tau}) \text{ or } e^{-t/\tau} = 2/3$$

$$\text{or} \quad \frac{t}{\tau} = \ln\left(\frac{3}{2}\right) \text{ or } \frac{t}{RC} = \ln\left(\frac{3}{2}\right)$$

$$\text{or} \quad C = \frac{t}{R\ln(3/2)} = \frac{2\times10^{-6}}{10\ln(3/2)} = 0.5\ \mu\text{F}$$

ILLUSTRATION 5.73

Consider the circuit shown in figure. Find out the steady-state current in the 4 Ω resistor. Assume the internal resistance of 8 V battery to be negligible.



Sol. The equivalent resistance between points A and B is

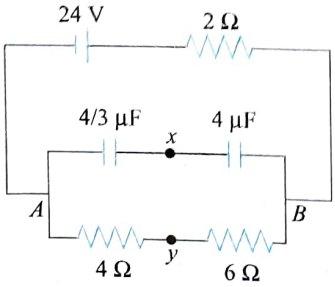
$$\frac{1}{R_{AB}} = \frac{3}{4} + \frac{1}{4} = 1 \text{ or } R_{AB} = 1\ \Omega$$

In steady state, the current will not pass through 0.5 μF capacitor as it offers infinite resistance to steady-state current or direct current. So the total resistance offered by the circuit is 1 + 3 = 4 Ω.

The current from the battery is $I = 8/4 = 2$ A. Hence, the potential difference between points A and B is $I \times R_{AB} = 2$ V. Therefore, the current through 4 Ω resistor is 2/4 = 0.5A.

ILLUSTRATION 5.74

Find out the potential difference between the points x and y in figure.



Sol. Given $E = 24$ V, $R = 2\ \Omega$, $R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$, $C_1 = (4/3)$ μF, $C_2 = 4$ μF. As the capacitor offers a very high resistance to the current in the steady state, so the current is prevented to pass through the capacitors. Now, the total resistance in the circuit is

$$R_{eq} = R + R_1 + R_2 = 2 + 4 + 6 = 12\ \Omega$$

Hence, the net current in the circuit is

$$\frac{E}{R_{eq}} = \frac{24}{12} = 2\ \text{A}$$

Therefore, the terminal potential difference across is

$$AB = 24 - 2 \times 2 = 20\ \text{V}.$$

Note: As the capacitors and resistances are in parallel, the potential difference of 20 V is available to both the capacitors and the resistors.

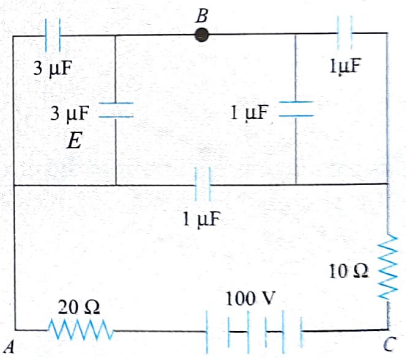
$$V_B - V_y = 20 \times \frac{6}{10} = 12\ \text{V} \quad \text{...(i)}$$

$$V_B - V_x = \frac{20 \times 4/3}{4/3 + 4} = 5\ \text{V} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), $V_x - V_y = 12 - 5 = 7$ V

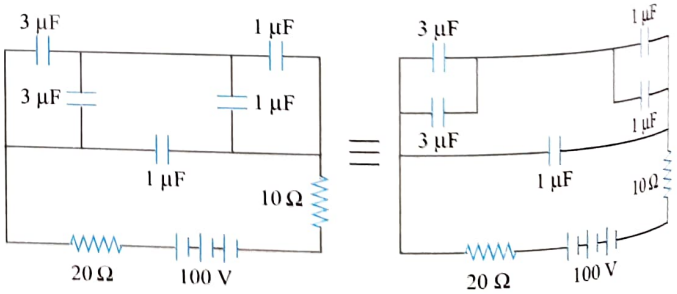
ILLUSTRATION 5.75

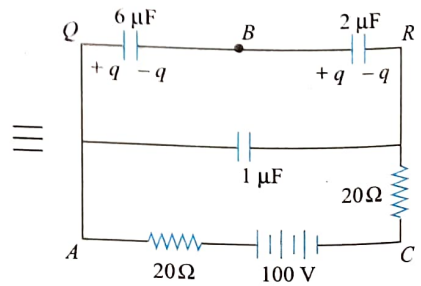
In the diagram shown in figure, find the potential difference between the points A and B and between the points B and C in the steady state.



Sol. Applying Kirchhoff's law in loop $AQBRCA$, we get

$$-\frac{q}{6} - \frac{q}{2} + 100 = 0 \text{ or } q = 150\ \mu\text{C}$$

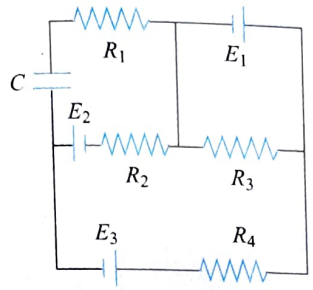




Therefore, the potential difference between AB is $150/6 = 25$ V and potential difference between BC is $100 - 25 = 75$ V.

ILLUSTRATION 5.76

In the given circuit (figure), $E_1 = 3E_2 = 2E_3 = 6$ V, $R_1 = 2R_4 = 6\ \Omega$, $R_3 = 2R_2 = 4\ \Omega$, $C = 5\ \mu$F. Find the current in R_3 and the energy stored in the capacitor.

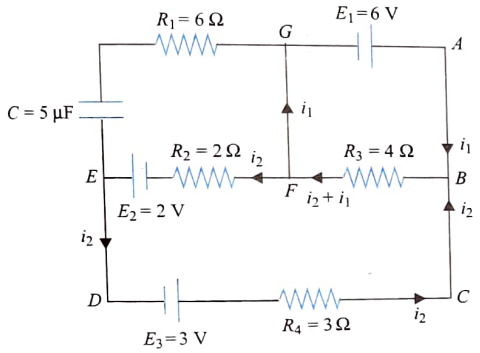


Sol. Applying Kirchhoff's law in $ABFGA$

$$6 - (i_1 + i_2)\,4 = 0 \qquad \ldots(i)$$

Applying Kirchhoff's law in $BCDEFB$

$$i_2 \times 3 - 3 - 2 + 2i_2 + (i_2 + i_1)\,4 = 0 \qquad \ldots(ii)$$



Putting the value of $4\,(i_1 + i_2) = 6$ in Eq. (ii), we get

$$3i_2 - 5 + 2i_2 + 6 = 0 \text{ or } i_2 = -\frac{1}{5}\text{ A}$$

Substituting this value in Eq. (i), we get

$$i_1 = 1.5 - \left(-\frac{1}{5}\right) = 1.7\text{ A}$$

Therefore, current in R_3 is $i_1 + i_2 = 1.5$ A. To find the potential difference across the capacitor, we have

$$V_E - 2 - 0.2 \times 2 = V_G$$

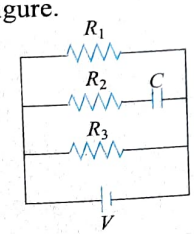
or $$V_E - V_G = 2.4\text{ V}$$

Therefore, energy stored in capacitor is

$$\frac{1}{2}CV^2 = \frac{1}{2} \times 5 \times 10^{-6} \times (2.4)^2 = 1.44 \times 10^{-5}\text{ J}$$

ILLUSTRATION 5.77

Calculate the current in branches containing R_1, R_2, and R_3 in the circuit shown in figure.



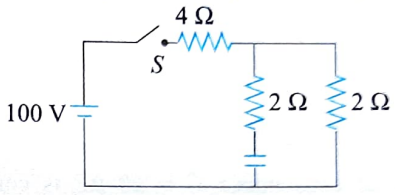
Sol. As battery connected is an ideal battery, the potential difference across R_1 and R_3 is independent of time and remains constant. So current in them remains constant. Variation in current will occur only in R_2. So

$$i_1 = \frac{V}{R_1} = \text{constant},\quad i_2 = i_0 e^{-t/R_2 C}$$

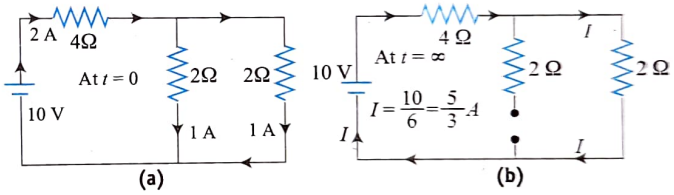
where $$i_0 = \frac{V}{R_2} \text{ and } i_3 = \frac{V}{R_3}$$

ILLUSTRATION 5.78

Calculate the current flowing through the capacitor branch in the circuit shown in figure, initially and finally.



Sol. At time $t = 0$, when the switch is closed it becomes as shown in Fig. (a). So the current flowing through the capacitor branch is 1 A. When the capacitor is fully charged, i.e., at $t = \infty$, the circuit becomes as in Fig. (b). This means that no current flows through the capacitor branch.



ENERGY DISSIPATED WHILE CHARGING

Total work done by battery in fully charging the capacitor is

$$W_b = QE \text{ but, } Q = CE$$

Hence $W_b = (CE)E = CE^2$

Final energy stored in the capacitor is

$$U = \frac{1}{2}CE^2$$

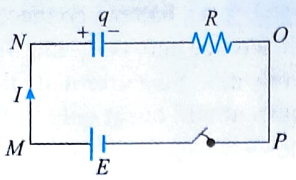
We see that the final energy stored in the capacitor is half of the work done by the battery. Now question arises, where the other half has gone. It might have been dissipated in the resistor in the form of heat. Let us check how much energy is dissipated in the resistor. For this, we have

$$H = \int_0^\infty I^2 R\,dt = \int_0^\infty I_0^2 e^{-2t/\tau} R\,dt = I_0^2 R\left(\frac{e^{-2t/\tau}}{-2/\tau}\right)_0^\infty$$

$$= \frac{-I_0^2 R\tau}{2}(0 - 1) = \left(\frac{E}{R}\right)^2 \frac{R}{2} RC = \frac{1}{2}CE^2$$

Hence, the other half is dissipated in the resistor in the form of heat. One point to notice here is that final energy dissipated in the resistor is independent of its value R.

Note: In closed loop *MNOPM*



$$E = \frac{q}{C} - IR = O$$

or $$E = \frac{q}{C} + IR \quad \ldots(i)$$

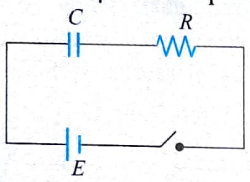
Multiplying Eq. (i) with I, we get

$$EI = \frac{q}{C}I + I^2R = \frac{q}{C}\frac{dq}{dt} + I^2R$$

The term on the left-hand side is the power supplied by battery, which is used in two ways as indicated on right-hand side. The first term on the RHS $\left(\frac{q}{C}\frac{dq}{dt} = \frac{d}{dt}\left(\frac{q^2}{2C}\right)\right)$ indicates the rate at which energy is being stored in the capacitor, and second term (I^2R) is the rate of dissipation of energy in the resistor.

ILLUSTRATION 5.79

A capacitor of capacitance $C = 10\ \mu F$ is connected to a resistance $R = 2\ \Omega$ and a battery of emf $E = 5$ V of negligible internal resistance. After 20 μs of completing the circuit, find



(a) power delivered by the battery
(b) power dissipated as heat
(c) rate of energy stored in the capacitor

Sol. Here $C = 10\ \mu F$, $R = 2\ \Omega$, $E = 5$ V, time constant, $\tau = RC = 2 \times 10 \times 10^{-6}\ s = 20\ \mu s$

(a) Current at $t = 20\ \mu s$ is

$$I = \frac{E}{R}e^{-t/\tau} = \frac{5}{2}e^{-20/20} = \frac{5}{2e}$$

Power delivered by battery is

$$P_1 = EI = 5 \times \frac{5}{2e} = \frac{25}{2e} = 4.6\ W$$

(b) Power dissipated as heat is

$$P_2 = I^2R = \left(\frac{5}{2e}\right)^2 2 = \frac{25}{2e^2} = 1.7\ W$$

(c) Energy on the capacitor is

$$U = \frac{q^2}{2C}$$

Rate of energy stored in the capacitor is

$$P_3 = \frac{dU}{dt} = \frac{2q}{2C}\frac{dq}{dt} = \frac{q}{C}\frac{dq}{dt} = \frac{Q(1-e^{-t/\tau})}{C}I$$

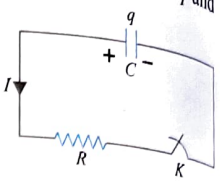
$$= \frac{EC}{C}\left(1-e^{-t/\tau}\right)I = EI\left(1-e^{-t/\tau}\right)$$

$$= \frac{5\times5}{2e}\left(1-e^{-20/20}\right)\ W = \frac{25}{2e} - \frac{25}{2e^2} = 2.9$$

Note: We see that $P_1 = P_2 + P_3$.

DISCHARGING OF CAPACITOR

Consider a capacitor is given a charge Q, and then it is connected to a resistor R. Let at $t = 0$, switch is closed. At $t = 0$, $q = Q$. Let at any time t, charge on the capacitor is q and current in the circuit is I as shown in figure. We want to find q and I. Here $I = -dq/dt$. Applying Kirchhoff's law, we get



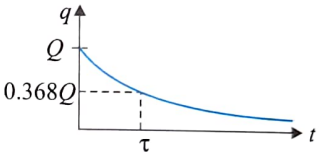
$$\frac{q}{C} = IR = -\frac{dq}{dt}R$$

or $$\int_Q^q \frac{dq}{q} = -\int_0^t \frac{dt}{RC} \text{ or } \ln\left(\frac{q}{Q}\right) = -t/RC$$

or $$q = Qe^{-t/\tau}$$

At $t = \tau$, we get $q = Qe^{-\tau/\tau} = Q/e = 0.368Q$

So time constant (τ) may also be defined as the time in which charge decreases to 36.8% of its initial maximum value in a discharging circuit. The variation of charge q with time t is shown in figure.



Current or rate of discharging:

$$I = \frac{-dq}{dt} = -Qe^{-t/\tau}\left(-\frac{1}{\tau}\right)$$

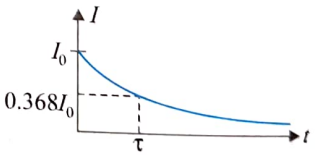
$$= \frac{Q}{\tau}e^{-t/\tau} = \frac{Q}{RC}e^{-t/\tau} = \frac{V}{R}e^{-t/\tau}$$

(where $V = Q/C$ is initial potential of capacitor)

$$I = I_0e^{-t/\tau} \text{ (where } I_0 = V/R \text{ is the maximum current)}$$

At $t = \tau$, we get $I = I_0e^{-\tau/\tau} = I_0/e = 0.368I_0$

So time constant (τ) may also be defined as the time in which current decreases to 36.8% of its initial maximum value in a discharging circuit. The variation of charge I with time t is shown in figure.



At $t = 0$, the capacitor is acting as a battery of emf Q/C. As the time passes, the charge on the capacitor decreases and the potential difference across the capacitor decreases. Hence, the capacitor acts as a battery of decreasing emf.

ENERGY DISSIPATED WHILE DISCHARGING

Let us see how much energy is dissipated in the resistor. For this

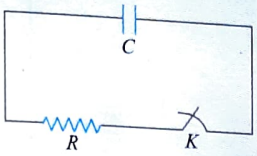
$$H = \int_0^\infty I^2R\,dt = \int_0^\infty \left(\frac{Q}{RC}\right)^2 e^{-2t/\tau}R\,dt = \frac{Q^2}{RC^2}\left(\frac{e^{-2t/\tau}}{-2/\tau}\right)_0^\infty$$

$$= -\frac{Q^2}{RC^2}\frac{\tau}{2}\left[e^{-2t/\tau}\right]_0^\infty = -\frac{Q^2RC}{RC^2\times2}(0-1) = \frac{1}{2}\frac{Q^2}{C}$$

which is the initial energy stored in the capacitor. Hence, here the whole of the energy stored in the capacitor is dissipated in the form of heat in the resistor.

ILLUSTRATION 5.80

A resistor and a capacitor are connected as shown in figure. The capacitor is originally given a charge of 5.0 μC and then discharged by closing the switch at $t = 0$.



(a) At what time will the charge be equal to 0.50 μC?

(b) What is the current at this time?

Sol.

(a) For the time t, we have

$$t = -RC \ln \frac{q}{Q} = -10 \times 10^6 \times 1.0 \times 10^{-6}$$

$$\ln \frac{0.50}{5.0} = 23 \text{ s}$$

This is 2.3 times the constant $RC = 10$ s

(b) $Q = 5.0\ \mu C = 5.010^{-6}$ C

$$I = \frac{Q}{RC} e^{-t/RC} = \frac{5.0 \times 10^{-6}}{10} e^{-23/10} = 5.0 \times 10^{-8} \text{ C}$$

ILLUSTRATION 5.81

A capacitor charged to 50 V is discharged by connecting the two plates. The connections are made at $t = 0$. At $t = 10$ ms, the potential difference across the plates is found to be 1.0 V. Find the potential difference at $t = 20$ ms.

Sol. We know that in a discharging circuit, charge on the capacitor at any time is given by $q = Qe^{-t/\tau}$. Because $q \propto V$, so same relation can be written in the form of potential difference.

$$V = V_0 e^{-t/\tau} \quad \text{where} \quad V_0 = 50 \text{ V}$$

Given at $t = 10$ ms, $V = 1$ V, putting in the above equation

$$1 = 50 e^{-10\text{ ms}/\tau} \qquad \ldots(i)$$

Let at $t = 20$ ms, $V = V_2$, then

$$V_2 = 50(e^{-10\text{ ms}/\tau})^2 \qquad \ldots(ii)$$

From Eqs. (i) and (ii),

$$V_2 = 50\left(\frac{1}{50}\right)^2 = 0.02 \text{ V}$$

Alternatively: If we take the instant $t = 10$ ms as $t = 0$, then potential difference at this instant, which is 1 V, becomes V_0. And we have to find potential difference at the end of next 10 ms.

$$V_2 = (1.0)e^{-10\text{ ms}/\tau} \qquad \ldots(iii)$$

From Eqs. (i) and (iii), $V_2 = \frac{1}{50}$ V $= 0.02$ V

ILLUSTRATION 5.82

A 5 μF capacitor having initial charge of 20 μC is discharged through a wire of resistance 5 Ω. Find the heat dissipated in the wire between 25 μs and 50 μs after the connections are made.

Sol. $\tau = RC = 5 \times 5 \times 10^{-6} = 25 \times 10^{-6}$ s $= 25$ μs

Current in the wire or circuit at any time is given by

$$i = \frac{Q}{RC} e^{-t/\tau}$$

So heat dissipated is

$$U = \int_{t_1}^{t_2} i^2 R dt = \frac{Q^2 R}{R^2 C^2} \int_{25}^{50} e^{-2t/\tau} dt$$

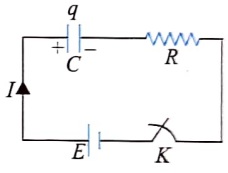
$$= \frac{Q^2}{RC^2}\left(-\frac{\tau}{2}\right)\left(e^{-2t/\tau}\right)_{25}^{50}$$

$$= -\frac{Q^2}{2C}\left[e^{-2\times 50/25} - e^{-2\times 25/25}\right] = \frac{Q^2}{2C}\left[e^{-2} - e^{-4}\right]$$

$$= \frac{(20\times 10^{-6})^2}{2\times 5\times 10^{-6}}\left[e^{-2} - e^{-4}\right] = 4.7 \times 10^{-6} \text{ J}$$

CHARGING A CAPACITOR HAVING SAME INITIAL CHARGE

Consider a situation where a charged capacitor is connected to a resistance and a battery. Let initially at $t = 0$, charge on the capacitor is $q = q_0$ before it is connected to a source of emf E as shown in figure. Let the switch is closed at $t = 0$, and let at any time t, charge on the capacitor is q and current in the circuit is I; we want to find q and I. Applying Kirchhoff's law, we get



$$E = \frac{q}{C} + IR = \frac{q}{C} + \frac{dq}{dt} R$$

or $$\int_{q_0}^{q} \frac{dq}{EC - q} = \int_0^t \frac{dt}{RC}$$

or $$\left[\ln(EC - q)\right]_{q_0}^{q} = -\frac{t}{RC}$$

or $$\ln\left(\frac{EC - q}{EC - q_0}\right) = -\frac{t}{RC}$$

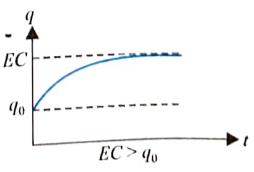
or $$\frac{EC - q}{EC - q_0} = e^{-t/RC}$$

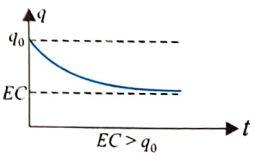
or $$q = q_0 e^{-t/\tau} + EC(1 - e^{-t/\tau}) \qquad \ldots(i)$$

$$= EC - (EC - q_0)e^{-t/\tau} \qquad \ldots(ii)$$

$$= EC\left[1 - \left(\frac{EC - q_0}{EC}\right)e^{-t/\tau}\right] \qquad \ldots(iii)$$

The variation of q versus t is shown in figure for two cases: (i) $EC > q_0$ and (ii) $EC < q_0$





$$I = \frac{dq}{dt} = -\frac{q_0}{\tau} e^{-t/\tau} + \frac{EC}{\tau} e^{-t/\tau}$$

$$= \left(\frac{EC - q_0}{\tau}\right) e^{-t/\tau}$$

If $EC > q_0$, I is positive, which means capacitor is charging. Capacitor will attain final charge EC and I becomes zero finally.

If $EC < q_0$, I is negative, which means capacitor is discharging. Capacitor will attain final charge EC and I becomes zero finally.

Note: Remember the integration

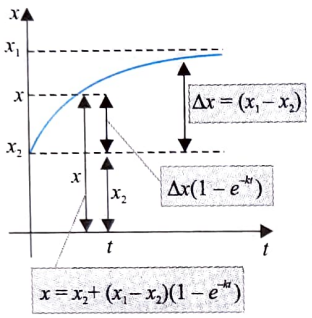
If $\int_{x_0}^{x} \frac{dx}{a-bx} = \int_0^t c\,dt$, then $x = \frac{a}{b} - \left(\frac{a}{b} - x_0\right)e^{-bct}$

If x increases from x_2 to x_1 exponentially, the x-t equation is

$$x = x_2 + (x_1 - x_2)(1 - e^{-kt})$$
$$= x_1(1 - e^{-kt}) + x_2 e^{-kt}$$

In previous derivation, $x_2 = q_0$ and $x_1 = C\varepsilon$; hence, charge as a function of time can be written as

$$q = C\varepsilon\left(1 - e^{-t/RC}\right) + q_0 e^{-t/RC}$$



The expression is the same as the one we have calculated in the previous section. Sometimes a physical quantity x decreases from x_1 to x_2, exponentially, then the x-t equation is like

$$x = x_2 + (x_1 - x_2)e^{-kt}$$

Here, k is a constant.

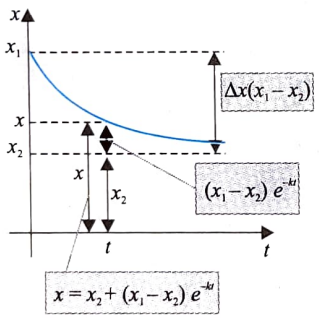
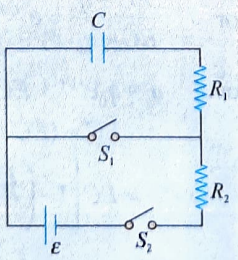


ILLUSTRATION 5.83

The capacitor shown in figure has been charged to a potential difference of V volt so that it carries a charge CV with both the switches S_1 and S_2 remaining open. Switch S_1 is closed at $t = 0$. At $t = R_1C$, switch S_1 is opened and S_2 is closed. Find the charge on the capacitor at $t = 2R_1C + R_2C$.



Sol. **Method 1:** First, capacitor will discharge through R_1.

At $t = R_1C$, $q_1 = CV\left(e^{-R_1C/R_1C}\right) = \frac{CV}{e}$

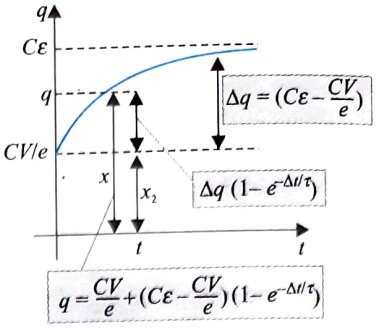
Now it will be charged through $R_1 + R_2$. So

$$\int_{q_1}^{q} \frac{dq}{C\varepsilon - q} = \int_{R_1C}^{2R_1C + R_2C} \frac{dt}{(R_1 + R_2)C}$$

On solving we get

$$q = C\varepsilon\left(1 - \frac{1}{e}\right) + \frac{CV}{e^2}$$

Method 2: Just after opening the switch S_1, the charge on the capacitor is $q_0 = CV/e$.



When the switch S_1 is opened and S_2 is closed, the capacitor is finally charged to $q_{final} = C\varepsilon$. Hence, capacitor is charged from q_0 to q_f exponentially. In this case, the time after S_2 is closed is

$$\Delta t = (2R_1C + R_2C) - R_1C = (R_1C + R_2C) = (R_1 + R_2)C$$

The time constant of the circuit after switch S_2 is closed is

$$\tau = (R_1 + R_2)C$$

Hence, required value of charge is

$$q = \frac{CV}{e} + \left(C\varepsilon - \frac{CV}{e}\right)\left(1 - e^{-\frac{(R_1 + R_2)C}{(R_1 + R_2)C}}\right)$$
$$= \frac{CV}{e} + \left(C\varepsilon - \frac{CV}{e}\right)(1 - e^{-1}) = C\varepsilon\left(1 - \frac{1}{e}\right) + \frac{CV}{e^2}$$

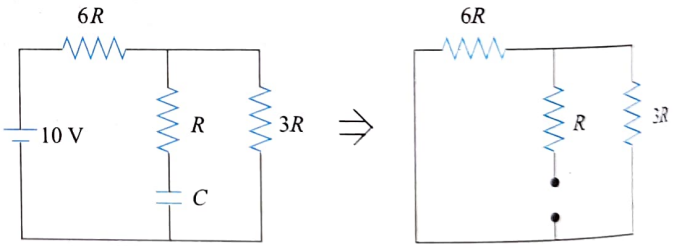
EQUIVALENT TIME CONSTANT

To find the equivalent time constant of a circuit, the following steps are taken:

(i) Short-circuit the battery.

(ii) Find net resistance across the capacitor (say R_{net}).

(iii) $\tau_C = (R_{net})C$



For example, in the circuit shown in figure, after short-circuiting the battery, $3R$ and $6R$ are in parallel, so their combined resistance is

$$\frac{(6R)(3R)}{6R + 3R} = 2R$$

Now this $2R$ is in series with the remaining R. Hence,

$$R_{net} = 2R + R = 3R \quad \text{or} \quad \tau_C = (R_{net})\,C = 3RC$$

Alternative method of finding current in the circuit and the charge on the capacitor at any time t

In a complicated C-R circuit, it is easy to find current in the circuit and charge stored in the capacitor at time $t = 0$ and $t = \infty$. But to find the current and the charge at time t, the following steps may be taken.

(i) Find equivalent time constant (τ_C) of the circuit.

(ii) Find steady-state charge q_0 (at time $t = \infty$) on the capacitor.

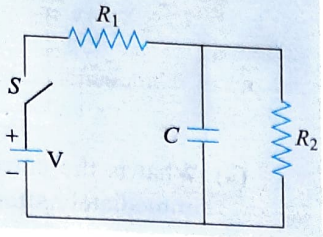
(iii) Charge on the capacitor at any time t is $q = q_0(1 - e^{-t/\tau_C})$.

By differentiating above equation with respect to time, we can find the current through the capacitor at time t. Then by using

Kirchhoff's laws, we can calculate currents in other parts of the circuit also.

ILLUSTRATION 5.84

In the given circuit, the switch S is closed at time $t = 0$. The charge Q on the capacitor at any instant t is given by $Q(t) = Q(1 - e^{-\alpha t})$. Find the value of Q_0 and α in terms of given parameters as shown in the circuit in figure.

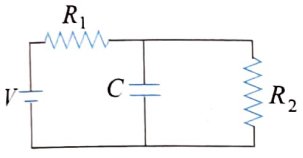


Sol. Q_0 is the steady-state charge stored in the capacitor.

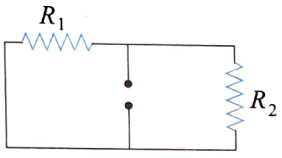
$\Rightarrow Q_0 = C$ [potential difference across the capacitor in steady state]

$= C$ [steady state current through R_2] (R_2)

$$= C\left(\frac{V}{R_1 + R_2}\right)R_2 = \frac{CVR_2}{R_1 + R_2}, \alpha \text{ is } \frac{1}{\tau_c} = \frac{1}{CR_{net}}$$



(a)



(b)

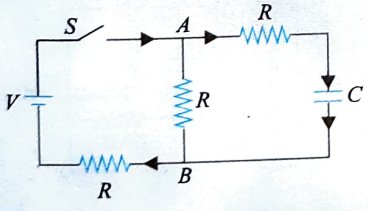
Here, R_{net} is the equivalent resistance across capacitor after short-circuiting the battery. Thus,

$$R_{net} = \frac{R_1R_2}{R_1 + R_2} \text{ (as } R_1 \text{ and } R_2 \text{ are in parallel)}$$

$$\therefore \quad \alpha = \frac{1}{C\left(\frac{R_1R_2}{R_1 + R_2}\right)} = \frac{R_1 + R_2}{CR_1R_2}$$

ILLUSTRATION 5.85

In the circuit shown in figure, the battery is an ideal one, with emf V. The capacitor is initially uncharged. The switch S is closed at time $t = 0$.

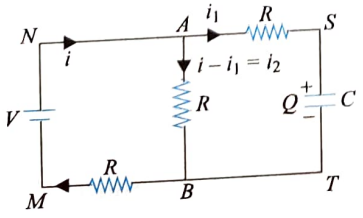


(a) Find the charge Q on the capacitor at time t.

(b) Find the current in AB at time t; what is its limiting value as $t \to \infty$.

Sol. Let at any time t charge on capacitor C be Q and currents are as shown in figure. Since charge Q will increase with time t,

$$i_1 = \frac{dQ}{dt}$$



(a) Applying Kirchhoff's second law in the loop $MNABM$, we get

$$V = (i - i_1)R + iR \text{ or } V = 2iR - i_1R \quad \ldots(i)$$

Similarly, applying Kirchhoff's second law in loop $MNSTM$, we have

$$V = i_1R + \frac{Q}{C} + iR \quad \ldots(ii)$$

Eliminating i from Eqs. (i) and (ii), we get

$$V = 3i_1R + \frac{2Q}{C} \text{ or } 3i_1R = V - \frac{2Q}{C} \text{ or } i_1 = \frac{1}{3R}\left(V - \frac{2Q}{C}\right)$$

or $$\frac{dQ}{dt} = \frac{1}{3R}\left(V - \frac{2Q}{C}\right) \text{ or } \frac{dQ}{V - \frac{2Q}{C}} = \frac{dt}{3R}$$

or $$\int_0^Q \frac{dQ}{V - \frac{2Q}{C}} = \int_0^t \frac{dt}{3R}$$

This equation gives

$$Q = \frac{CV}{2}(1 - e^{-2t/3RC})$$

(b) $$i_1 = \frac{dQ}{dt} = \frac{V}{3R}e^{-2t/3RC}$$

From Eq. (i), we get

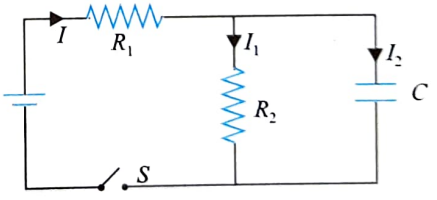
$$i = \frac{V + i_1R}{2R} = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R}$$

Current through AB is

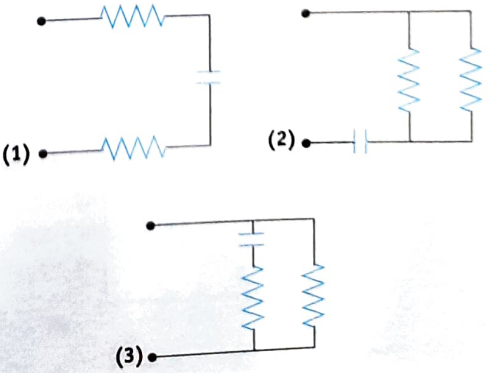
$$i_2 = i - i_1 = \frac{V}{2R} - \frac{V}{6R}e^{-2t/3RC} = \frac{V}{2R} \text{ as } t \to \infty$$

CONCEPT APPLICATION EXERCISE 5.6

1. Consider the circuit shown in figure. If the switch is closed at $t = 0$, then calculate the values of I, I_1, and I_2 at
   (a) $t = 0$ (b) $t = \infty$



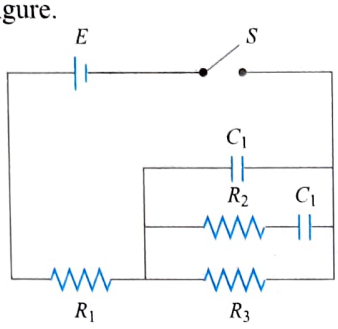
2. Figure shows three sections of the circuit that are to be connected in turn to the same battery via a switch. The resistors are all identical, as are capacitors. Rank the sections according to



(a) the final (equilibrium) charge on the capacitor _____.

(b) the time required for the capacitor to reach 50% of its final charge, greatest first _____.

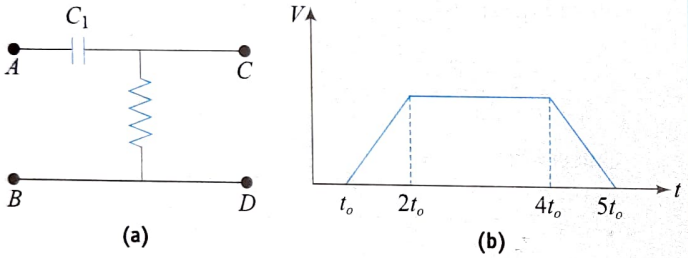
3. Determine the current through the battery in the circuit shown in figure.



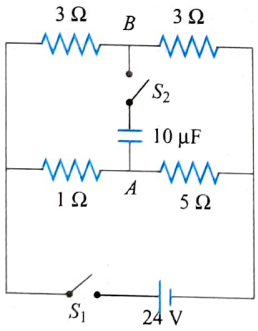
(a) Immediately after the switch S is closed _____.

(b) After a long time _____..

4. A varying voltage is applied to the clamps AB as shown in figure such that the voltage across the capacitor plates varies as shown in the figure. Plot the time depending on voltage across the clamps CD.



5. Consider the network shown in figure; initially, the switch S_1 is closed and S_2 is open.

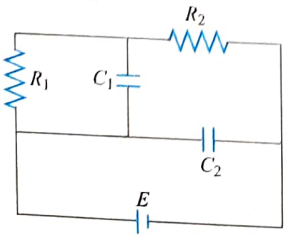


(a) Calculate $V_A - V_B$

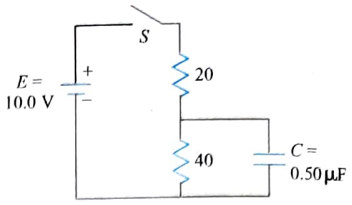
(b) When S_2 is also closed, what is $V_A - V_B$

(i) just after closing **(ii)** after long time

6. In the circuit shown in figure, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $C_1 = 1\ \mu F$, $C_2 = 2\ \mu F$, and $E = 6V$. Calculate the charge on each capacitor in the steady state.



7. In the circuit shown in figure, switch S is closed at time $t = 0$.



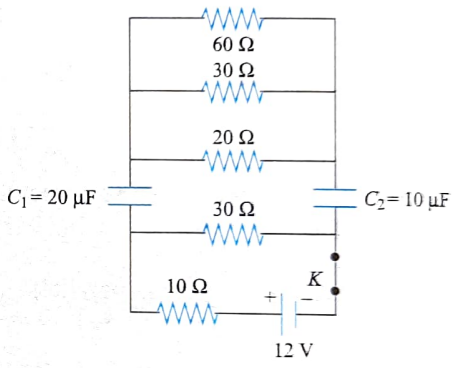
(a) What is the current I_0 leaving the battery at $t = 0$, immediately after the switch is closed?

(b) What is the current I "long time" later?

(c) What charge has accumulated on the capacitor after this long time?

(d) If, finally, switch S is opened again, how long will it take after the switch is opened for the capacitor to lose 80% of its charge?

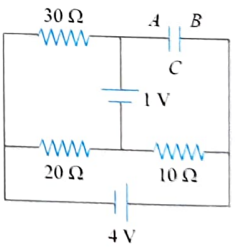
8. For the circuit arrangement shown in figure,



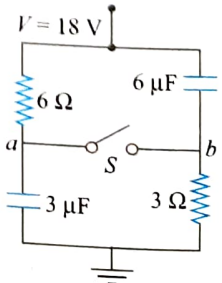
(a) find the potential difference across each capacitor in the steady-state condition.

(b) find the current through the 60 Ω resistor just after the instant when the key K is opened.

9. Find the potential difference between the plates of the capacitor C in the circuit shown in figure. The internal resistances of sources can be neglected.



10. **(a)** What is the potential difference between points a and b in figure when switch S is open ?

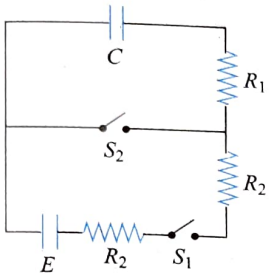


(b) Which point, a or b, is at the higher potential?

(c) What is the final potential of point b when switch S is closed?

(d) How much does the charge on each capacitor change when S is closed?

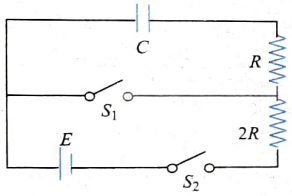
11. For a circuit shown in figure switch S_1 is closed at $t=0$, then at $t=(2R_2+R_1)C$, S_1 is opened and S_2 is closed.



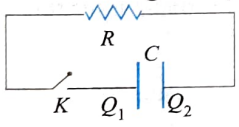
(a) Find the charge on capacitor at $t=(2R_2+2R_1)C$.

(b) Find current through R_2 (adjacent to the battery) at $t=(3R_1+2R_2)C$.

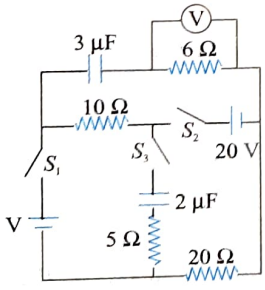
12. The given R-C circuit has two switches S_1 and S_2. Switch S_2 is closed, and S_1 is open till the capacitor is fully charged to q_0. Then S_2 is opened and S_1 is closed simultaneously till the charge on capacitor remains $q_0/2$ for which it takes time t_1. Now S_1 is again opened, and S_2 is closed till charge on capacitor becomes $3q_0/4$. It takes time t_2 (see figure for reference). Find the ratio t_1/t_2.



13. The plates of a capacitor of capacitance C are given the charges Q_1 and Q_2 as shown in figure. Now the switch is closed at $t=0$. Find the charges on plates after time t.



14. Only switch S_1 closed in figure.



(a) What is the steady-state reading of the voltmeter?

(b) What is the charge on the 3 μF capacitor?

(c) How much power does the 12 V battery supply in the steady state after all the switches are closed?

(d) What is the charge on 2 μF capacitor?

ANSWERS

1. (a) $I_1=0, I=I_2=\dfrac{\varepsilon}{R_1}$ (b) $I_2=0, I=I_1=\dfrac{\varepsilon}{R_1+R_2}$

3. (a) E/R_1 (b) E/R_1+R_3

5. (a) 8 V (b) (i) 0, (ii) 8 V **6.** 2 μC, 12 μC

7. (a) 0.5 A (b) 1/6 A (c) 10/3 μC (d) $20\times10^{-6}\ln(5)$

8. (a) $V_1=3$ V, $V_2=6$ V (b) 0.0375 A **9.** –1 V

10. (a) 18 V (b) Point a (c) 6 V (d) 54 μC, –36 μC

11. (a) $\dfrac{CE(e-1)}{e^2}$ (b) No current in R_2 **12.** 1/3

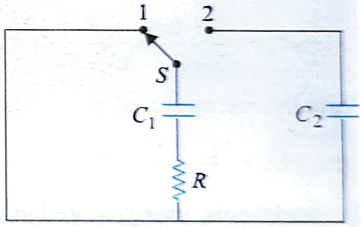
13. $\left(\dfrac{Q_1+Q_2}{2}\right)+\left(\dfrac{Q_1-Q_2}{2}\right)e^{-t/RC}, \left(\dfrac{Q_1+Q_2}{2}\right)-\left(\dfrac{Q_1-Q_2}{2}\right)e^{-t/RC}$

14. (a) 0 (zero) (b) 36 μC (c) $\dfrac{16}{5}$ W (d) $\dfrac{88}{3}$ μC

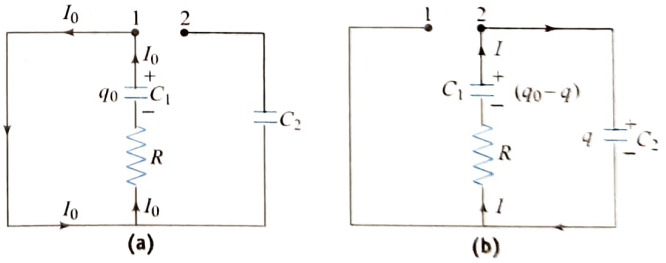
Solved Examples

EXAMPLE 5.1

A charged capacitor C_1 is discharged through a resistance R by putting switch S in position 1 of the circuit as shown in figure. When the discharge current reduces to i_0, the switch is suddenly shifted to position 2. Calculate the amount of heat liberated in resistor R starting from this instant. Also calculate current I through the circuit as a function of time.



Sol. Let the charge on capacitor C_1 be q_0 when the switch was shifted from position 1 to position 2. Just before shifting of switch, the circuit was as shown in Fig. (a).



$$\frac{q_0}{C_1}-I_0R=0 \text{ or } q_0=I_0RC_1$$

When the switch is shifted from position 1 to position 2, the capacitor C_1 continues to be discharged while C_2 starts charging. Let at time t, after shifting of switch to position 2, charge on capacitor C_2 be q and let current through the circuit be I.

Therefore, charge remaining on C_1 is equal to (q_0-q) as shown in Fig. (b). Applying Kirchhoff's voltage law on the circuit shown in Fig. (b), we get

$$\frac{q}{C_2}+IR-\frac{(q_0-q)}{C_1}=0$$

or $$IR = \frac{q_0 - q}{C_1} - \frac{q}{C_2} = \frac{(q_0C_2 - qC_2) - qC_1}{C_1C_2}$$

But current, $I = dq/dt$ (rate of increase of charge on C_2)

or $$R\frac{dq}{dt} = \frac{q_0C_2 - q(C_1 + C_2)}{C_1C_2}$$

or $$\frac{dq}{q_0C_2 - q(C_1 + C_2)} = \frac{dt}{RC_1 \times C_2}$$

But at $t = 0$, $q = 0$,

$$\int_0^q \frac{dq}{q_0C_2 - q(C_1 + C_2)} = \int_0^t \frac{dt}{RC_1C_2}$$

From the above equation, we get

$$q = \left(\frac{q_0C_2}{C_1 + C_2}\right)\left[1 - e^{-\left(\frac{C_1 + C_2}{RC_1C_2}\right)t}\right]$$

Substituting $q_0 = I_0RC_1$, we get

$$q = I_0\frac{RC_1C_2}{C_1 + C_2}\left[1 - e^{-\left(\frac{C_1 + C_2}{RC_1C_2}\right)t}\right]$$

But current

$$I = \frac{dq}{dt} = I_0e^{-\left(\frac{C_1 + C_2}{RC_1C_2}\right)t}$$

In a steady state, the common potential difference across capacitors is given by

$$V = \frac{q_0 + 0}{C_1 + C_2} = \frac{I_0RC_1}{C_1 + C_2} \quad \left(\text{using } V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)$$

Initially energy stored in C_1 was

$$U_1 = \frac{q_0^2}{2C_1} = \frac{1}{2}I_0^2R^2C_1$$

In steady state, energy stored in two capacitors is

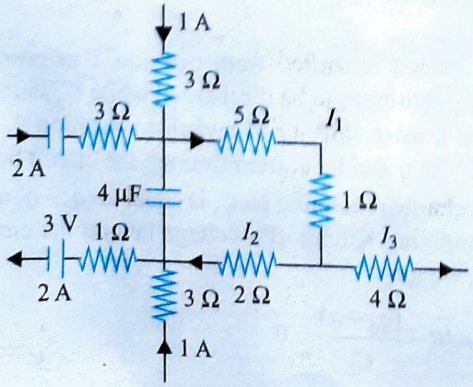
$$U_2 = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)\frac{I_0^2R^2C_1^2}{(C_1 + C_2)^2} = \frac{I_0^2R^2C_1^2}{2(C_1 + C_2)}$$

Heat generated across resistor R is equal to the loss of energy stored in capacitors during redistribution of charge, i.e.,

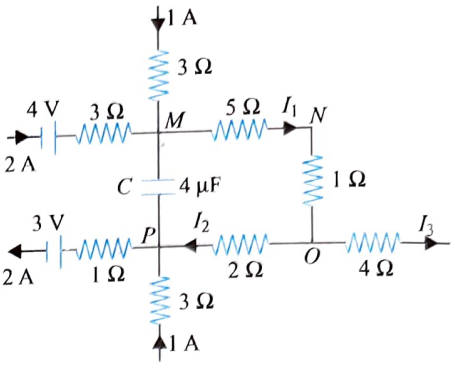
$$U_1 = U_2 = \frac{I_0^2R^2C_1C_2}{2(C_1 + C_2)}$$

EXAMPLE 5.2

A part of a circuit is in steady state along with the current flowing in the branches. Value of each resistance is shown in figure. Calculate the energy stored in the capacitor C (4 μF).



Sol. Applying Kirchhoff's first law at junction M, we get the current $I_1 = 3$ A. Applying Kirchhoff's first law at junction P, we get the current $I_2 = 1$ A.



Moving the loop from $MNOP$, we get

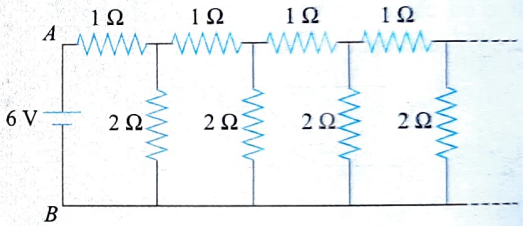
$$V_M - 5I_1 - I_1 - 2I_2 = V_P \text{ or } V_M - V_P = 6I_1 + 2I_2 = 20 \text{ V}$$

Energy stored in the capacitor is

$$\frac{1}{2}CV^2 = \frac{1}{2} \times 4 \times 10^{-6} \times 20 \times 20 = 8 \times 10^{-4} \text{ J}$$

EXAMPLE 5.3

An infinite ladder network of resistance is constructed with 1 Ω and 2 Ω resistances, as shown in figure.

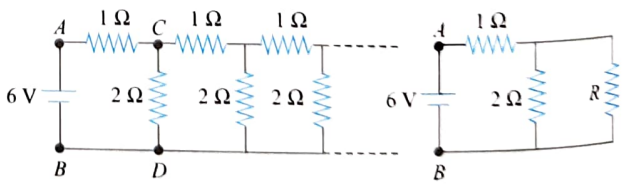


The 6 V battery between A and B has negligible internal resistance.

(i) Show that the effective resistance between A and B is 2 Ω.

(ii) What is the current that passes through the 2 Ω resistance nearest to the battery?

Sol. Let the effective resistance between points C and D be R, then the circuit can be redrawn as shown in figure.



The effective resistance between A and B is

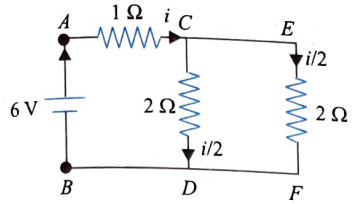
$$R_{eq} = 1 + \frac{2 \times R}{R + 2}$$

This resistance R_{eq} can be taken as R because if we add one identical item in infinite item, then the result will be the same. Therefore,

$$1 + \frac{2 \times R}{R + 2} = R \text{ or } R + 2 + 2R = R^2 + 2R$$

or $R^2 - R - 2 = 0$ or $(R + 1)(R - 2) = 0$ or $R = 2\ \Omega$

Now we can simplify the circuit by joining 2 Ω resistance parallel across C and D.



The current supplied by battery

$$i = \frac{6}{2}\,3\text{ A}$$

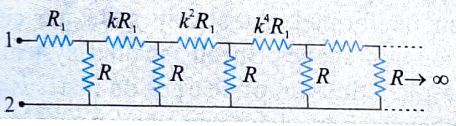
This current will be divided equally in the resistances connected across CD and EF.

Hence current is the resistance connected across C and D.

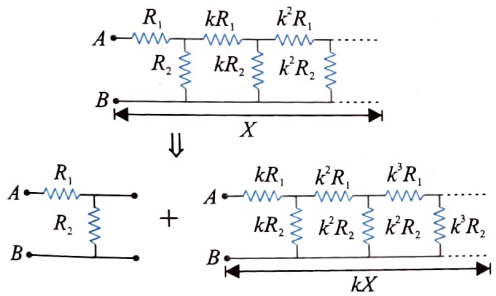
$$i_{CD} = \frac{i}{2} = \frac{3}{2}\text{ A}$$

EXAMPLE 5.4

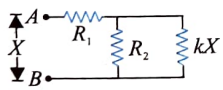
The circuit diagram shown in the figure consists of a large number of elements (each element has two resistors R_1 and R_2). The resistance of the resistors in each subsequent element differs by a factor of $k = 1/2$ from the resistance of the resistors in previous elements. Find the equivalent resistance between A and B shown in figure.



Sol. When each element of circuit is multiplied by a factor k, then equivalent resistance also becomes k times. Let the equivalent resistance between A and B be x.



So the equivalent circuit becomes



Equivalent resistance across A and B

$$R_{AB} = X = R_1 + \frac{R_2(KX)}{R_2 + (KX)}$$

$$\Rightarrow \quad KX^2 + R_2X = R_1R_2 + KR_1X + KR_2X \qquad \text{...(i)}$$

As $k = \frac{1}{2}$

Now Eq. (i) becomes

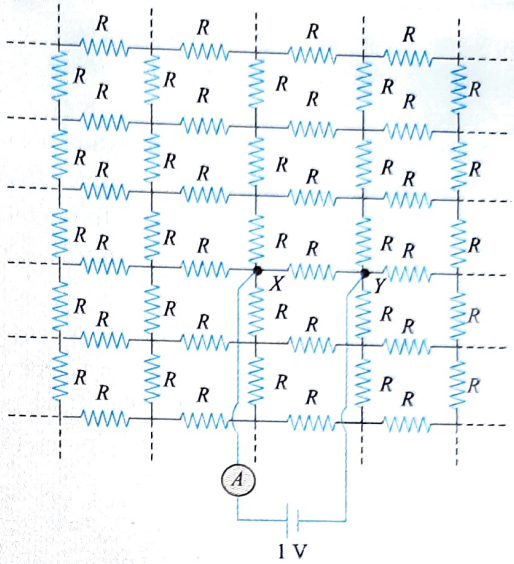
$$X^2 + (R_2 - R_1)X - 2R_1R_2 = O$$

$$\Rightarrow \quad X = \frac{-(R_2 - R_1) \pm \sqrt{(R_2 - R_1)^2 + 8R_1R_2}}{2}$$

or $$X = \frac{(R_1 - R_2) + \sqrt{R_1^2 + R_2^2 + 6R_1R_2}}{2}$$

EXAMPLE 5.5

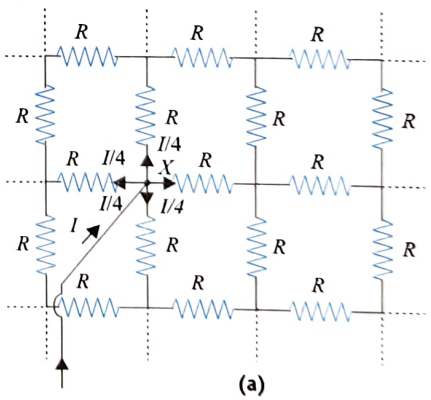
Each component in the infinite network shown in figure has a resistance $R = 4\ \Omega$. A battery of emf 1 V and negligible internal resistance is connected between any two neighboring points, say X and Y.

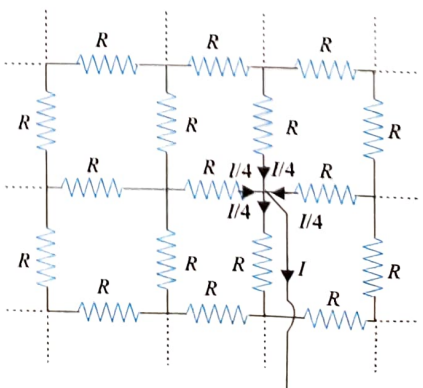


(i) Find current shown by the ammeter.

(ii) If the resistance R between X and Y is removed, then find current shown by ammeter.

Sol. Understanding of symmetry and superposition could greatly help in solving the given circuit. A battery is connected between any two neighboring points X and Y. Consider first the distribution of current at X as shown in Fig. (a). A current I enters the circuit at X and distributes in the infinite network. By symmetry, the current will be divided equally in the four resistances connected to X. This is because of the mutual equivalence of the four possible directions in which the current distributes at X.

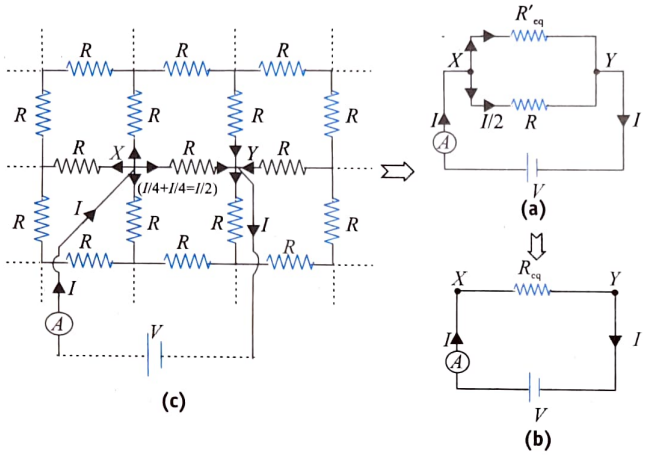




Now consider the neighboring point Y, which is connected to the negative terminal of the battery. By symmetry and independence of our earlier discussion, if a current I has to leave the circuit at Y, currents each $I/4$ will be directed toward B through the four resistors connected to Y, as shown in Fig. (b).

Let us now superpose the two cases so that a current I enters at X and leaves at Y. Superposition of Figs. (a) and (b) is shown in Fig. (c). In the resistance R between X and Y, the two currents in Figs. (a) and (b), being in the same direction, add up and a total current $I/2$ passes through it. Thus, current I divides at X such that $I/2$ goes through this R and the remaining $I/2$ through the rest of circuit, and at Y, these currents are combined resulting in current I leaving the network.

It implies that we can consider the network to consist of two resistances connected in parallel between X and Y. One of these is the resistance R between X and Y, and the other is the equivalent resistance of the rest of circuit. This is shown in Fig. (a).



From Fig. (a), we have

$$V = \frac{I}{2}R$$

And from Fig. (b), we have

$$V = IR_{eq}$$

$$IR_{eq} = \frac{I}{2}R$$

or $$R_{eq} = \frac{R}{2}$$

Hence, the equivalent resistance of the network between X and Y or any two neighboring points is $R/2$.

$$V = IR_{eq};\ I = \frac{V}{R_{eq}}$$

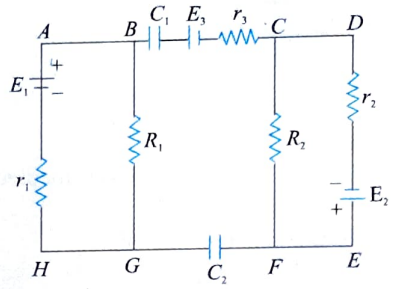
$$R_{eq} = \frac{R}{2};\ I = \frac{2V}{R}$$

Given $V = 1$ V and $R = 4\ \Omega$

$$I = \frac{2}{4} = 0.5 \text{ A}$$

EXAMPLE 5.6

In the circuit shown in figure, calculate charge on capacitors C_1 and C_2 in steady state.



$R_1 = 8\ \Omega$, $R_2 = 5\ \Omega$, $C_1 = 6\ \mu$F, $C_2 = 3\ \mu$F, $E_1 = 5$ V, $r_1 = 2\ \Omega$, $E_2 = 24$ V, $r_2 = 3\ \Omega$, $E_3 = 14$ V, and $r_3 = 2\ \Omega$

Sol. In steady state, no current flows through capacitors; therefore, there are four unknown quantities in the given circuit:

(i) current in left mesh $ABGHA$

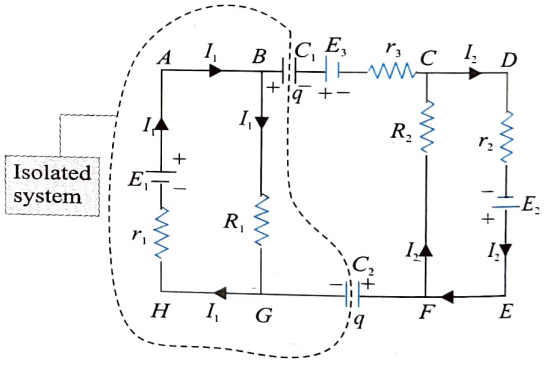
(ii) current in right mesh $CDEFC$

(iii) charge q_1 on capacitor C_1

(iv) charge q_2 on capacitor C_2

But by applying Kirchhoff's voltage law, three unique equations can be formed. At steady state, no current passes through capacitor branches. In an isolated system, net charge should be zero, i.e., the sum of charges on plates is zero. Hence, charge on both capacitors should be equal. So

$$q_1 = q_2 = q \text{ (say)}$$



Considering this fact, in steady state, circuit will be as shown in figure. Applying Kirchhoff's voltage law in mesh $ABGHA$, we get

$$I_1R_1 + I_1r_1 - E_1 = 0$$

or $$I_1 = 0.5 \text{ A}$$

For mesh $CDEFC$,

$$I_2r_2 - E_2 + I_2R_2 = 0$$

or $$I_2 = 3 \text{ A}$$

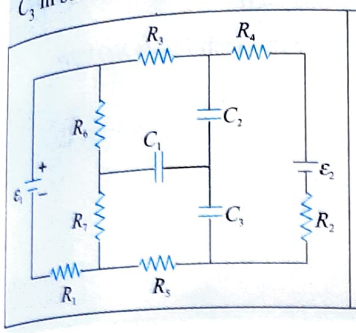
Now applying Kirchhoff's voltage law in mesh $BCFGB$, we get

$$+\frac{q}{C_1} + E_3 - I_2R_2 + \frac{q}{C_2} - I_1R_1 = 0$$

or $$q = 10\ \mu\text{C}$$

EXAMPLE 5.7

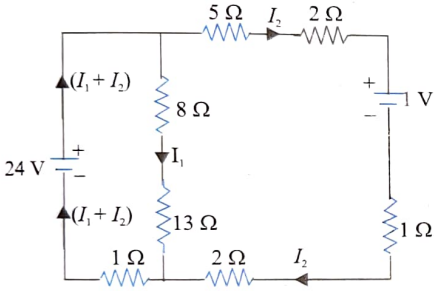
Analyze the circuit, find the charge in capacitors C_1, C_2, and C_3 in steady state.



$R_1 = 1\ \Omega$, $R_2 = 1\ \Omega$,
$R_3 = 5\ \Omega$, $R_4 = 2\ \Omega$,
$R_5 = 2\ \Omega$, $R_6 = 8\ \Omega$,
$R_7 = 13\ \Omega$, $C_1 = 1\ \mu F$,
$C_2 = 3\ \mu F$, $C_3 = 2\ \mu F$,
$\varepsilon_1 = 24\ V$, $\varepsilon_2 = 1\ V$

Sol. In steady state, no current flows through capacitor. Therefore, to calculate current in resistors, the circuit can be analyzed after removing capacitors.

Let in steady state, a discharging current $(I_1 + I_2)$ flow through 24 V battery. Current distribution (according to Kirchhoff's current law) will be as shown in figure.



Applying Kirchhoff's voltage law on left mesh, we get

$$8I_1 + 13I_1 + 1(I_1 + I_2) - 24 = 0$$

or $$22I_1 + I_2 = 24 \quad ...(i)$$

For right mash,

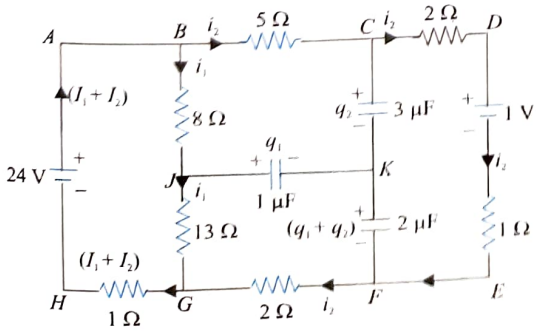
$$5I_2 + 2I_2 + 1 + I_2 + 2I_2 - 13I_1 - 8I_1 = 0$$

or $$10I_2 - 21I_1 + 1 = 0 \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$I_1 = 1\ \text{amp} \quad \text{and} \quad I_2 = 2\ \text{amp}$$

If in steady state the charges on 1 μF and 3 μF capacitors are q_1 and q_2, respectively, then the charge on 2 μF capacitor will be $q_1 - q_2$. Hence, in steady state, the circuit will be as shown in figure.



Applying Kirchhoff's voltage law on mesh *BCKJB*,

$$5I_2 + \frac{q_2}{3} - \frac{q_1}{1} - 8I_1 = 0 \quad ...(iii)$$

For mesh *JKFGJ*,

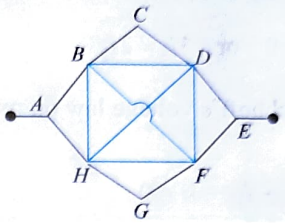
$$\frac{q_1}{1} + \frac{(q_1 + q_2)}{2} + 2I_2 - 13I_1 = 0 \quad ...(iv)$$

In Eqs. (iii) and (iv), substituting $I_1 = 1$ A and $I_2 = 2$ A, we get

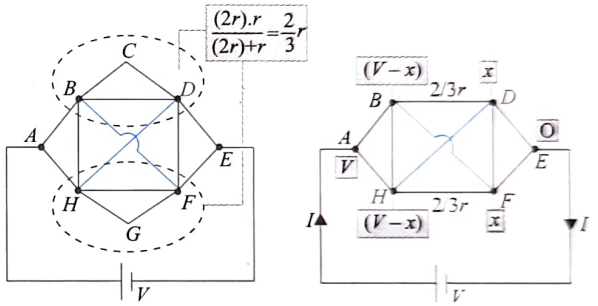
$$q_1 = 4\ \mu C \quad \text{and} \quad q_2 = 6\ \mu C$$

EXAMPLE 5.8

Identical resistors each of resistance r are connected as shown in figure. Calculate equivalent resistance between A and E.



Sol. **Method 1:** Let the potentials of nodes A and B be V and O, respectively.



From symmetry, it is clear that the current in the resistances AB and AH will be same. Similarly, the current in resistances DE and FE will be same. Hence, potential difference across these resistances will be same.

Let the potentials of points D and F be x, then the potentials of nodes B and H will be $(V - x)$. The potential difference across the resistances BH and DF is zero. The net resistance can be given as $R_{eq} = V/I$.

Nodal equation at A is

$$I = \frac{V - (V - x)}{r} + \frac{V - (V - x)}{r} = \frac{2x}{r} \quad ...(i)$$

Nodal equation at B is

$$\frac{(V - x) - V}{r} = \frac{(V - x) - x}{(2/3r)} + \frac{(V - x) - x}{r} \quad ...(ii)$$

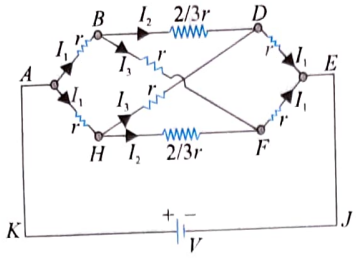
From (i) and (ii),

$$I = \frac{2}{r}\left(\frac{5}{12}V\right) \quad \text{or} \quad \frac{V}{I} = \frac{6}{5}r = R_{eq}$$

Method 2: If a battery of emf V is connected across A and E, circuit becomes as shown in figure. Therefore, currents in AB, AH, DE, and FE are identical (I_1, say).

Currents in BD and HF are identical to each other (I_2, say). Currents in BF and HD are identical to each other (I_3, say). No current flows through BH and DF. Hence, currents through different resistors will be as shown in figure. Applying Kirchhoff's voltage law at junction B, we get

$$I_1 = I_2 + I_3 \quad \text{...(i)}$$



Applying Kirchhoff's voltage law on mesh *BFHB*, we get

$$I_3 r - \frac{2}{3} r I_2 = 0 \quad \text{or} \quad 3I_3 = 2I_2 \quad \text{...(ii)}$$

Now applying Kirchhoff's voltage law on mesh *AHFEJKA*, we get

$$I_1 r + I_2\left(\frac{2}{3} r\right) + I_1 r - V = 0 \quad \text{...(iii)}$$

From Eqs. (i) and (ii), we get $I_2 = 1.5\, I_3$ and $I_1 = 2.5\, I_3$. Substituting these values in Eq. (iii), we get

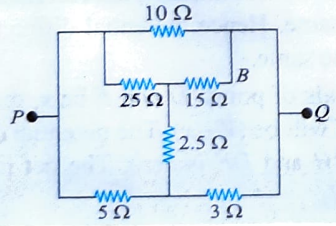
$$6I_3 r = V \quad \text{or} \quad \frac{V}{I_3} = 6r$$

Total current drawn from battery is $I = 2I_1 = 5I_3$. Therefore, equivalent resistance is

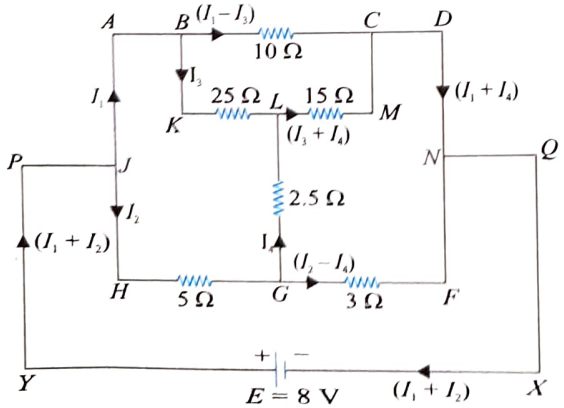
$$\frac{V}{5I_3} = \frac{1}{5} \times 6r = 1.2r$$

EXAMPLE 5.9

If a battery of emf 8 V and negligible internal resistance is connected between terminals *P* and *Q* of the circuit shown in figure, calculate the current through 2.5 Ω resistance and hence calculate the equivalent resistance of the circuit.



Sol. Let the current drawn by the circuit from battery be $I = (I_1 + I_2)$ and current distribution be as shown in figure.



Applying Kirchhoff's voltage law in mesh *BCMKB*,

$$10(I_1 - I_3) - 15(I_3 + I_4) - 25I_3 = 0 \quad \text{...(i)}$$

Applying Kirchhoff's voltage law in mesh *ABKLGHA*,

$$25I_3 - 25I_4 - 5I_2 = 0 \quad \text{...(ii)}$$

Applying Kirchhoff's voltage law in mesh *LMCDEFGL*,

$$15(I_3 + I_4) - 3(I_2 - I_4) + 25I_4 = 0 \quad \text{...(iii)}$$

Applying Kirchhoff's voltage law in mesh *PJHFNQXYP*,

$$5I_2 + 3(I_2 - I_4) - 8 = 0 \quad \text{...(iv)}$$

From above equations,

$I_1 = I_2 = 1$ A

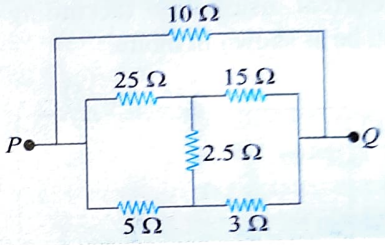
$I_3 = 0.2$ A

$I_4 = 0$

Total current drawn from battery is $I = I_1 + I_2 = 2$ A. Therefore, equivalent resistance is

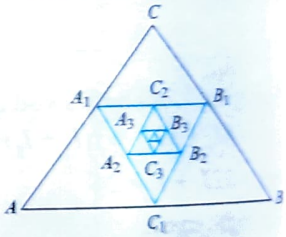
$$R = \frac{E}{I} = \frac{8}{2} = 4\ \Omega$$

Note: Given circuit can also be drawn as shown in figure. Now 10 Ω resistance is in parallel with a Wheatstone bridge.



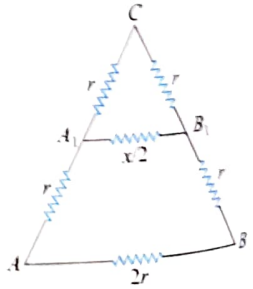
EXAMPLE 5.10

Find the resistance R_{AB} of the frame made of a thin wire. Assume that the number of successively embedded equilateral triangles (with sides decreasing to half) tends to infinity (see figure). Side *AB* is equal to *a*, and the resistance per unit length of wire is λ.



Sol. Let $R_{AB} = x$ be equivalent resistance of system between *A* and *B*. As the resistance of a conductor is directly proportional to length, the equivalent resistance between A_1 and B_1 will be $x/2$. Therefore, the equivalent circuit becomes as given in figure. Effective resistance of $2r$ and $x/2$ is

$$R_1 = \frac{2r\,\frac{x}{2}}{2r + \frac{x}{2}} = \frac{2rx}{4r + x}$$



Now R_1 is in series with AA_1 and BB_1; therefore, their effective resistance is

$$R_2 = R_1 + 2r = \frac{2rx}{4r + x} + 2r$$

R_2 is in parallel with $2r$ (of *AB*), so the net effective resistance across *AB* is

$$x = \frac{R_2 \times 2r}{R_2 + 2r} = \frac{\left(\frac{2rx}{4r + x} + 2r\right)2r}{\left(\frac{2rx}{4r + x} + 2r\right) + 2r}$$

$$3x^2 + 4rx - 8r^2 = 0 \text{ or } x = \frac{-4r \pm \sqrt{16r^2 + 4\times3\times8r^2}}{2\times3}$$

or

As x cannot be negative, we have

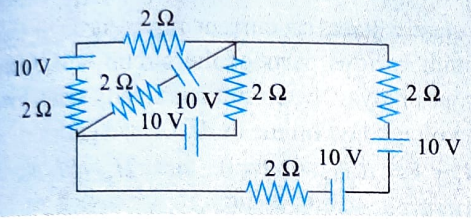
$$x = \frac{-4r + \sqrt{16r^2 + 96r^2}}{6} = \frac{(2\sqrt{7} - 2)}{3} r$$

But $2r = a\lambda$ or $r = \frac{a\lambda}{2}$, so

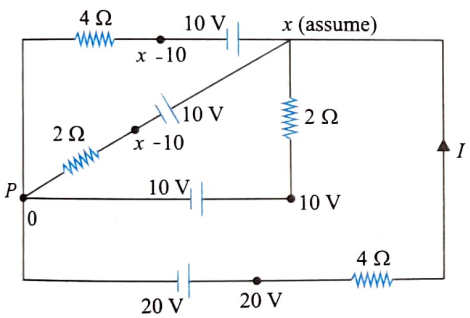
$$x = \frac{2(\sqrt{7} - 1)}{3} \times \frac{a}{2}\lambda = 0.55a\lambda$$

EXAMPLE 5.11

In the given circuit in figure, all batteries have emf 10 V and internal resistance negligible. All resistors are in ohm. Calculate the current in the rightmost 2 Ω resistor.



Sol. The simplified circuit is shown in figure. We have to find I.



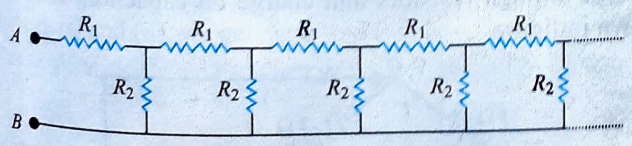
Let the potential of point P be zero. Potential at other points is shown in figure. Applying Kirchhoff's current law at x, we get

$$\frac{x-10}{4} + \frac{x-10}{2} + \frac{x-20}{4} + \frac{(x-10)}{2} = 0 \text{ or } x = \frac{35}{3} \text{ V}$$

$$I = \frac{20 - \frac{35}{3}}{4} = \frac{25}{12} \text{ A}$$

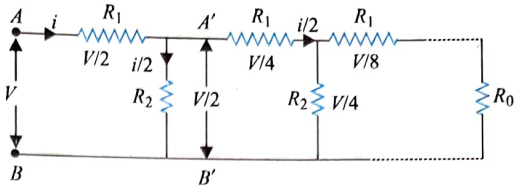
EXAMPLE 5.12

Consider an infinite ladder of network shown in figure. A voltage is applied between points A and B. If the voltage is halved after each section, find the ratio of R_1/R_2.



If this infinite ladder is terminated by a resistance, then find the value of the resistance such that the equivalent resistance of the ladder between A and B becomes independent of the number of the sections in betweeen.

Sol. Voltage across AB is V, voltage across $A'B'$ is $V/2$ i.e., voltage across R_2 is $V/2$.

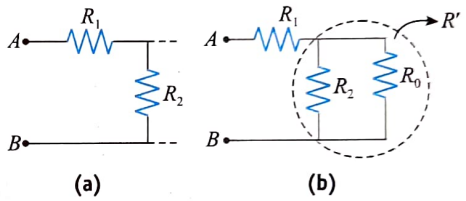


Now from Kirchhoff's law, it is obvious that voltage across R_1 is

$$V - \frac{V}{2} = \frac{V}{2}$$

$$R_1 i = R_2 \frac{i}{2} \text{ or } \frac{R_1}{R_2} = \frac{1}{2}$$

The equivalent resistance of the given infinite ladder between A and B becomes independent of the number of units in between. It means, if we remove all the resistances, other than terminal resistance 'R_0', the equivalent resistance across A and B should also be 'R_0'. Here the repeating unit is shown in Fig. (a). If we connect one unit across A and B, as shown in Fig. (b) the equivalent resistance across A and B should remain 'R_0'.



$$R' = R_0 = R_1 + \frac{R_0 R_2}{R_0 + R_2}$$

R_1 is in series with it, so equivalent resistance between A and B is

$$R_0 = R_1 + \frac{R_0 R_2}{R_0 + R_2}$$

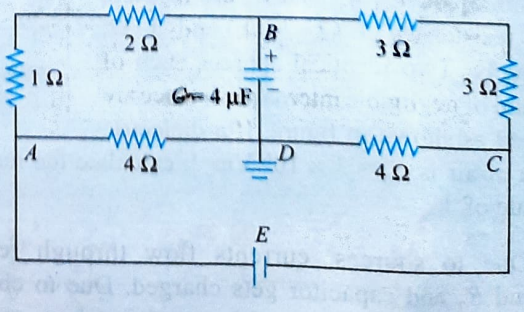
According to proposition $R_0 = R_1 + \frac{R_0 R_2}{R_0 + R_2}$

Solving for R_0, we get $R_0 = \frac{R_1}{2}\left[1 + \sqrt{\left(1 + \frac{4R_2}{R_1}\right)}\right]$

Thus, the circuit may be terminated after a few sections if resistance R_0 is connected in parallel as shown in figure.

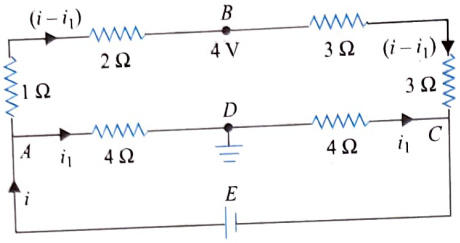
EXAMPLE 5.13

Analyze the circuit given in figure in the steady-state condition. Charge on the capacitor in this state is $q_0 = 16\ \mu C$.



(a) Find the current in each branch.
(b) Find the emf of the battery.
(c) If in the beginning, the battery is removed and nodes A and C are shortened, then find the duration in which charge on the capacitor becomes 5.92 μC.

Sol. For parts (a) and (b), we have



For BAD, $V_B + 2(i - i_1) + 1(i - i_1) - 4i_1 = V_D$

or $\quad 3i - 7i_1 = -4$

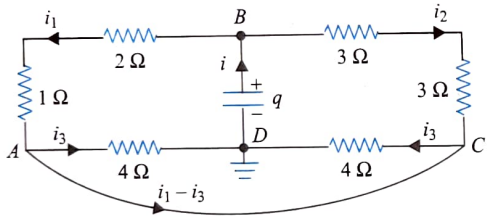
For BCD, $V_B - 6(i - i_1) + 4i_1 = V_D$

$\quad 6i - 10i_1 = 4$ or $i_1 = 3$ A, $i = 17/3$ A

Current in $AC = i_1 = 3$A. In ABC, $i - i_1 = 8/3$ A

$\quad E = 8i_1 = 24$ V

$\quad i = 2i_3 = i_1 + i_2$ ...(i)



$$\frac{q}{C} = 2i_1 + i_1 + 4i_3 = 3i_1 + 4i_3 \quad \text{...(ii)}$$

Also $$\frac{q}{C} = 3i_2 + 3i_2 + 4i_3 = 6i_2 + 4i_3 \quad \text{...(iii)}$$

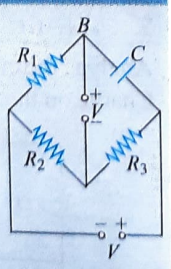
From Eqs. (ii) and (iii), we have $6i_2 = 3i_1$ or $i_1 = 2i_2$

Solve to get $\quad i = \frac{q}{4C}$ or $\frac{-dq}{dt} = \frac{q}{4C}$ or $q = q_0 e^{-t/4C}$

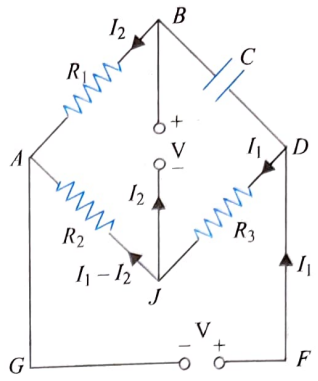
or $\quad 5.92 = 16e^{-\frac{t}{4\times4\times10^{-6}}}$ or $t = 16$ μs

EXAMPLE 5.14

In the circuit shown in figure, C is a parallel plate air capacitor having plate of area $A = 50$ cm^2 each and distance $d = 1$ mm apart. R_1, R_2, and R_3 are resistors having resistances 3 Ω, 2 Ω, and 1 Ω, respectively. Two identical sources each of emf V and of negligible internal resistance are connected as shown in figure. If a dielectric strength of air is $E_0 = 3 \times 10^6$ Vm^{-1}, calculate the maximum safe value of V.



Sol. Due to sources, currents flow through resistance R_1, R_2, and R_3 and capacitor gets charged. Due to charge, an electric field is established in the capacitor whose magnitude cannot exceed dielectric strength E_0 of air. Maximum safe value of V corresponds to the maximum possible charge on capacitor. Let the maximum possible charge on capacitor be q_0. Then the electric field inside the capacitor is $E_0 = q_0/A\varepsilon_0$,



where $q_0 = A\varepsilon_0 E_0 = 15{,}000\varepsilon_0$ and $C = \varepsilon_0 A/d = 5\varepsilon_0$ farad.

Since, in steady state, no current flows through the capacitor, current through various parts of the circuit will be as shown in figure. Now we analyze the circuit in a steady state. First applying Kirchhoff's voltage law on mesh $ABJA$, we get

$$-I_2R_1 + V + R_2(I_1 - I_2) = 0 \quad \text{or} \quad 2I_1 - 5I_2 = -V \quad \text{...(i)}$$

For mesh $AJDFGA$,

$$-R_2(I_1 - I_2) - R_3I_1 + V = 0 \text{ or } 3I_1 - 2I_2 = V \quad \text{...(ii)}$$

From Eqs. (i) and (ii): $I_1 = \frac{7V}{11}$ and $I_2 = \frac{5V}{11}$

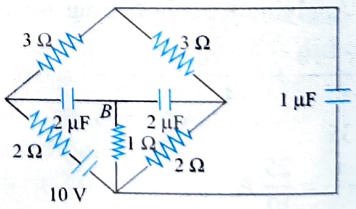
Now applying Kirchhoff's voltage law on mesh $BDJB$, we get

$$\frac{q}{C} + I_1R_3 - V = 0 \quad \text{or} \quad q = \frac{4CV}{11} = \frac{20}{11}\varepsilon_0 V$$

But the maximum possible value of q is $q_0 = 15{,}000\ \varepsilon_0$. Therefore, the maximum safe value of V is $11q_0/20\varepsilon_0 = 8250$ V $= 8.25$ kV.

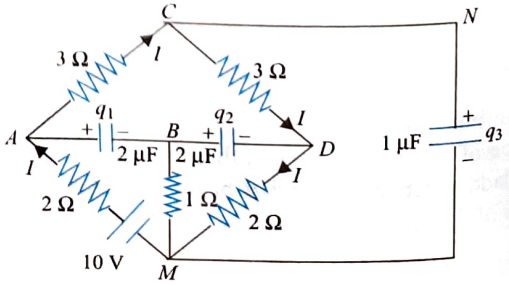
EXAMPLE 5.15

The circuit shown in figure is in steady state.



(i) Find the energy stored in the capacitors shown in figure.
(ii) Find the rate at which battery supplies energy.

Sol. Since, in steady state, no current flows through the capacitors, the current through 1 Ω resistor becomes zero. Current through resistors and charge on capacitors will be as shown in figure.



Applying *KVL* on mesh *MACDA*, we get

$$2I + 3I + 3I + 2I - 10 = 0 \quad \text{or} \quad I = 1 \text{ A}$$

Mesh *MABM*: $10 - 2I - \dfrac{q_1}{2\times10^{-6}} = 0 \quad \text{or} \quad q_1 = 16\ \mu\text{C}$

Mesh *MBDM*: $-\dfrac{q_2}{2\times10^{-6}} - 2I = 0 \quad \text{or} \quad q_2 = -4\ \mu\text{C}$

Mesh *MDCNM*: $2I + 3I - \dfrac{q_3}{(1\times10^{-6})} \quad \text{or} \quad q_3 = 5\ \mu\text{C}$

Energy stored in capacitors is

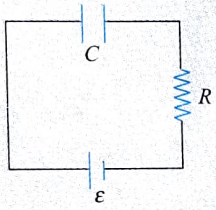
$$U = \sum \frac{q^2}{2C} = \frac{q_1^2}{2\times(2\times10^{-6})} + \frac{q_2^2}{2\times(2\times10^{-6})} + \frac{q_3^2}{2\times(1\times10^{-6})}$$

$$= 80.5\times10^{-6}\ \text{J}$$

Rate of supply of energy is $P = EI = 10\times1 = 10$ W

EXAMPLE 5.16

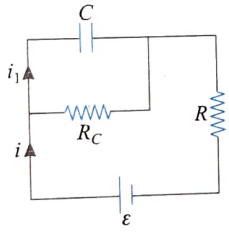
Consider a parallel plate capacitor of capacitance C with partially conducting medium between its plates having a resistance R_C. If this capacitor is connected to a battery of emf ε and a resistor R as shown in figure, find



(a) charge on the capacitor as a function of time.

(b) current through R as a function of time.

Sol. The equivalent circuit can be drawn as shown. Applying Kirchhoff's law for the two loops,



$$(i - i_1)R_C + iR = \varepsilon \qquad \ldots(i)$$

$$\frac{q}{C} = R_C(i - i_1) \qquad \ldots(ii)$$

Eliminating i between (i) and (ii), we get

$$\frac{q}{C} = \left(1 + \frac{R}{R_C}\right) + i_1 R = \varepsilon$$

For capacitor C, $\dfrac{dq}{dt} = i_1$

$$\frac{dq}{dt} = \frac{CE - q(1+\alpha)}{RC} \qquad \left(\text{where } \alpha = \frac{R}{R_C}\right)$$

Solving, we get

$$q = \frac{EC}{1+\alpha}(1 - e^{-t(1+\alpha)/RC})$$

Adding (i) and (ii), we get

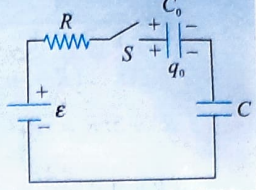
$$\frac{Q}{C} + iR = E \quad \text{or} \quad I = \frac{CE - q}{RC}$$

Substituting for q, we get

$$I = \frac{E}{R}\left[1 - \frac{1}{1+\alpha}(1 - e^{-t(1+\alpha)/RC})\right] \qquad \left(\text{where } \alpha = \frac{R}{R_C}\right)$$

EXAMPLE 5.17

The switch S is closed at $t = 0$. The capacitor C is uncharged but C_0 has a charge q_0 at $t = 0$. Calculate the current $i(t)$ in the circuit.



Sol. Let q_0 and q be the instantaneous charges on C_0 and C, respectively. Applying *KVL* to the circuit, we have

$$\frac{q_0}{C_0} + \frac{q}{C} + iR = \varepsilon$$

Differentiating this equation, we get

$$\frac{1}{C_0}\frac{dq_0}{dt} + \frac{1}{C}\frac{dq}{dt} + R\frac{di}{dt} = 0$$

or $\quad i\left[\dfrac{1}{C_0} + \dfrac{1}{C}\right] = -R\dfrac{di}{dt} \quad \text{or} \quad i\left[\dfrac{C + C_0}{CC_0}\right] = -R\dfrac{di}{dt}$

$$\left(\text{where } i = \frac{dq_0}{dt} = \frac{dq}{dt}\right)$$

or Integrating this expression, we have

$$\int_{i_0}^{i(t)} \frac{di}{i} = \int_0^t \frac{dt}{RC_{eq}} \quad \text{or} \quad [\log_e i]_{i_0}^{i(t)} = -\frac{t}{RC_{eq}}$$

or $\quad i(t) = i_0 e^{-(t/R)C_{eq}} \qquad \ldots(ii)$

where i_0 is the initial current.

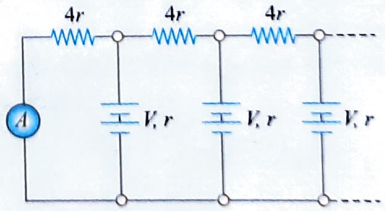
Further, $i_0 R + \dfrac{q_0}{C_0} = \varepsilon \quad \text{or} \quad i_0 = \left(E - \dfrac{q_0}{C_0}\right)\Big/R \qquad \ldots(iii)$

Substituting i_0 from Eq. (ii) into Eq. (i), we get

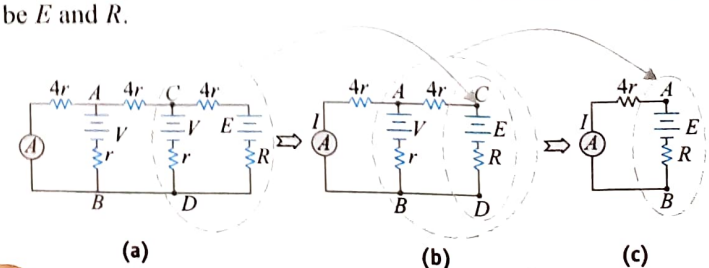
$$i(t) = \left[\frac{E - (q_0/C_0)}{R}\right] e^{-\left(\frac{t}{RC_{eq}}\right)}, \text{ where } C_{eq} = \frac{CC_0}{(C + C_0)}$$

EXAMPLE 5.18

The circuit shown in figure extends to the right into infinity. Each battery has the electromotive force V (unknown) and the internal resistance r (known). Each resistor has the resistance $4r$. The reading of the ideal ammeter shown in the diagram is I. Find the value of V in terms of I and r.



Sol. Let net emf and net resistance across CD be E and R, respectively. Then net emf and resistance across AB will also be E and R.



Calculating the value of R and E from Fig. (a)

$$R = \frac{1}{\frac{1}{r} + \frac{1}{(R+4r)}} \Rightarrow R = \frac{(R+4r)r}{(R+5r)}$$

$$R^2 + 5rR = rR + 4r^2 \Rightarrow R^2 + 4rR = 4r^2$$

$$(R+4r) = \frac{4r^2}{R} \quad \ldots(i)$$

and $$E = \frac{\frac{E}{R+4r} + \frac{V}{r}}{\frac{1}{R+4r} + \frac{1}{r}}$$

$$= \frac{Er + V(R+4r)}{r+R+4r} = \frac{Er + V(R+4r)}{r+5r}$$

or $$E(R+5r) = Er + V(R+4r)$$

or $$E(R+4r) = V(R+4r) \quad \text{or} \quad E = V$$

Also $$I = \frac{E}{R+4r} = \frac{ER}{4r^2} = \frac{VR}{4r^2}$$

or $$R = \frac{4r^2 I}{V} \quad \ldots$$

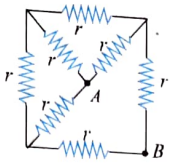
From (i) and (iii), $$\frac{4r^2 I}{V} + 4r = \frac{4r^2}{4r^2 I} V = \frac{V}{I}$$

or $$4r^2 I + 4rV = \frac{V^2}{I}$$

or $$V^2 - 4rIV - 4r^2 I^2 = 0 \quad \text{or} \quad V = 2Ir(1+\sqrt{2})$$

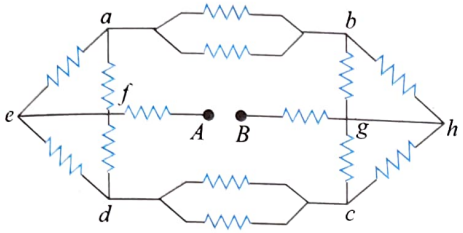
Single Correct Answer Type

1. Equivalent resistance between A and B in figure is



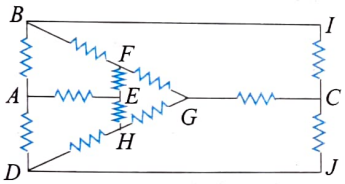
(1) $8r/7$ (2) $7r/8$
(3) $3r/4$ (4) r

2. Each of the resistors shown in figure has resistance R. Find the equivalent resistance between A and B.



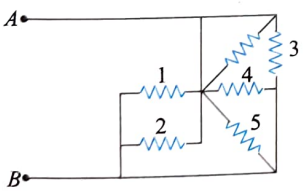
(1) $\frac{7R}{4}$ (2) $\frac{5R}{4}$
(3) $\frac{9R}{4}$ (4) $\frac{11R}{4}$

3. Find the equivalent resistance between A and E (resistance of each resistor is R).



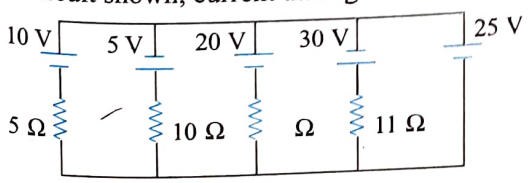
(1) $\frac{7}{12}R$ (2) $\frac{7}{13}R$
(3) $\frac{7}{15}R$ (4) $\frac{8}{13}R$

4. The circuit shown has resistors of equal resistance R. Find the equivalent resistance between A and B, when the key is closed.



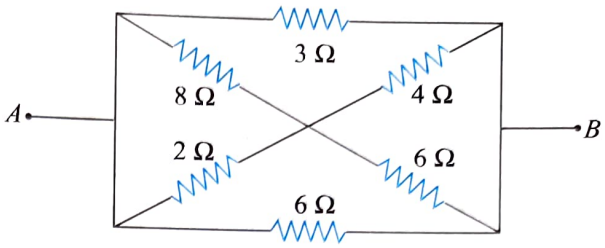
(1) $\frac{11R}{12}$ (2) $\frac{13R}{12}$
(3) $\frac{R}{5}$ (4) $\frac{15R}{12}$

5. In the circuit shown, current through 25 V cell is



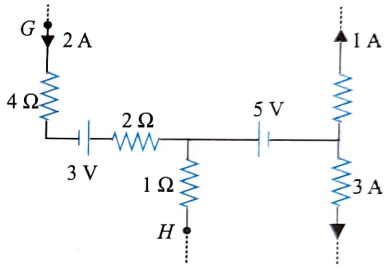
(1) 7.2 A (2) 10 A
(3) 12 A (4) 14.2 A

6. The equivalent resistance between A and B in the network in figure is



(1) $\frac{4}{3}\ \Omega$ (2) $\frac{3}{2}\ \Omega$
(3) $3\ \Omega$ (4) $2\ \Omega$

7. In the part of a circuit shown in figure, the potential difference $(V_G - V_H)$ between points G and H will be



(1) 0 V (2) 15 V
(3) 7 V (4) 3 V

8. N identical cells are connected to form a battery. When the terminals of the battery are joined directly (short-circuited), current I flows in the circuit. To obtain the maximum value of I,
(1) all the cells should be joined in series
(2) all the cells should be joined in parallel
(3) two rows of $N/2$ cells each should be joined in parallel
(4) $\sqrt{N}$ rows of $\sqrt{N}$ cells each should be joined in parallel, given that $\sqrt{N}$ is an integer

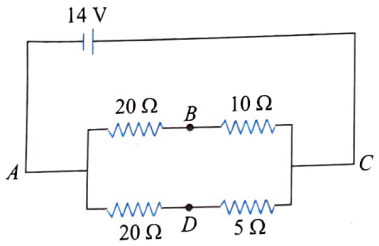
9. n identical cells, each of emf ε and internal resistance r, are joined in series to form a closed circuit. One cell A is joined with reversed polarity. The potential difference across each cell, except A, is

(1) $\frac{2\varepsilon}{n}$ (2) $\frac{n-1}{n}\varepsilon$
(3) $\frac{n-2}{n}\varepsilon$ (4) $\frac{2n}{n-2}\varepsilon$

10. In question 9, the potential difference across A is

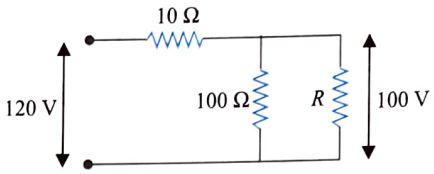
(1) $\frac{2\varepsilon}{n}$ (2) $\varepsilon\left(1-\frac{1}{n}\right)$
(3) $2\varepsilon\left(1-\frac{1}{n}\right)$ (4) $\varepsilon\left(\frac{n-2}{n}\right)$

11. What resistor should be connected in parallel with the 20 Ω resistor in branch ADC in the circuit shown in figure so that potential difference between B and D may be zero?



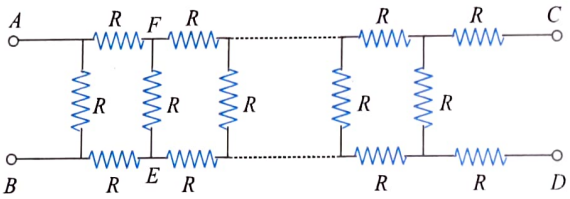
(1) 20 Ω (2) 10 Ω
(3) 5 Ω (4) 5 Ω

12. Find out the value of resistance R in figure.



(1) 100 Ω (2) 200 Ω
(3) 50 Ω (4) 150 Ω

13. In figure, find the value of resistor to be connected between C and D, so that the resistance of the entire circuit between A and B does not change with the number of elementary sets.

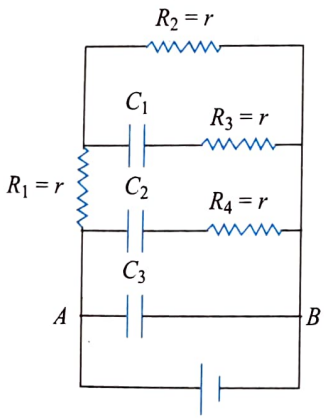


(1) R (2) $R(\sqrt{3}-1)$
(3) $3R$ (4) $R(\sqrt{3}+1)$

14. A wire of length L and three identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in a time t. A number N of similar cells is now connected in series with a wire of the same material and cross section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t. The value of N is

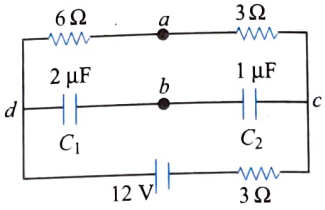
(1) 4 (2) 6
(3) 8 (4) 9

15. The equivalent resistance between points A and B in figure at steady state will be



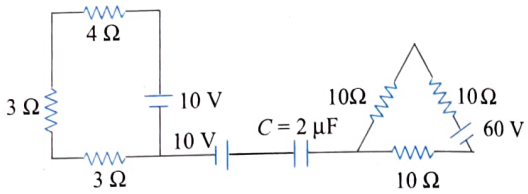
(1) $2r$ (2) $\frac{3}{5}r$
(3) $\frac{5}{3}r$ (4) none of these

16. What is the charge stored on each capacitor C_1 and C_2 in the circuit shown in figure?



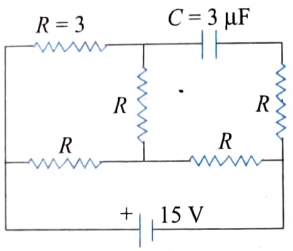
(1) 6 μC, 6 μC (2) 6 μC, 3 μC
(3) 3 μC, 6 μC (4) 3 μC, 3 μC

17. In the circuit shown in figure, find the maximum energy stored on the capacitor. Initially, the capacitor was uncharged.



(1) 150 μC (2) 100 μC
(3) 50 μC (4) zero

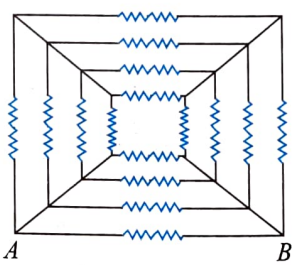
18. In the circuit shown in figure, the cell is ideal with emf 15 V. Each resistance is of 3 Ω. The potential difference across the capacitor in steady state is



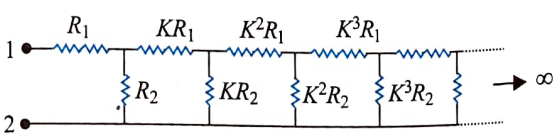
(1) 0 (2) 9 V
(3) 12 V (4) 15 V

19. Sixteen resistors, each of resistance 16 Ω, are connected in the circuit as shown in figure. The net resistance between A and B is

(1) 1 Ω (2) 2 Ω
(3) 3 Ω (4) Ω



20. The circuit diagram shown in figure consists of a large number of elements (each element has two resistors). The resistance of the resistors in each subsequent element differs by a factor of $K = 1/2$ from the resistances of the resistors in the previous elements. The equivalent resistance between A and B shown in figure is



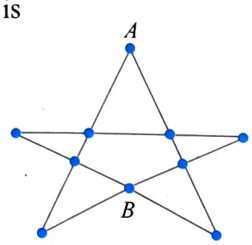
(1) $\frac{R_1 - R_2}{2}$

(2) $\frac{(R_1 - R_2) + \sqrt{6R_1R_2}}{2}$

(3) $\frac{(R_1 - R_2) + \sqrt{R_1^2 + R_2^2 + 6R_1R_2}}{2}$

(4) none of these

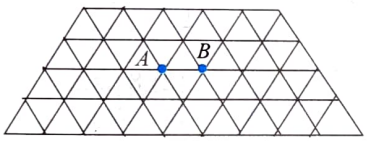
21. The resistance of all the wires between any two adjacent dots is R. The equivalent resistance between A and B as shown in figure is



(1) $\frac{7}{3}R$ (2) $\frac{7}{6}R$

(3) $\frac{14}{8}R$ (4) none of these

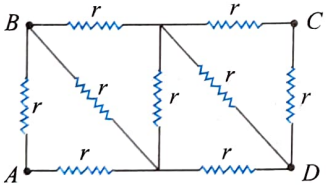
22. There is an infinite wire grid with cells in the form of equilateral triangles. The resistance of each wire between neighboring joint connections is R_0. The net resistance of the whole grid between the points A and B as shown in figure is



(1) R_0 (2) $\frac{R_0}{2}$

(3) $\frac{R_0}{3}$ (4) $\frac{R_0}{4}$

23. For the circuit shown in figure, the equivalent resistance between A and C is



(1) $\frac{12}{11}r$ (2) $\frac{13}{11}r$

(3) $\frac{14}{11}r$ (4) $\frac{15}{11}r$

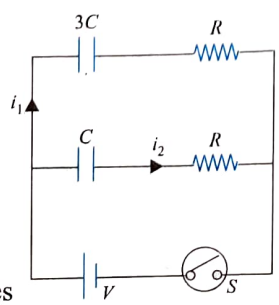
24. In the circuit shown, switch S is closed at $t = 0$. Let i_1 and i_2 be the currents at any finite time t, then the ratio i_1/i_2.

(1) is constant

(2) increases with time

(3) decreases with time

(4) first increases and then decreases

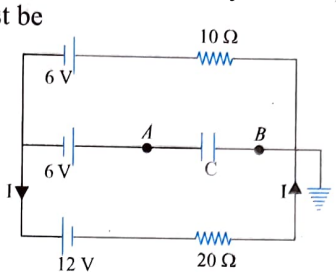


25. To get maximum current through a resistance of 2.5 Ω, one can use m rows of cells, each row having n cells. The internal resistance of each cell is 0.5 Ω. What are the values of n and m, if the total number of cells is 45.

(1) 3, 15 (2) 5, 6

(3) 9, 5 (4) 15, 3

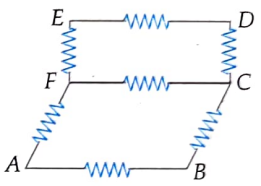
26. In the given circuit, with steady current, the potential of point A must be



(1) 1 V (2) 3 V

(3) 4 V (4) 2 V

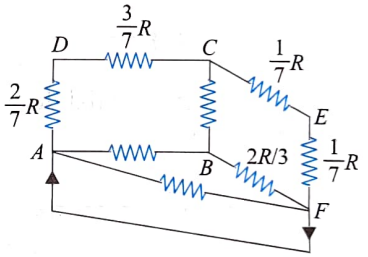
27. If the potential difference between A and D is V, what will be potential difference between F and C. Each of the resistance is R.



(1) $V/3$ (2) $V/4$

(3) $V/6$ (4) $V/8$

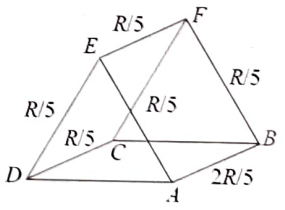
28. The current enters at A and leaves at F. The values of some resistances are shown. What should be the value of resistance AB so that no current will flow through CB?



(1) $\left(\frac{5}{7}\right)R$ (2) $\frac{R}{3}$

(3) $\left(\frac{5}{3}\right)R$ (4) $\left(\frac{5}{9}\right)R$

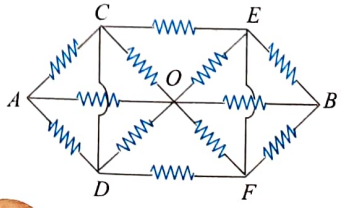
29. The current enters at A and comes out at D. Some of the resistances are shown. What should be resistance of wire CB so that it draws double of the current that enters the wire BF.



(1) $\left(\frac{9}{10}\right)R$ (2) $\left(\frac{8}{9}\right)R$

(3) $\left(\frac{7}{9}\right)R$ (4) $\left(\frac{3}{20}\right)R$

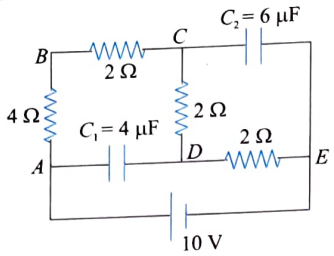
30. Find the equivalent resistance between A and B. Each resistor has same resistance R.



(1) $\frac{8}{5}R$ (2) $\frac{6}{5}R$

(3) $\frac{7}{5}R$ (4) $\frac{4}{5}R$

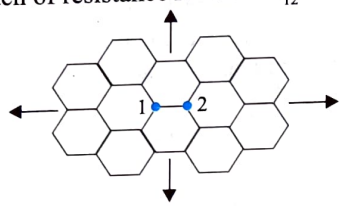
31. Find the potential difference across C_2.



(1) 6 V (2) 4 V

(3) zero (4) 2 V

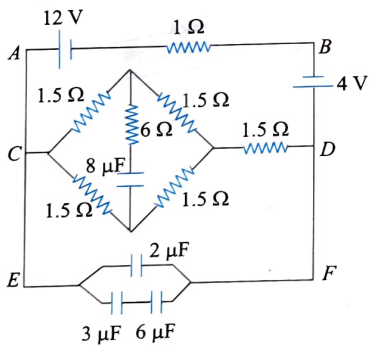
32. The given infinite grid consists of hexagonal cells of six resistors each of resistance R. Then R_{12} =



(1) $\frac{R}{3}$ (2) $\frac{2R}{3}$

(3) $\frac{4R}{3}$ (4) $\frac{3R}{4}$

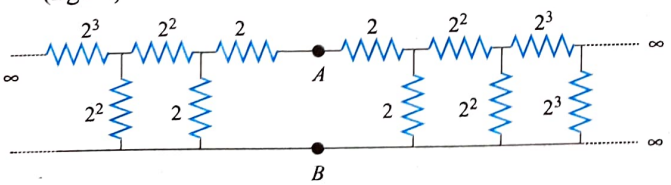
33. In the given circuit, find the potential difference across the 6 μF capacitor in steady state.



(1) 4 V (2) 2 V

(3) 6 V (4) none of these

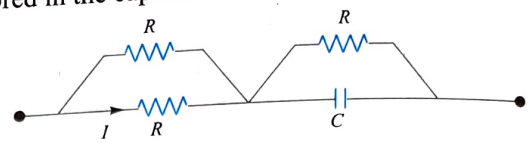
34. The equivalent resistance of the combination across AB (figure) is



(1) $\left(\frac{3+\sqrt{17}}{4}\right)$ (2) $3+\sqrt{17}$

(3) $\frac{3+\sqrt{17}}{2}$ (4) $2(3+\sqrt{17})$

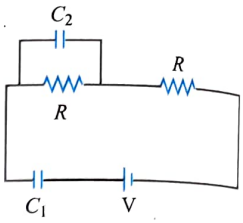
35. The capacitor shown in figure is in steady state. The energy stored in the capacitor is



(1) CI^2R^2 (2) $2CI^2R^2$

(3) $4CI^2R^2$ (4) none of these

36. Charge on the capacitor having capacitance C_2 in steady state (figure) is



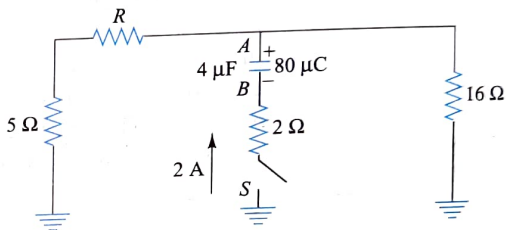
(1) Zero

(2) $(C_1 + C_2)V$

(3) C_2V

(4) C_1V

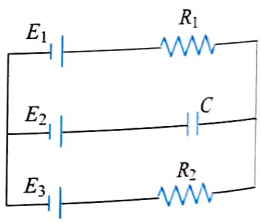
37. An 80 μC charge is given to the 4 μF capacitor in the circuit shown in figure so that the upper plate A is positively charged. An unknown resistance R is connected in the left limb. As soon as the switch S in the central limb is closed, a current of 2 A flows through the 2 Ω resistor in the central limb. The capacitive time constant for the circuit is



(1) 56 μs (2) 8 μs

(3) 200 μs (4) 40 μs

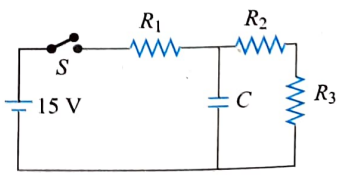
38. In the circuit shown (figure), the batteries have emf $E_1 = E_2 = 1$ V, $E_3 = 2.5$ V, and the resistance $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, Capacitance $C = 10\ \mu F$. The charge on the left plate of the capacitor C at steady state is



(1) + 2 μC (2) – 4 μC

(3) – 5 μC (4) + 12 μC

39. Figure shows a battery with emf 15 V in a circuit with $R_1 = 30\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 20\ \Omega$ and capacitance $C = 10\ \mu F$. The switch S is initially in the open position and is then closed at time $t = 0$. What will be the final steady-state charge on capacitor?



(1) 75 μC (2) 50 μC

(3) 10 μC (4) none of these

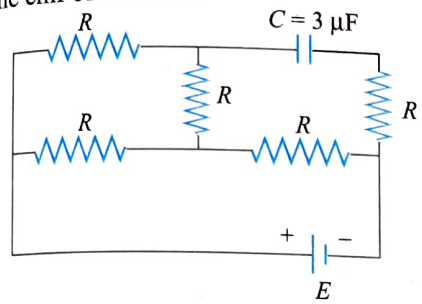
40. A piece of conducting wire of resistance R is cut into $2n$ equal parts. Half the parts are connected in series to form a bundle and remaining half in parallel to form another bundle. These

bundles are then connected to give the maximum resistance. The maximum resistance of the combination is

(1) $\frac{R}{2}\left(1+\frac{1}{n^2}\right)$ (2) $\frac{R}{2}(1+n^2)$

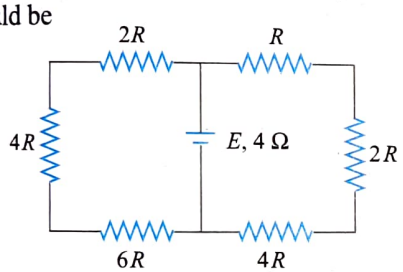
(3) $\frac{R}{2(1+n^2)}$ (4) $R\left(n+\frac{1}{n}\right)$

41. In the given circuit (figure), the potential difference across the capacitor is 12 V. Each resistance is of 3 Ω. The cell is ideal. The emf of the cell is



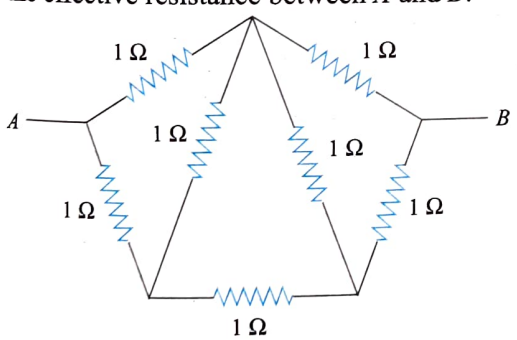
(1) 15 V (2) 9 V
(3) 12 V (4) 24 V

42. A battery of internal resistance 4 Ω is connected to the network of the resistance as shown in figure. If the maximum power can be delivered to the network, the magnitude of resistance in Ω should be



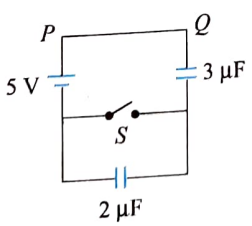
(1) 19/21 Ω (2) 84/19 Ω
(3) 12 Ω (4) 7 Ω

43. Find the effective resistance between A and B.



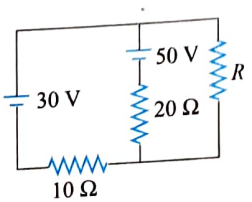
(1) 2 Ω (2) 1 Ω
(3) 8/7 Ω (4) 7 Ω

44. In figure, the charge that flows from P to Q when the switch S is closed is



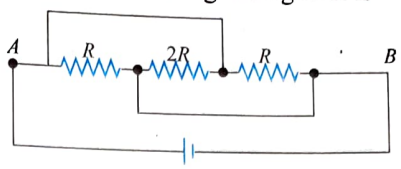
(1) 3 μC (2) 6 μC
(3) 9 μC (4) 15 μC

45. In the circuit shown (figure), the value of R in ohm that will result in no current through the 30 V battery is



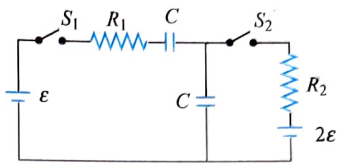
(1) 10 Ω (2) 25 Ω
(3) 30 Ω (4) 40 Ω

46. In figure, the current flowing through $2R$ is



(1) from left to right (2) from right to left
(3) no current (4) none of these

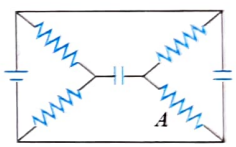
47. In the circuit shown (figure), switch S_2 is closed first and is kept closed for a long time. Now S_1 is closed. Just after that instant, the current through S_1 is



(1) $\frac{\varepsilon}{R_1}$ toward right (2) $\frac{\varepsilon}{R_1}$ toward left

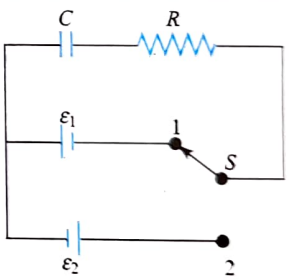
(3) zero (4) $\frac{2\varepsilon}{R_1}$

48. Each resistor in the following circuit (figure) has a resistance of 2 MΩ and the capacitors have capacitances of 1 μF. The battery voltage is 3 V. The voltage across the resistor A in the following circuit in steady state is



(1) 0 V (2) 0.5 V
(3) 0.75 V (4) 1.5 V

49. Initially, switch S is connected to position 1 for a long time (figure). The net amount of heat generated in the circuit after it is shifted to position 2 is



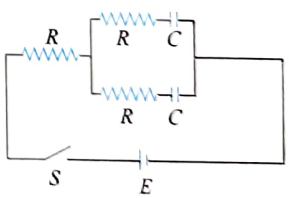
(1) $\frac{C}{2}(\varepsilon_1+\varepsilon_2)\varepsilon_2$

(2) $C(\varepsilon_1+\varepsilon_2)\varepsilon_2$

(3) $\frac{C}{2}(\varepsilon_1+\varepsilon_2)^2$

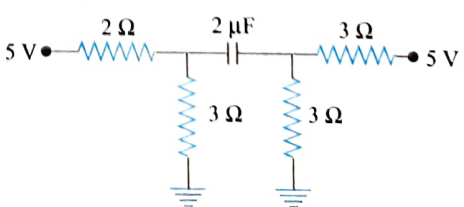
(4) $C(\varepsilon_1+\varepsilon_2)^2$

50. The switch S in the circuit diagram (figure) is closed at $t=0$. The charge on capacitors at any time t is



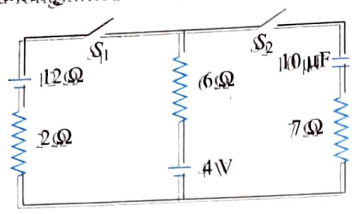
(1) $q_{(t)} = EC(1-e^{-2t/3RC})$
(2) $q_{(t)} = EC(1-e^{-t/2RC})$
(3) $q_{(t)} = EC(1-e^{-t/3RC})$
(4) $q_{(t)} = EC(1-e^{-3t/2RC})$

51. The charge on the capacitor in steady state in the circuit shown (figure) is



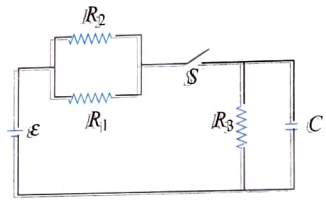
(1) 0.5 μC (2) 1 μC
(3) 2 μC (4) 4 μC

52. In the circuit shown (Figure), if switches S_1 and S_2 have been closed for a long time, then the charge on the capacitor



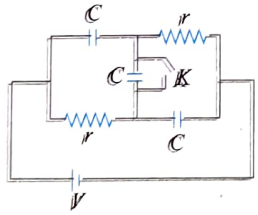
(1) is 100 μC
(2) increases to 120 μC if one-third of the gap of the capacitor's plates is filled with a dielectric ($K = 2$) of same area
(3) both (1) and (2)
(4) charge on the capacitor remains unchanged if one-third of the gap of the capacitor's plates is filled with a dielectric ($K = 2$) of same area

53. The circuit shown in figure consists of a battery of emf $\mathcal{E} = 10$ V, a capacitor of capacitance $C = 1.0$ μF, and three resistors of values $R_1 = 2\ \Omega$, $R_2 = 2\ \Omega$, and $R_3 = 1\ \Omega$. Initially, the capacitor is completely uncharged and the switch S is open. The switch S is closed at $t = 0$. Then



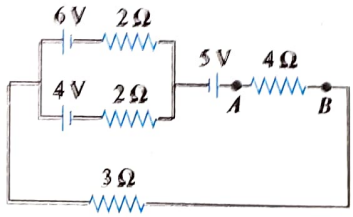
(1) the current through resistor R_3 at the moment the switch closed is zero
(2) the current through resistor R_3 a long time after the switch closed is 5 A
(3) the ratio of current through R_1 and R_2 is always constant
(4) all of these

54. In the circuit shown (figure), what is the change of total electrical energy stored in the capacitors when the key is pressed?



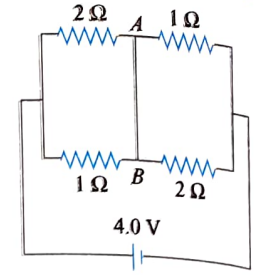
(1) $\frac{CV^2}{12}$ (2) $\frac{7CV^2}{8}$
(3) $\frac{5CV^2}{4}$ (4) $\frac{3CV^2}{8}$

55. In the circuit shown (figure), calculate the current through 6 V battery.



(1) (1/4) A (2) (1/8) A
(3) (1/2) A (4) none of these

56. In figure, points A and B are connected by a perfectly conducting wire. Calculate the current through AB



(1) 2 A
(2) 1 A
(3) 1.5 A
(4) 2.5 A

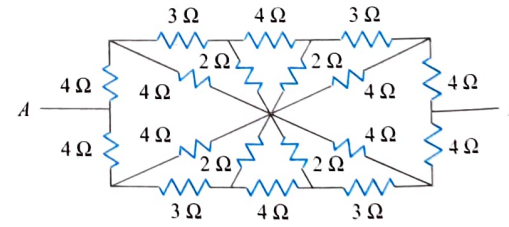
57. To get the maximum current through a resistance of 2.5 Ω, one can use m rows of cells, each row having n cells. The internal resistance of each cell is 0.5 Ω. What are the values of n and m, if the total number of cells is 45?
(1) 3, 15 (2) 5, 9
(3) 9, 5 (4) 15, 3

58. A cell of emf ε and internal resistance r is charged by a current i, then
(1) the cell stores chemical energy at the rate of εi
(2) the cell stores chemical energy at the rate of i^2r
(3) the cell stores chemical energy at the rate of $\varepsilon i - i^2r$
(4) the storage of chemical energy rate cannot be calculated

59. Two long coaxial and conducting cylinders of radius a and b are separated by a material of conductivity σ and a constant potential difference V is maintained between them by a battery. Then the current per unit length of the cylinder flowing from one cylinder to the other is
(1) $\frac{4\pi\sigma}{\ln(b/a)}V$ (2) $\frac{4\pi\sigma}{(b/a)}V$
(3) $\frac{2\pi\sigma}{\ln(b/a)}V$ (4) $\frac{2\pi\sigma}{(b+a)}V$

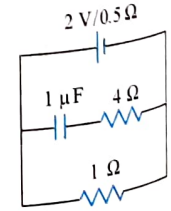
60. A 50 V battery is supplying current of 10 A when connected to a resistor. If the efficiency of the battery at this current is 25%, then the internal resistance of the battery is
(1) 2.5 Ω (2) 3.75 Ω
(3) 1.25 Ω (4) 5 Ω

61. A number of resistors are connected as shown in the figure. The equivalent resistance between A and B is



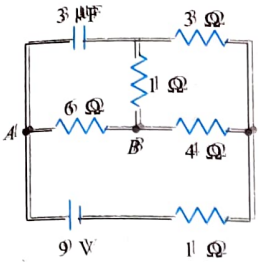
(1) 6 Ω (2) 12 Ω
(3) 9 Ω (4) 15 Ω

62. The charge on the capacitor as in figure is



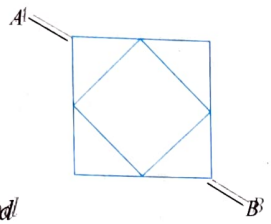
(1) 2 μC (2) $\frac{2}{3}$ μC
(3) $\frac{4}{3}$ μC (4) zero

63. Find the potential drop across the capacitor in the given circuit.



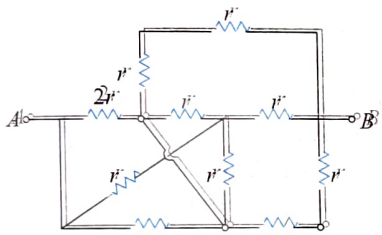
(1) 6 V (2) 6.5 V
(3) 7 V (4) none of these

64. A wire has linear resistance ρ (Ωm^{-1}). Find the resistance R between points A and B if the side of the big square is d.



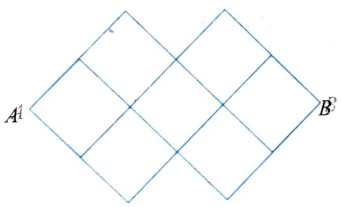
(1) $\frac{\rho d}{\sqrt{2}}$ (2) $\sqrt{2}\rho d$
(3) $2\rho d$ (4) none of these

65. The equivalent resistance between A and B in the arrangement of resistances as shown is



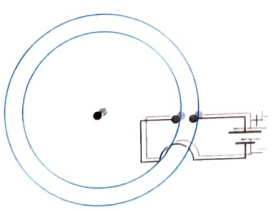
(1) $4r$ (2) $3r$
(3) $2.5r$ (4) r

66. In the shown wire frame, each side of a square (the smallest square) has a resistance R. The equivalent resistance of the circuit between the points A and B is



(1) R (2) $2R$
(3) $4R$ (4) $8R$

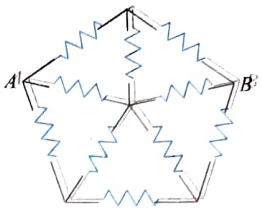
67. A spherical shell, made of material of electrical conductivity $10^9/\pi$ $(\Omega m)^{-1}$, has thickness $t = 2$ mm and radius $R = 10$ cm. In an arrangement, its inside surface is kept at a lower potential than its outside surface. The resistance offered by the shell is equal to



(1) $5\pi \times 10^{12}$ Ω (2) 2.5×10^{11} Ω
(3) 5×10^{12} Ω (4) 5×10^{11} Ω

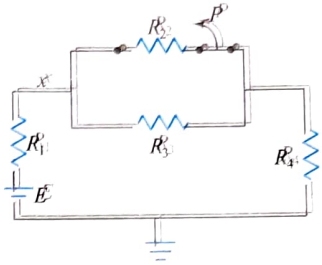
68. Two cylindrical rods of uniform cross-sectional area A and $2A$, having free electrons per unit volume $2n$ and n, respectively, are joined in series. A current I flows through them in steady state. Then the ratio of drift velocity of free electron in left rod to drift velocity of electron in the right rod (V_L/V_R) is
(1) 1/2 (2) 1
(3) 2 (4) 4

69. The effective resistance between A and B of the shown network, where resistance of each resistor is R, is



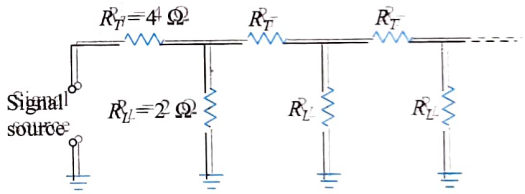
(1) $\frac{8R}{11}$ (2) $\frac{6R}{11}$
(3) $\frac{6R}{5}$ (4) none of these

70. If the switch at point P is opened (shown in the figure), choose the correct option.



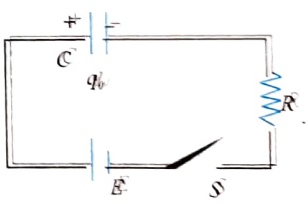
(1) The current in R_1 would not change.
(2) The potential difference between point x and the ground would increase.
(3) The current provided by the battery would increase.
(4) The emf produced by the battery (assumed to have no internal resistance) would change.

71. Figure shows a circuit model for the transmission of an electrical signal, such as cable TV, to a large number of subscribers. Each subscriber connects a load resistance R_L between the transmission line and the ground. Assume the ground to be at zero potential and to have negligible resistance. The resistance of the transmission line between the connection points of different subscribers is modeled as the constant resistance R_T. The equivalent resistance across the signal source is



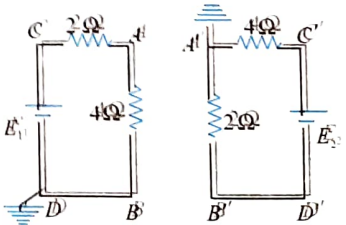
(1) 10 Ω (2) 5 Ω
(3) $\sqrt{55}$ Ω (4) $\sqrt{65}$ Ω

72. A capacitor having initial charge $q_0 = CE/2$ is connected to a cell of emf E through a resistor R as shown. Find the total heat generated in the circuit after the switch S is closed.



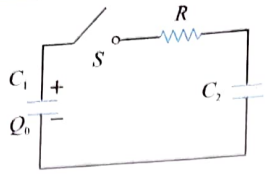
(1) $(1/12)CE^2$ (2) $(1/8)CE^2$
(3) $(1/4)CE^2$ (4) none of these

73. Given that current through $CA = 1$ A, current through $C'A' = 2$ A. Now if A is connected to A' and B is connected B'. Find currents through CA and $C'A'$, respectively.



(1) 1 A, 3 A (2) 3 A, 1 A
(3) 1 A, 1 A (4) 3 A, −3 A

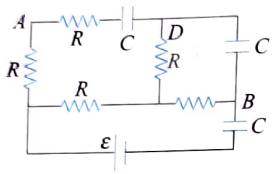
74. In the circuit shown, the capacitor C_1 is initially charged with charge q_0. The switch S is closed at time $t = 0$. The charge on C_2 after time t is



(1) $\frac{q_0C_1}{C_1+C_2}\left(1-e^{-\frac{t(C_1+C_2)}{C_1C_2R}}\right)$ (2) $\frac{q_0C_2}{C_1+C_2}\left(1-e^{-\frac{t(C_1+C_2)}{C_1C_2R}}\right)$

(3) $\frac{q_0C_1}{C_1+C_2}\left(1-e^{-\frac{t}{C_2R}}\right)$ (4) $\frac{q_0C_2}{C_1+C_2}\left(1-e^{-\frac{t}{C_2R}}\right)$

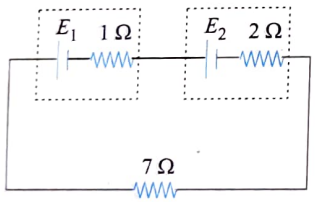
75. Consider the circuit shown in the figure. Find the charge on capacitor C between A and D in steady state.



(1) $C\varepsilon$ (2) $C\varepsilon/2$

(3) $C\varepsilon/3$ (4) zero

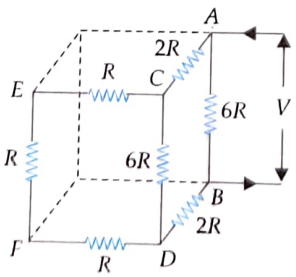
76. What should be the value of E_1/E_2 so that current flowing through 7 Ω resistor could be increased by short-circuiting the battery of emf E_2?



(1) $\frac{E_1}{E_2} < 4$ (2) $\frac{E_1}{E_2} > 3$

(3) $\frac{E_1}{E_2} > 4$ (4) $E_1 = E_2$

77. In the circuit as shown, $V_A - V_B = V$. The resistance of each wire AB, AC, etc. are shown in the figure. Find the current in AC.



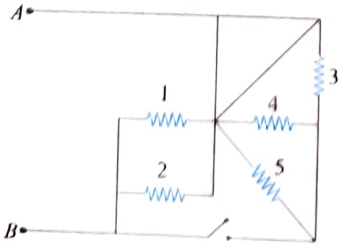
(1) $2V/3R$

(2) $4V/11R$

(3) $V/6R$

(4) none of these

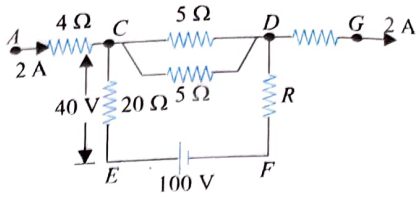
78. The circuit has resistors of equal resistance R. The equivalent resistance between A and B, when key is closed is R_1 and when key is open is R_2. Find the ratio of R_1 and R_2.



(1) 11/12 (2) 2/5

(3) 1/5 (4) none of these

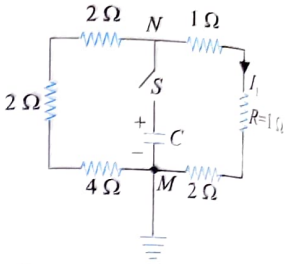
79. In the given circuit, find the value of R:



(1) 20 Ω (2) 25 Ω

(3) 15 Ω (4) 10 Ω

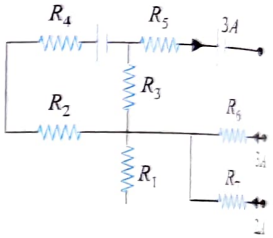
80. A capacitor of capacity 6 μF and initial charge 160 μC is connected with key S and resistance as shown in figure. Point M is earthed. If key is closed at $t = 0$, then the current (I_1) through resistance R (= 1 Ω) at $t = 16$ μs is



(1) 10/3e A (2) 10/e A

(3) 20/3e A (4) none

81. In the given circuit, if I_1 and I_2 be the current in resistances R_1 and R_2, respectively, then



(1) $I_1 = 3$ A, $I_2 = 2$ A

(2) $I_1 = 0$, $I_2 = 2$ A

(3) $I_1 = 2$ A and I_2 cannot be determined with given data

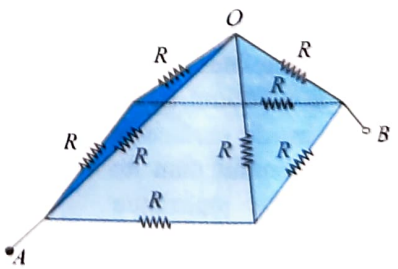
(4) none of these

82. n identical cells are joined in series with its two cells A and B in the loop with reversed polarities. Emf of each shell is ε and internal resistance r. Potential difference across cell A or B is (here $n > 4$)

(1) $\frac{2\varepsilon(n-2)}{n}$ (2) $\frac{2\varepsilon(n+2)}{n}$

(3) $\frac{4\varepsilon}{n}$ (4) $\frac{2\varepsilon}{n}$

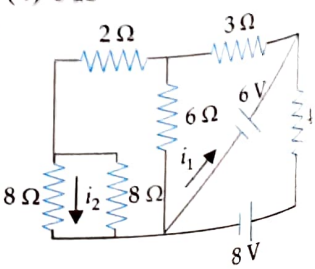
83. In the given figure, $R = 3$ Ω. Determine equivalent resistance between points A and B.



(1) 3 Ω (2) 2 Ω

(3) 1.5 Ω (4) 6 Ω

84. In the circuit as shown in figure, the ratio of current i_1/i_2 is



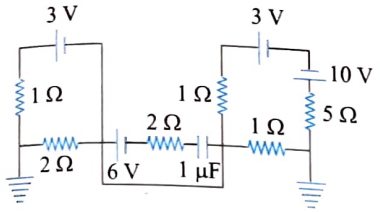
(1) 4/3

(2) 1/2

(3) 2

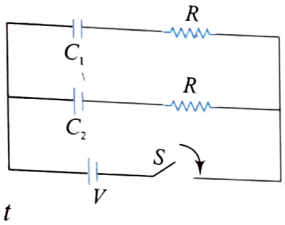
(4) none of these

85. For the circuit shown in the figure, determine the charge of the capacitor in steady state.



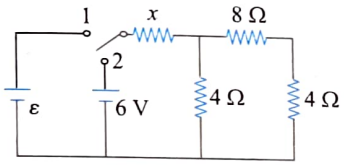
(1) 4 C (2) 6 C
(3) 1 C (4) Zero

86. In the circuit shown in figure $C_1 = 2C_2$. Switch S is closed at time $t = 0$. Let i_1 and i_2 be the currents flowing through C_1 and C_2 at any time t, then the ratio i_1/i_2



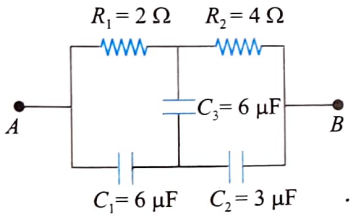
(1) is constant
(2) increases with increase in time t
(3) decreases with increase in time t
(4) first increases then decreases

87. When the switch 1 is closed, the current through the 8 Ω resistance is 0.75 A. When the switch 2 is closed (only), the current through the resistance marked x is 1 A.



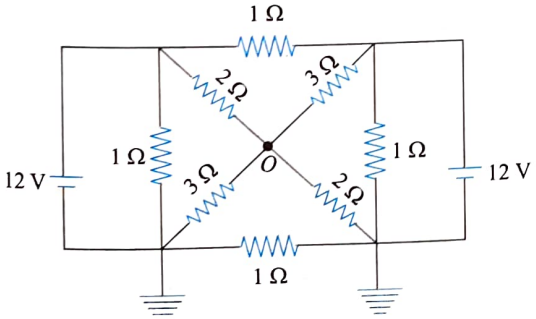
The value of x is
(1) 5 V (2) 18 V
(3) 10 V (4) 15 V

88. What is the equivalent capacitance between A and B in the circuit at steady state shown.



(1) 6 F (2) 1.5 F
(3) zero (4) 2 μF

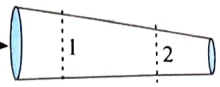
89. The potential of the point O is



(1) 3.5 V (2) 6.5 V
(3) 3 V (4) 6 V

Multiple Correct Answers Type

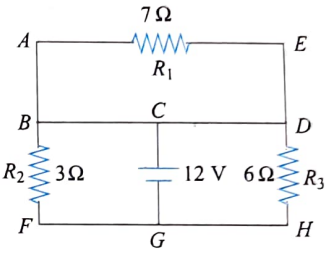
1. Consider a conductor of variable cross section in which current is flowing from cross section 1 to 2. Then



(1) current passing through both the cross sections is the same
(2) current through 1 is less than that through 2
(3) drift velocity of electrons at 1 is less than that at 2
(4) drift velocity is same at both the cross sections

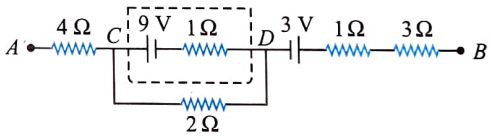
2. The charge flowing in a conductor varies with time as $Q = at - bt^2$. Then, the current
(1) decreases linearly with time
(2) reaches a maximum and then decreases
(3) falls to zero at time $t = a/2b$
(4) changes at a rate $-2b$

3. A single battery is connected to three resistances as shown in figure.



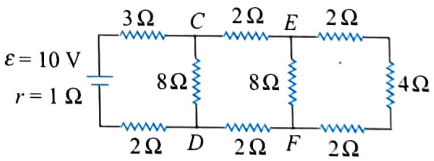
(1) The current through 7 Ω resistance is 4 A.
(2) The current through 3 Ω resistance is 4 A.
(3) The current through 6 Ω resistance is 2 A.
(4) The current through 7 Ω resistance is 0.

4. The potential difference between points A and B in the circuit shown in figure is 16 V. Then



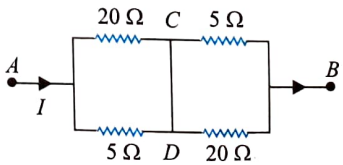
(1) the current through the 2 Ω resistance is 3.5 A
(2) the current through the 4 Ω resistance is 2.5 A
(3) the current through the 3 Ω resistance is 1.5 A
(4) the potential difference between the terminals of the 9 V battery is 7 V

5. In the circuit shown in figure, the cell has emf 10 V and internal resistance 1 Ω.



(1) The current through the 3 Ω resistor is 1 A.
(2) The current through the 3 Ω resistor is 0.5 A.
(3) The current through the 4 Ω resistor is 0.5 A.
(4) The current through the 4 Ω resistor is 0.25 A.

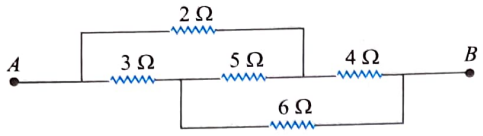
6. When some potential difference is maintained between A and B, current I enters the network at A and leaves at B (see figure).



(1) The equivalent resistance between A and B is 8 Ω.
(2) C and D are at the same potential.

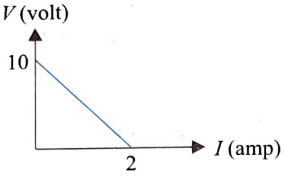
(3) No current flows between C and D.
(4) Current $3I/5$ flows from D to C.

7. In the circuit shown in figure, some potential difference is applied between A and B. The equivalent resistance between A and B is R.



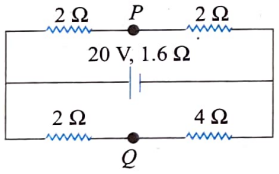
(1) No current flows through the 5 Ω resistor.
(2) $R = 15\ \Omega$
(3) $R = 12.5\ \Omega$
(4) $R = \frac{18}{5}\ \Omega$

8. A battery of emf E and internal resistance r is connected across a resistance R. Resistance R can be adjusted to any value greater than or equal to zero. A graph is plotted between the current passing through the resistance (I) and potential difference (V) across it. Select the correct alternatives.



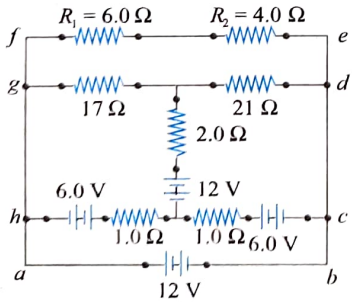
(1) Internal resistance of the battery is 5 Ω.
(2) Emf of the battery is 10 V.
(3) Maximum current that can be taken from the battery is 2 A.
(4) V–I graph can never be a straight lines as shown in figure.

9. In the given circuit (figure),



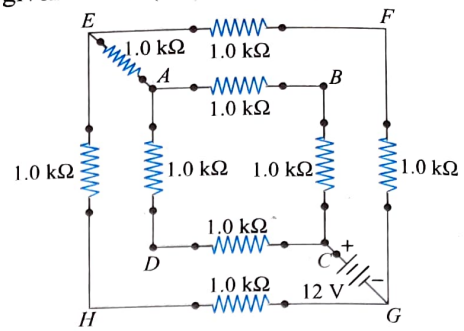
(1) the current through the battery is 5 A
(2) P and Q are at the same potential
(3) P is 2 V higher than Q
(4) Q is 2 V higher than P

10. In the circuit shown in figure, mark the correct options.



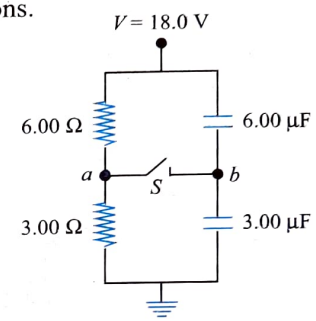
(1) Potential drop across R_1 is 3.2 V.
(2) Potential drop across R_2 is 5.4 V.
(3) Potential drop across R_1 is 7.2 V.
(4) Potential drop across R_2 is 4.8 V.

11. In the given circuit (as shown in figure),



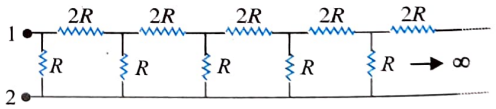
(1) the equivalent resistance between C and G is 3 kΩ
(2) the current provided by the source is 4 mA
(3) the current provided by the source is 8 mA
(4) voltage across points G and E is 4 V

12. Study the following circuit diagram in figure and mark the correct options.

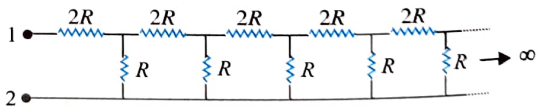


(1) The potential of point a with respect to point b in the figure when switch S is open is –6 V.
(2) The points a and b are at the same potential, when S is opened.
(3) The charge flowing through switch S when it is closed is 54 μC.
(4) The final potential of b with respect to ground when switch S is closed is 8 V.

13. Two circuits (as shown in figure) are called Circuit A and Circuit B. The equivalent resistance of Circuit A is x and that of Circuit B is y between 1 and 2.



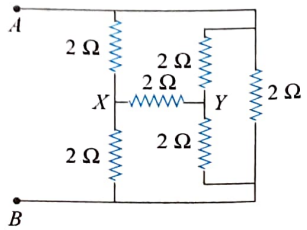
Circuit A



Circuit B

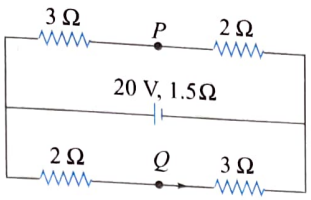
(1) $y > x$
(2) $y = (\sqrt{3}+1)R$
(3) $xy = 2R^2$
(4) $y - x = 2R$

14. For the circuit shown in figure, select the correct statements from the following options.



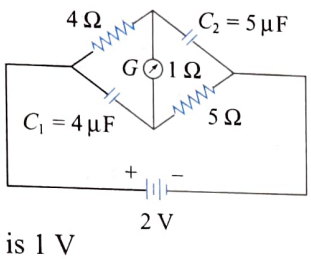
(1) x and y are equipotential points.
(2) Effective resistance between A and B is 2 Ω.
(3) Effective resistance between A and B is 1 Ω.
(4) None of the above

15. In the given circuit (figure),



(1) the current through the battery is 5.0 A
(2) P and Q are at the same potential
(3) P is 2.5 V higher than Q
(4) Q is 2.5 V higher than P

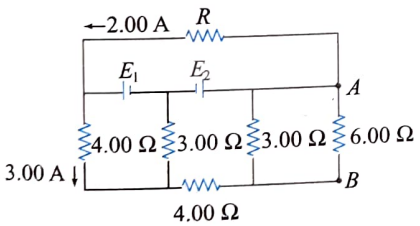
16. In the circuit shown (figure), the cell is ideal with emf = 2 V. The resistance of the coil of the galvanometer G is 1Ω. Then



(1) no current flows in G
(2) 0.2 A current flows in G
(3) potential difference across C_1 is 1 V
(4) potential difference across C_2 is 1.2 V

17. A parallel plate capacitor of capacitance 10 μF is connected to a cell of emf 10 V and is fully charged. Now a dielectric slab ($k = 3$) of thickness equal to the gap between the plates is completely filled in the gap, keeping the cell connected. During the filling process,
(1) the increase in charge on the capacitor is 200 μC
(2) the heat produced is zero
(3) energy supplied by the cell = increase in stored potential energy + work done on the person who is filling the dielectric slab
(4) energy supplied by the cell = increase in stored potential energy + work done on the person who is filling the dielectric slab + heat produced

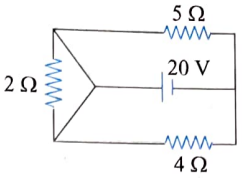
18. In the circuit shown in figure, E_1 and E_2 are two ideal sources of unknown emfs. Some currents are shown. Potential difference appearing across 6 Ω resistance is $V_A - V_B = 10$ V.



(1) The current in the 4.00 Ω resistor is 5 A.
(2) The unknown emf E_1 is 36 V.
(3) The unknown emf E_2 is 54 V.
(4) The resistance R is equal to 9 Ω.

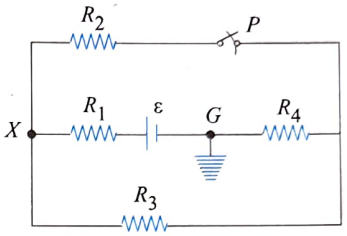
19. Consider a resistor of uniform cross-sectional area connected to a battery of internal resistance zero. If the length of the resistor is doubled by stretching it, then
(1) the current will become four times
(2) the electric field in the wire will become half
(3) the thermal power produced by the resistor will become onefourth
(4) the product of the current density and conductance will become half

20. In the circuit shown in the figure,



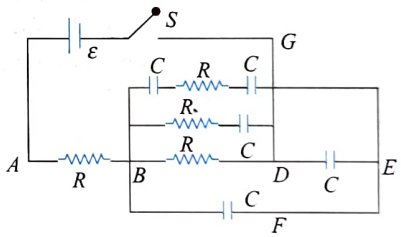
(1) current passing through 2 Ω resistance is zero
(2) current passing through 4 Ω resistance is 5 A
(3) current passing through 5 Ω resistance is 4 A
(4) potential difference across 2 Ω resistance is zero

21. If the switch at point P is opened (which was initially closed), choose the correct option(s).



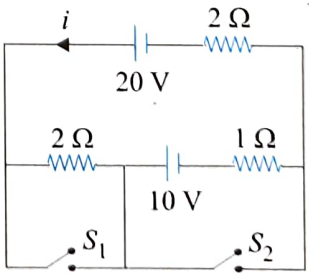
(1) The current in R_1 would not change.
(2) The potential difference between point X and the ground would increase.
(3) The current provided by the battery would decrease.
(4) The emf produced by the battery (assumed to have no internal resistance) would change.

22. Current through the battery in the circuit shown in the figure



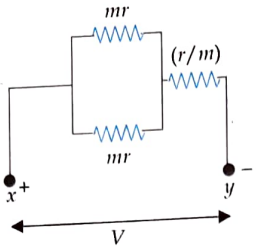
(1) immediately after the switch S is closed is ε/R
(2) immediately after the switch S is closed is $\varepsilon/2R$
(3) after long time is $\varepsilon/2R$
(4) after long time is $3\varepsilon/4R$

23. In the circuit shown in figure,



(1) $i = \frac{5}{3}$ A when S_1 is closed and S_2 is open
(2) $i = 5$ A when S_1 is open and S_2 is closed
(3) $i = 2$ A when S_1 and S_2 both are open
(4) $i = 10$ A when both S_1 and S_2 are closed

24. In the given circuit, the value of m is varying. The correct statements about the circuit are



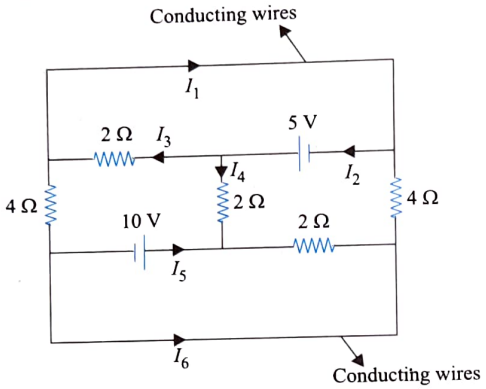
(1) The condition for maximum current flowing from x is $m = 2$.

(2) The maximum current is $V/\sqrt{2}\,r$.

(3) The condition for maximum current flowing from x is $m = \sqrt{2}$.

(4) The maximum current is $3V/2r$.

25. In the given circuit, mark the correct statement/statements.



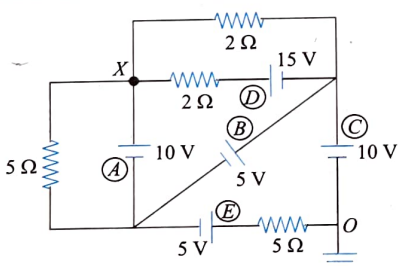
(1) The current through 10 V battery is 35/4 A.

(2) The current through 5 V battery is 5/4 A.

(3) The current through upper conducting wire is 15/8 A.

(4) The current through lower conducting wire is 25/8 A.

26. In the circuit, various resistances and five batteries are connected as shown in the figure. If point O is earthed,



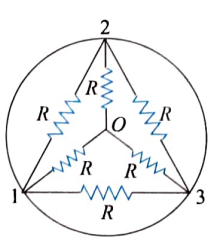
(1) the current in the battery marked as (A) is 1/2 A

(2) the current in the battery marked as (B) is 1 A

(3) the potential of point x marked in circuit is 15 V

(4) the current through the battery marked as (C) is 2 A

27. Inside a superconducting ring, six identical resistors each of resistance R are connected as shown in figure. The equivalent resistance(s)



(1) between 1 and 3 is zero.

(2) between 1 and 3 is $R/2$

(3) between 1 and 2, 2 and 3, 3 and 1 are all equal

(4) between 1 and 2, 2 and 3, 3 and 1 are all not equal

28. In question 28, the equivalent resistance(s)

(1) between 0 and 2 is $R/3$

(2) between 0 and 1 is $R/3$

(3) between 0 and 1 is zero

(4) between 1 and 2, 2 and 3, 3 and 1 are all not equal

29. In question 28, imagine a battery of emf E between the points 0 and 1, with its positive terminal connected with O.

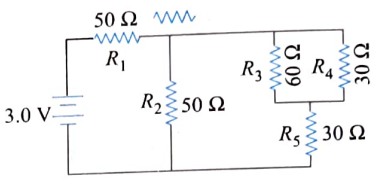
(1) The current entering at O is equally divided into three resistances.

(2) The current in the other three resistances R_{12}, R_{13}, R_{23} is zero.

(3) The resistances R_{02} and R_{03} have equal magnitudes of current while the resistance R_{01} has different current.

(4) Potential $V_2 = V_3 > V_1$.

30. In the circuit shown, the resistances are given in ohms and the battery is assumed ideal with emf equal to 3.0 V.



(1) The resistor R_5 that dissipates maximum power.

(2) The resistor R_1 that dissipates maximum power.

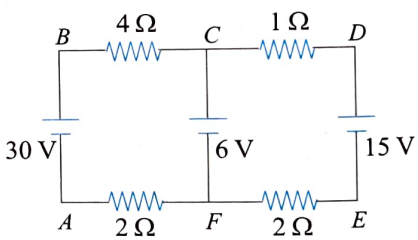
(3) The potential difference across resistor R_3 is 0.4 V.

(4) The current passing through 3 V battery is 40 mA.

Linked Comprehension Type

For Problems 1–3

Figure shows a network of four resistances and three batteries



1. Choose the correct alternative.

(1) The potential difference is $V_C - V_F = 6$ V.

(2) No current flows in the branch CF.

(3) Current flows in the branch from F to C.

(4) Both (1) and (3)

2. Mark the incorrect statement.

(1) The current flowing in the left loop is independent of the right loop.

(2) The current flowing in the right loop is independent of the left loop.

(3) Both 30 V and 15 V batteries do not produce current in the branch CF.

(4) Both (1) and (2)

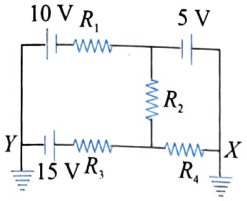
3. Which of the batteries is getting charged.

(1) only 6 V (2) both 6 V and 15 V

(3) only 15 V (4) none

For Problems 4–6

Consider the circuit shown in figure.



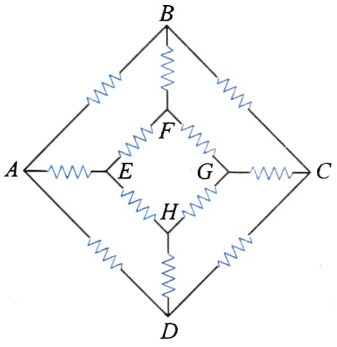
4. Current through R_2 is zero if $R_4 = 2\ \Omega$ and $R_3 = 4\ \Omega$. In this case
   (1) current through $R_3 = 2.5$ A
   (2) current through $R_4 = 3$ A
   (3) both (1) and (2) are correct
   (4) both (1) and (2) are wrong

5. Assuming $R_1 = 2\ \Omega$, current passing through resistance R_1 is
   (1) 2 A (2) 2.5 A
   (3) 3.5 A (4) zero

6. Assuming $R_1 = 2\ \Omega = R_4$, $R_3 = 4\ \Omega$, current passing through the circuit if resistance R_2 is removed is (remove ground connection at point X)
   (1) 2 A (2) 3 A
   (3) 1 A (4) 2.5 A

For Problems 7–9

In the network shown in figure, each resistance is R.



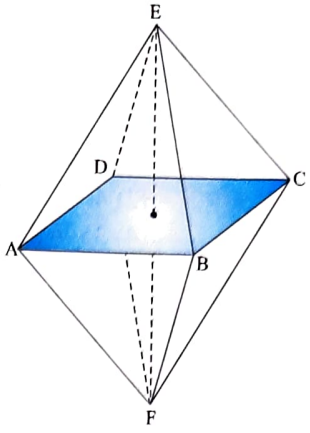
7. The equivalent resistance between A and B is
   (1) $\frac{3R}{4}$ (2) $\frac{5R}{6}$
   (3) $\frac{7R}{12}$ (4) $\frac{4R}{3}$

8. The equivalent resistance between A and G is
   (1) $\frac{3R}{4}$ (2) $\frac{5R}{6}$
   (3) $\frac{7R}{12}$ (4) $\frac{4R}{3}$

9. The equivalent resistance between A and C is
   (1) $\frac{3R}{4}$ (2) $\frac{5R}{6}$
   (3) $\frac{7R}{12}$ (4) $\frac{4R}{3}$

For Problems 10–12

Consider 12 resistors arranged symmetrically in shape of bipyramid $ABCDEF$. Here $ABCD$ is a square. Point E, point F, and center of square are in the same straight line perpendicular to the plane of square. The resistance of each resistor is R.



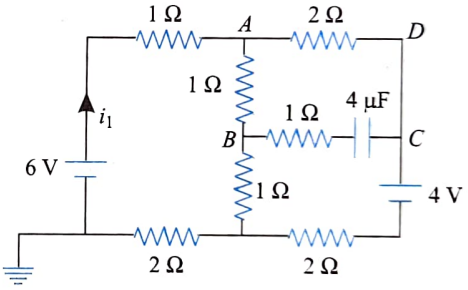
10. The effective resistance between E and F is
    (1) $R/2$ (2) $R/3$
    (3) R (4) none of these

11. The effective resistance between A and C is
    (1) $R/2$ (2) $R/3$
    (3) R (4) none of these

12. The effective resistance between A and B is
    (1) $7R/12$ (2) $5R/7$
    (3) $5R/12$ (4) none of these

For Problems 13–15

Consider the circuit shown in figure. The circuit is in steady state.



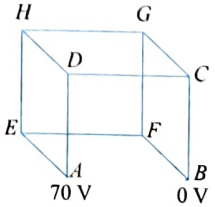
13. The value of i_1 is
    (1) 7/9 A (2) 14/13 A
    (3) 14/3 A (4) 17/23 A

14. The potential of point B is
    (1) 27/34 V (2) 46/13 V
    (3) 1/2 V (4) 61/49V

15. The charge in capacitor is
    (1) 2 μC (2) 4 μC
    (3) 6 μC (4) 8 μC

For Problems 16–18

In the arrangement, 11 wires each of resistance 5 Ω are used as sides of cube. A potential difference of 70 V is maintained between corners A and B of cube.



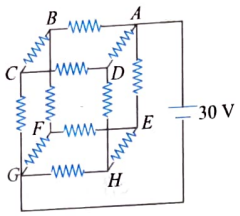
16. Current flowing in wire HG is
(1) 4 A (2) 2 A
(3) 5 A (4) 10 A

17. Current flowing in wire EF is
(1) 4 A (2) 2 A
(3) 5 A (4) 10 A

18. Potential of junction G is
(1) 30 V (2) 40 V
(3) 20 V (4) 35 V

For Problems 19–21

A resistor circuit is constructed such that 12 resistors are arranged to form a cube as shown in figure. Each resistor has a resistance of 2 Ω. The potential difference of 30 V is applied across two of the opposing points as shown.



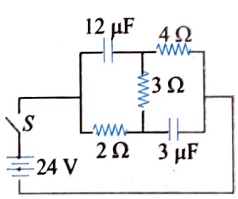
19. The points having the same potential are
(i) B, D, E (ii) C, F, H (iii) C, E
(1) Only (i) is correct.
(2) (i), (ii), and (iii) are correct.
(3) Only (ii) is correct.
(4) (i) and (ii) both are correct.

20. If we replace resistors between A and B and resistors between G and H by wires of zero resistance, then the points having the same potential are
(i) D, E, C, F (ii) A, B (iii) G, H
(1) Only (i) is correct.
(2) Only (ii) is correct.
(3) Only (iii) is correct.
(4) (i), (ii), and (iii) are correct.

21. In question 20, the potential difference between the points C and G is
(1) 15 V (2) 10 V
(3) 20 V (4) 7.5 V

For Problems 22–23

The following two questions refer to the circuit shown. Assume that the capacitors are initially uncharged.

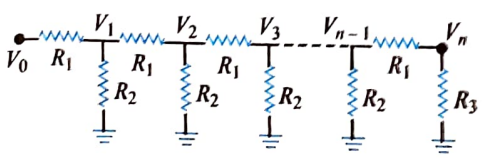


22. At time $t = 0$, the switch S in the circuit is closed. At that instant, what is the equivalent resistance of the circuit?
(1) 9.00 (2) 1.20
(3) 1.80 (4) 0.92

23. After the switch has been closed for a long time, how much charge is on the positive plate of the 3 μF capacitor?
(1) 24 μC (2) 32 μC
(3) 56 μC (4) 72 μC

For Problems 24–26

A network of resistances is constructed with R_1 and R_2 as shown in figure. The potential at the points 1, 2, 3, ⋯, N are V_1, V_2, V_3, ⋯, V_n, respectively, each having a potential k times smaller than the previous one.



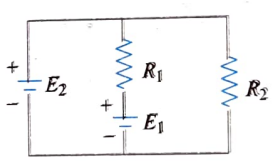
24. The ratio R_1/R_2 is
(1) $k^2 - \frac{1}{k}$ (2) $\frac{k}{k-1}$
(3) $k - \frac{1}{k^2}$ (4) $\frac{(k-1)^2}{k}$

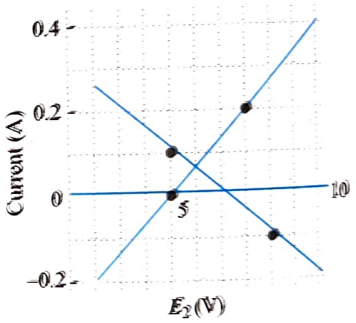
25. The ratio R_2/R_3 is
(1) $\frac{(k-1)^2}{k}$ (2) $k^2 - \frac{1}{k}$
(3) $\frac{k}{k-1}$ (4) $k - \frac{1}{k^2}$

26. The current that passes through the resistance R_2 nearest to the V_0 is
(1) $\frac{(k-1)^2}{k}\frac{V_0}{R_3}$ (2) $\frac{(k+1)^2}{k}\frac{V_0}{R_3}$
(3) $\left(k + \frac{1}{k^2}\right)\frac{V_0}{R_3}$ (4) $\left(\frac{k-1}{k^2}\right)\frac{V_0}{R_3}$

For Problems 27–29

In the circuit given in the figure, both batteries are ideal. emf E_1 of battery 1 has a fixed value, but emf E_2 of battery 2 can be varied between 1.0 V and 10.0 V. The graph gives the currents through the two batteries as a function of E_2 but are not marked as which plot corresponds to which battery. But for both plots, current is assumed to be negative when the direction of the current through the battery is opposite to the direction of that battery's emf (direction of emf is from negative to positive).





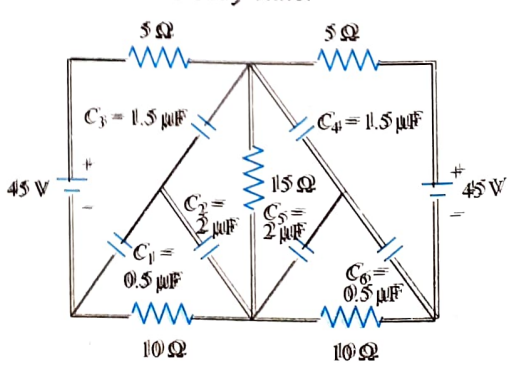
27. The value of emf E_1 is
(1) 8 V (2) 6 V
(3) 4 V (4) 2 V

28. The resistance R_1 has value
(1) 10 Ω (2) 20 Ω
(3) 30 Ω (4) 40 Ω

29. The resistance R_2 is equal to
(1) 10 Ω (2) 20 Ω
(3) 30 Ω (4) 40 Ω

For Problems 30–32

The circuit shown is in a steady state.



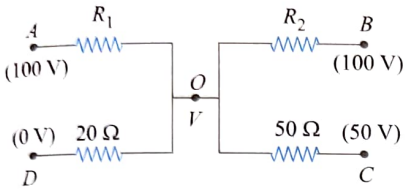
30. The charge in capacitor C_1 is
(1) 20 μC (2) 30 μC
(3) 40 μC (4) 10 μC

31. The charge in capacitor C_2 is
(1) 30 μC (2) 10 μC
(3) 20 μC (4) 40 μC

32. The charge in capacitor C_3 is
(1) 10 μC (2) 30 μC
(3) 20 μC (4) 40 μC

For Problems 33–35

In the circuit as shown, it is given that $R_1/R_2 = 1/2$ and potential at O is V.



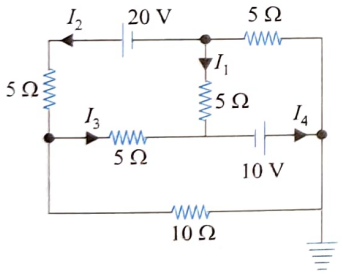
33. Find the value by which V should be greater, if $V < 75$ V:
(1) 100/9 V (2) 20 V
(3) 100/7 V (4) 100/3 V

34. If the value of potential at O is 20 volts, find the value of R_2.
(1) 150 Ω (2) 300 Ω
(3) 450 Ω (4) 600 Ω

35. What should be the value of resistance R_1 so that potential at O is always of the positive value?
(1) 80 Ω (2) 16 Ω
(3) 240 Ω (4) any value

For Problems 36–37

In the circuit shown in the figure



36. The value of current I_1 is
(1) 2 A (2) zero
(3) 2.5 A (4) none of these

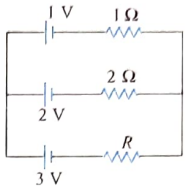
37. The value of current I_4 is
(1) 2A (2) zero
(3) 2.5 A (4) none of these

Matrix Match Type

1. Match Column I with Column II:

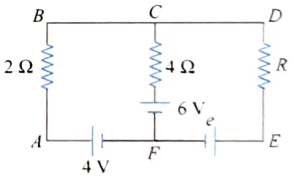
Column I	Column II
i. Electrical conductivity of conductor depends on	a. dimensions (length, area of cross section etc.)
ii. Conductance of a conductor depends on	b. temperature
iii. For a given conductor of given dimensions and at a given temperature, current density depends on	c. nature of conductor
iv. For a given potential difference applied across a conductor of given length, current in it will depend on	d. electric field strength

2. For the circuit shown in figure, all the three batteries are ideal. Match the column.



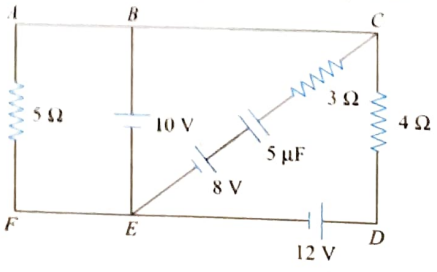
Column I	Column II
i. The value of R such that current through 3 V battery is zero.	a. 1 Ω
ii. The value of R such that current through 1 V battery is zero.	b. 2 Ω
iii. The value of R such that current through 2 V battery is zero.	c. 3 Ω
iv. The value of R for which current through 3 V battery is 5/8 A.	d. not possible for any value of R

3. A circuit is shown in figure. R is a nonzero variable but finite resistance. e is some unknown emf with polarities as shown. Match the columns.



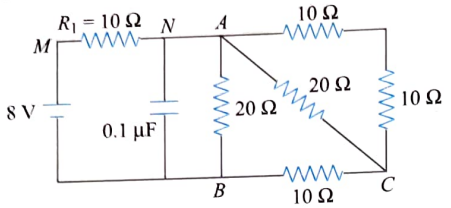
Column I	Column II
i. Current passing through 4 Ω resistance can be zero	a. possible if $e = 6$ V
ii. Current passing through 4 Ω resistance can be from F to C	b. possible if $e > 6$ V
iii. Current passing through 4 Ω resistance can be from C to F	c. possible if $e < 6$ V
iv. Current passing through 2 Ω resistance will be from B to A	d. possible for any value of e from zero to infinity

4. A network consisting of three resistors, three batteries, and a capacitor is shown in figure. In steady state



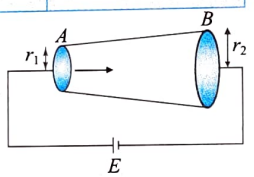
Column I	Column II
i. Current in branch EB is	a. 10 μC
ii. Current in branch CB is	b. 0.5 A
iii. Current in branch ED is	c. 1.5 A
iv. Charge on capacitor is	d. 5 μC

5. A capacitor of capacitance 0.1 μF is connected to a battery of emf 8 V (as shown in figure) under the steady-state condition.



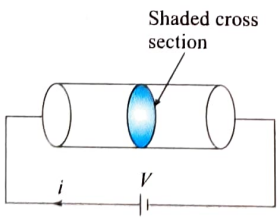
Column I	Column II
i. Charge on the capacitor	a. 0.4 μC
ii. Current in AC branch	b. 0.2 A
iii. Current in AB branch	c. 0.1 A
iv. Current in R connected between M and N	d. 0.4 A

6. A battery of emf E is connected across a conductor as shown in figure. As one observes from A to B, match the following.



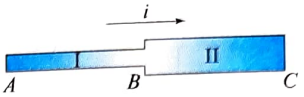
Column I	Column II
i. Current	a. increases
ii. Drift velocity of electron	b. decreases
iii. Electric field	c. remains same
iv. Potential drop across the length	d. cannot be determined

7. Column I gives physical quantities based on a situation in which an ideal cell of emf V is connected across a cylindrical rod of uniform cross-sectional area and conductivity (s) as shown in the figure. E, J, ϕ, and i are electric field at, current density through, electric flux through, and current through the shaded cross section, respectively, as shown in figure. Physical quantities in Column II are related to those in Column I. Match the expressions in Column I with the statements in Columns II.



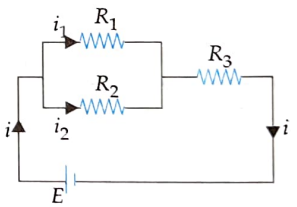
Column I	Column II
i. $\frac{\phi}{i}$	a. Conductivity of the rod
ii. $\frac{E}{J}$	b. Resistance of the rod
iii. $\sigma\phi V$	c. Resistivity of the rod
iv. $\frac{V}{\sigma\phi}$	d. Power delivered to the rod

8. Column I gives physical quantities of a situation in which current i passes through two rods I and II of equal length that are joined in series. The ratio of free electron density (η), resistivity (ρ), and cross-sectional area (A) of both rods are in ratio $n_1:n_2 = 2:1$; $r_2 = 2:1$ and $A_1:A_2 = 1:2$, respectively. Column II gives corresponding results. Match the ratios in Column I with the values in Column II.



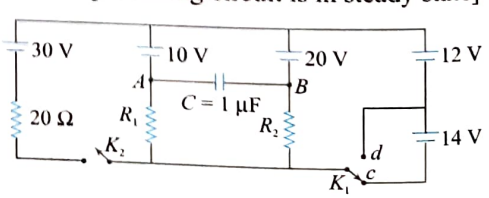
Column I	Column II
i. $\frac{\text{Drift velocity of free electron in rod I}}{\text{Drift velocity of free electron in rod II}}$	a. 0.5
ii. $\frac{\text{Electric field in rod I}}{\text{Electric field in rod II}}$	b. 1
iii. $\frac{\text{Potential difference across rod I}}{\text{Potential difference across rod II}}$	c. 2
iv. $\frac{\text{Average time taken by free electron to move from } A \text{ to } B}{\text{Average time taken by free electron to move from } B \text{ to } C}$	d. 4

9. In the given circuit, match the statements of Column I and Column II.



Column I	Column II
i. Current I depends upon	a. E, R_1, R_2, R_3
ii. Currents i_1 depends upon	b. E, R_1, R_2
iii. Ration (i_1/i_2) depends upon	c. E, R_3
iv. Ration (i/i_1) depends upon	d. R_1, R_2

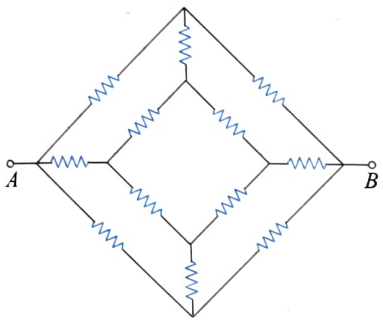
10. A circuit involving five ideal cells, three resistances (R_1, R_2, and 20 Ω), and a capacitor of capacitance $C = 1$ μF is shown. Match the conditions in Column I with results given in Column II [assuming circuit is in steady state].



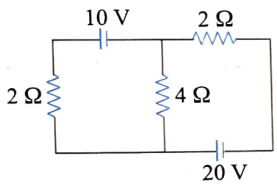
Column I	Column II
i. K_2 is open and K_1 is in position C	a. Potential at point A is greater than potential at B
ii. K_2 is open and K_1 is in position D	b. Current through R_1 is downward
iii. K_2 is closed and K_1 is in position C	c. Current through R_2 is upward
iv. K_2 is closed and K_1 is in position D	d. Charge on capacitor is 10 μC

Numerical Value Type

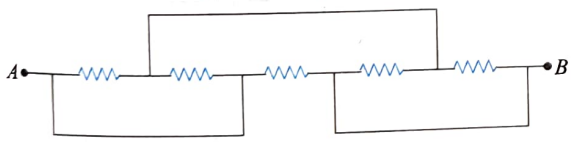
1. The figure shows a network of resistor each heaving value 12 Ω. Find the equivalent resistance between points A and B (in Ω).



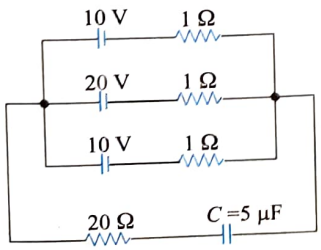
2. Find the current (in A) in 4 Ω resistance in circuit shown in figure



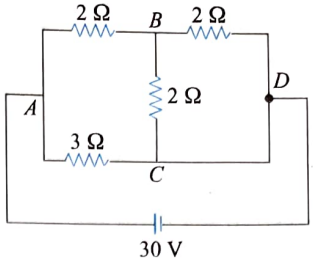
3. In the given arrangement of resistances, find the equivalent resistance (in Ω) between points A and B in the figure. Each resistance is R of magnitude equal to 10 Ω.



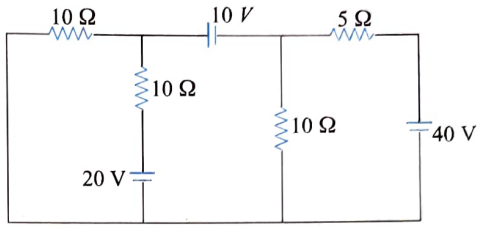
4. In the given network in the figure, find the charge on the capacitor (in μC).



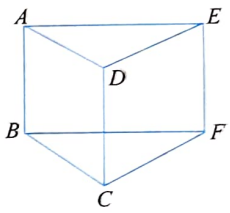
5. Find current in the branch CD (in A) of the circuit shown in figure.



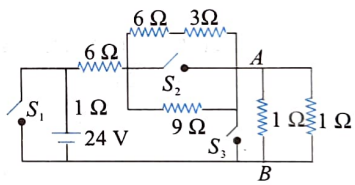
6. Find the current in 5 Ω resistance (in A) in circuit shown in figure.



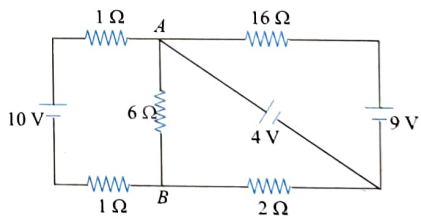
7. Nine wires each of resistance $r = 5$ Ω are connected to make a prim as shown in figure. Find the equivalent resistance of the arrangement across AB (in Ω).



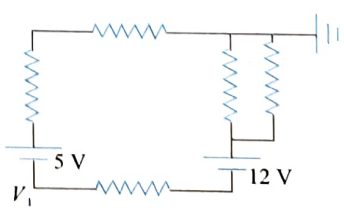
8. If the switches S_1, S_2, and S_3 in figure are arranged such that the current through the battery is minimum, find the voltage across points A and B (in V).



9. Find the potential difference (in V) between points A and B shown in the circuit.



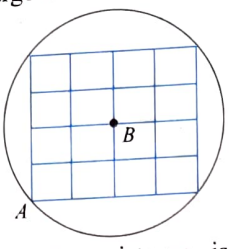
10. In the circuit shown, each resistance is 2 Ω. The potential V_1 is as indicated in the circuit. What is the magnitude of V_1 in volt?



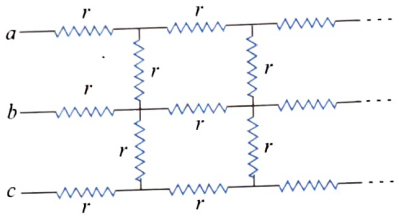
11. The area of cross section, length, and density of a piece of metal of atomic weight 60 are 10^{-6} m^2, 1 m, and 5×10^3 kgm^{-3}, respectively. If every atom contributes one free

electron, find the drift velocity (in mms^{-1}) of electrons in the metal when a current of 16 A passes through it. Avogadro's number is $N_A = 6 \times 10^{23}$/mol and charge on an electron is $e = 1.6 \times 10^{-19}$ C.

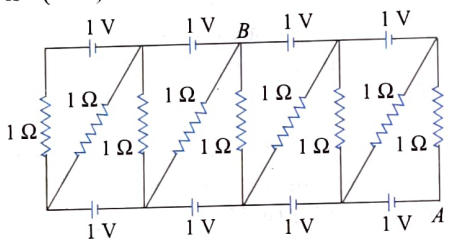
12. A finite square grid, each link having resistance r, is fitted in a resistance-less conducting circular wire. Determine the equivalent resistance between A and B (in Ω) if $r = (80/7)$ Ω.



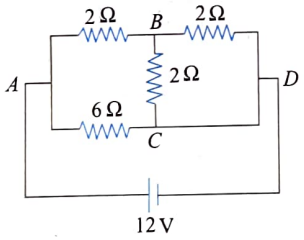
13. In the infinite grid, the value of each resistance is $r = 2(\sqrt{5}-1)$ Ω. Find the equivalent resistance between the points a and c (in Ω).



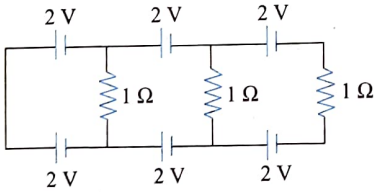
14. The potential difference $V_A - V_B$ for the circuit shown in figure is $-(22/x)$ V. Find the value of x.



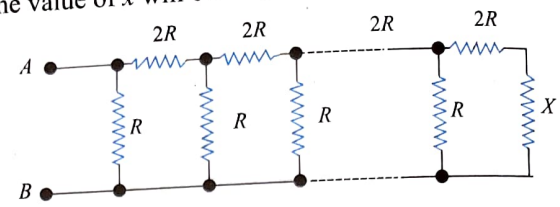
15. Find current in the branch CD of the circuit (in ampere).



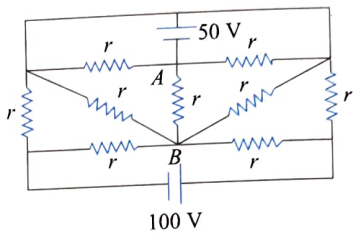
16. Find the current (in A) in the rightmost resistor shown in figure.



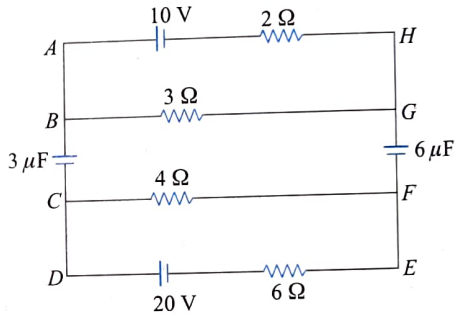
17. At what value of the resistance in the circuit shown in the figure will the total resistance between points A and B be independent of the number of cells? If $R = (\sqrt{3}+1)\,\Omega$, then the value of x will be "....." Ω.



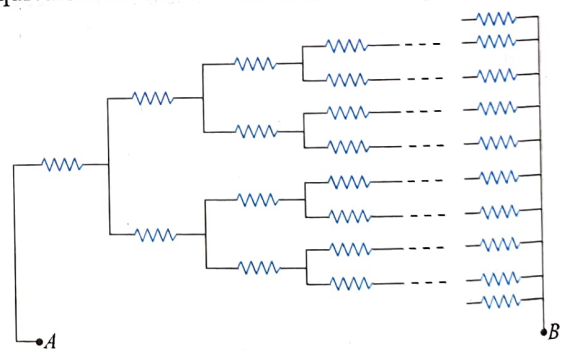
18. In the given circuit, each resistance is $r = 18.75$ Ω. The current (in A) in the resistance connected across A and B is '....' A.



19. In the circuit shown in figure, find the steady state charge in 6 μF capacitor (in μC).



20. In an infinite network of resistances,each resistance of value $R = 2$ Ω, are arranged as shown in the figure. Find the equivalent resistance between A and B (in Ω).



Archives

JEE Main

Single Correct Answer Type

1. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 be the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 be the time taken for the charge to reduce to one-fourth of its initial value. Then the ratio t_1/t_2 will be

(1) 1 (2) $\frac{1}{2}$

(3) $\frac{1}{4}$ (4) 2 **(AIEEE 2010)**

2. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly

(1) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$ (2) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$

(3) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$ (4) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$

(AIEEE 2010)

3. A resistor 'R' and 2 μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. ($\log_{10}2.5 = 0.4$)

(1) $1.3 \times 10^4\ \Omega$ (2) $1.7 \times 10^5\ \Omega$

(3) $2.7 \times 10^6\ \Omega$ (4) $3.3 \times 10^7\ \Omega$ **(AIEEE 2011)**

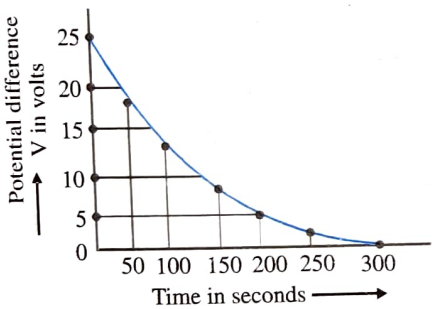
4. If a wire is stretched to make it 0.1% longer, its resistance will

(1) increase by 0.05% (2) increase by 0.2%

(3) decrease by 0.2% (4) decrease by 0.05%

(AIEEE 2011)

5. The figure shows an experimental plot discharging of a capacitor in an RC circuit. The time constant τ of this circuit lies between:

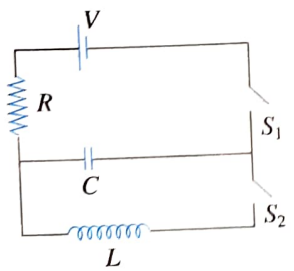


(1) 150 sec and 200 sec (2) 0 and 50 sec

(3) 50 sec and 100 sec (4) 100 sec and 150 sec

(AIEEE 2012)

6. In an LCR circuit as shown below, both switches are open initially. Now switch S_1 is closed and S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is capacitive time constant). Which of the following statement is correct?



(1) At $t = \tau$, $q = CV/2$

(2) At $t = 2\tau$, $q = CV(1 - e^2)$

(3) At $t = \tau/2$, $q = CV(1 - e^1)$

(4) Work done by the battery is half of the energy dissipated in the resistor. **(JEE Main 2013)**

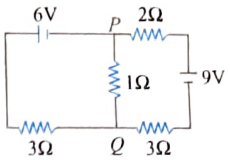
7. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} ms^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to

(1) $1.6 \times 10^{-8}\ \Omega$ m (2) $1.6 \times 10^{-7}\ \Omega$ m

(3) $1.6 \times 10^{-6}\ \Omega$ m (4) $1.6 \times 10^{-5}\ \Omega$ m

(JEE Main 2015)

8. In the circuit shown, the current in the 1 Ω resistor is:



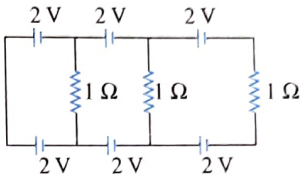
(1) 1.3 A, from P to Q

(2) 0 A

(3) 0.13 A, from Q to P

(4) 0.13 A, from P to Q **(JEE Main 2015)**

9. In the given circuit the current in each resistance is

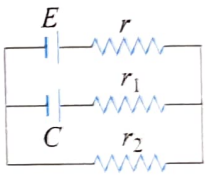


(1) 0.5 A (2) 0 A

(3) 1 A (4) 0.25 A

(JEE Main 2017)

10. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be



(1) $CE\frac{r_2}{(r+r_2)}$ (2) $CE\frac{r_1}{(r_1+r)}$

(3) CE (4) $CE\frac{r_1}{(r_2+r)}$

(JEE Main 2017)

11. Two batteries with emf 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between

(1) 11.7 V and 11.8 V (2) 11.6 V and 11.7 V

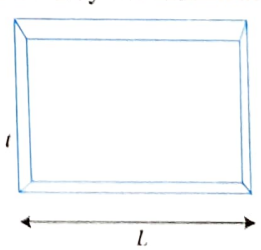
(3) 11.5 V and 11.6 V (4) 11.4 V and 11.5 V

(JEE Main 2018)

JEE ADVANCED

Single Correct Answer Type

1. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure, is



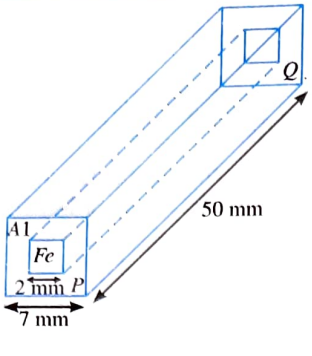
(1) directly proportional to L

(2) directly proportional to t

(3) independent of L

(4) independent of t **(IIT-JEE 2010)**

2. In an aluminum (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are 2.7×10^{-8} Ωm and 1.0×10^{-7} Ωm, respectively. The electrical resistance between the two faces P and Q of the composite bar is:

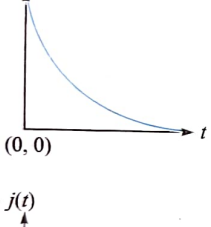


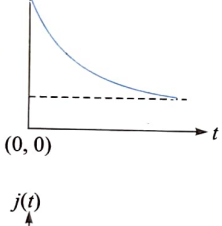
(1) $\frac{2475}{64}\mu\Omega$ (2) $\frac{1875}{64}\mu\Omega$

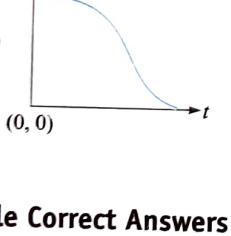
(3) $\frac{1875}{49}\mu\Omega$ (4) $\frac{2475}{132}\mu\Omega$

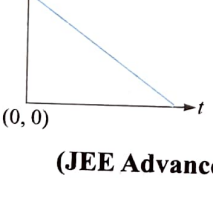
(JEE Advanced 2015)

3. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time $t = 0$, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density $j(t)$ at any point in the material?

(1) 

(2) 

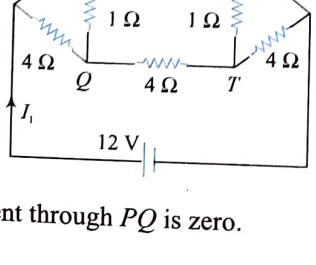
(3) 

(4) 

(JEE Advanced 2016)

Multiple Correct Answers Type

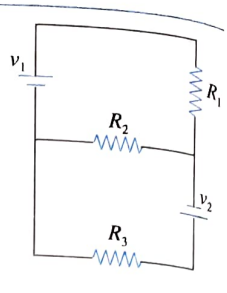
1. For the resistance network shown in the figure, choose the correct option(s)



(1) The current through PQ is zero.
(2) $I_1 = 3$ A
(3) The potential at S is less than at Q.
(4) $I_2 = 2$ A

(JEE Advanced 2012)

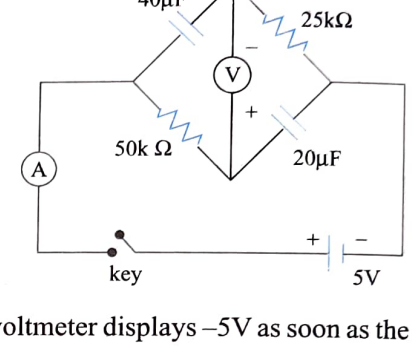
2. Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if



(1) $V_1 = V_2$ and $R_1 = R_2 = R_3$
(2) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$
(3) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$
(4) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

(JEE Advanced 2014)

3. In the circuit shown below, the key is pressed at time $t = 0$. Which of the following statement(s) is (are) true?

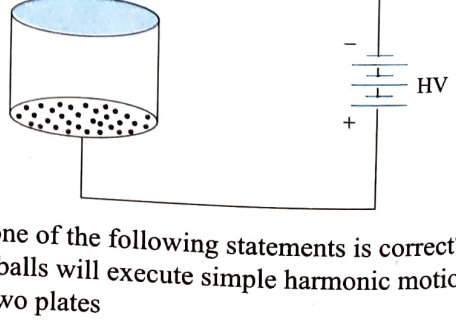


(1) The voltmeter displays –5V as soon as the key is pressed, and displays +5 V after a long time
(2) The voltmeter will display 0 V at time $t = ln\ 2$ seconds
(3) The current in the ammeter becomes $1/e$ of the initial value after 1 second
(4) The current in the ammeter becomes zero after a long time

Linked Comprehension Type

For Problems 1–2

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius $r << h$. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity) **(JEE Advanced 2016)**



1. Which one of the following statements is correct?
(1) The balls will execute simple harmonic motion between the two plates

(2) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

(3) The balls will bounce back to the bottom plate carrying the same charge they went up with

(4) The balls will stick to the top plate and remain there

2. The average current in the steady state registered by the ammeter in the circuit will be

(1) proportional to V_0^2

(2) proportional to $V_0^{1/2}$

(3) proportional to the potential V_0

(4) zero

For Problems 3–4

Consider a simple RC circuit as shown in Fig. 1.

Process 1: In the circuit the switch S is closed at $t = 0$ and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time $T >> RC$). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2: In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time $T >> RC$. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T >> RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in process 1.

These two processes are depicted in Fig. 2.

(JEE Advanced 2017)

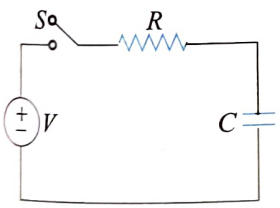


Figure 1

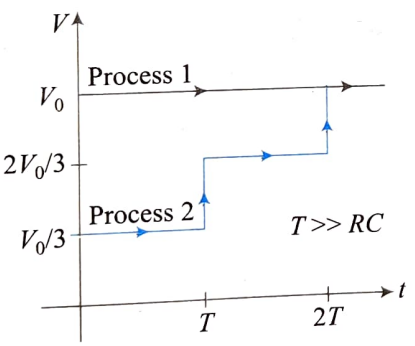


Figure 2

3. In process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by:

(1) $E_C = E_D$ (2) $E_C = 2E_D$

(3) $E_C = \frac{1}{2}E_D$ (4) $E_C = E_D \ln 2$

4. In process 2, total energy dissipated across the resistance E_D is:

(1) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$ (2) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$

(3) $E_D = \frac{1}{2}CV_0^2$ (4) $E_D = 3CV_0^2$

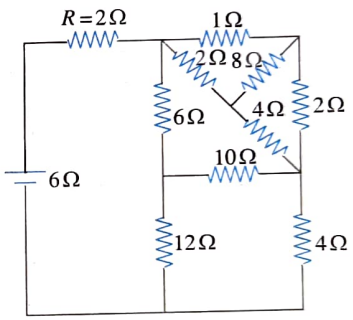
Numerical Value Type

1. At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them becomes 4 volt? [take ln 5 = 1.6, ln 3 = 1.1]

(IIT-JEE 2010)

2. In the following circuit, the current through the resistor R (= 2 Ω) is I Amperes. The value of I is:

(JEE Advanced 2015)



Answers Key

EXERCISES

Single Correct Answer Type

1. (2)	2. (4)	3. (1)	4. (3)	5. (3)
6. (1)	7. (3)	8. (2)	9. (1)	10. (3)
11. (1)	12. (1)	13. (2)	14. (2)	15. (1)
16. (1)	17. (4)	18. (3)	19. (3)	20. (3)
21. (2)	22. (3)	23. (4)	24. (2)	25. (4)
26. (4)	27. (2)	28. (3)	29. (4)	30. (4)
31. (2)	32. (2)	33. (2)	34. (1)	35. (2)
36. (1)	37. (4)	38. (3)	39. (1)	40. (1)
41. (1)	42. (1)	43. (3)	44. (2)	45. (3)
46. (2)	47. (2)	48. (4)	49. (3)	50. (3)
51. (2)	52. (3)	53. (4)	54. (3)	55. (3)
56. (2)	57. (4)	58. (3)	59. (3)	60. (2)
61. (1)	62. (3)	63. (2)	64. (1)	65. (4)
66. (2)	67. (4)	68. (2)	69. (1)	70. (2)
71. (2)	72. (2)	73. (4)	74. (2)	75. (4)
76. (3)	77. (3)	78. (2)	79. (2)	80. (3)
81. (3)	82. (1)	83. (2)	84. (3)	85. (2)
86. (2)	87. (2)	88. (4)	89. (4)	

Multiple Correct Answers Type

1. (1),(3)	**2.** (1),(3),(4)	**3.** (2),(3),(4)
4. (1),(3),(4)	**5.** (1),(4)	**6.** (1),(2),(4)
7. (1),(4)	**8.** (1),(2),(3)	**9.** (1),(4)
10. (3),(4)	**11.** (1),(2),(4)	**12.** (1),(3)
13. (1),(2),(3),(4)	**14.** (1),(3)	**15.** (1),(4)
16. (2),(3),(4)	**17.** (1),(2),(3)	**18.** (1),(2),(3),(4)
19. (2),(3)	**20.** (1),(2),(3),(4)	**21.** (2),(3)
22. (1),(3)	**23.** (2),(3),(4)	**24.** (2),(3)
25. (2),(3)	**26.** (1),(3),(4)	**27.** (1),(3)
28. (1),(2)	**29.** (1),(2)	**30.** (2),(3),(4)

Linked Comprehension Type

1. (1)	**2.** (3)	**3.** (1)	**4.** (4)	**5.** (2)
6. (4)	**7.** (3)	**8.** (2)	**9.** (1)	**10.** (1)
11. (1)	**12.** (3)	**13.** (2)	**14.** (2)	**15.** (4)
16. (2)	**17.** (3)	**18.** (1)	**19.** (4)	**20.** (4)
21. (1)	**22.** (4)	**23.** (3)	**24.** (4)	**25.** (3)
26. (4)	**27.** (2)	**28.** (2)	**29.** (4)	**30.** (4)
31. (3)	**32.** (2)	**33.** (3)	**34.** (4)	**35.** (4)
36. (2)	**37.** (1)			

Matrix Match Type

1. i. → b.,c.; ii. → a.,b.,c.; iii. → d.; iv. → a.,b.,c.
2. i. → d.; ii. → d.; iii. → a.; iv. → b.
3. i. → b.; ii. → a., b., c.; iii. → b.; iv. → a., b., c., d.
4. i. → c.; ii: → b.; iii. → b.; iv. → a.
5. i. → a.; ii. → c.; iii. → b.; iv. → d.
6. i. → c.; ii. → b.; iii. → b.; iv. → b.
7. i. → c.; ii. → c.; iii. → d.; iv. → b.
8. i. → a.; ii. → c.; iii. → d.; iv. → c.
9. i. → a., b., c., d.; ii. → a., b., c., d.; iii. → d.; iv. → d.
10. i.→ a., b., d.; ii. → a., b., c., d.; iii. → a., b., d.; iv. → a., b., c., d.

Numerical Value Type

1. (9)	**2.** (3)	**3.** (5)	**4.** (60)	**5.** (15)
6. (4.80)	**7.** (3)	**8.** (1)	**9.** (6)	**10.** (9)
11. (2)	**12.** (6)	**13.** (8)	**14.** (9)	**15.** (4)
16. (0)	**17.** (2)	**18.** (2)	**19.** (4)	**20.** (4)

ARCHIVES

JEE Main

Single Correct Answer Type

1. (3)	**2.** (4)	**3.** (3)	**4.** (2)	**5.** (4)
6. (2)	**7.** (4)	**8.** (3)	**9.** (2)	**10.** (1)
11. (3)				

JEE Advanced

Single Correct Answer Type

1. (3) **2.** (2) **3.** (1)

Multiple Correct Answers Type

1. (1),(2),(3),(4) **2.** (1),(2),(4) **3.** (1),(2),(3),(4)

Linked Comprehension Type

1. (2) **2.** (1) **3.** (1) **4.** (1)

Numerical Value Type

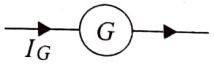
1. (2) **2.** (1)

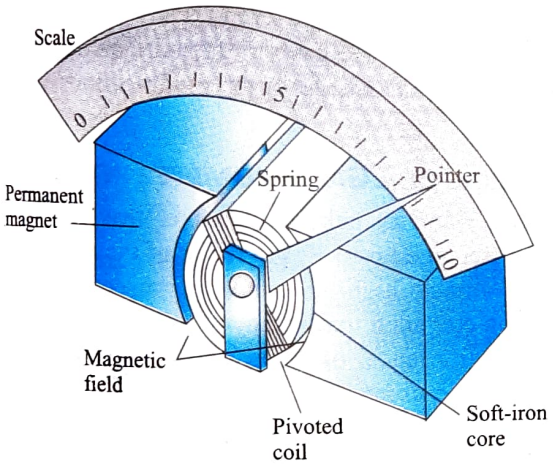
6 Electrical Measuring Instruments

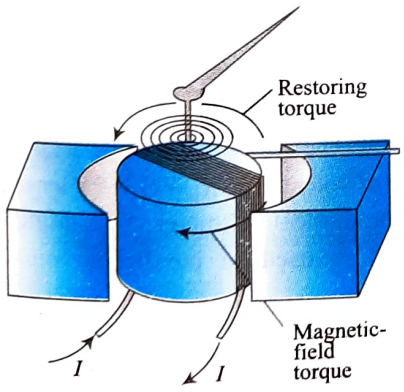
GALVANOMETER

A galvanometer is a device that measures the magnitude and direction of current in a circuit. It is a very sensitive device, which can be used to measure only small currents, say, in μA. If we pass a current through a galvanometer more than the specified amount, it may get damaged. The maximum current that can be passed through a galvanometer is known as the full-scale deflection current (say, I_G). The resistance of a galvanometer is very small.

In a galvanometer, there is a needle attached to a coil, which is subject to a magnetic field. Attached to the coil is a spring similar to the hairspring on the balance wheel of a watch.





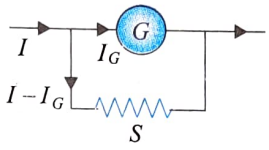


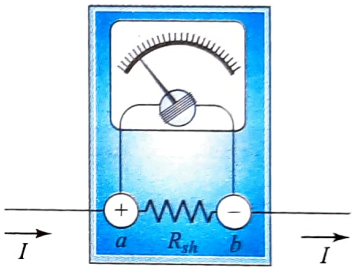
In equilibrium position, with no current in the coil, the needle is at zero. When there is current in the coil, the magnetic field exerts a torque on it, which is proportional to the current. This torque is responsible for the rotation of the coil and hence the needle deflects. As the coil turns, the spring exerts a restoring torque that is proportional to the angular displacement.

AMMETER

A galvanometer cannot be used directly to measure big currents. To measure large currents, we have to carry out some modifications in the galvanometer. What we get after modifications is known as an **ammeter**.

Suppose we want to make an ammeter that can measure current up to I. Such an ammeter is said to have a range of 0–I. For this we have to connect a small resistance (known as **shunt**) in parallel with the galvanometer as shown in figure.





The value of S is so selected that only I_G current passes through the galvanometer and the remaining I–I_G through the shunt. Let the resistance of the galvanometer be G. Since G and S are in parallel, the potential difference across them should be same, i.e.,

$$I_G G = S(I - I_G) \text{ or } S = \frac{GI_G}{I - I_G}$$

The above equation gives the value of S to be connected in parallel with the galvanometer to convert it into an ammeter of range 0–I.

Note:

- **Resistance of ammeter:** G and S are in parallel, so their equivalent resistance is given by

$$R_A = \frac{SG}{S+G}$$

  Since S is a very small resistance (it has to be small if we want most of the current to pass through it), R_A is even smaller than S. In general, the resistance of an ammeter is very small. For an ideal ammeter, resistance is zero.
- An ammeter is used in series to measure the current.
- The reading of an ammeter is generally lesser than the actual current in the circuit. It is because when we connect an ammeter in the circuit to measure the current, the ammeter introduces its own resistance in the circuit, which results in an increase in the resistance of the circuit and decrease in the current.
- The lower the value of shunt resistance, the higher the range of ammeter.

- **How an ammeter reads the current:** The current through the galvanometer is responsible for the deflection of the needle of the galvanometer. We have to use this fraction of current (I_G) in measuring the actual current I.

  As the potential difference across G is the same as that across S, from the equation $I_G G = (I - I_G)S$, we get

  $$I_G = \left(\frac{S}{G+S}\right) I \text{ or } \boxed{I_G \propto I}$$

  Hence, I_G is proportional to I or the deflection of the needle is proportional to the current I. If the value of current I is changed, then the deflection of the needle also changes. The scale can be graduated to read the value of I directly.

ILLUSTRATION 6.1

A galvanometer has a resistance of 50 Ω and its full-scale deflection current is 50 μA. What shunt resistance should be added so that the ammeter can have a range of 0 – 5 mA?

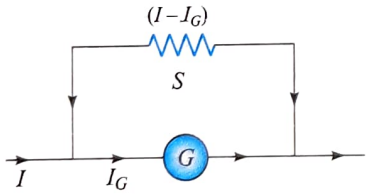
Sol. Given $I_G = 50\ \mu A$. The upper limit gives the maximum current to be measured, which is $I = 5$ mA. The galvanometer resistance is $G = 50\ \Omega$. Now

$$S = \frac{I_G G}{I - I_G} = \frac{50\times10^{-6}\times50}{5\times10^{-3} - 50\times10^{-6}} = \frac{50\times10^{-6}\times50}{5\times10^{-3}} \approx 0.5\ \Omega$$

ILLUSTRATION 6.2

What is the value of the shunt that passes 10% of the main current through a galvanometer of 99 Ω?

Sol. Given $G = 99\ \Omega$ and $I_G = (10/100)\, I = 0.1I$



Now $S = \dfrac{I_G G}{(I - I_G)} = \dfrac{0.1I\times99}{(I - 0.1I)} = \dfrac{0.1}{0.9}\times99 = 11\ \Omega$

ILLUSTRATION 6.3

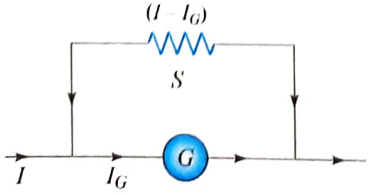
The deflection in a moving coil galvanometer falls from 50 divisions to 10 divisions when a shunt of 12 Ω is applied. What is the resistance of the galvanometer? Assume the main current to remain same.

Sol. In case of a galvanometer, $I \propto \theta$.

Given $\dfrac{I_G}{I} = \dfrac{10}{50} = \dfrac{1}{5}$

i.e., $I_G = \dfrac{1}{5} I$

Now as in case of a shunted galvanometer as shown in figure,

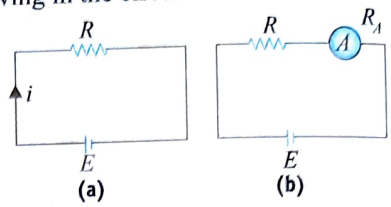


$$(I - I_G)\, S = I_G G \text{ or } \left(I - \frac{1}{5} I\right)\times12 = \frac{1}{5} IG$$

or $G = 4\times12 = 48\ \Omega$

ILLUSTRATION 6.4

Consider a circuit as shown in Fig. (a). We want to measure the current i flowing in the circuit.



For this, we connect an ammeter of resistance R_A as shows in Fig. (b). Find the percentage error in the current.

Sol. Actual current is $i = E/R$. Let the current measured by the ammeter be i_i, then $i_i = E/(R + R_A)$, clearly L_i is less than i. Percentage error is

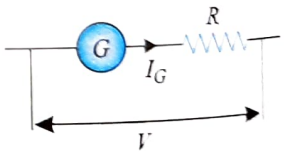
$$\frac{i - i_i}{i}\times100 = \left(\frac{E/R - E/(R+R_A)}{E/R}\right)\times100\left(\frac{R_A}{R+R_A}\right)\times100$$

VOLTMETER

A voltmeter an instrument used to find the potential difference across any two points in a circuit. A galvanometer can directly measure small potential differences only. To measure high potential differences, we have to do some modifications in the galvanometer. What we get after modifications is known as **voltmeter**.

CONVERSION OF A GALVANOMETER INTO VOLTMETER

Suppose we want to make a voltmeter that can measure the potential difference up to V; the range of the voltmeter is 0–V. For this, a suitable high resistance is connected in series with the galvanometer such that when a potential difference of V is applied, only a current I_G passes through the galvanometer as shown in figure. We can write



$$V = I_G(G + R) \text{ or } R = \frac{V}{I_G} - G$$

The above equation gives the value of R in terms of V.

Note:

- **Resistance of voltmeter:** $R_V = G + R$. Generally, R_V is very high, and for an ideal voltmeter, R_V is infinite.
- A voltmeter is used in parallel to measure potential difference.
- The higher the value of R, the higher the range of voltmeter.
- **How a voltmeter reads the potential difference:** Let V be the potential difference across a resistor to be measured. We have the relation,

  $$I_G = \frac{V}{R+G} \Rightarrow \boxed{I_G \propto V}$$

  We know that the deflection of the needle is proportional to the current I_G and hence to V. The scale can be graduated to read the potential difference directly.

ILLUSTRATION 6.5

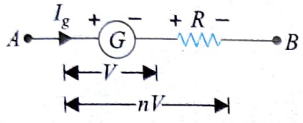
A galvanometer has a resistance of 50 Ω and its full-scale deflection current is 50 μA. What resistance should be added to it so that it can have a range of 0–5 V?

Sol. Given $I_G = 50\ \mu A$. Maximum voltage to be measured is $V = 5$ V. The galvanometer resistance $G = 50\ \Omega$. Now,

$$R = \frac{V}{I_G} - G = \frac{5}{50 \times 10^{-6}} - 50 \simeq 100\ k\Omega$$

ILLUSTRATION 6.6

A voltmeter has a resistance G ohm and range V volt. Calculate the resistance to be used in series with it to extend its range to n V volt.



Sol. The maximum current through the galvanometer is

$$I_g = \frac{V}{G}$$

With a multiplier resistor in series with galvanometer the potential difference across the entire branch is

$$nV = I_g G + I_g R$$

$$= \left(\frac{V}{G}\right)G + \left(\frac{V}{G}\right)R = V + \left(\frac{V}{G}\right)R$$

or $R = (n-1)G$

ILLUSTRATION 6.7

A galvanometer has a resistance of 30 Ω, and a current of 2 mA is needed for a full-scale deflection. What is the resistance and how is it to be connected to convert the galvanometer **(i)** into an ammeter of 0.3 A range and **(ii)** into a voltmeter of 0.2 V range?

Sol. Given $G = 30\ \Omega$ and $I_G = 2$ mA

(i) To convert the galvanometer into an ammeter of range 0.3 A,

$$(I - I_G)S = I_G G \text{ or } (0.3 - 0.002)S = 0.002 \times 30$$

or $$S = \frac{0.002 \times 30}{0.298} = 0.2013\ \Omega$$

(ii) To convert the galvanometer into a voltmeter of range 0.2 V,

$$V = I_G(R + G) \text{ or } 0.2 = 2 \times 10^{-3}(30 + R) \text{ or } R = 70\ \Omega$$

ILLUSTRATION 6.8

The scale of a galvanometer is divided into 150 equal divisions. The galvanometer has a current sensitivity of 10 divisions per mA and a voltage sensitivity of 2 divisions per mV. How can the galvanometer be designed to read **(i)** 6 A per division and **(ii)** 1 V per division?

Sol. Full-scale voltage = 150 divisions/2 divisions per mV

$= 75$ mV

Full-scale current = 150 divisions/10 divisions per mA

$= 15$ mA

So resistance of galvanometer is

$$G = \frac{\text{Full-scale voltage}}{\text{Full-scale current}} = \frac{75 \times 10^{-3}}{15 \times 10^{-3}} = 5\ \Omega$$

(i) Range of ammeter is $I = 150 \times 6 = 900$ A. So,

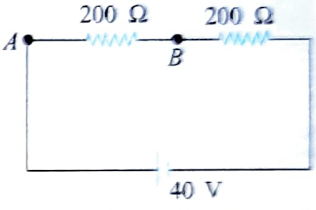
$$S = \frac{I_G G}{I - I_G} = \frac{15 \times 10^{-3} \times 5}{(900 - 15 \times 10^{-3})} = 8.3 \times 10^{-5}\ \Omega$$

(ii) Range of voltmeter is $V = 150 \times 1 = 150$ V. So,

$$R = \frac{V}{I_G} - G = \frac{150}{15 \times 10^{-3}} - 5 = 9995\ \Omega$$

ILLUSTRATION 6.9

(i) In figure, find the potential difference between the points A and B.



(ii) Now we wish to measure this potential difference by using a voltmeter of resistance 2 kΩ. Find the reading of the voltmeter and percentage error.

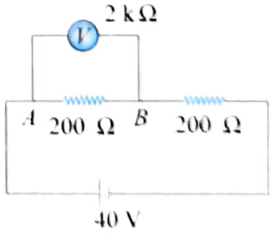
(iii) Solve part (ii) if the voltmeter were of resistance 20 kΩ. What conclusion do you draw from the results you get in the above parts?

Sol.

(i) As both the resistances are same, 40 V will be divided equally among both the resistances. Hence, the potential difference across A and B is 20 V.

(ii) Equivalent resistance of 200 Ω and 2 kΩ is

$$R_1 = \frac{200 \times 2000}{200 + 2000} = \frac{2000}{11}\ \Omega$$



Reading of voltmeter = potential difference across AB

$$= V_1 = 20\left[\frac{\frac{2000}{11}}{\frac{2000}{11} + 200}\right] = 19.05\ V$$

$$\text{Percentage error} = \frac{20 - 19.05}{20} \times 100 = 4.75\%$$

(iii) In this case

$$R_1 = \frac{200 \times 20{,}000}{200 + 20{,}000} = \frac{20{,}000}{101}\ \Omega$$

Reading of voltmeter
= potential difference across AB

$$= V_2 = 40\left[\frac{\frac{20,000}{101}}{\frac{20,000}{101}+200}\right] = 19.90 \text{ V}$$

$$\text{Percentage error} = \frac{20-19.9}{20}\times 100 = 0.5\%$$

In case (iii), percentage error is less than that in case (ii). It means the more the resistance of the voltmeter, the more accurate the reading.

ILLUSTRATION 6.10

A voltmeter reads 5.0 V at full-scale deflection and is graded according to its resistance per volt at full-scale deflection as 5000 ΩV^{-1}. How will you convert it into a voltmeter that reads 20 V at full-scale deflection? Will it still be graded as 5000 ΩV^{-1}? Will you prefer this voltmeter to one that is graded 2000 ΩV^{-1}?

Sol. Resistance per volt at full-scale deflection is 5000 ΩV^{-1}.
Reading of voltmeter at full-scale deflection is 5 V.
Therefore, resistance of voltmeter G is $5000 \times 5 = 25000\ \Omega$.
Also current for maximum deflection is

$$I_g = \frac{1\text{V}}{5000\,\Omega} = 0.0002 \text{ A}$$

Range of voltmeter to be changed to $V = 20$ V

$$\text{Now, } R = \frac{V}{I_g} - G = \frac{20}{0.0002} - 25,000$$

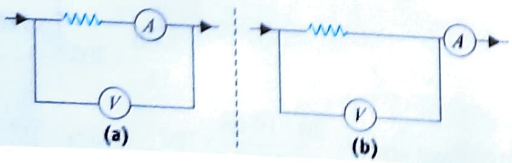
$$= 100,000 - 25,000 = 75,000\ \Omega$$

Thus, 7500 Ω resistor is to be connected in series.

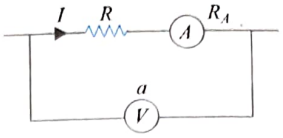
Resistance of 20 V voltmeter is 75000 + 25000 = 100,000 Ω. Its grading becomes 100000/20 = 5000 ΩV^{-1}, which is same as in the earlier case. A voltmeter with grading 2000 Ω V^{-1} will have less resistance and is, therefore, not preferred.

ILLUSTRATION 6.11

You are given two resistors X and Y whose resistances are to be determined using an ammeter of resistance 0.5 Ω and a voltmeter of resistance 20 kΩ. It is known that X is in the range of a few ohms while Y is in the range of several thousand ohms. In each case, which of the following two connections (figure) would you choose for resistance measurement? Justify your answer.



Sol. Let us measure a resistance R using circuit (a).



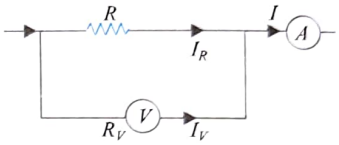
True value = R

$$\text{Measured value} = \frac{V}{I} = R + R_A$$

$$\text{Percentage error} = \frac{R + R_A - R}{R}\times 100$$

$$= \frac{R_A \times 100}{R} = \frac{0.5\times 100}{R} = \frac{50}{R}$$

So the more the value of R, the lesser the percentage error.
Hence, circuit (A) should be used to measure Y. Now, let us measure R using circuit (b).



True value = R

$$\text{Measured value} = \frac{V}{I} = \frac{V}{I_R + I_V} = \frac{V}{\frac{V}{R}+\frac{V}{R_V}} = \frac{RR_V}{R+R_V}$$

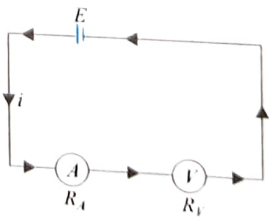
$$\text{Percentage error} = \frac{R - \frac{RR_V}{R+R_V}}{R}\times 100 = \frac{1}{1+R_V/R}\times 100$$

The lesser the value of R, the lesser the percentage error.
Hence, circuit (b) should be used to measure X.

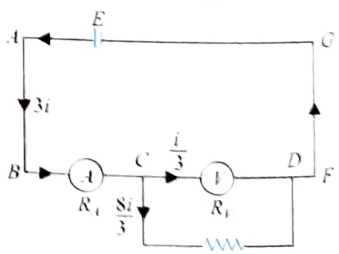
ILLUSTRATION 6.12

A voltmeter of resistance R_V and an ammeter of resistance R_A are connected in series across a battery of negligible internal resistance. When a resistance R is connected in parallel to voltmeter, reading of ammeter increases three times while that of voltmeter reduces to one third. Find R_V and R_A in terms of R.

Sol. Let a battery of the emf E is connected in series with voltmeter and ammeter as shown in figure [case (i)].



Case (i)



Case (ii)

In case (ii) the reading of ammeter increases three times and the reading of the voltmeter reduces to one third, it means main current increases three times ($3i$) while current through voltmeter will reduce to one third ($i/3$). Hence, the remaining $3i - i/3 = 8i/3$ passes through R as shown in figure.

The potential difference across C and D

$$V_C - V_D = \left(\frac{i}{3}\right)R_V = \left(\frac{8i}{3}\right)R \text{ or } R_V = 8R \quad \text{...(i)}$$

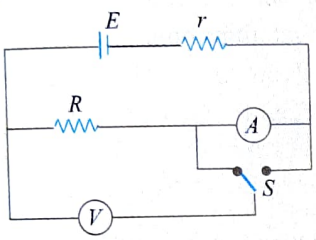
As in the second case, the main current becomes three times. It means the total resistance becomes 1/3 times or

$$R_{eq} = R_A + \frac{RR_V}{R+R_V} = \frac{1}{3}(R_V + R_A) \quad \text{...(ii)}$$

From (i) and (ii) we get, $R_A = \frac{8R}{3}$

ILLUSTRATION 6.13

The emf E and the internal resistance r of the battery shown in figure are 6.0 V and 2.5 Ω respectively. The external resistance R is 25 Ω The resistances of the ammeter and voltmeter are 5.0 Ω and 225 Ω, respectively.

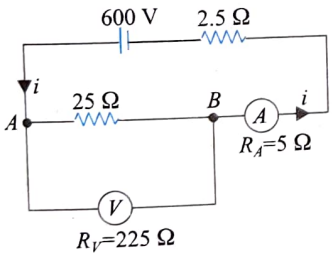


(i) Find the reading of the ammeter and voltmeter

(ii) If the switch is thrown to other side then find the reading of the ammeter and voltmeter at this situation.

Sol.

(a) When switch is at position '1'



The equivalent resistance of the circuit

$$R_{eq} = 2.5 + 5.0 + \left(\frac{25\times225}{225+25}\right)$$

$$= 2.5 + 5.0 + 22.5 = 30.0\ \Omega$$

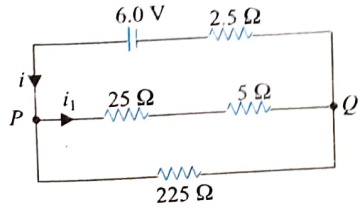
The current drawn from battery $i = \frac{6.0}{30.0} = \frac{1}{5} = 0.2\text{ A}$

This is the reading of ammeter.

The reading of the voltmeter = potential difference across AB

$= i \times R_{AB} = 0.2 \times 22.5 = 4.5$ V

(b) When the switch is at position '2'



The equivalent resistance

$$R_{eq} = 2.5 + \frac{30\times225}{(30+225)} = \frac{2955}{102}\ \Omega \simeq 29\ \Omega$$

The current supplied by battery $i = \frac{6.0}{2.9} \simeq 0.2\text{ A}$

The current in ammeter branch

$$i_1 = 0.2 \times \frac{225}{(30+225)} \simeq 0.18\text{ A}$$

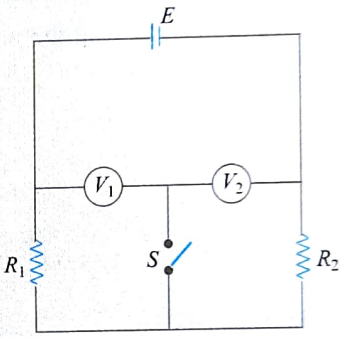
This is also the reading of the ammeter.

The reading of voltmeter should be equal to the potential difference across the branch having ammeter.

Hence $V_{PQ} = 0.18\times(20+5) = 4.4$ V

ILLUSTRATION 6.14

Two voltmeters V_1 and V_2 of resistances 5000 Ω and 3000 Ω, respectively are connected with the resistances $R_1 = 3000\ \Omega$, $R_2 = 5000\ \Omega$ and an ideal battery of emf $E = 400$ V is connected across the voltmeters as shown in figure, then



(a) Find the reading of voltmeters V_1 and V_2 when

(i) switch S is open

(ii) switch S is closed

(b) Current through the switch S, when it is closed.

Sol.

(a) (i) At the time when switch S is open, V_1 and V_2 are in series, connected to 400 V battery. Potential will drop in direct ratio of their resistors.

$$\frac{V_1}{V_2} = \frac{R_{V_1}}{R_{V_2}} = \frac{5000}{3000} = \frac{5}{3}$$

Hence, $V_1 = \frac{5}{8}\times400 = 250$ V

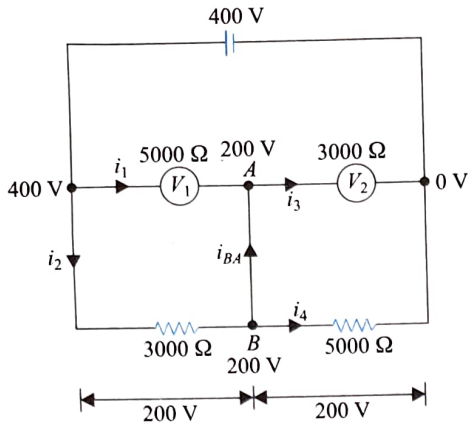
And, $V_2 = \frac{3}{8}\times400 = 150$ V

(ii) When the switch S is closed then V_1 and R_1 are in parallel. Similarly, V_2 and R_2 are also in parallel. Now, they are in series and they come out to be equal. So, 400 V will equally distribute between them.

$$\therefore\ V_1 = V_2 = \frac{400}{2} = 200\text{ V}$$

(b) From figure it can be observed, $i_2 = \frac{400-200}{3000} = \frac{1}{15}$ A

and $i_4 = \frac{200-0}{5000} = \frac{1}{25}$ A

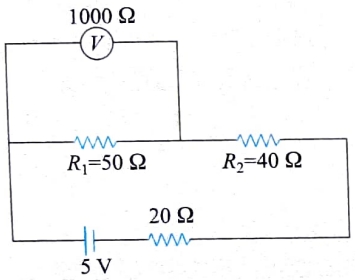


If we apply junction law at B, then current through switch

$$i_{BA} = i_2 - i_4 = \frac{2}{75}\text{A} \text{ in upward direction.}$$

ILLUSTRATION 6.15

Two resistances $R_1 = 50\ \Omega$ and $R_2 = 40\ \Omega$A are connected with a battery of e.m.f 5 V and internal resistance 20 Ω as shown in figure. A voltmeter of resistance 1000 Ω is used to measure the potential difference across R_1. Find the percentage error is made in the reading.



Sol. When voltmeter is not connected the current in the circuit is given as

$$i = \frac{E}{r + R_1 + R_2} = \frac{5}{20 + 50 + 40} = \frac{5}{10} = \frac{1}{22}\text{A}$$

Potential difference across resistance (R_1),

$$V_1 = i \times R_1 = \frac{1}{22} \times 50 = \frac{25}{11}\text{V}$$

When the voltmeter is connected across R_1 then the voltmeter resistance is taken in parallel with R_1 thus total resistance is of this part of circuit is given as

$$R' = \frac{1000 \times 50}{1000 + 50} = \frac{1000}{21}\ \Omega$$

Current in the circuit, $i' = \dfrac{5}{\left(20 + 40 + \dfrac{1000}{21}\right)} = \dfrac{21}{452}\text{A}$

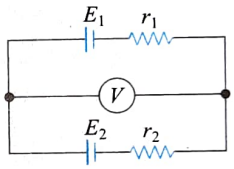
Potential difference measured by voltmeter,

$$V_2 = i' \times R' = \frac{21}{452} \times \frac{1000}{21} = \frac{250}{113}\text{V}$$

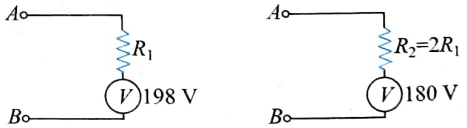
Percentage error in reading, $e = \dfrac{\frac{25}{11} - \frac{250}{113}}{\frac{25}{11}} \times 100 = \frac{300}{113}\%$

CONCEPT APPLICATION EXERCISE 6.1

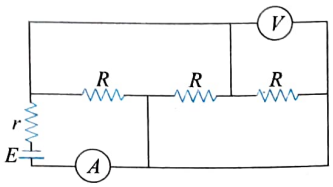
1. A 100 V voltmeter having an internal resistance of 20 kΩ is connected in series with a large resistance R across a 110 V line. What is the magnitude of resistance R if the voltmeter reads 5 V?
2. A galvanometer with a coil of resistance 12.0 Ω shows full-scale deflection for a current of 2.5 mA. How will you convert the meter into
   (a) an ammeter of range 0 to 7.5 A?
   (b) a voltmeter of range 0 to 1.0 V?
3. What shunt resistance is required to make a 1.00 mA, 20.0 Ω meter into an ammeter with a range of 0 to 50.0 mA?
4. It is required to measure the resistance of a circuit operating at 120 V. There is only one galvanometer of current sensitivity 10^{-6} A per division. How should the galvanometer be connected in the circuit to operate an ammeter? What minimum resistance can be measured with such a galvanometer if its full-scale has 40 divisions?
5. Two cells of emf 1.3 V and 1.5 V respectively are arranged as shown in figure. The voltmeter reads 1.45 V. The voltmeter is assumed to be ideal. Find the ratio of internal resistances of the batteries $\left(\frac{r_1}{r_2} = ?\right)$



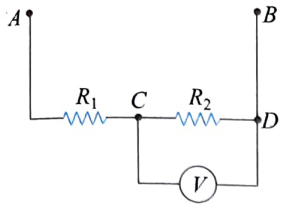
6. A voltmeter connected in series with a resistance R_1 to a circuit indicates a voltage $V_1 = 198$ V. When a series resistor $R_2 = 2R_1$ is used, the voltmeter indicates a voltage $V_2 = 180$ V. If the resistance of the voltmeter is $R_V = 900\ \Omega$, find the applied voltage across A and B.



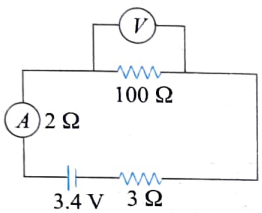
7. A moving coil galvanometer of resistance 20 Ω gives a full-scale deflection when a current of 1 mA is passed through it. It is to be converted into an ammeter reading 20 A on full scale. But the shunt of 0.005 Ω, only is available. What resistance should be connected in series with the galvanometer coil?
8. In the circuit shown in figure ammeter and voltmeter are ideal If $E = 4$ V, $R = 9\ \Omega$ and $r = 1\ \Omega$, find the readings of ammeter and voltmeter.



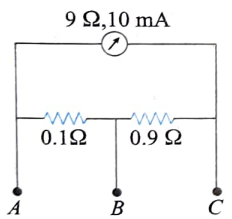
9. Resistances R_1 and R_2, each 60 Ω, are connected in series. The potential difference between points A and B is 120 V. Find the reading of voltmeter connected between points C and D if its resistance $R_V = 120$ Ω.



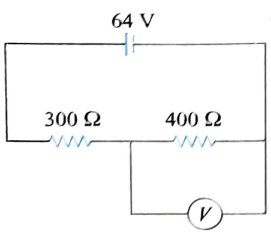
10. A cell of emf 3.4 V and internal resistance 3 Ω is connected to an ammeter having resistance 2Ω and to an external resistance of 100 Ω. When a voltmeter is connected across the 100 Ω resistance, the ammeter reading is 0.04 A. Find the voltage reading by the voltmeter and its resistance. Had the voltmeter been an ideal one what would have been its reading?



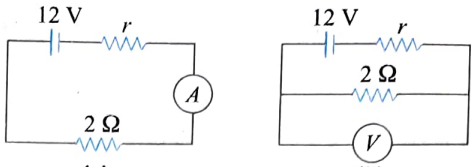
11. A milliammeter of range 10 mA and resistance 9 Ω is joined in a circuit as shown. The meter gives full scale deflection for current I when A and B are used as its terminals, i.e. current enters at A and leaves at B (C is left isolated). Find the value of I.



12. In the circuit, a voltmeter reads 32 V when it is connected across 400 Ω resistance. Calculate what the same voltmeter will read when it is connected across the 300 Ω resistance?



13. A galvanometer (coil resistance 99 Ω) is converted into an ammeter using a shunt of 1 Ω. and connected as shown in Fig. (a). The ammeter reads 3 A. The same galvanometer is converted into a voltmeter by connecting a resistance of 101 Ω. in series. This voltmeter is connected as shown in Fig. (b). Its reading is found to be 4/5 of the full-scale reading. Find:

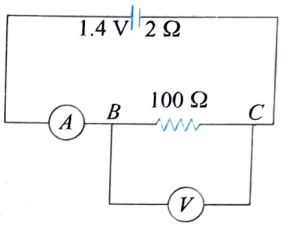


(a) Internal resistance r of the cell

(b) Range of the ammeter and voltmeter

(c) Full scale deflection current of the galvanometer.

14. A battery of emf 1.4 V and internal resistance 2 Ω is connected to a resistor of 100 Ω resistance through an ammeter. The resistance of the ammeter is 4/3 Ω. A voltmeter has also been connected to find the potential difference across the resistor.



(i) The ammeter reads 0.02 A. What is the resistance of the voltmeter?

(ii) The voltmeter reads 1.1 V. What is the error in the reading?

ANSWERS

1. 420 kΩ

2. (i) By applying shunt resistance of 4.0×10^{-3} Ω

(ii) By applying series resistance of 3988 Ω

3. 0.408 Ω **4.** Series, 3 MΩ **5.** 3 **6.** 220 V

7. 80 Ω **8.** 1 A, 3 V **9.** 48 V **10.** 400 Ω, 3.2 V, 3.238 V

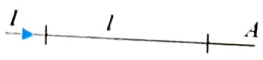
11. 1 A **12.** 24 V **13.** (a) 1.01 Ω (b) 5 A, 10 V (c) 0.05 A

14. (i) 200 Ω (ii) –0.233 V

POTENTIOMETER

When we measure the current or potential difference using an ammeter or voltmeter, we do not get exact values, because when we connect the ammeter or voltmeter in the circuit, it disturbs the original circuit. But potentiometer is an instrument that can measure current or potential difference accurately. Basically, a potentiometer does not draw any current from the original circuit and hence does not disturb the original circuit.

It is based on the principle that if a constant current is passed through a wire of uniform cross section, the potential difference across any segment of the wire is proportional to its length. Let a current I pass through a wire of uniform cross-sectional area A as shown below.



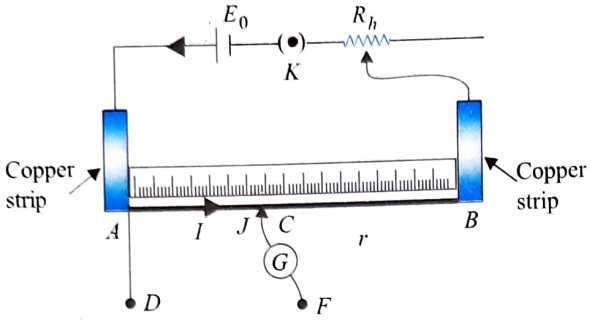
Consider a segment of length l of the wire. Resistance of this segment is given by $R = \rho l/A$. Potential difference across this segment is given by

$$V = IR = I\rho l/A \text{ or } V = kl \text{ or } k = V/l$$

where $k = I\rho/A$ is the potential difference per unit length known as **potential gradient**.

CONSTRUCTION OF POTENTIOMETER

A potentiometer consists of a wire AB of uniform cross section, generally 1–10 m long, fixed on a wooden board. Let the resistance of AB be r.



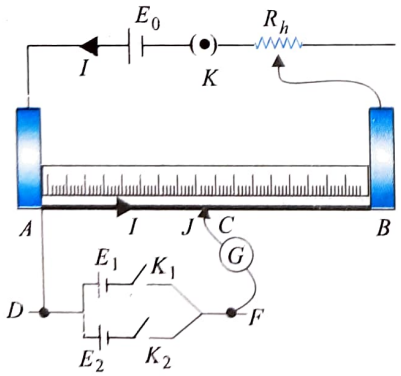
The ends A and B are connected to a battery E_0 (known as driving battery), a switch K, and a rheostat R_h. On closing the switch, current I is established in the wire AB, i.e.,

$$I = \frac{E_0}{R_h + r}$$

Current I can be adjusted by varying the value of R_h. A jockey J can slide freely on AB. The jockey touches the wire at C. A scale is fitted on the wooden board parallel to AB so that length AC can be read. Between points D and F, we can insert any cell, resistance, or any other device of which we need the current or voltage information. After inserting such a device, the jockey is slid on the wire, and the location of C is so selected that finally there is no current through the galvanometer. In this condition, the applied potential difference across D and F will be equal to the potential difference across the segment AC.

USES OF POTENTIOMETER

Comparison of emf of two cells: Consider two cells of emfs E_1 and E_2 are connected across points D and F along with switches K_1 and K_2 as shown in figure. Initially, both the switches are open.



First close switch K_1 and move the jockey on the wire AB until the galvanometer shows no deflection. When $AC = l_1$, deflection in the galvanometer becomes zero. Then from the principle of potentiometer, we get

$$E_1 = kl_1 \quad \text{...(i)}$$

where k is the potential gradient of AB. We can say that E_1 is balanced at length l_1.

Similarly, on opening switch K_1 and closing K_2, let E_2 be balanced at length l_2, then

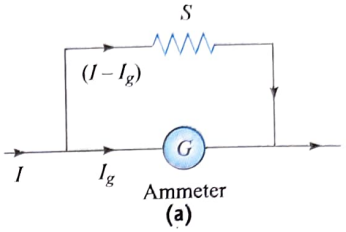
$$E_2 = kl_2 \quad \text{...(ii)}$$

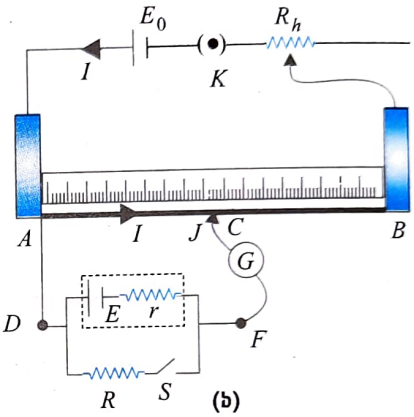
From Eqs. (i) and (ii),

$$\frac{E_2}{E_1} = \frac{l_2}{l_1}$$

Thus we can compare the emfs of the cells.

Determination of internal resistance of a cell: Suppose we have a cell of emf E whose internal resistance r is to be measured.





Connect the cell in the circuit of the potentiometer as shown in figure. First keep switch S open and slide the jockey so that there is no deflection. Let the balancing length be l_1. Then

$$E = kl_1 \quad \text{...(i)}$$

Now close switch S and again find the balancing length. Let the balancing length be l_2. A separate current, say, I_1 is established in the lower circuit. I_1 is given by

$$I_1 = \frac{E}{R + r}$$

Let the terminal potential difference across the cell be V, then

$$V = kl_2 \text{ or } I_1 R = kl_2 \text{ or } \frac{E}{R+r} R = kl_2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\frac{R+r}{R} = \frac{l_1}{l_2} \text{ or } r = \left(\frac{l_1 - l_2}{l_2}\right) R$$

Thus, we can measure the internal resistance of a cell.

Note: We can also write

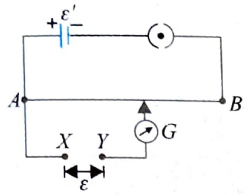
$$r = \left(\frac{l_1 - l_2}{l_2}\right) R = \left(\frac{kl_1 - kl_2}{kl_2}\right) R \text{ or } r = \left(\frac{E - V}{V}\right) R$$

Sensitivity of potentiometer: The sensitivity of a potentiometer means the smallest potential difference that can be measured with its help. It can be increased by decreasing the potential

gradient. The same can be achieved by increasing the length of the potentiometer wire and by reducing the current in the potentiometer wire circuit with the help of a rheostat if the potentiometer wire is of fixed length.

ILLUSTRATION 6.16

For the potentiometer circuit shown in figure, points X and Y represent the two terminals of an unknown emf ε. A student observed that when the jockey is moved from the end A to the end B of the potentiometer wire, the deflection in the galvanometer remains in the same direction. What are the two possible faults in the circuit that could result in this observation?



If the galvanometer deflection at the end B is **(i)** more **(ii)** less than that of the end A, which of the two faults, listed above, would be there in the circuit? Give reasons in support of your answer in each case.

Sol. The two possible faults due to which deflection in the galvanometer remains in the same direction are:

(a) The positive terminal of ε (unknown emf) is not connected to the point A where the positive terminal of ε' (driving cell) has been connected.

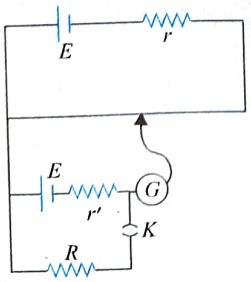
(b) If emf of the driving cell is less than the emf of the unknown cell ($\varepsilon' < \varepsilon$)

(i) If the galvanometer deflection at the end B is more than at the end A. The fault in the circuit is the one that is listed (a) above. This is due to the reason that the –ve terminal of ε' is joined to negative terminal of ε. The two emfs thus support each other and their resultant emf is the sum of the two.

(ii) If the galvanometer deflection at the end B is less than that at the end A, the fault in the circuit is the one that is listed (b) above. This is due to the reason that the resultant emf is the difference of the two emf s as the fault (a) is not there and the two cells oppose each other, ε' is unable to balance ε.

ILLUSTRATION 6.17

The circuit shown in figure shows the use of potentiometer to measure the internal resistance of a cell.



(a) When the key is open, how does the balance point change, if the current from the driver cell decreases?

(b) When the key is closed, how does the balance point change, if R is increased, keeping the current from the driver cell constant?

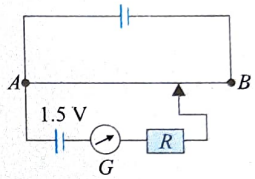
Sol.

(a) If current from driver cell decreases, then potential gradient decreases. So the balancing length should increase. Hence, balance point shifts toward right.

(b) If R is increased, current in the lower circuit decreases, due to which terminal potential difference of battery increases. To balance increased terminal potential difference, balancing length should increase, hence balance point shifts toward right.

ILLUSTRATION 6.18

A potentiometer wire of length 1 m is connected to a driver cell of emf 3 V shown in figure. When a cell of 1.5 V emf is used in the secondary circuit, the balance point is found to be at 60 cm. On replacing this cell and using a cell of unknown emf, the balance point shifts to 80 cm.



(i) Calculate unknown emf of the cell.

(ii) Explain with reason, whether the circuit works, if the driver cell is replaced by a cell of emf 1 V.

(iii) Does the high resistance R, used in the secondary circuit affect the balance point? Justify your answer.

Sol.

(i) As, $\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$ or $\varepsilon_2 = \varepsilon_1\left(\frac{l_2}{l_1}\right)$

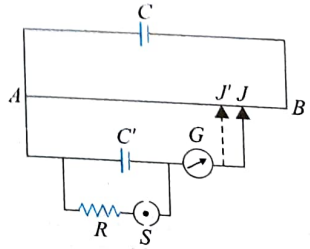
Hence, $\varepsilon_2 = (1.5\text{ V})\left(\frac{80\text{ cm}}{60\text{ cm}}\right) = 2.0\text{ V}$

(ii) The driver cell must have higher emf than that of the cell in the secondary circuit. Since emfs (1.5 V and 2.0 V) of the cells in the secondary circuit are greater than 1 V, the balance points for them cannot be obtained on AB and the circuit does not work.

(iii) Since no current flows in the secondary circuit a: the balance point, the high resistance R in this circuit does not affect the balance point.

ILLUSTRATION 6.19

Figure shows a potentiometer circuit for determining the mal resistance of a cell. When switch S is open, the balance point is found to be at 76.3 cm of the wire. When switch S is closed and the value of R is 4.0 Ω, the balance point shifts to 60.0 cm. Find the internal resistance of cell C'.



Sol. Let ε be the emf of the cell C' and r its internal resistance. Let $l = AJ$ be the balance length when switch S is open. When a resistance R is introduced by closing the switch a current begins to flow through the cell C' and resistance R. The potential difference between the terminals of the cell falls and the balance length increases to $l' = AJ'$.

The terminal resistance of the cell, $r = \dfrac{E-V}{I}$

where V is the terminal voltage of C' and I is the current in the circuit involving C' and R. Also $I = \dfrac{V}{R}$.

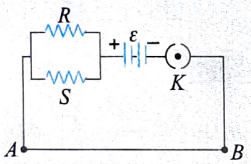
Hence $r = \left(\dfrac{E}{V} - 1\right)R$

But $\dfrac{E}{V} = \dfrac{l}{l'}$. Hence $r = R\left(\dfrac{l-l'}{l'}\right)$

$$r = 4.0 \times \left(\frac{76.3 \times 60.0}{60.0}\right) = 1.1\ \Omega$$

ILLUSTRATION 6.20

A potentiometer wire has a length L and resistance R_0. It is connected to a battery and a resistance combination as shown in figure. Obtain an expression for potential drop per unit length of this potentiometer wire. What is the maximum emf of the "test cell" for which one can get a 'balance point' on this potentiometer wire? What precaution should one take while combining this "test cell" in the circuit?



Sol. Equivalent resistance of parallel combination of R and S,

$$R_P = \frac{RS}{R+S}$$

Total resistance in the circuit, $R = R_{AB} + R_P = R_0 + \dfrac{RS}{(R+S)}$

Current in the circuit, $I = \dfrac{\varepsilon}{R} = \dfrac{\varepsilon}{R_0 + \dfrac{RS}{(R+S)}}$

Total potential difference across AB, $V = IR_0 = \dfrac{\varepsilon R_0}{R_0 + \dfrac{RS}{(R+S)}}$

Thus, potential difference per unit length,

$$k = \frac{V}{L} = \frac{\varepsilon R_0}{L\left[R_0 + \dfrac{RS}{(R+S)}\right]}$$

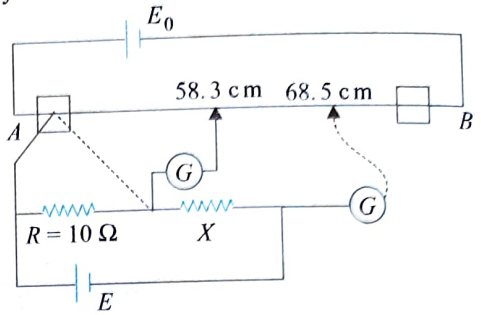
Maximum emf of the test cell $kL = \dfrac{\varepsilon R_0}{\left[R_0 + \dfrac{RS}{(R+S)}\right]}$

In fact, the emf of the test cell should be less than $\dfrac{\varepsilon R_0}{\left[R_0 + \dfrac{RS}{(R+S)}\right]}$.

The test cell should be connected with its positive terminal at the end A of the potentiometer wire AB.

ILLUSTRATION 6.21

Figure shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor $R = 10.0\ \Omega$ is found to be 58.3 cm, while that with the unknown resistance X is 68.5 cm. Determine the value of X. What would you do if you fail to find a balance point with the given cell E?



Sol. Here, $l_1 = 58.3$ cm, $l_2 = 68.5$ cm, $R = 10\ \Omega$, $X = ?$

Let I be the current in the potentiometer wire and E_1 and E_2 be the potential drops across R and X, respectively. Then

$$\frac{E_2}{E_1} = \frac{IX}{IR} = \frac{X}{R} \quad \text{or} \quad X = \frac{E_2}{E_1}R \qquad \ldots(i)$$

But $\dfrac{E_2}{E_1} = \dfrac{l_2}{l_1}$

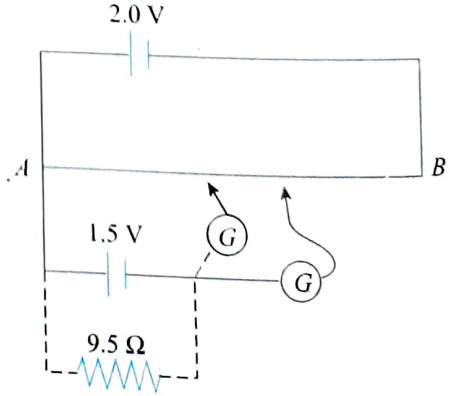
From Eq. (i),

$$X = \frac{l_2}{l_1}R = \frac{68.5}{58.3} \times 10.0 = 11.75\ \Omega$$

If there is no balance point with the given cell of emf E, it means the potential drop across R or X is greater than the potential drop across the potentiometer wire AB. In order to obtain the balance point, the potential drops across R and X are to be reduced, which is possible by reducing the current. For that, either a suitable resistance should be put in series with R and X or a cell of smaller emf should be used. Another possible way is to increase the potential drop across the potentiometer wire by increasing the voltage of driver cell.

ILLUSTRATION 6.22

Figure shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm, length of the potentiometer. Determine the internal resistance of the cell.

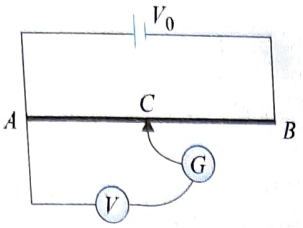


Sol. Here, $l_1 = 76.3$ cm, $l_2 = 64.8$ cm, $r = ?$, $R = 9.5\ \Omega$

Now, $r = \left(\dfrac{l_1 - l_2}{l_2}\right) R = \left(\dfrac{76.3 - 64.8}{64.8}\right) \times 9.5 = 1.68\ \Omega$

ILLUSTRATION 6.23

A voltage V_0 is applied to a potentiometer whose sliding contact is exactly in the middle. A voltmeter V is connected between the sliding contact and one fixed end of the potentiometer. It is assumed that the resistance of the voltmeter is not very high in comparison to the resistance of the potentiometer wire. What voltage will the voltmeter show: higher than, less than, or equal to $V_0/2$?



Sol. Let R be the resistance of the whole potentiometer wire and R_V be the resistance of the voltmeter. Then the total resistance across section AC is

$$R_{AC} = \frac{R_V (R/2)}{R_V + (R/2)} = \frac{R}{2(1 + R/2R_V)} < \frac{R}{2}$$

The resistance of section CB is equal to $R/2$. So the reading of the voltmeter will be less than $V_0/2$.

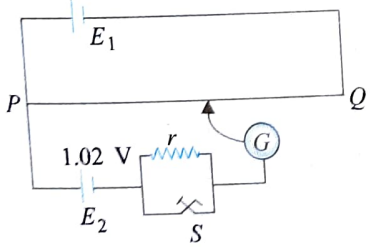
ILLUSTRATION 6.24

Potentiometer wire PQ of 1 m length is connected to a standard cell E_1. Another cell E_2 of emf 1.02 V is connected with a resistance r and a switch S as shown in the circuit diagram. With switch S open, the null position is obtained at a distance of 51 cm from P.

(i) Calculate the potential gradient of the potentiometer wire.

(ii) Find the emf of cell E_1.

(iii) When switch S is closed, will the null point move toward P or toward Q? Give reason for your answer.



Sol.

(i) Potential gradient is

$$k = \frac{V}{l} = \frac{1.02}{0.51} = 2\ \text{Vm}^{-1}$$

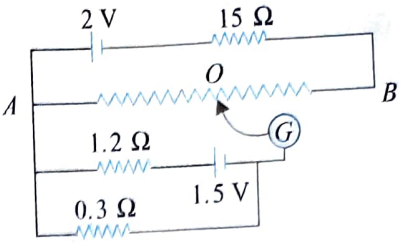
(ii) The emf of the cell is

$$E_1 = kl = 2 \times 1 = 2\ \text{V}$$

(iii) When the switch S is closed, there is no shift in the position of the null point because it depends on the potential gradient along the potentiometer wire (which depends on the emf of battery E_1 and the resistance of the potentiometer wire circuit and length of potentiometer) and the emf of cell E_2, which does not change when the switch S is closed.

ILLUSTRATION 6.25

In figure, AB is a 1 m long uniform wire of 10 Ω resistance. Other data are shown in the figure. Calculate **(i)** potential gradient along AB and **(ii)** length AO when the galvanometer shows no deflection.



Sol.

(i) Current in wire AB is $I = 2/(15 + 10) = 2/25$ A

Potential difference across AB is

$$V = IR = 2/25 \times 10 = 0.8\ \text{V}$$

Potential gradient along AB is

$$k = V/l = 0.8/1 = 0.8\ \text{Vm}^{-1}$$

(ii) Current through 0.3 Ω is

$$\frac{1.5}{1.2 + 0.3} = 1\ \text{A}$$

Potential difference across 0.3 Ω is $1 \times 0.3 = 0.3$ V

Let l_1 be the length AO, then

$$0.3 = 0.8 \times l_1$$

or $l_1 = 0.3/0.8 = 0.375$ m

ILLUSTRATION 6.26

Cells A and B and a galvanometer G are connected to a slide wire OS by two sliding contacts C and D as shown in figure. The slide wire is 100 cm long and has a resistance of 12 Ω. With $OD = 75$ cm, the galvanometer gives no deflection when OC is 50 cm. If D is moved to touch the end of wire S, the value of OC for which the galvanometer shows no deflection is 62.5 cm. The emf of cell B is 1.0 V.

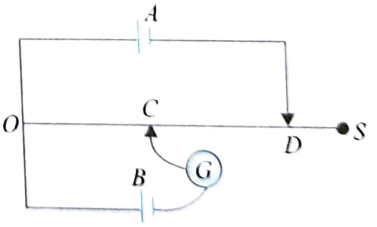
Calculate

(i) the potential difference across O and D when D is at 75 cm mark from O

(ii) the potential difference across OS when D touches S

(iii) internal resistance of cell A

(iv) the emf of cell A



Sol. Resistance of wire OD is

$$\frac{12}{100} \times 75 = 9\ \Omega$$

Let E and r be the emf and internal resistance of cell E, respectively.

(i) Since 1 V is balanced across 50 cm, so potential gradient of wire is 1/50 Vcm^{-1}. Therefore, voltage drop across wire OD of length 75 cm is $(1/50) \times 75 = 1.5$ V.

(ii) Now potential gradient of wire is $1/62.5\ \text{Vcm}^{-1}$. Therefore, voltage drop across wire OS of length 100 cm is $(1/62.5) \times 100 = 1.6$ V.

(iii) For first case

$$\left(\frac{E}{9+r}\right)\times 9 = 1.5 \quad \ldots(\text{i})$$

For second case

$$\left(\frac{E}{12+r}\right)\times 12 = 1.6 \quad \ldots(\text{ii})$$

(iv) On solving Eqs. (i) and (ii), we get $r = 3\ \Omega$ and $E = 2$ V.

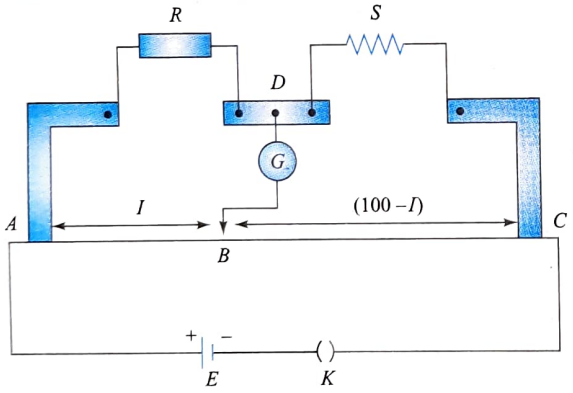
METER BRIDGE OR SLIDE WIRE BRIDGE

A slide wire bridge is a particular application of Wheatstone bridge and is used for (a) measuring an unknown resistance and (b) comparing two unknown resistances. A slide wire bridge works on the principle of Wheatstone bridge.

CONSTRUCTION

A slide wire bridge consists of a uniform wire AC, usually of eureka or manganin of 1 m length. It is stretched on a wooden board between two copper strips. A meter scale is fitted on the board parallel to the length of the wire. Another copper strip is fitted on the wooden board in order to provide two gaps.

In one of the gaps (say left gap), a resistance box R is connected, while in other gap (right gap), an unknown resistance S is connected. A cell E and a key K are connected across the ends A and C as shown in figure.



CHECKING OF CONNECTIONS

Close the key K and put the jockey at the end A of the wire and see the direction of deflection in the galvanometer. Now remove the jockey from A and put it at the end B of the wire and note the direction of deflection in the galvanometer. If the direction of deflection reverses, the connections are correct.

WORKING

Close the key K and take out some suitable low resistance R from the resistance box. Now move the jockey gently on wire AC till the galvanometer shows no deflection. Let this point be B on the wire. Let $AB = l$, then $BC = (100 - l)$.

Let the resistance of the wire between A and B be P and the resistance of the wire between B and C be Q. If r is the resistance of the wire of unit length, then

$$P = lr \text{ and } Q = (100 - l)\, r$$

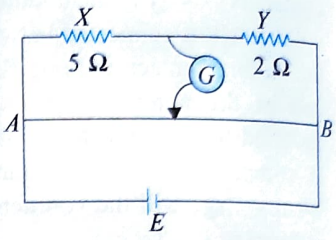
According to the principle of Wheatstone bridge,

$$\frac{P}{Q} = \frac{R}{S} \text{ or } \frac{lr}{(100-l)r} = \frac{R}{S} \text{ or } S = \left(\frac{100-l}{l}\right)R$$

If λ and R are known, S can be calculated.

ILLUSTRATION 6.27

In the simple potentiometer circuit, where the length AB of the potentiometer wire is 1 m, the resistors X and Y have values of 5 Ω and 2 Ω, respectively. When X is shunted by a wire, the balance point is found to be 0.625 m from A. What is the resistance of the shunt?



Sol. Let R be the resistance of the shunted wire. The effective resistance of R and 5 Ω in parallel is $5R/(5 + R)$.

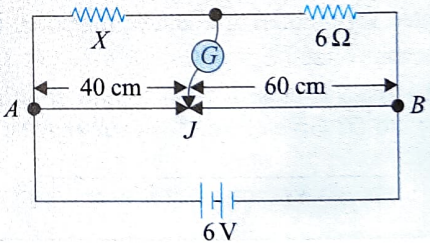
At balance point,

$$\frac{5R/(5+R)}{2} = \frac{0.625}{1-0.625} = \frac{0.625}{0.0375} = \frac{5}{3}$$

On solving, we get $R = 10\ \Omega$.

ILLUSTRATION 6.28

In the given circuit, a meter bridge is shown in a balanced state. The bridge wire has a resistance of $1\ \Omega\text{cm}^{-1}$. Find the value of the unknown resistance X and the current drawn from the battery of negligible internal resistance.



Sol. For the balanced bridge, the ratio of the two resistances is equal to the ratio of the lengths of the two parts AJ and JB of the wire, i.e.,

$$\frac{X}{6\ \Omega} = \frac{40\ \text{cm}}{60\ \text{cm}} \quad \text{or} \quad X = 4\ \Omega$$

No current flows through the galvanometer G. The resistances of the parts AJ and JB are 40 Ω and 60 Ω, respectively. If R is the equivalent resistance between the points A and B, then we have

$$\frac{1}{R} = \frac{1}{(X+6)\ \Omega} + \frac{1}{(40+60)\ \Omega}$$

or $$R = \frac{100}{11}\ \Omega$$

$$i = \frac{V}{R} = \frac{6\ \text{V}}{(100/11)\Omega} = 0.66\ \text{A}$$

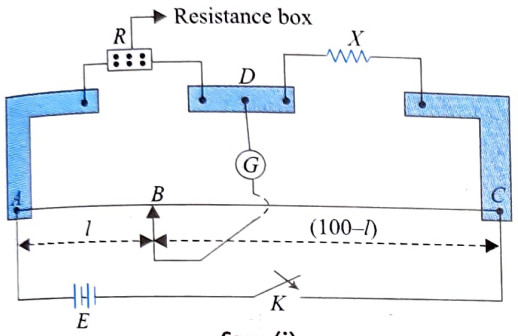
ILLUSTRATION 6.29

In a meter bridge experiment, null point is found at length 20 cm. In same experiment when the known resistance R is shunted by 10 Ω resistance, null point is found to be shifted by 10 cm. Find the unknown resistance X.

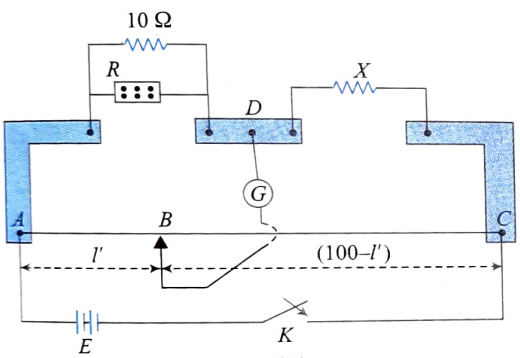
Sol. **In first case:**

$$\frac{R}{X}=\frac{l}{100-l}$$

$$\therefore\quad X=\left(\frac{100-l}{l}\right)R \text{ or } X=\left(\frac{100-20}{20}\right)R=4R \qquad ...(i)$$



Case (i)



Case (ii)

In second case: When known resistance R is shunted, the net resistance in this slot will decrease (say R'). Therefore, its new null point length should also decrease i.e. null point will shift towards left.

$$\therefore\quad \frac{R'}{X}=\frac{l'}{100-l'}=\frac{20-10}{100-(20-10)}=\frac{1}{9}$$

or $X=9R'$ ...(ii)

From Eqs. (i) and (ii), we have $4R=9R'$ ...(iii)

R' is resultant of R and 10 Ω. in parallel $\frac{1}{R'}=\frac{1}{10}+\frac{1}{R}$

or $R'=\frac{10R}{10+R}$ ...(iv)

From (iii) and (iv) we get, $4R=9\left[\frac{10R}{10+R}\right]$

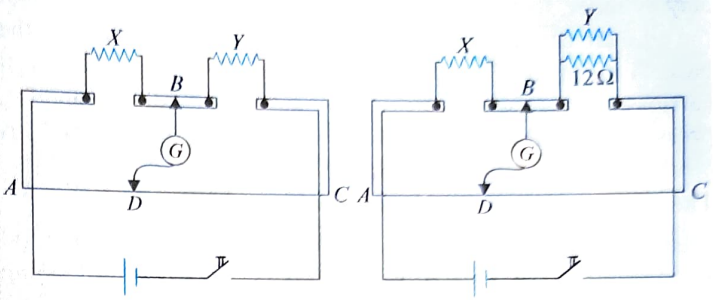
Solving this equation, we get $R=\frac{50}{4}\Omega$

Now, from Eq. (i), the unknown resistance

$$X=4R=4\left(\frac{50}{4}\right) \text{ or } X=50\,\Omega$$

ILLUSTRATION 6.30

Figure shows a meter bridge (which is nothing but a practical Wheatstone bridge), consisting of two resistors X and Y together in parallel with a meter long constantan wire of uniform cross section.



With the help of a movable contact D, one can change the ratio of resistance of the two segments of the wire until a sensitive galvanometer G connected across B and D shows no deflection. The null point is found to be at a distance of 33.7 cm. The resistor Y is shunted by a resistance of 12 Ω, and the null point is found to shift by a distance of 18.2 cm. Determine the resistance of X and Y.

Sol. As the wire is of uniform cross section, the resistances of the two segments AD and DC of the wire are in the ratio of the lengths of AD and DC. According to the condition of balance of Wheatstone bridge,

$$\frac{X}{Y}=\frac{l_1}{l_2}$$

Here l_1 = 33.7 cm and l_2 = 100 – 33.7 = 66.3 cm

$$\frac{X}{Y}=\frac{33.7}{66.3} \qquad ...(i)$$

As resistance Y' is due to a parallel combination of resistance Y and a resistance of 12 Ω,

$$\frac{1}{Y'}=\frac{1}{Y}+\frac{1}{12}=\frac{12+Y}{12Y} \text{ or } Y'=\frac{12Y}{12+Y}$$

Since Y' is less than Y, the ratio X/Y' will be greater than X/Y and the null point should shift toward end C.

$$\frac{X}{Y'}=\frac{33.7+18.2}{66.3-18.2}=\frac{51.9}{48.1}$$

or $$\frac{X(12+Y)}{12Y}=\frac{51.9}{48.1}$$

or $$12+Y=\frac{51.9}{48.1}\times 12\times\frac{Y}{X}=\frac{51.9}{48.1}\times 12\times\frac{66.3}{33.7}$$

$$=25.47\,\Omega$$

or $Y=25.47-12=13.47\,\Omega$

Putting this value in Eq. (i), we get

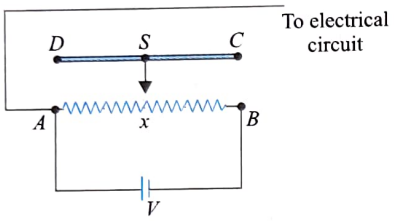
$$X=\frac{33.7}{66.3}\times Y=\frac{33.7}{66.3}\times 13.47=6.85\,\Omega$$

RHEOSTAT

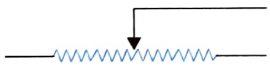
It is a device which is used for changing the strength of the current in an electrical circuit and dividing the potential. It consists of a nichrome wire with high specific resistance and low temperature-coefficient of resistance, wound on a hollow china-clay cylinder. Its turns are insulated from each other. The ends of the wire are connected to the binding-screws A and B fixed at the base. A metal rod CD is fixed above the cylinder parallel to it. It carries a sliding metallic strip S. The strip can be slide to and for, pressing the coil of the wire. It is called the 'sliding contact'. A binding screw C is fixed at one end of the rod.

The rheostat can be used in two ways:

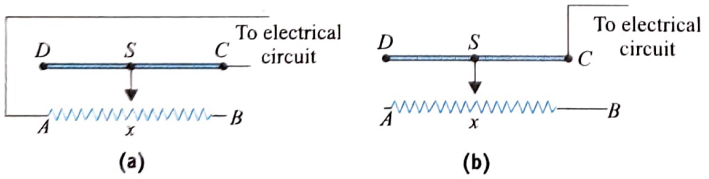
(i) **As a potential-divider:** A battery, whose potential difference V is to be divided, is connected across the binding screws A and B; and the electric circuit is connected across A and C (or across B and C) as before (figure)



If the distance of the contact-point X of the sliding-contact S be three-fourth length of the wire AB from a screw A, then the potential difference between A and X should be three-fourth the total potential difference i.e., 3 V/4. Hence a potential difference of 3 V/4 will be established across the circuit. It means by sliding the contact point X from A to B any desired potential difference from 0 to V can be established across the circuit. In the electric circuit, the symbol of potential divider is:



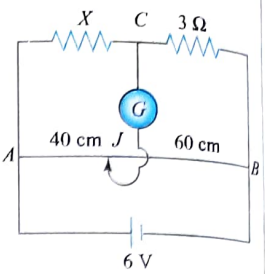
(ii) **As a current-controller:** For using rheosted, as one wire of the circuit is connected to one of the binding screws A and B fixed to the base and the other wire is connected to the binding screw C fixed to the rod as shown in Fig. (a) and (b) below.



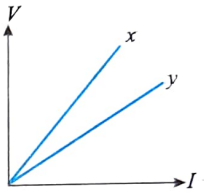
Hence, in the first position [Fig. (a)] the resistance of the length AX of the rheostat wire is included in the circuit, and in the second position [Fig. (b)] the resistance of the length XB of the wire is included. By sliding the sliding-contact on the rod CD, the value of resistance included in the circuit, and hence the current in the circuit, can be changed. In the circuit, the rheostat is indicated by the symbol.

CONCEPT APPLICATION EXERCISE 6.2

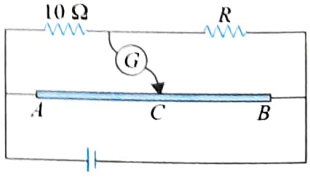
1. What will be the effect on the accuracy of the result if we replace a single-wire potentiometer by a potentiometer having 12 wires, the length of each wire being 1 m?
2. In the circuit shown in figure, a meter bridge is in its balance state. The meter bridge wire has a resistance of 1 Ω cm^{-1}. Calculate the value of the unknown resistance X and the current drawn from the battery of negligible internal resistance.



3. The variation of potential difference V with length l in case of two potentiometers X and Y is as shown in figure. Which of these two will you prefer for comparing the emfs of the two cells and why?

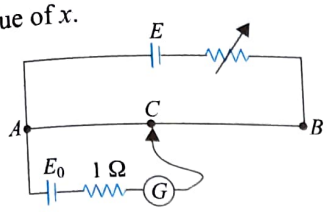


4. Two unknown resistances X and Y are placed on the left and right gaps of a meter bridge. The null point in the galvanometer is obtained at a distance of 80 cm from left. A resistance of 100 Ω is now connected in parallel across X. The null point is then found by shifting the sliding contact toward left by 20 cm. Calculate X and Y.
5. In an experiment with a potentiometer, the null point is obtained at a distance of 60 cm along the wire from the common terminal with a Leclanche cell. When a shunt resistance of 1 Ω is connected across the cell, the null point shifts to a distance of 30 cm from the common terminal. What is the internal resistance of the cell?
6. In the experiment of calibration of voltmeter, a standard cell of emf 1.1 V is balanced against 440 cm of potentiometer wire. The potential difference across the ends of a resistance is found to balance against 220 cm of the wire. The corresponding reading of the voltmeter is 0.5 V. Find the error in the reading of voltmeter.
7. The resistance wire AB in the balancing setup shown in figure is 10 cm long. When AC = 40 cm, no deflection occurs in the galvanometer. Find the unknown resistance R.

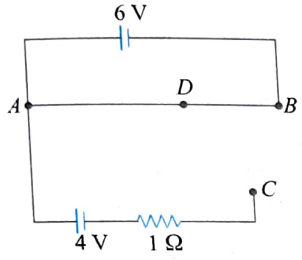


8. Figure shows a potentiometer arrangement with $R_{AB} = 10\Omega$ and rheostat of variable resistance x. For $x = 0$ null deflection point is found at 20 cm from A For unknown

value of x null deflection point was at 30 cm from A, then find the value of x.



9. A 6 V battery of negligible internal resistance is connected across a uniform wire AB of length 100 cm. The positive terminal of another battery of emf 4 V and internal resistance 1 Ω is joined to the point A as shown in figure. Take the potential at B to be zero.



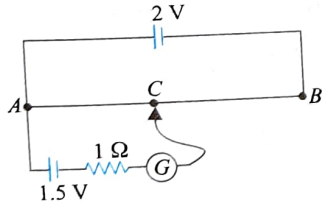
(a) What are the potentials at the points A and C?

(b) At which point D of the wire AB, the potential is equal to the potential at C?

(c) If the points C and D are connected by a wire, what will be the current through it?

(d) If the 4 V battery is replaced by 7.5 V battery, what would be the answers of parts (a) and (b)?

10. A battery of emf 2 V and negligible internal resistance is connected across a uniform wire of length 10 m and resistance 30 Ω. The appropriate terminals of a cell of emf 1.5 V and internal resistance 1 Ω is connected to one end of the wire and the other terminal of the cell is connected through a sensitive galvanometer to a slider on the wire. What is the length of the wire that will be required to produce zero deflection of the galvanometer? How will the balancing length change?



(a) When a coil of resistance 5 Ω is placed in series with the accumulator.

(b) The cell of 1.5 V is shunted with 5 Ω resistor?

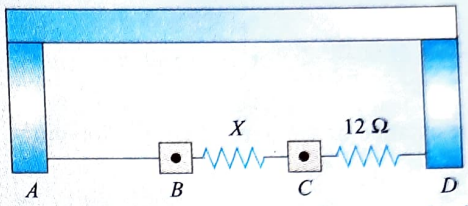
ANSWERS

1. Accuracy increases **2.** 2 Ω
3. The potentiometer Y will be preferred
4. $X = \frac{500}{3}\Omega$ $Y = \frac{125}{3}\Omega$ **5.** 1 Ω **6.** –0.05 V
7. 15 Ω **8.** 5 Ω
9. (a) 6 V, 2V (b) AD = 66.7 cm (c) Zero
   (d) 6 V, –1.5 V, No such point will exist
10. 7.5 m (a) 8.75 m (b) 6.25 m

Solved Examples

EXAMPLE 6.1

A thin uniform wire AB of length 1 m, an unknown resistance X, and a resistance of 12 Ω are connected by thick conducting strips, as shown in figure. A battery and a galvanometer (with a sliding jockey connected to it) are also available. Connections are to be made to measure the unknown resistance X using the principle of Wheatstone bridge. Answer the following questions.



(i) Are there positive and negative terminals on the galvanometer?

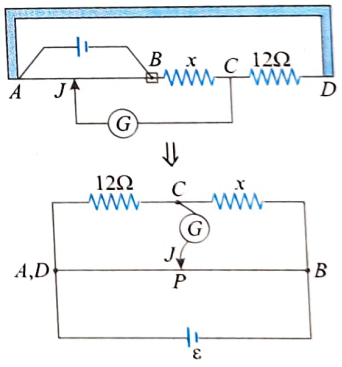
(ii) Copy the figure in your answer book and show the battery and the galvanometer (with jockey) connected at appropriate points.

(iii) After appropriate connections are made, it is found that no deflection takes place in the galvanometer when the sliding jockey touches the wire at a distance of 60 cm from A. Obtain the value of the resistance X.

Sol.

(i) No. There are no positive and negative terminals on the galvanometer.

(ii) We can connect battery and galvanometer as shown in the figure.

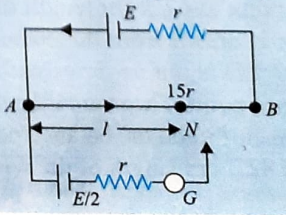


(iii) As bridge is balanced $\frac{R_{AP}}{R_{PB}} = \frac{60\text{ cm}}{40\text{ cm}} = \frac{12}{x}$

$$\Rightarrow \quad x = \frac{12 \times 4}{6} = 8.0\ \Omega$$

EXAMPLE 6.2

Consider the potentiometer circuit arranged as in figure. The potentiometer wire is 600 cm long, and its resistance is 15r.



(a) At what distance from point A should the jockey touch the wire to get zero deflection in the galvanometer?

(b) If the jockey touches the wire at a distance of 560 cm from A, what will be the current in the galvanometer?

Sol.

(a) Current in AB, $I = \dfrac{E}{r+15r} = \dfrac{E}{16r}$

Potential gradient, $k = \dfrac{15rI}{600} = \dfrac{15r}{600} \times \dfrac{E}{16r} = \dfrac{E}{640}$

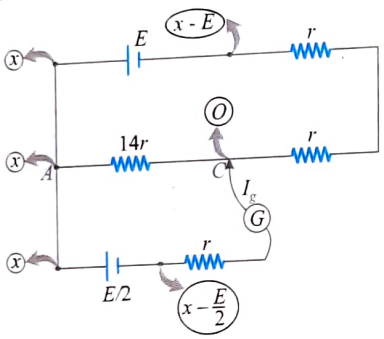
Let $E/2$ be balanced at l, then

$$\frac{E}{2} = kl = \frac{E}{640} l \quad \text{or} \quad l = 320 \text{ cm}$$

(b) The resistance of the wire of length 560 cm

$$R_{AC} = 15r \times \frac{560}{600} = 14r$$

Let us draw the circuit diagram corresponding to this situation.



Let us assume the potential at point C to be zero and x at A. Now writing potential at different nodes and applying Kirchhoff's junction law at C.

$$\frac{x-0}{14r} + \frac{(x-E)-0}{2r} + \frac{\left(x-\dfrac{E}{2}\right)-0}{r} = 0$$

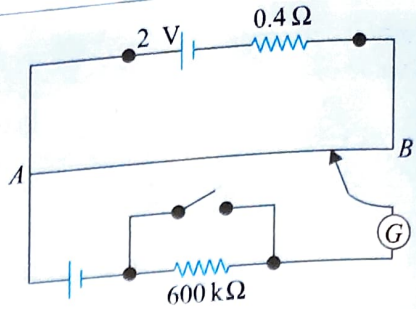
which gives $x = \dfrac{7E}{11}$

Hence current through galvanometer branch

$$I_g = \frac{x - \dfrac{E}{2}}{r} = \frac{\dfrac{7E}{11} - \dfrac{E}{2}}{r} = \frac{3}{22}\frac{E}{r}$$

EXAMPLE 6.3

Figure shows a potentiometer with a cell of emf 2.0 V and internal resistance 0.4 Ω maintaining a potential drop across the resistor wire AB. A standard cell that maintains a constant emf of 1.02 V (for very moderate currents up to a few μA) gives a balance point at 67.3 cm length of the wire. To ensure very low current is drawn from the standard cell, a very high resistance of 600 kΩ is put in series with it, which is shorted close to the balance point. The standard cell is then replaced by a cell of unknown emf ε and the balance point found, similarly, turns out to be at 82.3 cm length of the wire.



(a) What is the value of ε?

(b) What purpose does the high resistance of 600 kΩ have?

(c) Is the balance point affected by this high resistance?

(d) Is the balance point affected by internal resistance of the driver cell?

(e) Would the method work in the above situation if the driver cell of the potentiometer had an emf of 1.0 V instead of 2.0 V?

(f) Would the circuit work well for determining an extremely small emf, say of the order of a few mV (such as the typical emf of a thermocouple)? If not, how will you modify the circuit?

Sol.

(a) At balance point, $\dfrac{\varepsilon}{\varepsilon_{\text{standard}}} = \dfrac{l}{l_{\text{standard}}}$

$$\text{or} \quad \varepsilon = \frac{l \times \varepsilon_{\text{standard}}}{l_{\text{standard}}} = \frac{82.3 \times 1.02}{67.3} = 1.2474 \text{ V}$$

(b) To reduce the current through the galvanometer when the movable contact is far from the balance point.

(c) No.

(d) Yes.

(e) No. If ε is greater than the emf of the driver cell of the potentiometer, there will be no balance point on the wire AB.

(f) The circuit, as it is, would be unsuitable, because the balance point (for e of the order of a few mV) will be very close to the end A and the percentage error in measurement will be very large. The circuit is modified by putting a suitable resistor R in series with the wire AB so that potential drop across AB is only slightly greater than the emf to be measured. Then the balance point will be at larger length of the wire and the percentage error will be much smaller.

EXAMPLE 6.4

A potentiometer wire has a length of 10 m and resistance 4 Ωm^{-1}. An accumulator of emf 2 V and a resistance box are connected in series with it. Calculate the resistance to be introduced in the box so as to get a potential gradient of (a) 0.1 V/m and (b) 0.1 mVm^{-1}.

Sol. Let us assume R be the resistance to be introduced in the box. The current in the potentiometer wire is given by

$$I = \frac{E}{R + l\rho}$$

where ρ is the resistance per unit length of the wire and l is the length of the wire. Now, potential gradient in potentiometer wire,

$$k = I\rho = \frac{E\rho}{R + I\rho}$$

Here, $l = 10$ m, $\rho = 4\ \Omega m^{-1}$

(i) For $k = 0.1\ Vm^{-1}$, we have

$$0.1 = \frac{2 \times 4}{R + 10 \times 4} = \frac{8}{R + 40} \quad \text{or} \quad R = \frac{4}{0.1} = 40\ \Omega$$

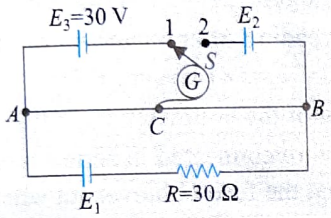
(ii) For $k = 0.1\ mVm^{-1} = 0.1 \times 10^{-3}\ Vm^{-1}$, we have

$$0.1 \times 10^{-3} = \frac{2 \times 4}{R + 10 \times 4}$$

or $$10^{-4} = \frac{8}{R + 40} \quad \text{or} \quad R = 79960\ \Omega$$

EXAMPLE 6.5

In the circuit shown in Figure AB is a uniform 'wire of length $L = 5$ m. It has a resistance of 2 Ω/m. When $AC = 2.0$ m, it was found that the galvanometer shows zero reading when switch S is placed in either of the two positions 1 or 2. Find the emf E_1.



Sol. When the switch is at position '1'. The potential difference across AC,

$$V_{AC} = E_3 = 30\,V \qquad ...(i)$$

When the switch is at position '2'. The potential difference across CB,

$$V_{CB} = E_2 \qquad ...(ii)$$

From (i) and (ii), $\frac{E_2}{30} = \frac{V_{CB}}{V_{AC}}$

It means, $\frac{E_2}{30} = \frac{3}{2}$ $\left[\because \frac{V_{CB}}{V_{AC}} = \frac{l_{CB}}{l_{AC}} = \frac{3}{2}\right]$

or $E_2 = 45$ V

The potential difference across the wire AB, $V_{AB} = 30 + 45 = 75$ V

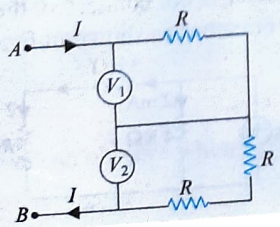
And resistance of the wire AB, $R_{AB} = 2 \times 5 = 10\ \Omega$

∴ Current through AB is $i_{AB} = \frac{V_{AB}}{R_{AB}} = \frac{75}{10} = 7.5$ A

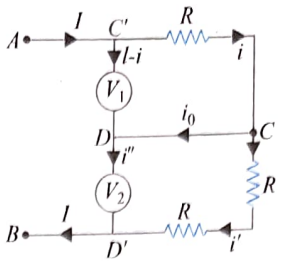
Hence emf of cell E_1, $E_1 = 7.5 \times (R_{AB} + R) = 300$ V

EXAMPLE 6.6

In the circuit shown in figure a current $I = 30$ mA enters through A and leaves through B. Reading of the identical voltmeters V_1 and V_2 are 20 V and 30 V respectively. Find the magnitude of resistance R.



Sol. Let us assume the resistance of each voltmeter be R_V. Now distributing the currents in each part of the circuits,



The potential difference across the voltmeter V_1 is 20 V.

$$Ri = 20 \qquad ...(i)$$

And the potential difference across the voltmeter V_2 is 30 V.

$$2Ri' = 30 \qquad ...(ii)$$

Dividing Eq. (ii) with (i), we get $\frac{2i'}{i} = \frac{3}{2} \Rightarrow i' = \frac{3}{4}i$

The current through CD, $i_0 = i - i' = i - \frac{3i}{4} = \frac{i}{4}$

∴ Current through the voltmeter V_2, $i'' = (I - i) + \frac{i}{4} = I - \frac{3i}{4}$

As the potential difference across DD' and CD' should be equal.

Hence, $$R_V\left(I - \frac{3i}{4}\right) = 2Ri' = 30 \qquad ...(iii)$$

From (i) and (iii), $\frac{I - i}{I - \frac{3i}{4}} = \frac{2}{3}$

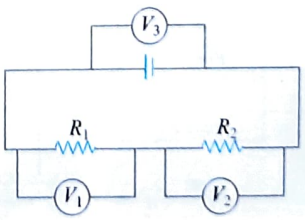
$$\Rightarrow \quad I = \frac{3i}{2} \Rightarrow i = \frac{2}{3} \times 150 = 100\ mA$$

$$Ri = 20 \Rightarrow R \times 100 \times 10^{-3} = 20$$

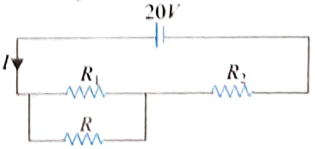
Hence, $R = 200\ \Omega$

EXAMPLE 6.7

In the circuit shown in the figure, two resistors R_1 and R_2 have been connected in series with an ideal cell. If a voltmeter is connected across R_1 its reading is $V_1 = 4.0$ volt and if the same voltmeter is connected across R_2 its reading is $V_2 = 5.0$ volt. The reading of the voltmeter when it is connected across the cell is $V_3 = 20$ volt. Find the actual potential difference across R_1 in the circuit.



Sol. As the cell connected in the circuit is ideal hence the reading of voltmeter is $V_3 = 20$ V



It means the emf of the cell should be 20 volts. Let the resistance of the voltmeter be R. The voltmeter is connected across R_1.

The current I through circuit, $I = \dfrac{20}{\dfrac{R_1 R}{R_1 + R} + R_2}$

$\therefore$ The reading of voltmeter (V_1) = The potential difference across R_1 (or R)

$$V_1 = 4 = \left(\frac{20}{\dfrac{R_1 R}{R_1 + R} + R_2} \right) \cdot \frac{R_1 R}{R_1 + R}$$

$$\Rightarrow \quad \frac{R_1 R_2 + R R_2}{R R_1} = 4 \qquad \ldots(i)$$

Now voltmeter is connected across R_2

The reading of voltmeter (V_2) = The potential difference across R_2 (or R)

$$V_2 = 5 = \left(\frac{20}{\dfrac{R_2 R}{R_2 + R} + R_1} \right) \cdot \frac{R_2 R}{R_2 + R}$$

$$\Rightarrow \quad \frac{R_1 R_2 + R_1 R}{R_2 R} = 3 \qquad \ldots(ii)$$

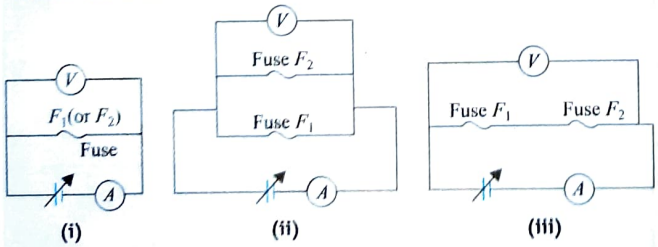
Eliminating R between (i) and (ii), we get $\dfrac{R_1}{R_2} = \dfrac{4}{5}$

The actual voltage across R_1,

$$V_1 = \frac{20 \cdot R_1}{R_1 + R_2} = \frac{20}{1 + \dfrac{R_2}{R_1}} = \frac{20}{1 + \dfrac{5}{4}} = \frac{80}{9} \text{ volt}$$

EXAMPLE 6.8

A variable voltage source is connected with a voltmeter, an ammeter and two fuses F_1 and F_2 in three different arrangements. In first arrangement when the fuse F_1 is connected across a source of variable voltage and the voltage is increased gradually, the fuse blows out just when the reading of the voltmeter and ammeter reaches 4 V and 2.0 A respectively [see Fig. (i)]. The same experiment is repeated with another fuse F_2 and the reading of the voltmeter and ammeter when it blows out is 12 V and 3 A respectively.



(a) If two fuses are connected in parallel as shown in Fig. (ii) and voltage is increased gradually. Find the reading of the ammeter when any one of the fuses blows out.

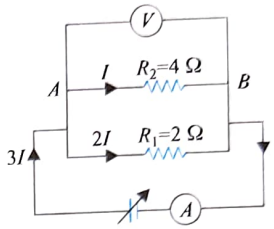
(b) If two fuses are connected in series as shown in Fig. (ii) and voltage is increased gradually. Find the reading of the voltmeter at the point one of the fuses blows out.

Sol. It is given that the fuses F_1 and F_2 blow out just when the reading of the ammeters is 2.0 A and 3.0 A respectively. It means the maximum current that the two fuses can tolerate are $i_{1,\max} = 2.0$ A and $i_{2,\max} = 3.0$ A respectively.

Resistance of fuse F_1, $R_1 = \dfrac{4.0\text{ V}}{2.0\text{ A}} = 2\,\Omega$

And the resistance of fuse F_2, $R_2 = \dfrac{12\text{ V}}{3\text{ A}} = 4\,\Omega$

(a) Now two fuses are connected in parallel as shown in Fig. (ii). From Fig. (i), it is clear that the fuse F_1 blows out when the current through the fuses F_1 reaches 2.0 A.

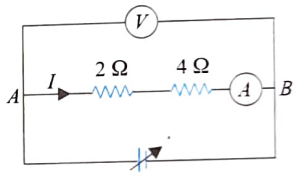


Hence from figure $2I = 2.0\text{ A} \Rightarrow I = 1.0\text{ A}$

Hence the reading of ammeter = The current supplied by the battery $= 3I = 3 \times 1.0\text{ A} = 3.0\text{ A}$

And reading of the voltmeter $V_{AB} = 1.0 \times 4 = 4.0\text{ V}$

(b) If two fuses are connected in series as shown in Fig. (iii). it is clear that the fuse F_1 blows out when $I = 2.0$A



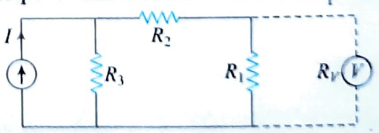
Hence the reading of the voltmeter = The potential difference across middle branch (V_{AB})

$\therefore \quad V_{AB} = (4+2) \times 2 = 12\text{ V}$

And the reading of ammeter = 2.0 A

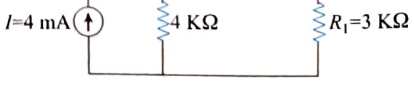
EXAMPLE 6.9

Three resistors $R_1 = R_2 = 3$ kΩ and $R_3 = 6$ kΩ have been connected to a constant current source as shown in figure. The current source supplies current $I = 4$ mA to the circuit. A voltmeter having resistance $R_V = 6$ kΩ is connected, as shown, to measure the potential difference across R_1.



Find the percentage error in the measurement of potential difference (V_1) across R_1 caused due to finite resistance of the voltmeter.

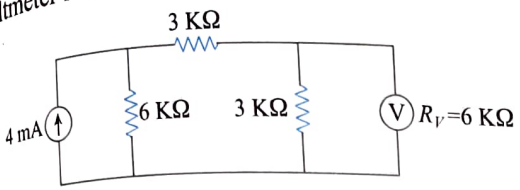
Sol. When voltmeter is not connected, the current will divide equally in both the branches as shown in figure.

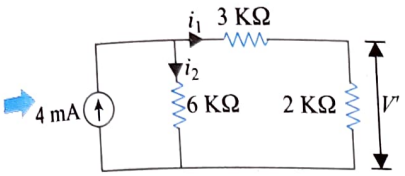


The potential difference (V_1) across R_1

$V_1 = (3\,k\Omega)(2\,mA) = 6$ volt

When voltmeter is connected across R_1,





From figure we can observe the potential difference across 6 KΩ and 5 kΩ should be equal:

$5i_1 = 6i_2$ ...(i)

Also, $i_1 + i_2 = 4\,mA$ ...(ii)

From (i) and (ii), we get $i_1 = \frac{24}{11}$ mA

Hence potential difference across R_1 $V' = 2 \times \frac{24}{11} = \frac{48}{11}$ volt

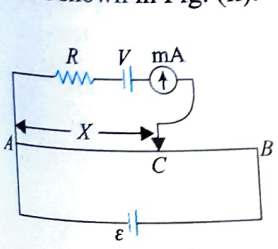
The percentage error in the measurement of potential difference

$$= \left|\frac{V - V'}{V}\right| \times 100\ \%$$

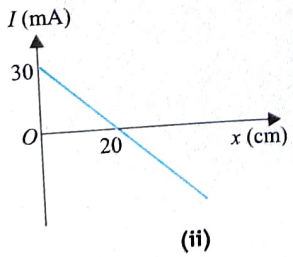
$$\%\ \text{error} = \frac{6 - \frac{48}{11}}{6} \times 100 = \frac{14}{33} \times 100 = 25.9\%$$

EXAMPLE 6.10

AB is a uniform wire of length L = 100 cm is connected across a cell of emf ε = 15 volt. A unknown resistance R, cell of emf V and a milliammeter (which can show deflection in both directions) is connected to the circuit as shown in Fig. (i). Contact C can be slide on the wire AB. Distance $AC = x$. The current (I) through the milliammeter is taken positive when the cell of emf V is discharging and it is negative when the cell gets charged. The variation of the current I vs x has been shown in Fig. (ii).



(i)



(ii)

Neglect internal resistance of the cells. Find

(a) the value of e.m.f of unknown cell 'V'

(b) unknown resistance 'R'

(c) the current recorded be milliammeter 'I' when x = 100 cm

Sol.

(a) From Fig. (ii), we can observe when $I = 0$

x = 20 cm

It means the potential difference across AC,

$$V_{AC} = \frac{\varepsilon x}{L} = \frac{15 \times 20}{100} = 3.0 \text{ volt}$$

Since there is no current through the milliammeter. Hence

$V_{AC} = V = 3.0$ volt

(b) When $x = 0$; $I = 30$ mA

$\therefore$ $3.0 = IR$

$3.0 = 30 \times 10^{-3} R$

$R = 100\ \Omega$

(c) When x = 100 cm

$V_{AB} = \varepsilon = 15$ volt

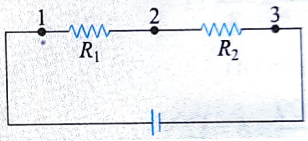
$\therefore$ $100I = 15 - 3$

$I = 0.12\ A = 120$ mA

This current is in negative direction.

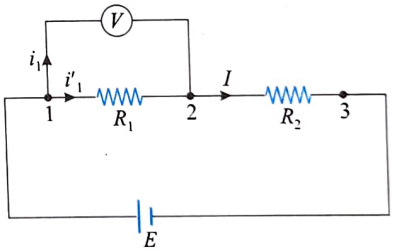
EXAMPLE 6.11

In the circuit shown in the figure, the battery is ideal and resistance R_2 = 100 Ω. A voltmeter having internal resistance 200 Ω, if connected across 1-2, it reads V_{12} = 4 V and when connected across 2-3, it reads V_{23} = 6 V. What will be the reading of the voltmeter when it is connected between the points 1-3? Also find the value of R_1.



Sol. Let the emf of the battery be E.

Current through the voltmeter when it is connected acrosss 1-2,



$$i_1 = \frac{V_{12}}{R_V} = \frac{4}{200} = \frac{1}{50} \text{A}$$

and current through R_1, $i_1 = \frac{4}{R_1}$

$\therefore$ Current through R_2, $i = \frac{4}{R_1} + \frac{1}{50}$

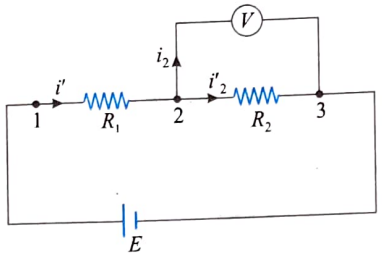
$\therefore$ Potential difference across R_2

$$V_2 = \left(\frac{4}{R_1} + \frac{1}{50}\right) \times 100 = \frac{400}{R_1} + 2$$

The emf of the battery

$$E = 4 + \left(\frac{400}{R_1} + 2\right) = 6 + \frac{400}{R_1} \quad \text{...(i)}$$

When the voltmeter is connected between 2-3



Current through voltmeter, $i_2 = \frac{6}{200}$ A

Current through R_2, $i_2 = \frac{6}{100}$ A

$\therefore$ Current through R_1

$$i = \frac{6}{200} + \frac{6}{100} = \frac{18}{200} = \frac{9}{100} \text{ A}$$

Hence emf of the battery

$$E = \frac{9R_1}{100} + 6 \qquad \text{...(ii)}$$

From (i) and (ii) $\frac{9R_1}{100} + 6 = 6 + \frac{400}{R_1}$

$$\Rightarrow \quad R_1 = \sqrt{\frac{40000}{9}} = \frac{200}{3}\ \Omega$$

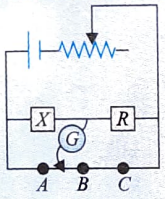
Put the above value in (ii)

$$E = \frac{9}{100} \times \frac{200}{3} + 6 = 12 \text{ V}$$

When connected across 1-3, the voltmeter will read $E = 12$ V.

EXAMPLE 6.12

An unknown resistance 'X' is to be determined using resistances R_1, R_2, or R_3. Their corresponding null points are A, B, and C. Find which of the above will give the most accurate reading and why? $R = R_1$ or R_2 or R_3

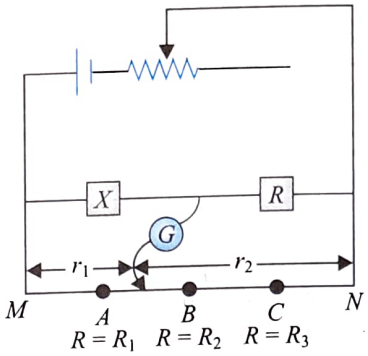


Sol. At all null points, the Wheatstone bridge will be balanced. Therefore,

$$\frac{X}{r_1} = \frac{R}{r_2} \text{ or } X = R\frac{r_1}{r_2}$$

where R is a constant and r_1 and r_2 are variables. The maximum fraction error is

$$\frac{\Delta X}{X} = \frac{\Delta r_1}{r_1} + \frac{\Delta r_2}{r_2}$$



Here $\Delta r_1 = \Delta r_2 = y$ (say), y can be considered as least count of the scale, then

$$\frac{\Delta X}{X} = y\left[\frac{r_2 + r_1}{r_1 r_2}\right]$$

For $\Delta X/X$ to be minimum, $r_1 \times r_2$ should be maximum [since $r_1 + r_2 = c$ (constant)].

Let $\quad E = r_1 \times r_2 = r_1 \times (c - r_1) = r_1 c - r_1^2$

$$\therefore \quad \frac{dE}{dr_1} = c - 2r_1 = 0 \text{ or } r_1 = \frac{c}{2}$$

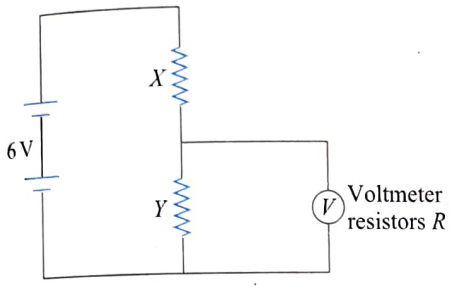
$$\text{or} \quad r_2 = \frac{c}{2} \quad \text{or} \quad r_1 = r_2$$

Thus, R_2 gives the most accurate value.

Exercises

Single Correct Answer Type

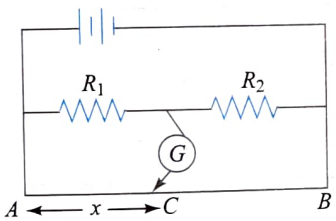
1. In the circuit shown in figure, resistors X and Y, each with resistance R, are connected to a 6 V battery of negligible internal resistance. A voltmeter, also of resistance R, is connected across Y.



What is the reading of the voltmeter?

(1) zero (2) between zero and 3 V
(3) 3 V (4) between 3 V and 6 V

2. In the shown arrangement of a meter bridge, if AC corresponding to null deflection of galvanometer is x, what would be its value if the radius of the wire AB is doubled?



(1) x (2) $x/4$
(3) $4x$ (4) $2x$

3. The length of a wire of a potentiometer is 100 cm, and the emf of its standard cell is E volt. It is employed to measure the emf of a battery whose internal resistance is 0.5 Ω. If the balance point is obtained at $l = 30$ cm from the positive end, the emf of the battery is

(1) $\frac{30E}{100}$ (2) $\frac{30E}{100.5}$

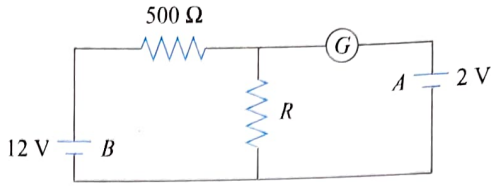
(3) $\frac{30E}{(100-0.5)}$ (4) $\frac{30\,(E-0.5i)}{100}$

where i is the current in the potentiometer wire.

4. In a meter bridge experiment, the null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y?

(1) 50 cm (2) 80 cm
(3) 40 cm (4) 70 cm

5. In the circuit shown in figure, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be

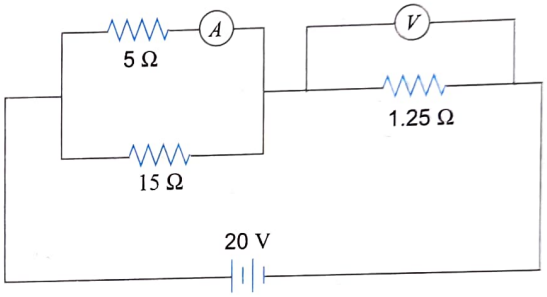


(1) 1000 Ω (2) 500 Ω
(3) 100 Ω (4) 200 Ω

6. The resistance of a galvanometer is 10 Ω. It gives full-scale deflection when 1 mA current is passed. The resistance connected in series for converting it into a voltmeter of 2.5 V will be

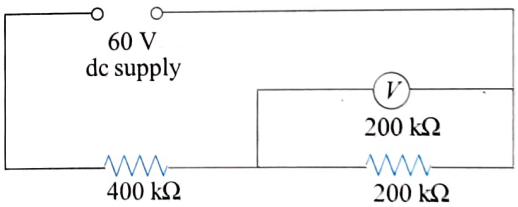
(1) 24.9 Ω (2) 249 Ω
(3) 2490 Ω (4) 24900 Ω

7. An ideal ammeter (zero resistance) and an ideal voltmeter (infinite resistance) are connected as shown in figure. The ammeter and the voltmeter readings are



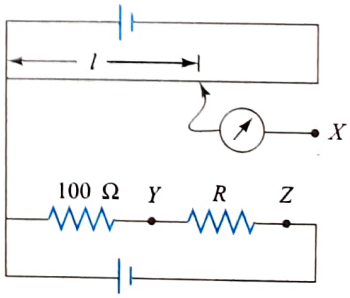
(1) 6.25 A, 3.75 V (2) 3.00 A, 5 V
(3) 3.00 A, 3.75 V (4) 6.00 A, 6.25 V

8. A constant 60 V dc supply is connected across two resistors of resistance 400 kΩ and 200 kΩ. What is the reading of the voltmeter, also of resistance 200 kΩ, when connected across the second resistor as shown in figure?



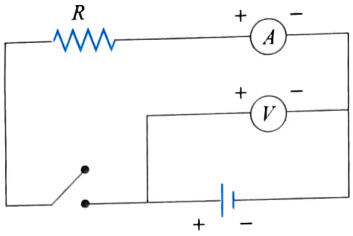
(1) 12 V (2) 15 V
(3) 20 V (4) 30 V

9. Figure shows a circuit that may be used to compare the resistance R of an unknown resistor with a 100 Ω standard. The distances l from one end of the potentiometer slider wire to the balance point are 400 mm and 588 mm when X is connected to Y and Z, respectively. The length of the slide wire is 1.00 m. What is the value of resistance R?



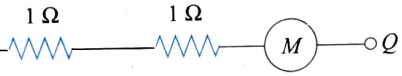
(1) 32 Ω (2) 47 Ω
(3) 68 Ω (4) 147 Ω

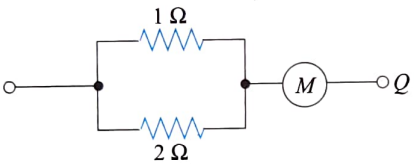
10. In the circuit shown in figure, an ideal ammeter and an ideal voltmeter are used. When the key is open, the voltmeter reads 1.53 V. When the key is closed, the ammeter reads 1.0 A and the voltmeter reads 1.03 V. The resistance R is

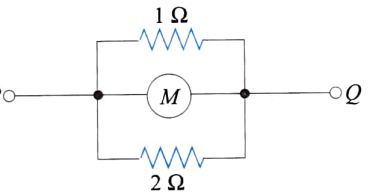


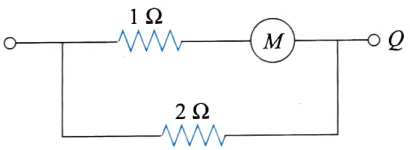
(1) 0.5 Ω (2) 1.03 Ω
(3) 1.53 Ω (4) 0.53 Ω

11. In which of the following arrangements of resistors does the meter M, which has a resistance of 2 Ω, give the largest reading when the same potential difference is applied between points P and Q?

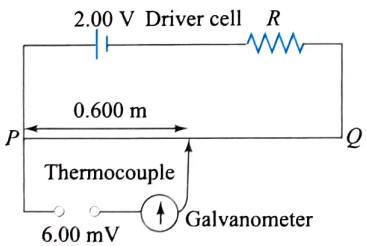
(1)


(2)


(3)


(4)


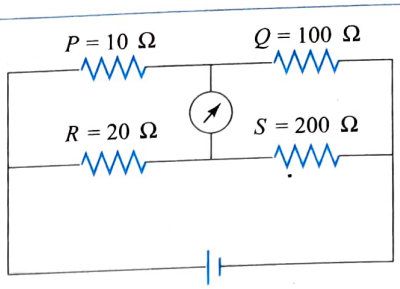
12. Figure shows a simple potentiometer circuit for measuring a small emf produced by a thermocouple.



The meter wire PQ has a resistance of 5 Ω, and the driver cell has an emf of 2.00 V. If a balance point is obtained 0.600 m along PQ when measuring an emf of 6.00 mV, what is the value of resistance R?

(1) 95 Ω (2) 995 Ω
(3) 195 Ω (4) 1995 Ω

13. Figure shows a balanced Wheatstone net. Now, it is disturbed by changing P to 11 Ω. Which of the following steps will not bring the bridge to balance again?

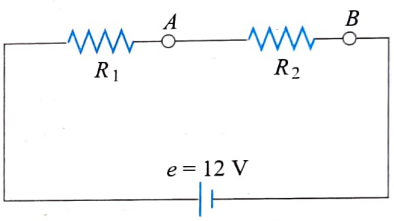


(1) increasing R by 2 Ω
(2) increasing S by 20 Ω
(3) increasing Q by 10 Ω
(4) making product $RQ = 2200\ (\Omega)^2$

14. In an experiment to measure the internal resistance of a cell by a potentiometer, it is found that the balance point is at a length of 2 m when the cell is shunted by a 5 Ω resistance and is at a length of 3 m when the cell is shunted by a 10 Ω resistance, the internal resistance of the cell is then

(1) 1.5 Ω (2) 10 Ω
(3) 15 Ω (4) 1 Ω

15. When an ammeter of negligible internal resistance is inserted in series with circuit, it reads 1 A. When a voltmeter of very large resistance is connected across R_1, it reads 3 V. But when the points A and B are short-circuited by a conducting wire, then the voltmeter measures 10.5 V across the battery. The internal resistance of the battery is equal to

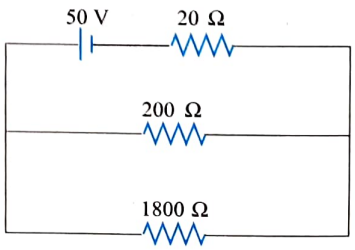


(1) $\frac{3}{7}\ \Omega$ (2) 5 Ω
(3) 3 Ω (4) none of these

16. An 80 Ω galvanometer deflects full-scale for a potential of 20 mV. A voltmeter deflecting full-scale of 5 V is to be made using this galvanometer. We must connect

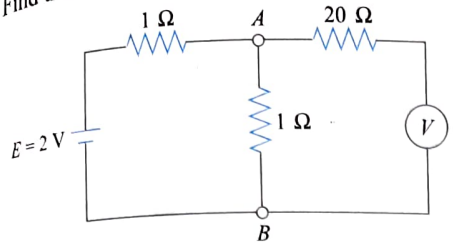
(1) a resistance of 19.92 kΩ parallel to the galvanometer
(2) a resistance of 19.92 kΩ in series with the galvanometer
(3) a resistance of 20 kΩ parallel to the galvanometer
(4) a resistance of 20 kΩ in series with the galvanometer

17. A voltmeter having a resistance of 1800 Ω is employed to measure the potential difference across 200 Ω resistance, which is connected to dc power supply of 50 V and internal resistance 20 Ω. What is the approximate percentage change in the potential difference across 200 Ω resistance as a result of connecting the voltmeter across it?



(1) 2.2%
(2) 5%
(3) 10%
(4) 20%

18. In the given circuit, the voltmeter and the electric cell are ideal. Find the reading of the voltmeter

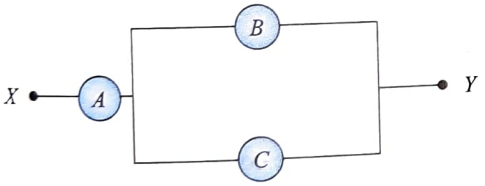


(1) 1 V
(2) 2 V
(3) 3 V
(4) none of these

19. The emf of the driver cell of a potentiometer is 2 V, and its internal resistance is negligible. The length of the potentiometer wire is 100 cm, and resistance is 5 Ω. How much resistance is to be connected in series with the potentiometer wire to have a potential gradient of 0.05 $mVcm^{-1}$?
(1) 1990 Ω
(2) 2000 Ω
(3) 1995 Ω
(4) none of these

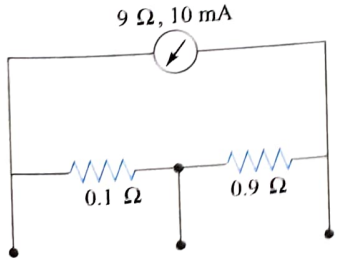
20. In the above question, if the balancing length for a cell of emf E is 60 cm, the value of E will be
(1) 3 mV
(2) 5 mV
(3) 6 mV
(4) 2000 mV

21. A, B, and C are voltmeters of resistance R, $1.5R$, and $3R$, respectively. When some potential difference is applied between X and Y, the voltmeter readings are V_A, V_B, and V_C, respectively. Then



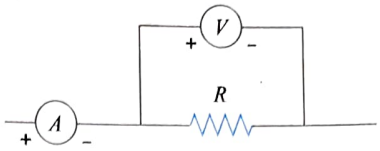
(1) $V_A = V_B = V_C$
(2) $V_A \neq V_B = V_C$
(3) $V_A = V_B \neq V_C$
(4) $V_B \neq V_A = V_C$

22. A milliammeter of range 10 mA and resistance 9 Ω is joined in a circuit as shown in figure. The meter gives full-scale deflection for current I when A and B are used as its terminals. If current enters at A and leaves at B (C is left isolated), the value of I is



(1) 100 mA
(2) 900 mA
(3) 1 A
(4) 1.1 A

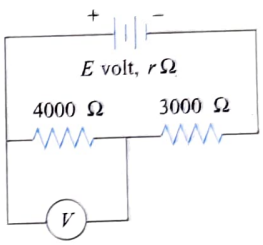
23. A candidate connects a moving coil voltmeter V, a moving coil ammeter A, and a resistor R as shown in figure. If the voltmeter reads 20 V and the ammeter reads 4 A, R is



(1) equal to 5 Ω
(2) greater than 5 Ω
(3) less than 5 Ω
(4) greater or less than 5 Ω depending upon its material

24. If a shunt 1/10 of the coil resistance is applied to a moving coil galvanometer, its sensitivity becomes
(1) 10 fold
(2) 11 fold
(3) $\frac{1}{10}$ fold
(4) $\frac{1}{11}$ fold

25. In figure, when an ideal voltmeter is connected across 4000 Ω resistance, it reads 30 V. If the voltmeter is connected across 3000 Ω resistance, it will read



(1) 20 V
(2) 22.5 V
(3) 35 V
(4) 40 V

26. A voltmeter has a resistance of G ohm and range V volt. The value of resistance used in series to convert it into voltmeter of range nV volt is
(1) nG
(2) $(n-1)G$
(3) G/n
(4) $G/(n-1)$

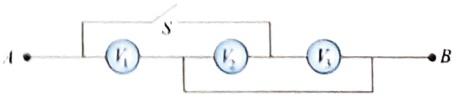
27. A galvanometer has a resistance of 3663 Ω. A shunt S is connected across it such that 1/34 of the total current passes through the galvanometer. The value of the shunt is
(1) 3663 Ω
(2) 111 Ω
(3) 107.7 Ω
(4) 3555.3 Ω

28. In Q. 27, the combined resistance of the shunt and the galvanometer is
(1) 3665 Ω
(2) 111 Ω
(3) 107.7 Ω
(4) 3555.3 Ω

29. In Q. 27, the external resistance that must be connected in series with the main circuit so that the total current in the main circuit remains unaltered even when the galvanometer is shunted is
(1) 3663 Ω
(2) 111 Ω
(3) 107.7 Ω
(4) 3555.3 Ω

30. An ammeter is obtained by shunting a 30 Ω galvanometer with a 30 Ω resistance. What additional shunt should be connected across it to double the range?
(1) 15 Ω
(2) 10 Ω
(3) 5 Ω
(4) none of these

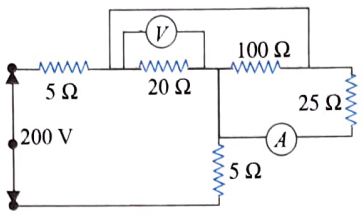
31. Three voltmeters are connected as shown.



A potential difference has been applied between A and B. On closing the switch S, readings of voltmeters?

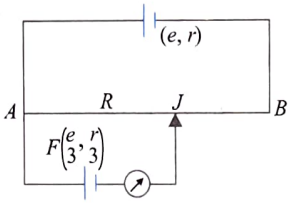
(1) V_1 increases
(2) V_1 decreases
(3) V_2 and V_3 both increase
(4) One of V_2 and V_3 increases and other decreases.

32. In figure the voltmeter and ammeter shown are ideal. Then voltmeter and ammeter readings, respectively, are



(1) 125 V, 3 A (2) 100 V, 4 A
(3) 120 V, 4 A (4) 120 V, 3 A

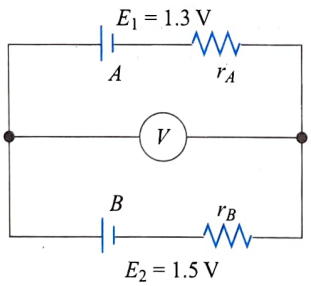
33. A potentiometer arrangement is shown in figure. The driver cell has emf e and internal resistance r.



The resistance of potentiometer wire AB is R. F is the cell of emf $e/3$ and internal resistance $r/3$. Balance point (J) can be obtained for all finite values of
(1) $R > r/2$ (2) $R < r/2$
(3) $R > r/3$ (4) $R < r/3$

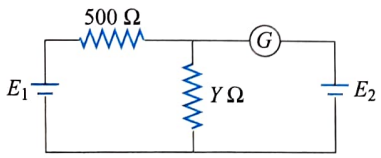
34. When a galvanometer is shunted with a 4 Ω resistance, the deflection is reduced to 1/5. If the galvanometer is further shunted with a 2 Ω wire, the new deflection will be (assuming the main current remains the same)
(1) 5/13 of the deflection when shunted with 4 Ω only
(2) 8/13 of the deflection when shunted with 4 Ω only
(3) 3/4 of the deflection when shunted with 4 Ω only
(4) 3/13 of the deflection when shunted with 4 Ω only

35. Two cells A and B of electromotive forces 1.3 V and 1.5 V, respectively, are arranged as shown in figure. The voltmeter (assumed ideal) reads 1.45 V, the internal resistances of cells A and B are r_A and r_B, respectively. Which of the following is correct ?



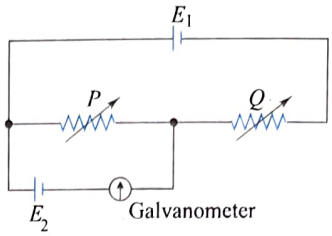
(1) $r_A = 2\,r_B$ (2) $r_A = 3\,r_B$
(3) $r_B = 2\,r_A$ (4) $r_B = 3\,r_A$

36. In the circuit shown in figure, the battery E_1 has an emf of 12 V and zero internal resistance, while the battery E_2 has an emf of 2 V. If the galvanometer G reads zero, then the value of the resistance Y is



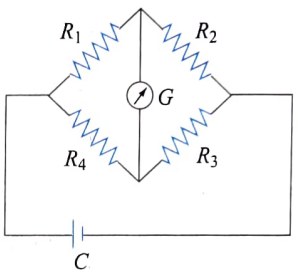
(1) 10 Ω (2) 100 Ω
(3) 500 Ω (4) 200 Ω

37. Two cells of emfs E_1 and E_2 and of negligible internal resistances are connected with two variable resistors as shown in figure. When the galvanometer shows no deflection, the values of the resistances are P and Q. What is the value of the ratio E_2/E_1?



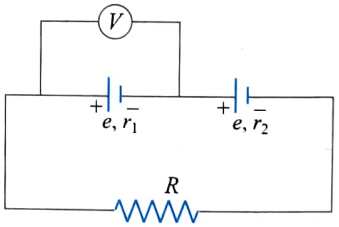
(1) $\frac{P}{Q}$ (2) $\frac{P}{P+Q}$
(3) $\frac{Q}{P+Q}$ (4) $\frac{P+Q}{P}$

38. The Wheatstone's bridge shown in the figure is balanced. If the positions of the cell C and the galvanometer G are now interchanged, G will show zero deflection



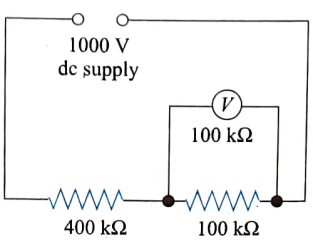
(1) in all cases
(2) only if all the resistances are equal
(3) only if $R_1 = R_3$ and $R_2 = R_4$
(4) only if $R_1/R_3 = R_2/R_4$

39. In figure, two cells have equal emf ε but internal resistances are r_1 and r_2. If the reading of the voltmeter is zero, then relation between R, r_1 and r_2 is



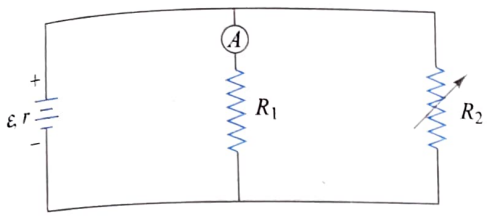
(1) $R = r_1 - r_2$ (2) $R = r_1 + r_2$
(3) $2r_1 - r_2$ (4) $r_1 r_2$

40. A constant voltage dc source is connected, as shown in figure, across two resistors of resistances 400 kΩ and 100 kΩ. What is the reading of the voltmeter, also of resistance 100 kΩ, when connected across the second resistor as shown?



(1) 111 V
(2) 250 V
(3) 125 V
(4) 333 V

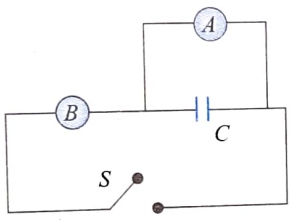
41. In the given circuit (figure) in which case will the ammeter reading not change when R_2 is varied?



(1) $R_1 = r$
(2) $R_1 = 2r$
(3) $R_1 > R_2$
(4) $r = 0$

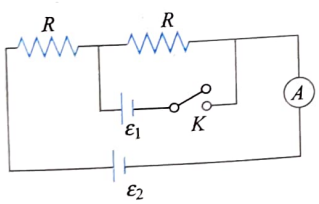
42. An ammeter and a voltmeter are joined in series to a cell. Their readings are A and V, respectively. If a resistance is now joined in parallel with the voltmeter, then
(1) both A and V will increase
(2) both A and V will decrease
(3) A will decrease, V will increase
(4) A will increase, V will decrease

43. A capacitor of capacitance C is connected to two voltmeters A and B (figure). A is ideal, having infinite resistance, while B has resistance R. The capacitor is charged and then the switch S is closed. The readings of A and B will be equal



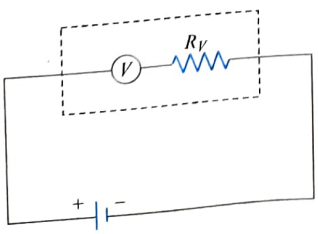
(1) at all times
(2) after time RC
(3) after time RC in 2
(4) only after a very long time

44. In the given circuit (figure), when key K is open, reading of ammeter is I. Now key K is closed, then the correct statement is



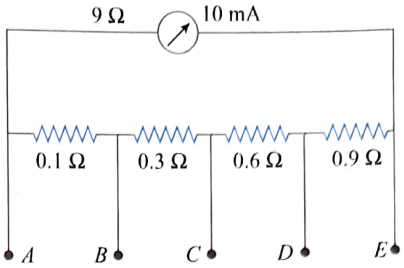
(1) If $\varepsilon_1 < IR$, reading of the ammeter is less than I.
(2) If $IR < \varepsilon_1$, reading of the ammeter is greater than I.
(3) If $\varepsilon_1 < 2IR$, reading of the ammeter will be zero.
(4) Reading of ammeter will not change.

45. A voltmeter with resistance R_V = 2500 Ω indicates a voltage of 125 V in the circuit shown in figure. What is the series resistance (R) to be connected with voltmeter in this circuit so that it indicates 100 V?



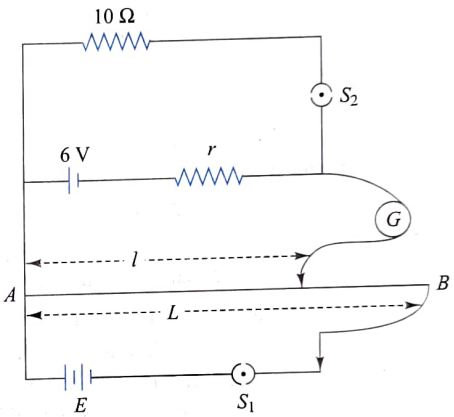
(1) 625 Ω
(2) 120 Ω
(3) 550 Ω
(4) Data insufficient

46. A milli-ammeter of range 10 mA and resistance 9 Ω are joined in a circuit as shown in the figure. The meter gives full scale deflection, when current in the main circuit is I, and A and D are used as terminals. The value of I is



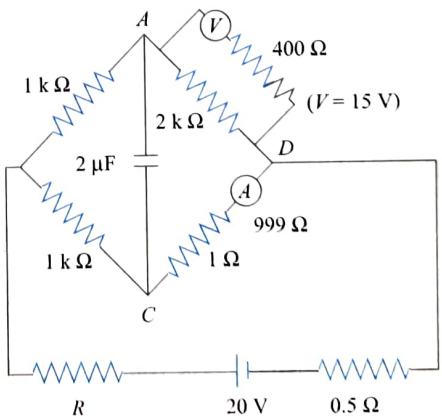
(1) 1.09 A
(2) 10.9 A
(3) zero
(4) 0.109 A

47. In the arrangement shown in figure, when the switch S_2 is open, the galvanometer shows no deflection for $l = L/2$. When the switch S_2 is closed, the galvanometer shows no deflection for $l = 5/12L$. The internal resistance (r) of 6 V cell and the emf E of the other battery are, respectively



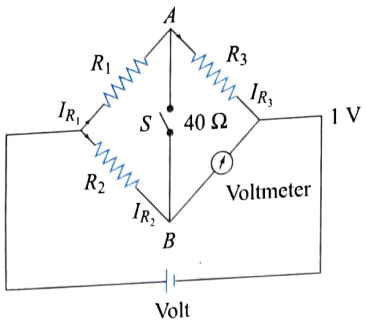
(1) 3 Ω, 8 V
(2) 2 Ω, 12 V
(3) 2 Ω, 24 V
(4) 3 Ω, 12 V

48. Calculate the energy stored in the capacitor of capacitance 2 μF. The voltmeter gives a reading of 15 V and the ammeter A reads 15 mA.



(1) 5 μJ
(2) 10 μJ
(3) 0.5 μJ
(4) zero

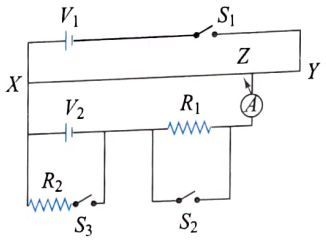
49. In the given circuit (figure), $R_1 \neq R_2$ and the reading of the voltmeter is the same, irrespective of whether the switch S is open or closed. Then, which of the following is correct?



(1) $I_{R_2} = I_r$ (2) $I_{R_1} = I_{R_2}$

(3) $I_{R_3} = I_r$ (4) None of these

50. In the circuit shown in figure, XY is a potentiometer wire 100 cm long. The circuit is connected up as shown. With switches S_2 and S_3 open, a balance point is found at Z. After switch S_1 has remained closed for some time, it is found that contact at Z must be moved toward Y to maintain a balance. Which of the following is the most likely reason for this?



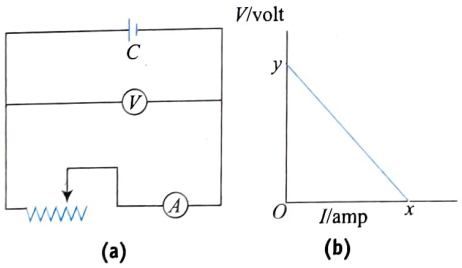
(1) The cell V_1 is running down.

(2) The cell V_2 is running down.

(3) The wire XZ is getting warm and its resistance is increasing.

(4) The resistor R_1 is getting warm and increasing in value.

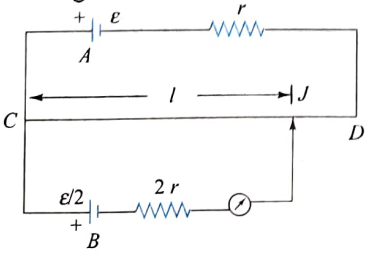
51. Figures show a circuit used in an experiment to determine the emf and internal resistance of the battery C. A graph was plotted of the potential difference V between the terminals of the battery against the current I, which was varied by adjusting the rheostat. The graph is shown in Fig. (b); x and y are the intercepts of the graph with the axes as shown. What is the internal resistance of the battery?



(1) x (2) y

(3) x/y (4) y/x

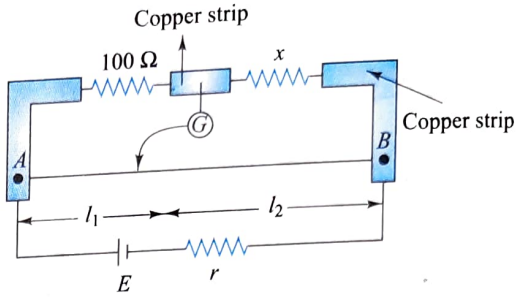
52. In the potentiometer arrangement (figure), the driving cell A has emf ε and internal resistance r. The emf of the cell B is to be rechecked has emf $\varepsilon/2$ and internal resistance $2r$. The potentiometer wire CD is 100 cm long. If balance is obtained, the length $CJ = l$ is



(1) $l = 50$ cm (2) $l > 50$ cm

(3) $l < 50$ cm (4) Balance cannot be obtained

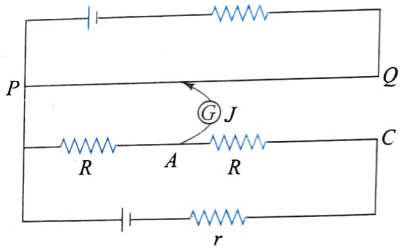
53. In a practical Wheatstone bridge circuit (figure), when one more resistance of 100 Ω is connected in parallel with unknown resistance x, then the ratio l_1 / l_2 becomes 2. l_1 is the balance length. AB is a uniform wire. Then the value of x must be



(1) 50 Ω (2) 100 Ω

(3) 200 Ω (4) 400 Ω

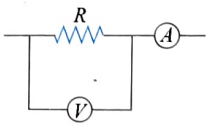
54. Circuit for the measurement of resistance by potentiometer is shown in figure. The galvanometer is first connected at point A and zero deflection is observed at length $PJ = 10$ cm. In second case, it is connected at point C and zero deflection is observed at a length 30 cm from P, then the unknown resistance X is

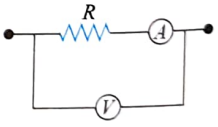


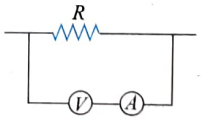
(1) 2 R (2) $R/2$

(3) $R/3$ (4) 3 R

55. Which of the following circuits (figure) gives the correct value of resistance, when computed by using $R = V/I$ where V and I are voltmeter and ammeter readings, respectively? The meters are not ideal.

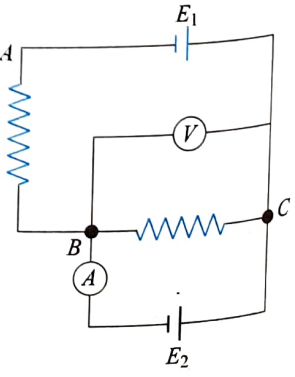
(1) 

(2) 

(3) 

(4) none of these

56. Two ideal batteries having emf E_1 and E_2 are connected as shown in figure. The values of resistances are chosen in such a way that ammeter reading is zero. The reading of voltmeter will be (consider the meters to be ideal)



(1) E_1

(2) E_2

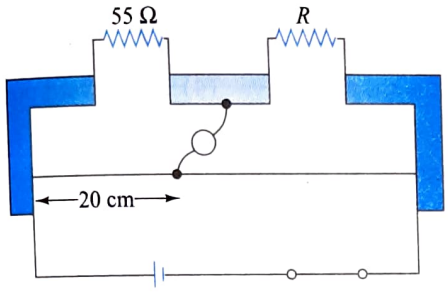
(3) In between E_1 and E_2
(4) Nothing can be predicted about voltmeter reading from the given information.

57. In a Wheatstone bridge, resistances P, Q, and R are connected in the three arms and the fourth arm is formed by two resistances S_1 and S_2 connected in parallel. The condition for the bridge to be balanced will be

(1) $\frac{P}{Q}=\frac{R(S_1+S_2)}{2S_1S_2}$ (2) $\frac{P}{Q}=\frac{R}{S_1+S_2}$

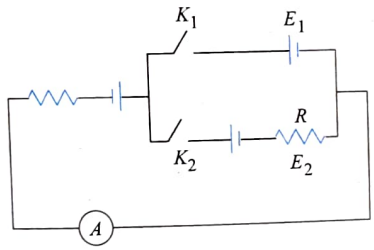
(3) $\frac{P}{Q}=\frac{2R}{S_1+S_2}$ (4) $\frac{P}{Q}=\frac{R(S_1+S_2)}{S_1S_2}$

58. Figure shows a meter bridge set up with null deflection in the galvanometer. The value of the unknown resistor R is



(1) 110 Ω (2) 55 Ω
(3) 13.75 Ω (4) 220 Ω

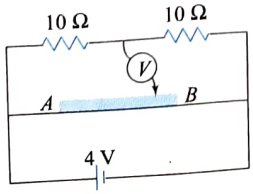
59. In the given arrangement, the reading of ammeter is same in each case when either K_1 or K_2 is closed. The reading of the ammeter is



(1) $\frac{E_1+E_2}{R}$ (2) $\frac{E_1-E_2}{R}$
(3) data insufficient (4) none of these

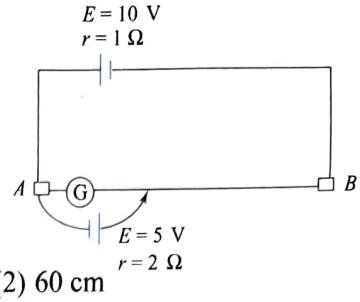
60. A galvanometer may be converted into ammeter or voltmeter. In which of the following cases the resistance of the device will be largest? (Assume the maximum range of galvanometer to be 1 mA)
(1) An ammeter of range 10 A
(2) A voltmeter of range 5 V
(3) An ammeter of range 5 A
(4) A voltmeter of range 10 V

61. A potentiometer wire AB as shown is 40 cm long of resistance 50 Ωm^{-1}; the free end of an ideal voltmeter is touching the potentiometer wire. What should be the velocity of the jockey as a function of time so that reading in voltmeter varies with time as (2 sin πt)?



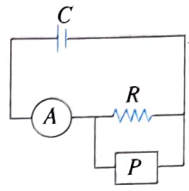
(1) 10 π sin πt cms^{-1} (2) 10 π cos πt cms^{-1}
(3) 20 π sin πt cms^{-1} (4) 20 π cos πt cms^{-1}

62. For the potentiometer arrangement shown in the figure, length of wire AB is 100 cm and its resistance is 9 Ω. Find the length AC for which the galvanometer G will show zero deflection.



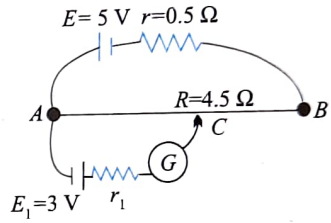
(1) 66.7 cm (2) 60 cm
(3) 50 cm (4) 33.3 cm

63. An ammeter A of finite resistance and a resistor R are joined in series to an ideal cell C. A potentiometer P is joined in parallel to R. The ammeter reading is I_0 and the potentiometer reading is V_0. P is now replaced by a voltmeter of finite resistance. The ammeter reading now is I and the voltmeter reading is V. Then



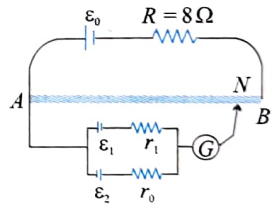
(1) $I > I_0$, $V < V_0$ (2) $I > I_0$, $V > V_0$
(3) $I = I_0$, $V < V_0$ (4) $I < I_0$, $V > V_0$

64. In the given potentiometer circuit, length of the wire AB is 3 m and resistance is R = 4.5 Ω. The length AC for no deflection in galvanometer is



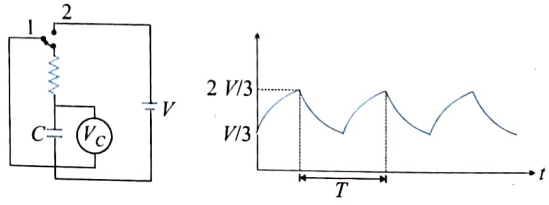
(1) 2 m
(2) 1.8 m
(3) dependent on r_1
(4) none of these

65. A battery of emf ε_0 = 12 V is connected across a 4 m long uniform wire having resistance 4 Ωm^{-1}. The cells of small emf ε_1 = 2 V and ε_2 = 4 V having internal resistances r_1 =2 Ω and r_2 = 6 Ω, respectively, are connected as shown in the figure. If galvanometer shows no deflection at point N, the distance of point N from point A is equal to



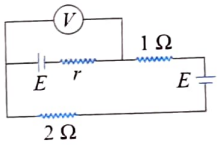
(1) 1/6 m (2) 1/3 m
(3) 25 cm (4) 50 cm

66. V_C is the ideal voltmeter in the figure. Resistance of the resistor shown is R. Initially the switch is in position 2 when charging of the capacitor starts. Initially the capacitor was uncharged. The switch in circuit shifts automatically from 2 to 1 when $V_C > 2V/3$ and goes back to 2 from 1 when $V_C < V/3$. The ideal voltmeter reads voltage across capacitor as plotted. What is the period T of the wave form in terms of R and C?



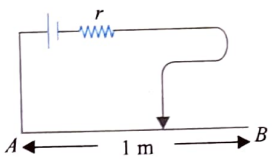
(1) $3RC \ln 3$ (2) $2RC \ln 2$
(3) $(RC/2) \ln 2$ (4) none of these

67. In the given circuit, the reading of ideal voltmeter is $E/2$. The internal resistance of the battery is



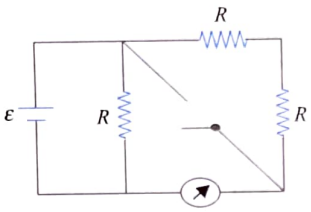
(1) 1 Ω (2) $\frac{2}{3}$ Ω
(3) $\frac{2}{5}$ Ω (4) $\frac{5}{2}$ Ω

68. In the following circuit, resistance of potentiometer wire is 100 Ω. Power consumption in potentiometer wire is same when jocky is placed at 10 cm from end A or end B. Internal resistance of cell (r) is



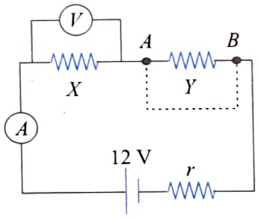
(1) 30 Ω (2) 6 Ω
(3) 60 Ω (4) 3 Ω

69. In the circuit shown, the reading of the ammeter is doubled after the switch is closed. Each resistor has a resistance $R = 1$ Ω and the ideal cell has an emf ε of 10 V. Then, the ammeter has a coil resistance equal to



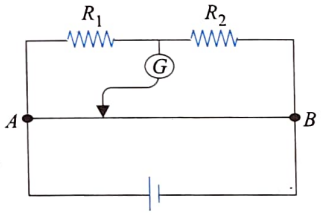
(1) 2 Ω (2) 1 Ω
(3) 2.5 Ω (4) none

70. When an ammeter of negligible internal resistance is inserted in series with circuit it reads 1 A. When the voltmeter of very large resistance is connected across X it reads 1 V. When the point A and B are shorted by a conducting wire, the voltmeter measures 10 V. The internal resistance of the battery (r) is equal to



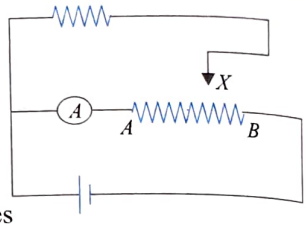
(1) zero (2) 0.5 Ω
(3) 0.2 Ω (4) 0.1 Ω

71. The figure shown gives values of R_1 and R_2; the balance point for jockey is at 30 cm from A. When R_2 is shunted by a resistance of 10 Ω, jockey has to slide 20 cm to obtain balance point. The values of R_1 and R_2 are



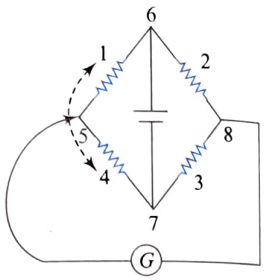
(1) $\frac{10}{5}$ Ω, 5 Ω (2) $\frac{40}{3}$ Ω, $\frac{40}{7}$ Ω
(3) $\frac{40}{7}$ Ω, $\frac{40}{3}$ Ω (4) none of these

72. In circuit shown in figure, terminal X is brought from point A to B. Then reading of ammeter



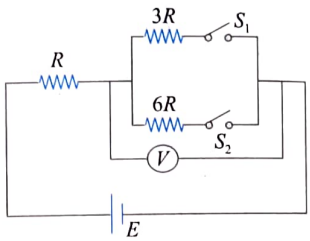
(1) increases
(2) decreases
(3) first increases then decreases
(4) first decreases then increases

73. Four wires of equal length 2 m are arranged as shown in the figure. Wires, 2, 3 and 4 are of equal cross-sectional area and wire 1 is of half the cross section of these wires. By how much distance pointer at point 5 must be moved to get null point?



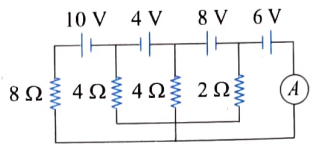
(1) 0.5 m toward point 6 (2) 0.5 m toward point 7
(3) 1 m toward point 6 (4) 1 m toward point 7

74. In the circuit shown in figure, reading of voltmeter is V_1 when only S_1 is closed, reading of voltmeter is V_2 when only S_2 is closed, and reading of voltmeter is V_3 when both S_1 and S_2 are closed. Then



(1) $V_3 > V_2 > V_1$ (2) $V_2 > V_1 > V_3$
(3) $V_3 > V_1 > V_2$ (4) $V_1 > V_2 > V_3$

75. Find the reading of the ideal ammeter connected in the given circuit. Assume that the cells have negligible internal resistance.

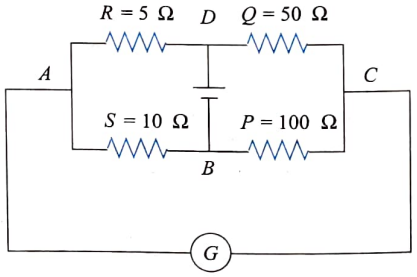


(1) 2.5 A (2) 5 A
(3) 2 A (4) none of these

Multiple Correct Answers Type

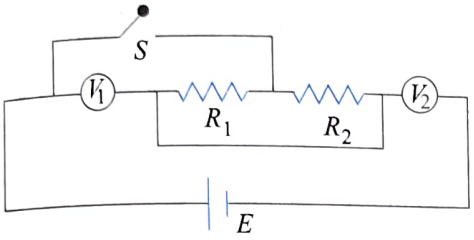
1. A voltmeter reads the potential difference across the terminals of an old battery as 1.40 V, while a potentiometer reads its voltage to be 1.55 V. The voltmeter resistance is 280 Ω.
(1) The emf of the battery is 1.4 V.
(2) The emf of the battery is 1.55 V.
(3) The internal resistance r of the battery is 30 Ω.
(4) The internal resistance r of the battery is 5 Ω.

2. Figure shows a balanced Wheatstone bridge.



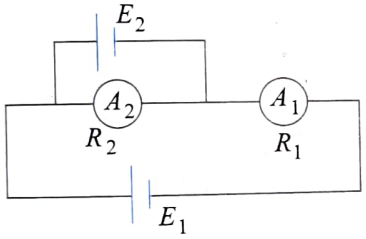
(1) If P is slightly increased, the current in the galvanometer flows from C to A.
(2) If P is slightly increased, the current in the galvanometer flows from A to C.
(3) If Q is slightly increased, the current in the galvanometer flows from C to A.
(4) If Q is slightly increased, the current in the galvanometer flows from A to C.

3. Two voltmeters and two resistances are connected as shown in figure. On closing the switch S, what will be the effect on the readings of the voltmeters?



(1) V_1 increases (2) V_1 decreases
(3) V_2 increases (4) V_2 decreases

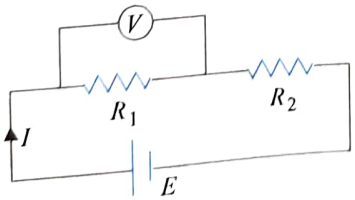
4. Two ideal batteries and two ammeters are arranged as shown in figure.



(1) Readings of both ammeters can be same if $E_1 > E_2$.
(2) Readings of both ammeters can be same if $E_2 > E_1$ provided $R_2 > R_1$.
(3) Readings of both ammeters can be same if $E_2 > E_1$ provided $R_2 < R_1$.
(4) If $E_2 > E_1$, then current in ammeters will flow in opposite directions.

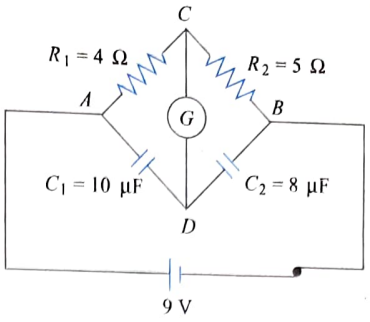
5. In the above question, if the polarity of E_2 is reversed, then
(1) current in both ammeters will flow in same direction
(2) current in both ammeters will flow in opposite directions
(3) current in both ammeters can be same if $R_1 > R_2$
(4) current in both can be same if $R_1 < R_2$

6. In figure, voltmeter is not ideal. If the voltmeter is removed from R_1 and then put across R_2, what will be the effect on current I? Given $R_1 > R_2$.



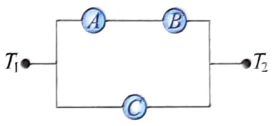
(1) decreases
(2) remains same
(3) increases
(4) I would have been same if voltmeters were ideal.

7. In the circuit shown in figure, the cell is ideal with emf 9 V. If the resistance of the coil of galvanometer is 1 Ω, then



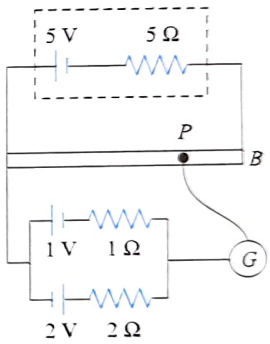
(1) no current flows in the galvanometer
(2) charge flowing through 8 μF is 40 μC
(3) potential difference across 10 μF is 5 V
(4) potential difference across 10 μF is 4 V

8. Three ammeters A, B, and C of resistances R_A, R_B, and R_C, respectively, are joined as shown in figure. When some potential difference is applied across the terminals T_1 and T_2, their readings are I_A, I_B, and I_C, respectively, then



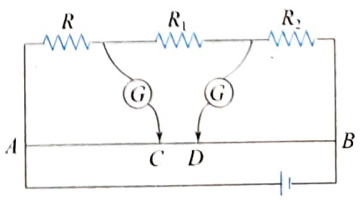
(1) $I_A = I_B$ (2) $I_A R_A + I_B R_B = I_C R_C$
(3) $\frac{I_A}{I_C} = \frac{R_C}{R_A}$ (4) $\frac{I_B}{I_C} = \frac{R_C}{R_A + R_B}$

9. A battery of emf $\varepsilon_0 = 5$ V and internal resistance 5 Ω is connected across a long uniform wire AB of length 1 m and resistance per unit length 5 Ωm^{-1}. Two cells of $\varepsilon_1 = 1$ V and $\varepsilon_2 = 2$ V are connected as shown in the figure.



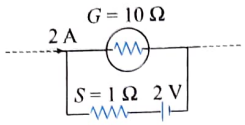
(1) The null point is at A.
(2) If the jockey is touched to point B, the current in the galvanometer will be going towards B.
(3) When jockey is connected to point A, no current flows through 1 V battery.
(4) The null point is at distance of 8/15 m from A.

10. The diagram shows a modified meter bridge, which is used for measuring two unknown resistances at the same time. When only the first galvanometer is used, for obtaining the balance point, it is found at point C. Now the first galvanometer is removed and the second galvanometer is used, which gives balance point at D. Using the details given in the diagram, find out the value of R_1 and R_2.



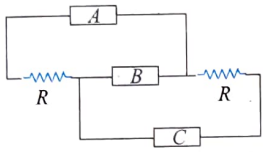
(1) $R_1 = 5R/3$ (2) $R_2 = 4R/3$
(3) $R_1 = 4R/3$ (4) $R_2 = 5R/3$

11. The galvanometer shown in the figure has resistance 10 Ω. It is shunted by a series combination of a resistance $S = 1$ Ω and an ideal cell of emf 2 V. A current 2 A passes as shown.



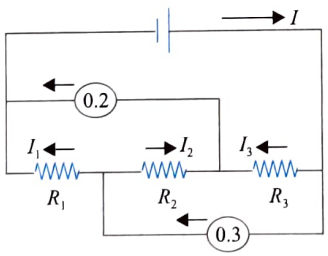
(1) The reading of the galvanometer is 1 A.
(2) The reading of the galvanometer is zero.
(3) The potential difference across the resistance S is 1.5 V.
(4) The potential difference across the resistance S is 2 V.

12. A circuit is shown below.



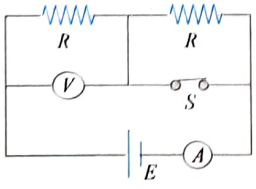
(1) If A is an ideal ammeter, B an ideal battery of voltage V, and C an ideal voltmeter, the ratio reading of C/reading of A is R.
(2) If A is a capacitor, B an ideal ammeter and C an ideal battery of voltage V, the potential difference across the capacitor is zero.
(3) If A is an ideal ammeter, B an ideal battery of voltage V, and C an ideal voltmeter, the ratio reading of C/reading of A is $R/2$.
(4) If A is a capacitor, B an ideal ammeter, and C an ideal battery of voltage V, the potential difference across the capacitor is 2 V.

13. A circuit contains an ideal battery, three resistors, and two ideal ammeters. The ammeters read 0.2 A and 0.3 A. After two of the resistors are switched, the readings of the ammeters did not change. Find the battery current.



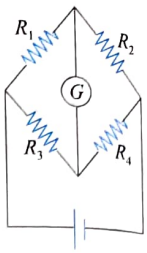
(1) If $R_2 = R_1$, $I_1 = I_2 = 0.15$ A and $I_3 = 0.05$ A.
(2) If $R_2 = R_1$, the battery current is $I = 0.20$ A.
(3) If $R_2 = R_3$, $I_2 = I_3 = 0.1$ A and $I_1 = 0.2$ A.
(4) If $R_2 = R_3$, the battery current is $I = 0.4$ A.

14. In the circuit shown, battery, ammeter, and voltmeter are ideal and the switch S is initially closed as shown. When switch S is opened,



(1) equivalent resistance across the battery increases
(2) power dissipated by left resistance R increases
(3) voltmeter reading decreases
(4) ammeter reading decreases

15. All the resistance in the given Wheatstone bridge have different values and the current through the galvanometer is zero. The current through the galvanometer will still be zero if,



(1) the emf of the battery is doubled
(2) all resistances are doubled
(3) resistances R_1 and R_2 are interchanged
(4) the battery and the galvanometer are interchanged

Linked Comprehension Type

For Problems 1–2

A cell of emf 3.4 V and internal resistance 3 Ω is connected to an ammeter having resistance 2 Ω and an external resistance of 100 Ω. When a voltmeter is connected across the 100 Ω resistance, the ammeter reading is 0.04 A.

1. Find the resistance of the voltmeter.
(1) 400 Ω (2) 200 Ω
(3) 300 Ω (4) 500 Ω

2. Had the voltmeter been an ideal one, what would have been its reading?
(1) 7.2 V (2) 1.8 V
(3) 0.5 V (4) 3.24 V

For Problems 3–4

A battery is connected to a potentiometer and a balance point is obtained at 84 cm along the wire. When its terminals are connected by a 5 Ω resistor, the balance point changes to 70 cm.

3. Calculate the internal resistance of the cell.
(1) 4 Ω (2) 2 Ω
(3) 5 Ω (4) 1 Ω

4. Find the new position of the balance point when 5 Ω resistor is changed by 4 Ω resistor.
(1) 26.5 cm (2) 52 cm
(3) 67.2 cm (4) 83.3 cm

For Problems 5–7

A potentiometer is an ideal voltmeter since a voltmeter draws some current through the circuit while potentiometer needs no current to work. A potentiometer works on the principle of emf comparison. In working condition, a constant current flows throughout the wire of a potentiometer using standard cell of emf e_1. The wire of potentiometer is made of uniform material and cross-sectional area, and it has uniform resistance per unit length. The potential gradient depends upon the current in the wire.

A potentiometer with a cell of emf 2 V and internal resistance 0.4 Ω is used across the wire AB. A standard cadmium cell of emf 1.02 V gives a balance point at 66 cm length of wire. The standard cell is then replaced by a cell of unknown emf e (internal resistance r), and the balance point found similarly turns out to be 88 cm length of the wire. The length of potentiometer wire AB is 1 m.

5. The value of e is
(1) 1.36 V (2) 2.63 V
(3) 1.83 V (4) none

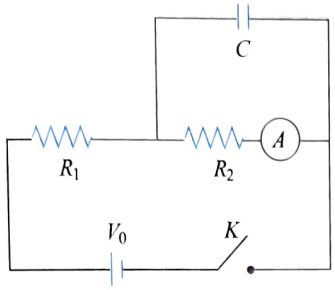
6. The reading of the potentiometer, if a 4 V battery is used instead of e, is

(1) 88.3 cm (2) 47.3 cm
(3) 95 cm (4) cannot be calculated

7. If the resistance is connected across the cell e, the balancing length will
(1) increase (2) decrease
(3) remain same (4) none

For Problems 8–10

In the connection shown in figure, initially switch K is open and the capacitor is uncharged. Then the switch is closed and the capacitor is charged up to the steady state and the switch is opened again. Determine the values indicated by the ammeter. (Given: $V_0 = 30$ V, $R_1 = 10$ kΩ, $R_2 = 5$ kΩ)



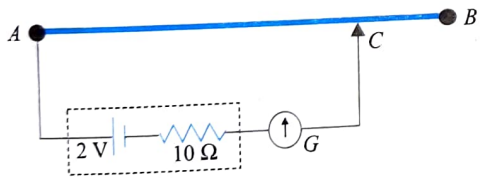
8. Just after closing the switch
(1) 2 mA (2) 3 mA
(3) 0 mA (4) none of these

9. A long time after the switch was closed
(1) 2 mA (2) 3 mA
(3) 6 mA (4) none of these

10. Just after reopening the switch
(1) 2 mA (2) 3 mA
(3) 6 mA (4) none of these

For Problems 11–13

The length of a potentiometer wire AB is 600 cm, and it carries a constant current of 40 mA from A to B. For a cell of emf 2 V and internal resistance 10 Ω, the null point is found at 500 cm from A. When a voltmeter is connected across the cell, the balancing length of the wire is decreased by 10 cm.



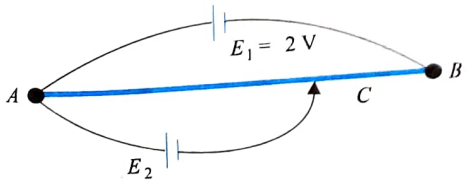
11. Potential gradient along AB is
(1) 1/5 Vm^{-1} (2) 2/5 Vm^{-1}
(3) 3/5 Vm^{-1} (4) 4/5 Vm^{-1}

12. Reading of the voltmeter is
(1) 2 V (2) 2.04 V
(3) 1.96 V (4) 1.0 V

13. Resistance of the voltmeter is
(1) 400 Ω (2) 500 Ω
(3) 510 Ω (4) 490 Ω

For Problems 14–15

AB is a potentiometer wire of length 100 cm. When a cell E_2 is connected across AC, where $AC = 75$ cm, no current flows from E_2. The internal resistance of the cell E_1 is negligible.

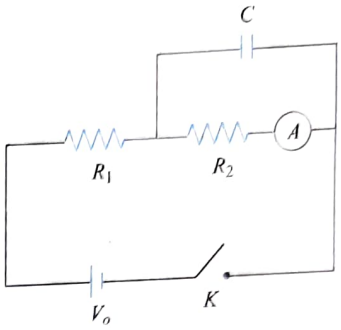


14. Find the potential gradient along AB.
(1) 0.01 Vcm^{-1} (2) 0.03 Vcm^{-1}
(3) 0.04 Vm^{-1} (4) 0.02 Vcm^{-1}

15. Find emf of the cell E_2.
(1) 2 V (2) 1.5 V
(3) 1 V (4) 1.75 V

For Problems 16–18

In the connection shown in the figure, initially the switch K is open and the capacitor is uncharged. Then the switch is closed, and the capacitor is charged up to the steady state and the switch is opened again. Determine the values indicated by the ammeter. [Given $V_0 = 30$ V, $R_1 = 10$ kΩ, $R_2 = 5$ kΩ]



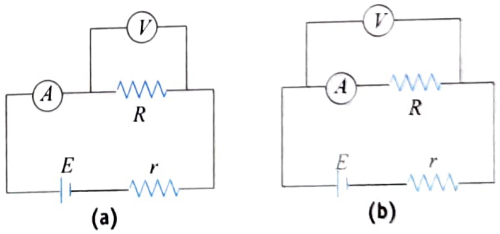
16. Just after closing the switch
(1) 2 mA (2) 3 mA
(3) 0 mA (4) none of these

17. A long time after the switch was closed
(1) 2 mA (2) 3 mA
(3) 6 mA (4) none of these

18. Just after reopening the switch
(1) 2 mA (2) 3 mA
(3) 6 mA (4) none of these

For Problems 19–21

The value of resistance of an unknown resistor is calculated using the formula $R = V/I$ where V and I are the readings of the voltmeter and the ammeter, respectively. Consider the circuits below. The internal resistances of the voltmeter and the ammeter (R_V and R_G, respectively) are finite and nonzero.



Let R_A and R_B be the calculated values in the two cases A and B, respectively.

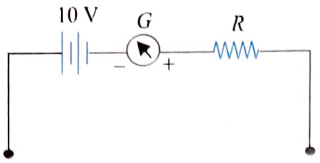
19. The relation between R_A and the actual value of R is
(1) $R > R_A$ (2) $R < R_A$
(3) $R = R_A$ (4) dependent upon E and r

20. The relation between R_B and the actual value of R is
(1) $R < R_B$ (2) $R > R_B$
(3) $R = R_B$ (4) dependent upon E and r

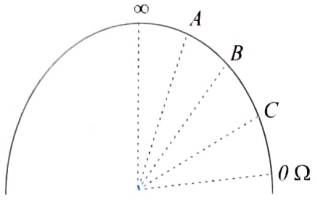
21. If the resistance of voltmeter is $R_V = 1$ kΩ and that of ammeter is $R_G = 1$ Ω, the magnitude of the percentage error in the measurement of R (the value of R is nearly 10 Ω) is
(1) zero in both cases
(2) nonzero but equal in both cases
(3) more in circuit A
(4) more in circuit B

For Problems 22–24

An ohm meter measures the resistance placed between its leads. This resistance reading is indicated by a galvanometer that operates on current. The ohm meter has an internal source of voltage to create the necessary current to operate the galvanometer and also has appropriate resistor to allow just right amount of current through galvanometer. A simple ohm meter is shown here.



When there is an infinite resistance, there is zero current through galvanometer and it points in middle. If the test leads of this meter are directly shorted (zero resistance), galvanometer will give full deflection.



Galvanometer specification:

Resistance of galvanometer = 100 Ω

Fullscale current = 0.5 mA

Total number of division = 40

22. What is the value of resistance (R) required in series with battery and galvanometer?

(1) 20 k Ω (2) 19.9 k Ω

(3) 19 k Ω (4) 9 k Ω

23. If an unknown resistance is placed between two leads and needle deflects up to B (mid of scale), then resistance is equal to

(1) 20 k Ω (2) 10 k Ω

(3) 5 k Ω (4) 25 k Ω

24. If 5 Ω is measured with help of ohm meter, the needle lies between

(1) ∞ and A (2) A and B

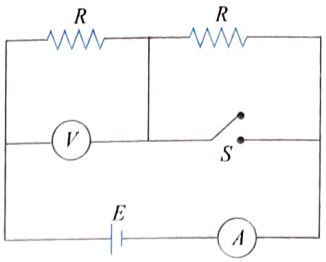
(3) B and C (4) C and zero

Matrix Match Type

1. In a potentiometer experiment

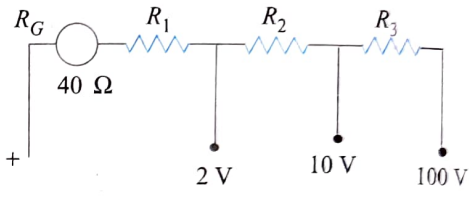
Column I	Column II
i. Deflection of galvanometer is in the same direction at the two ends of the wire	a. Accuracy in measurement increases
ii. A protective resistance added in series to the galvanometer	b. Accuracy in measurement decreases
iii. A short wire is used as a potentiometer	c. The emf of the battery in the primary circuit is less than the emf of the cell to be measured
iv. More length of potentiometer up to null point	d. Uncertainty in the location of balance

2. In the circuit shown in figure, the battery, ammeter, and voltmeter are ideal and the switch S is initially closed as shown. When S is opened, match the parameters of column I with the effects in column II.



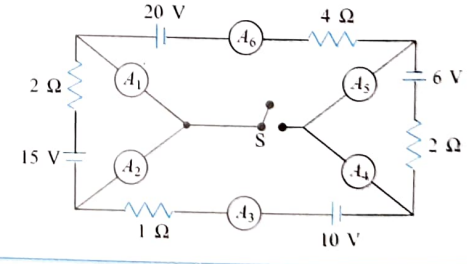
Column I	Column II
i. Equivalent resistance across the battery	a. Remains same
ii. Power dissipated by left resistance R	b. Increases
iii. Voltmeter reading	c. Decreases
iv. Ammeter reading	d. Becomes zero

3. Figure shows the internal wiring of a three-range voltmeter whose binding posts are marked "+", 2 V, 10 V, and 100 V. When the meter is connected to the circuit being measured, one connection is made to the post marked + and the other to the post marked with the desired voltage range. The resistance of the moving coil R_G is 40 Ω, and a current of 1 mA in the coil causes it to deflect full-scale. Then match the following:



Column I	Column II
i. Value of resistance R_1 in kΩ	a. 100
ii. Value of resistance R_3 in kΩ	b. 2
iii. Overall resistance of the meter in 100 V range in kΩ	c. 1.96
iv. Overall resistance of the meter in 2 V range in kΩ	d. 90

4. In the circuit shown, all the ammeters are ideal. Match the following based on the circuit

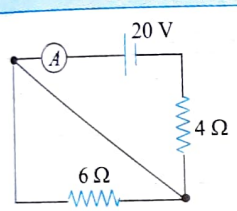
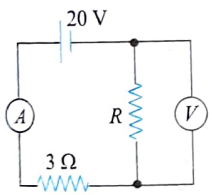
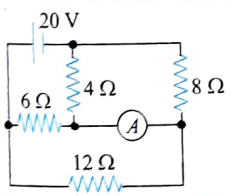
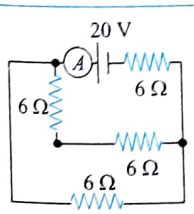


Column I	Column II
i. If switch S is open, the ammeter(s) that read(s) less than 10 A	a. A_1 and A_2
ii. If switch S is open, the ammeter(s) that read(s) equal current	b. A_4 and A_5
iii. If switch S is closed, the ammeter(s) that read(s) more than 5 A	c. A_3 and A_4
iv. If switch S is closed, the ammeter(s) that show(s) increase in the reading	d. A_6 and A_5

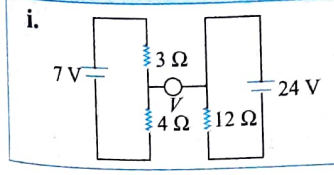
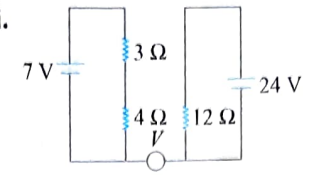
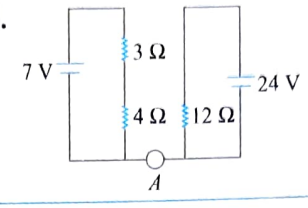
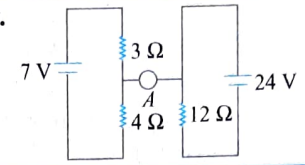
5. In an experiment for comparing emfs of two primary cells using potentiometer, some observations are given in Column I.

Column I	Column II
i. Deflection of galvanometer is in same direction at two ends of the wire	a. Accuracy in measurement increases
ii. A series protective resistance added in series to the galvanometer	b. The positive terminals of all batteries/cells are not connected at a point
iii. A short wire is used in potentiometer	c. emf of battery in primary circuit is less than the emf of cell to be measured
iv More length of wire up to null point	d. Uncertainity in the location of balance point increases

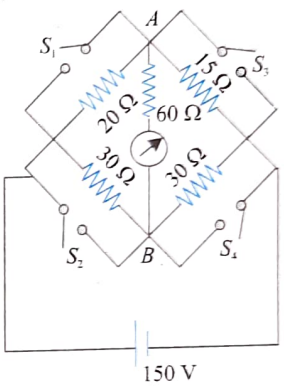
6. Column I has four circuits each having an ammeter. Column II has four values of current in the ammeter. The ammeter has zero resistance. The voltmeter in (V) has infinite resistance and a reading 8 V. The resistance R has not been specified. Match the circuit with its correct ammeter reading.

Column I	Column I
i. 	a. zero
ii. 	b. 2 A
iii. 	c. 4 A
iv. 	d. 5 A

7. Match the readings of the voltmeter and ammeter respectively shown in the figures.

Column I	Column II
i. 	a. zero
ii. 	b. 20 V
iii. 	c. 2 A
iv. 	d. 1 A

8. Consider the circuit shown. The resistance connected between the junction A and B is 60, including the resistance of the galvanometer. The switches have no resistance when shorted and infinite resistance when opened. All the switches are initially open and they are closed as given in Column I. Match the condition in Column I with the direction of current through galvanometer and the value of the current through the battery in Column II.

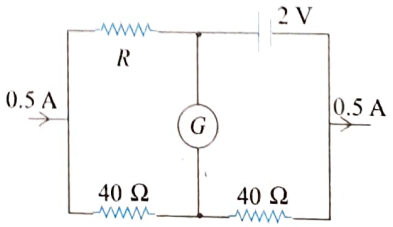


Column I	Column II
i. Only switch S_1 is closed	a. Current flows from A to B
ii. Only switch S_2 is closed	b. Current flows from B to A
iii. Only switch S_3 is closed	c. Current through the battery is 10 A
iv. Only switch S_4 is closed	d. Current through the battery is 13 A

Numerical Value Type

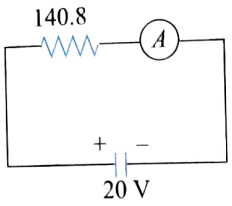
1. A galvanometer, together with an unknown resistance in series, is connected across two identical batteries of each 1.5 V. When the batteries are connected in series, the galvanometer records a current of 1 A, and when the batteries are connected in parallel, the current is 0.6 A. In this case, the internal resistance of the battery is 1/'*' Ω. What is the value of '*'?

2. In the circuit shown in figure, the internal resistance of the cell is negligible. For the value of $R = 40/x$ Ω, no current flows through the galvanometer. What is x?



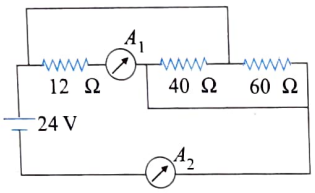
3. A 5 m potentiometer wire having 3 Ω resistance per meter is connected to a storage cell of steady emf 2 V and internal resistance 1 Ω. A primary cell is balanced against 3.5 m of it. When a resistance of $32/n$ Ω is put in series with the storage cell, the null point shifts to the center of the last wire, i.e., 4.5 m. What is 'n'?

4. The ammeter shown in figure consists of 480 Ω coil connected in parallel to 20 Ω shunt. Find the reading of the ammeter (in A).

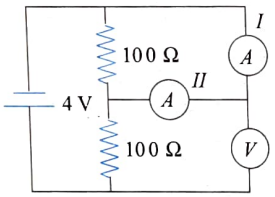


5. A potentiometer wire of length 10 m and resistance 30 Ω is connected in series with a battery of emf 2.5 V, internal resistance 5 Ω, and an external resistance R. If the fall of potential along the potentiometer wire is 50 μ Vmm^{-1}. Find the value of R (in Ω).

6. Find the reading of the ammeters A_1 (in ampere) connected as shown in the network.



7. In figure, the ammeter (I) reads a current of 10 mA, while the voltmeter reads a potential difference of 3 V. The ammeters are identical, and the internal resistance of the battery is negligible (consider all ammeters and voltmeters as nonideal).

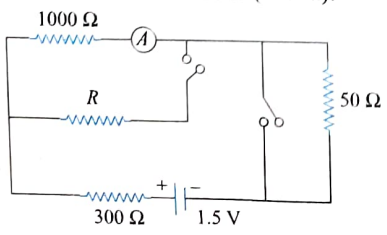


The resistance of ammeter is $m\times10^2$ Ω. What is the value of m?

8. The maximum current in a galvanometer can be 10 mA. Its resistance is 10 Ω. To convert it into an ammeter of 1 A, what resistance should be connected in parallel with galvanometer (in 10^{-1} Ω)?

9. The current in a conductor and the potential difference across its ends are measured by an ammeter and a voltmeter, respectively. The voltmeter draws negligible currents. The ammeter is accurate but the voltmeter has a zero error (that is, it does not read zero when no potential difference is applied). Calculate the zero error if the readings for two different conditions are 1.75 A, 14.4 V and 2.75 A, 22.4 V (in $\times 10^{-1}$ V).

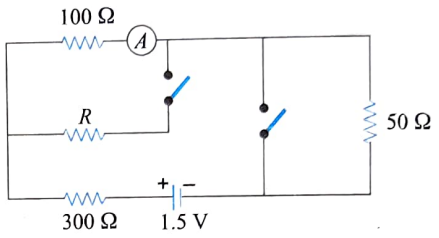
10. In the circuit shown in figure the reading of an ideal ammeter is the same with both switches open as with both closed then find the resistance R (in kΩ).



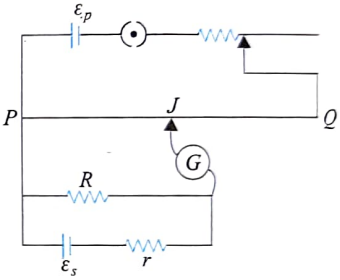
11. If resistance R_1 in resistance box is 300 Ω, then the balanced length is found to be 75.0 cm from end A. The diameter of unknown wire is 1 mm and length of the unknown wire is 31.4 cm. Find the specific resistance of the unknown wire. (μΩ-m)

12. An ammeter and a voltmeter are connected in series to a battery of emf E = 6.0 V. When a certain resistance is connected in parallel with the voltmeter, the reading of the voltmeter decreases two times, whereas the reading of the ammeter increases the same number of times. Find the voltmeter reading (in volts) after the connection of the resistance.

13. In the circuit shown in figure the reading of ideal ammeter connected as shown is same with both switches open as with both closed. Then find the resistance R (in Ω).



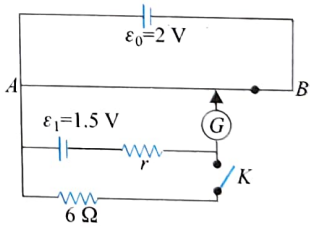
14. One of the circuits for the measurement of resistance by potentiometer is shown. The galvanometer is connected at point A and zero deflection, is observed at length PJ = 30 cm. In second case the secondary cell is changed.



Take ε_s = 10 V and r = 1 Ω in 1st reading and ε_s = 5 V and r = 2 Ω in 2nd reading.

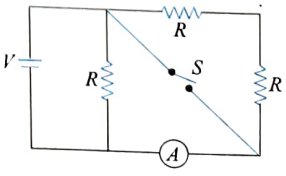
In second case, the zero deflection is observed at length PJ = 10 cm. What is the resistance R (in ohm)?

15. For the galvanometer of the potentiometer shown in the figure, the balance point is obtained at a distance 75 cm from A when the key is open.

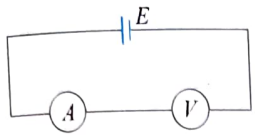


The second balance point is obtained at 60 cm from A when the key is closed. Find the internal resistance (in Ω) of the battery ε_1.

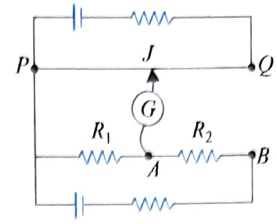
16. In the circuit shown, R = 10 Ω and V = 100 volt. With switch S open the reading of ammeter is one third its reading when 'S' is closed. Calculate the resistance of the ammeter (in Ω).



17. A non ideal ammeter and voltmeter are connected in series with an ideal cell of emf $E = 22$ V as shown in figure. Reading of the voltmeter is V_0. When a resistance is added in parallel to the voltmeter its reading becomes $V_0/10$ and the reading of the ammeter becomes 10 times the earlier value. Find the value of V_0 (in volts).



18. In the figure shown PQ is a potentiometer wire. When galvanometer is connected at A, it shows zero deflection when $PJ = x$. Now the galvanometer is connected to B and it shows zero deflection when $PJ = 3x$. Find the ratio of unknown resistance R_x and $R(R_x/R = ?)$



Archives

JEE MAIN

Single Correct Answer Type

1. Which of the following statements is false?
   (1) A rheostat can be used as a potential divider.
   (2) Kirchhoff's second law represents energy conservation.
   (3) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.
   (4) In a balanced Wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.
   (JEE Main 2017)

2. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full-scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into voltmeter of range 0–10 V is
   (1) 2.535×10^3 Ω (2) 4.005×10^3 Ω
   (3) 1.985×10^3 Ω (4) 2.045×10^3 Ω
   (JEE Main 2017)

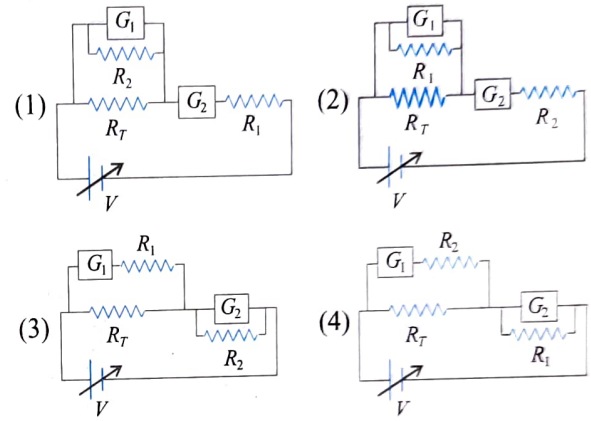
3. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combinations is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances ?
   (1) 910 Ω (2) 990 Ω
   (3) 505 Ω (4) 550 Ω
   (JEE Main 2018)

4. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.
   (1) 2.5 Ω (2) 1 Ω
   (3) 1.5 Ω (4) 2 Ω **(JEE Main 2018)**

JEE ADVANCED

Single Correct Answer Type

1. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is



(IIT-JEE 2010)

Multiple Correct Answers Type

1. Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $RC < R/2$, which of the following statement(s) about any one of the galvanometers is (are) true?
   (1) The maximum voltage range is obtained when all the components are connected in series
   (2) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer
   (3) The maximum current range is obtained when all the components are connected in parallel
   (4) The maximum current range is obtained when the two galvanometers are connected in series, and the combination is connected in parallel with both the resistorsg **(JEE Advanced 2016)**

Numerical Value Type

1. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0–30 V. If connected to a $2n/249$ Ω resistance, it becomes an ammeter of range 0–1.5 A. The value of n is
   (JEE Advanced 2014)

2. A moving coil galvanometer has 50 turns and each turn has an area 2×10^{-4} m^2. The magnetic field produced by the magnet inside the galvanometer is 0.02 T. The

torsional constant of the suspension wire is 10^{-4} Nm rad^{-1}. When a current flows through the galvanometer, a full scale deflection occurs if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0–1.0 A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in ohms, is..... .

(JEE Advanced 2018)

Answers Key

EXERCISES

Single Correct Answer Type

1. (2)	**2.** (1)	**3.** (1)	**4.** (1)	**5.** (3)
6. (3)	**7.** (2)	**8.** (1)	**9.** (2)	**10.** (2)
11. (3)	**12.** (2)	**13.** (2)	**14.** (2)	**15.** (1)
16. (2)	**17.** (1)	**18.** (1)	**19.** (3)	**20.** (1)
21. (1)	**22.** (3)	**23.** (2)	**24.** (4)	**25.** (2)
26. (2)	**27.** (2)	**28.** (3)	**29.** (4)	**30.** (1)
31. (3)	**32.** (2)	**33.** (1)	**34.** (1)	**35.** (2)
36. (2)	**37.** (2)	**38.** (1)	**39.** (1)	**40.** (1)
41. (4)	**42.** (4)	**43.** (1)	**44.** (2)	**45.** (1)
46. (4)	**47.** (2)	**48.** (4)	**49.** (1)	**50.** (1)
51. (4)	**52.** (2)	**53.** (2)	**54.** (1)	**55.** (4)
56. (2)	**57.** (4)	**58.** (4)	**59.** (2)	**60.** (4)
61. (4)	**62.** (1)	**63.** (1)	**64.** (4)	**65.** (3)
66. (2)	**67.** (1)	**68.** (1)	**69.** (1)	**70.** (3)
71. (3)	**72.** (4)	**73.** (1)	**74.** (2)	**75.** (3)

Multiple Correct Answers Type

1. (2),(3)	**2.** (2),(3)	**3.** (2),(3)
4. (1),(2),(4)	**5.** (2),(3)	**6.** (1),(4)
7. (1),(2),(4)	**8.** (1),(2),(4)	**9.** (1),(2)
10. (1),(2)	**11.** (1),(2),(4)	**12.** (1),(2)
13. (1),(3),(4)	**14.** (1),(3),(4)	**15.** (1),(2),(4)

Linked Comprehension Type

1. (1)	**2.** (4)	**3.** (4)	**4.** (3)	**5.** (1)
6. (4)	**7.** (2)	**8.** (3)	**9.** (1)	**10.** (1)
11. (2)	**12.** (3)	**13.** (4)	**14.** (4)	**15.** (2)
16. (3)	**17.** (1)	**18.** (1)	**19.** (1)	**20.** (1)
21. (4)	**22.** (2)	**23.** (1)	**24.** (4)	

Matrix Match Type

1. i. → c.; ii. → d.; iii. → b., d.; iv. → a.
2. i. → b.; ii. → c.; iii. → c.; iv. → c.
3. i. → c.; ii. → d.; iii. → a.; iv. → b.
4. i. → a., b., c., d.; ii. → a., b.; iii. → b., c.; iv. → b., c., d.
5. i. → b., c.; ii. → d.; iii. → d.; iv. → a.
6. i. → d.; ii. → c.; iii. → a.; iv. → d.
7. i. → a.; ii. → a.; iii. → a.; iv. → a.
8. i. → a., d.; ii. → b.; iii. → a., c.; iv. → a.

Numerical Value Type

1. (3)	**2.** (9)	**3.** (7)	**4.** (0.125)	**5.** (115)
6. (2)	**7.** (1)	**8.** (1)	**9.** (4)	**10.** (6)
11. (250)	**12.** (2)	**13.** (600)	**14.** (4)	**15.** (1.5)
16. (10)	**17.** (20)	**18.** (2)		

ARCHIVES

JEE Main

Single Correct Answer Type

1. (4)	**2.** (3)	**3.** (4)	**4.** (3)

JEE Advanced

Single Correct Answer Type

1. (3)

Multiple Correct Answers Type

1. (1),(3)

Numerical Value Type

1. (5) **2.** (5.55)

7 Heating Effects of Current

HEATING EFFECTS OF CURRENT

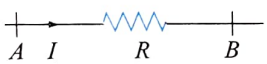
When an electric current is passed through a conductor, it becomes hot and its temperature starts rising. This is known as **heating effect of current** or **Joule's heating effect**. Here electric energy converts into heat energy. Various appliances such as geyser, iron, heater, fuse wire, etc. work on this basis.

CAUSE OF HEATING

When current is passed through a conductor, the electrons start drifting toward the positive end. They gain additional kinetic energy (KE) apart from thermal KE. These electrons suffer collisions with atoms/ions more violently and transfer their KE to atoms/ions. It increases the amplitude of vibrations of ions/atoms. Thus, the average KE of vibrations of atoms/ions increases, which shows up in the form of increased temperature. Here, the electric energy supplied by the source of emf is converted into heat.

HEAT PRODUCED BY AN ELECTRIC CURRENT

Suppose a current I is flowing in a resistor of resistance R (figure). The amount of charge passed through the resistor in time t is $q = It$. Decrease in the potential of this charge is given by $V = IR$, and decrease in the potential energy of the charge is given by $qV = I^2Rt$.



This decrease in the energy will appear in the form of heat energy. So the electric energy produced in a resistor of resistance R in time t in which a current I is flowing is given by $H = I^2RT$, which is Joule's law of heating.

Thus, Joule's law of heating states that the amount of heat produced in a conductor is directly proportional to (1) the square of the current, (2) the resistance of the conductor, and (3) time. Other forms of H are as follows:

$$H = I^2Rt = \frac{V^2}{R} = VIt$$

Joule's heating effect is irreversible. The resistor will become hot (and not cool down), irrespective of the direction of current. As $H \propto I^2$, heating effect of current is common to both dc and ac. This is why instruments and appliances such as filament bulb, heater, geyser, press, toaster, etc. work on both dc and ac.

ELECTRIC POWER PRODUCED IN THE CIRCUIT

The electric power generated in the circuit is the energy produced in the resistor per unit time. Thus,

$$P = \frac{H}{t} = I^2R = \frac{V^2}{R} = VI$$

UNITS OF ELECTRIC ENERGY AND ELECTRIC POWER

Electric energy can be expressed in units such as J, cal, kWh, etc.

$$1 \text{ cal} = 4.18 \text{ J} \simeq 4.2 \text{ J}$$

Relation between kWh and J:

$$1 \text{ kWh} = 1000 \text{ W} \times \text{h} = 1000 \text{ W} \times 3600 \text{ s}$$
$$= 3.6 \times 10^6 \text{ Ws} = 3.6 \times 10^6 \text{ J}$$

1 kWh is the energy consumed by an appliance of power 1 kW when it runs for 1 h.

Commercial unit: 1 kWh is one unit of electricity. To calculate the number of units, we can use the following relation:

$$\text{Number of units} = \frac{\text{watt} \times \text{hour}}{1000}$$

The energy dissipated in kWh can be calculated using the following relation:

$$E = \frac{V \text{ (in volt)} \times I \text{ (in ampere)} \times t \text{ (in hour)}}{1000}$$

Electric power can be expressed in units such as W, kW, MW, hp; they share the following relationships: 1 kW = 10^3 W, 1 MW = 10^6 W, 1 hp = 746 W.

Important Points:

- If the resistances are connected in series, then using $P = I^2R$, the power developed will be higher in the resistor of higher value as current will be same in all resistors.
- If the resistances are connected in parallel, then using $P = V^2/R$, the power developed will be higher in the resistor of lower value as potential will be same across all resistors.

ILLUSTRATION 7.1

A series battery of 6 lead accumulators, each of emf 2.0 V and internal resistance 0.50 Ω, is charged by a 100 V dc supply. What series resistance should be used in the charging circuit in order to limit the current to 0.8 A? Using the required resistor, obtain:

(a) the power supplied by the dc source
(b) the power dissipated as heat and
(c) chemical energy stored in the battery in 15 min.

Sol. We are given that emf of the charging supply, $\varepsilon_1 = 100$ V
The total emf of the battery which is being charged,

$$\varepsilon_2 = 2.0 \times 6 = 12 \text{ V}$$

Total internal resistance of the battery, $r = 0.50 \times 6 = 3.0\ \Omega$

when the battery is being charged, the emf of the battery (ε_2) acts in a direction opposite to that (ε_1) of the dc supply. Hence, effective emf in the charging circuit,

$$\varepsilon = \varepsilon_1 - \varepsilon_2 = 100 - 12 = 88 \text{ V}$$

Let R be the resistance required to limit the current (I) to 8.0 A. Clearly,

$$I = \frac{\varepsilon}{R+r} \quad \text{or} \quad 8 = \frac{88}{R+3} \quad \text{or} \quad R = 8\,\Omega$$

(a) Power supplied by the dc source, $P = \varepsilon_1 I$

Hence, $P = 100 \times 8 = 800$ W

(b) Power dissipated as heat,

$$P' = I^2(R+r) = (8)^2(8+3) = 704 \text{ W}$$

(c) Energy stored in the battery in 15 min (i.e., 15 × 60s)

$$= (P - P')(15 \times 60) = (800 - 704) \times 900 = 86400 \text{ J}$$

ILLUSTRATION 7.2

A dry cell of emf 1.5 V and internal resistance 0.10 Ω is connected across a resistor in series with a very low resistance ammeter. When the circuit is switched on, the ammeter reading settles to a steady value of 2.0 A.

(a) What is the steady rate of chemical energy consumption of the cell?

(b) What is the steady rate of energy dissipation inside the cell?

(c) What is the steady rate of energy dissipation inside the resistor?

(d) What is the steady power output of the source?

Sol.

(a) Rate of chemical energy consumption of the cell is

$$EI = 1.5 \times 2 = 3 \text{ W}$$

(b) Rate of energy dissipation inside the cell is

$$I^2r = (2)^2 \times 0.1 = 0.4 \text{ W}$$

(c) Rate of energy dissipation inside the resistor is

$$I^2R = EI - I^2r = 3 - 0.4 = 2.6 \text{ W}$$

(d) Power output of the source = $I^2R = 2.6$ W

ILLUSTRATION 7.3

Two wires of same mass, having ratio of lengths 1:2, density 1:3, and resistivity 2:1, are connected one by one to the same voltage supply. The rate of heat dissipation in the first wire is found to be 10 W. Find the rate of heat dissipation in the second wire.

Sol. Given

$$\frac{l_1}{l_2} = \frac{1}{2}, \frac{d_1}{d_2} = \frac{1}{3}, \frac{\rho_1}{\rho_2} = \frac{2}{1}$$

$$m = A_1 l_1 d_1 = A_2 l_2 d_2$$

$$\therefore \quad \frac{P_2}{P_1} = \frac{V^2/R_2}{V^2/R_1}$$

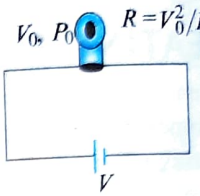
$$= \frac{R_1}{R_2} = \frac{\rho_1 l_1 A_2}{\rho_2 l_2 A_1} = \frac{\rho_1 l_1^2 d_1}{\rho_2 l_2^2 d_2}$$

$$= \frac{2}{1} \times \left(\frac{1}{2}\right)^2 \times \frac{1}{3} = \frac{1}{6}$$

or $$P_2 = \frac{P_1}{6} = \frac{10}{6} = \frac{5}{3} \text{ W}$$

Important Point:

Resistance of a bulb (or other appliances): Let a bulb be designed to operate on a voltage V_0 and its power indicated on it be P_0 (see figure). The resistance of the bulb is given by $R = V_0^2/P_0$.



Now let a potential difference of V is applied across this bulb, then power consumed is given by

$$P = \frac{V^2}{R} = \left(\frac{V}{V_0}\right)^2 P_0$$

If $V = V_0$, then $P = P_0$. The above formula is very convenient to calculate the power consumption when the applied voltage is different from the specified one.

An electric appliance consumes the specified power P_0 only if it runs at the specified voltage V_0. If the applied voltage V_A is greater than the specified voltage, the appliance may get damaged as in this situation, $I = V_A/R$ will exceed its current capacity $I_C = V_0/R$. Further, if an appliance is made to run at a voltage lower than the specified, then true power consumption will be less than the specified value.

ILLUSTRATION 7.4

A 100 W bulb is designed to operate on a potential difference of 230 V.

(a) Find the resistance of the bulb.

(b) Find the current drawn by the bulb if it is operated at a potential difference for which it is designed.

(c) Find the current drawn and power consumed by the bulb if it is connected to a 200 V supply.

Sol. Power rating of the bulb is $P_0 = 100$ W, voltage and rating of the bulb is $V_0 = 230$ V.

(a) Resistance of the bulb is

$$R = \frac{V_0^2}{P_0} = \frac{(230)^2}{100} = 529\,\Omega$$

(b) Current drawn is

$$I = \frac{V_0}{R} = \frac{230}{529} = \frac{10}{23} \text{ A}$$

(c) $I = \frac{200}{529}$ A $\Rightarrow P = I^2R = \left(\frac{200}{529}\right)^2 \times 529 = 75.6$ W

ILLUSTRATION 7.5

A 500 W heating unit is designed to operate from a 200 V line. By what percentage will its heat output drop if the line voltage drops to 160 V? Find the heat produced by it in 10 min.

Sol. Actual power consumed is

$$P = \left(\frac{V}{V_0}\right)^2 P_0 = \left(\frac{160}{200}\right)^2 500 = 320 \text{ W}$$

Heat output drop is 500 – 320 = 180 W. Percentage heat drop is

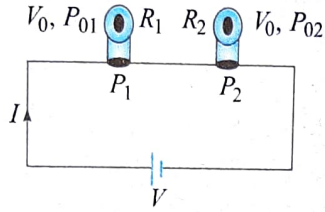
$$\frac{180}{500} \times 100 = 36\%$$

Thus, heat produced in 10 min (600 s) is given by

$$H = 320 \times 600 = 192\,000 \text{ J} = 192 \text{ kJ}$$

Important Points:

- **Two bulbs connected in series:** Suppose two bulbs of same voltage rating V_0 and power ratings P_{01} and P_{02} are connected in series. Here $P_{01} > P_{02}$. Suppose potential V is applied across them as shown in figure.



Resistances of the bulbs are given by

$$R_1 = \frac{V_0^2}{P_{01}}, R_2 = \frac{V_0^2}{P_{02}}$$

Suppose powers produced in them are P_1 and P_2, respectively. Then $P_1 = I^2R_1$ and $P_2 = I^2R_2$. Now

$$P_{01} > P_{02} \Rightarrow R_1 < R_2 \Rightarrow P_1 < P_2$$

It means the bulb having more power rating will consume less power when connected in series. So the total power produced is

$$P = P_1 + P_2 = \frac{V^2}{R_1 + R_2}$$

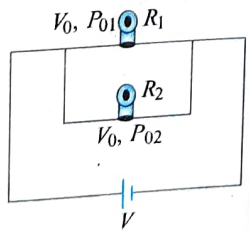
$$= \frac{V^2}{\frac{V_0^2}{P_{01}} + \frac{V_0^2}{P_{02}}} = \left(\frac{V}{V_0}\right)^2 \left(\frac{P_{01}P_{02}}{P_{01} + P_{02}}\right)$$

If $V = V_0$, then

$$P = \frac{P_{01}P_{02}}{P_{01} + P_{02}} \text{ or } \frac{1}{P} = \frac{1}{P_{01}} + \frac{1}{P_{02}}$$

Note: If any one bulb in a series gets fused, then others will not glow.

- **Two bulbs connected in parallel:** Suppose two bulbs of same voltage rating V_0 and power ratings P_{01} and P_{02} are connected in parallel. Here $P_{01} > P_{02}$. Suppose potential V is applied across them as shown in figure.



Resistances of the bulbs is

$$R_1 = \frac{V_0^2}{P_{01}}, \quad R_2 = \frac{V_0^2}{P_{02}}$$

Let powers produced in them be P_1 and P_2, respectively. Then $P_1 = V^2/R_1$ and $P_2 = V^2/R_2$.

$$P_{01} > P_{02} \Rightarrow R_1 < R_2 \Rightarrow P_1 > P_2$$

It means that the bulb having more power rating will consume more power when connected in parallel. So the total power produced is

$$P = P_1 + P_2 = \frac{V^2}{R_1} + \frac{V^2}{R_2} = \frac{V^2}{V_0^2}P_{01} + \frac{V^2}{V_0^2}P_{02}$$

$$= \left(\frac{V}{V_0}\right)^2 (P_{01} + P_{02})$$

If $V = V_0$, then $P = P_{01} + P_{02}$.

Note: If one of the bulbs in a parallel connection gets fused, then others will continue to glow.

ILLUSTRATION 7.6

Two bulbs are marked 220 V–100 W and 220 V–50 W.

(a) Which bulb will produce more illumination if they are connected in parallel to a 220 V supply?

(b) Which bulb will produce more illumination if they are connected in series to a 220 V supply?

(c) Also find the total power consumed by both the bulbs in each of the two parts above.

Sol.

(a) The first bulb will produce more power. In parallel, the more the power rating, the more the power produced.

(b) The second bulb will produce more power. In series, the more the power rating, the lesser the power produced.

(c) In the first part,

In the second part,

$$P = P_{01} + P_{02} = 100 + 50 = 150 \text{ W}.$$

In second part:

$$P = \frac{P_{01}P_{02}}{P_{01} + P_{02}} = \frac{100 \times 50}{150} = \frac{100}{3} \text{ W}$$

ILLUSTRATION 7.7

Two bulbs are rated 30 W–200 V and 60 W–200 V. They are connected with a 400 V power supply. Find which bulb will get fused if they are connected in **(a)** series and **(b)** parallel.

Sol.

(a) $R_1 = \frac{(200)^2}{30}\Omega, R_2 = \frac{(200)^2}{60}\Omega \Rightarrow R_1 > R_2$

Hence, voltage across the first bulb will be greater than 200 V. So it will get fused.

(b) In parallel, potential across both the bulbs will be same and equal to 400 V. So both will get fused.

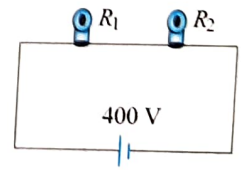


ILLUSTRATION 7.8

An electric tea kettle has two heating coils. When one of the coils is switched on, boiling begins in 6 min. When the other coil is switched on, boiling begins in 8 min. In what time will the boiling begin if both coils are switched on simultaneously **(a)** in series and **(b)** in parallel.

Sol. Let the power of first coil be P_1 and that of the second coil is P_2. Let H be the amount of heat required to boil water. Then $H = P_1 t_1 = P_2 t_2$, where $t_1 = 6$ min and $t_2 = 8$ min.

(a) When the coils are connected in series,

$$P = \frac{P_1 P_2}{P_1 + P_2}$$

$$t = \frac{H}{P} = H\left[\frac{1}{P_1} + \frac{1}{P_2}\right] = H\left[\frac{t_1}{H} + \frac{t_2}{H}\right]$$

$$= t_1 + t_2 = 6 + 8 = 14 \text{ min}$$

(b) When the coils are connected in parallel,

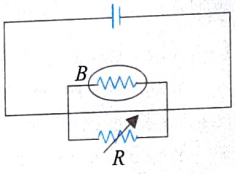
$$P = P_1 + P_2$$

$$t = \frac{H}{P} = \frac{H}{P_1 + P_2} = \frac{H}{\frac{H}{t_1} + \frac{H}{t_2}}$$

$$= \frac{t_1 t_2}{t_1 + t_2} = \frac{6 \times 8}{6 + 8} = 3.43 \text{ min}$$

ILLUSTRATION 7.9

A bulb B is connected to a source having constant emf and some internal resistance. A variable resistance R is connected in parallel to the bulb. If the resistance R is increased, how will it affect the



(a) Brightness of the bulb?

(b) Power spent by the source?

Sol. Let resistance of the bulb be R_b.

The equivalent of two parallel resistances is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_b} + \frac{1}{R} \Rightarrow R = \frac{RR_b}{R + R_b} = \frac{R_b}{1 + \frac{R_b}{R}}$$

If R is increased, R_{eq} must also increase. This increases the overall resistance of the circuit and the current through the cell drops.

As $i = \frac{\varepsilon}{r + R_{eq}}$ and $P_{battery} = \varepsilon i$

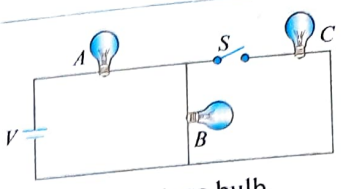
The potential drop across the bulb = potential difference across the battery

Hence $V_{battery} = \varepsilon - ir = V_{bulb}$

As the current i is decreasing, it means $V_{battery}$ should increase. Hence the potential difference across the bulb will increase.It means power spent by the cell decrease. Potential drop across the internal resistance of the cell drops and the potential difference across the bulb increases. Hence, the bulb becomes brighter.

ILLUSTRATION 7.10

A and B are two identical bulbs of 40 W connected to a $V = 12$ volt cell. Switch S is closed to connect a third bulb C in the circuit. What happens to brightness of bulb A? Answer for two cases.



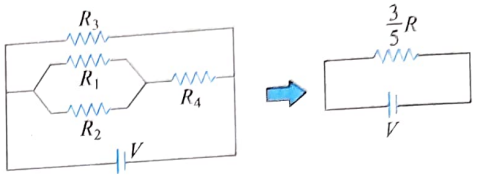
(a) Bulb C is a very high wattage bulb.

(b) Bulb C is a very low wattage bulb.

All the three bulbs have rated voltage of 12 volt.

Sol. B and C are in parallel.

Therefore equivalent resistance of B and C will be smaller than that of B.



Hence, the overall resistance decreases.

Current through A will increase and brightness of A will increase.

When C has high power rating, its resistance is very small. The equivalent of B and C is even smaller. Bulb A will nearly glow at its full brightness.

If C has low power rating, its resistance is too high.

Thus equivalent resistance of B and C will be only slightly less than that of B.

The brightness of A will increase slightly.

Important Points:

- Let a resistance R under a potential difference V dissipate power, then

$$P = \frac{V^2}{R}$$

So if the resistance is changed from R to R/n keeping V same, the power consumed will be

$$P' = \frac{V^2}{R/n} = n\frac{V^2}{R} = nP$$

That is, if for a given voltage, resistance is changed from R to R/n, power consumed changes from P to nP.

- If n equal resistances are connected in series with a voltage source, then power dissipated will be

$$P_s = \frac{V^2}{nR} \qquad [\text{as } R_S = nR]$$

And if the same resistances are connected in parallel with the same voltage source,

$$P_p = \frac{V^2}{R/n} = \frac{nV^2}{R} \qquad [\text{as } R_P = R/n]$$

$$\therefore \quad \frac{P_p}{P_s} = n^2$$

or $P_p = n^2 P_s$

That is, power consumed by n equal resistors in parallel is n^2 times that of the power consumed in series, if V remains same.

CONCEPT APPLICATION EXERCISE 7.1

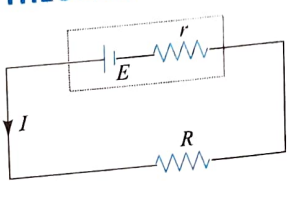
1. A heater joined in series with a 50 W bulb is connected to the mains. If the 50 W bulb is replaced by a 100 W bulb, then will the heater now give more heat, less heat, or same heat ? Why?
2. An electric bulb rated 220 V and 60 W is connected in series with another electric bulb rated 220 V and 40 W. The combination is connected across a source of emf 220 V. Which bulb will glow more?
3. We have a 30 W, 6 V bulb, which we want to glow by a supply of 120 V. What can be done for this?
4. Two wires of the same material and having the same uniform area of cross section are connected in an electric circuit. The masses of the wires are m and $2m$, respectively. When a current I flows through both of them connected in series, then find the ratio of heat produced in them in a given time.
5. Water boils in an electric kettle in 15 min after being switched on. Using the same main supply, should the length of the heating element be increased or decreased if the water is to be boiled in 10 min? Why?
6. A house is fitted with certain numbers of 100 W, 230 V incandescent lamps. The power to the house is fed by a generator producing the power at 240 V. The resistance of the wires from the generator to the house is 2 Ω. Find the maximum number of lamps that can be illuminated so that the voltage across none of the lamps drops below 230 V.
7. A house is fitted with seven tubelights of rating 220 V, 40 W each, two bulbs of rating 220 V, 60 W each, five fans each drawing a current of 0.4 A at 220 V, and a heater of resistance 48.4 Ω. The main line power supplied to the house is at 220 V. Calculate the bill for the month of January if tubelights and bulbs are used for 6 h daily, fans for 1 h daily, and heater for 10 h daily. The electricity is to cost ₹2 per unit.
8. Two bulbs are marked 200 V, 300 W and 200 V, 600 W, respectively. The bulbs are connected in series and the combination is connected to a 200 V supply.
   (a) Which bulb will produce more illumination?
   (b) Find the total power consumed by both the bulbs.
   (c) Find the total power consumed if both the bulbs were connected in parallel.
9. A 25 W and a 100 W bulb are joined in series and connected to the mains. Which bulb will glow brighter?

ANSWERS

1. Heater will give more heat
2. The bulb rates 220 V and 40 W will glow more
3. 22.8 Ω resistance should be connected in series with bulb
4. 1 : 2 **5.** Decreases **6.** 11 **7.** ₹ 796.08
8. (a) Ist bulb (b) 200 W (c) 900 W **9.** 25 W bulb

MAXIMUM POWER TRANSFER THEOREM

Suppose we want to find for what value of external resistance the maximum power will be drawn from a battery. For this, in the shown network (figure), let the



power developed in resistance R be

$$P = I^2R = \frac{E^2}{(R+r)^2}R \qquad \left(\text{as } I = \frac{E}{R+r}\right)$$

Now, for $dP/dR = 0$ (since P will be maximum if $dP/dR = 0$)

$$E^2\frac{(R+r)^2 - 2(R)(R+r)}{(R+r)^4} = 0$$

or $(r+R) = 2R$ or $r = R$

It means the power output is maximum, when the external resistance equals the internal resistance i.e., when $R = r$.

ILLUSTRATION 7.11

How will you connect (series and parallel) 24 cells each of internal resistance 1 Ω to get maximum power output across a load of 10 Ω?

Sol. Suppose there are m rows and each row has n cells. The total number of cells is $mn = 24$. Here

$$I = \frac{ne}{R + (nr/m)}$$

For maximum power, internal resistance equals the load resistance

$$\frac{nr}{m} = R \text{ or } n = 10m \text{ or } 10m^2 = 24 \text{ or } m = \sqrt{2.4} = 1.55$$

If $m = 1$, $n = 24$, $I = 24e/34$, then

$$P_1 = \left(\frac{24e}{34}\right)^2 \times 10 = 4.98e^2$$

If $m = 2$, $n = 12$, $I = 12e/16$, then

$$P_2 = \left(\frac{12e}{16}\right)^2 \times 10 = 5.625e^2$$

So we have two rows ($m = 2$) each containing 12 cells ($n = 12$) in series.

SOME APPLICATIONS

Fusing of bulb when it is switched on: Usually filament bulbs get fused when they are switched on. This is because with the rise in temperature, the resistance of the bulb increases and becomes constant in steady state. So the power consumed by the bulb (V^2/R) initially is more than that in steady state and hence the bulb glows more brightly in the beginning and may get fused.

ILLUSTRATION 7.12

Two wires made of tinned copper having identical cross section ($=10^{-6}$ m^2) and lengths 10 and 15 cm are to be used as fuses. Show that the fuses will melt at the same value of current in each case.

Sol. The temperature of the wire rises to a certain steady temperature when the heat produced per second by the current just becomes equal to the rate of loss of heat from its surface. Heat produced per second by the current is

$$P = I^2R = I^2\left(\frac{\rho l}{\pi r^2}\right) = \frac{I^2\rho l}{\pi r^2} \qquad \text{...(i)}$$

where l is the length, r is the radius, and ρ is the specific resistance. Let H be the heat lost per second per unit surface area of the wire. If we neglect the loss of heat from the end faces of the wire, then heat lost per second by the wire is $H \times$ surface area of wire $P' = H \times 2\pi r l$ ...(ii)

At steady state temperature, from (i) and (ii)

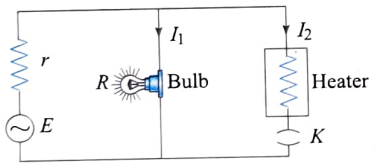
$$H \times 2\pi r l = \frac{I^2 \rho l}{\pi r^2}$$

or $$H = \frac{I^2 \rho}{2\pi^2 r^3} \quad \text{...(iii)}$$

From Eq. (iii) we note that the rate of loss of heat (H), which, in turn, depends on the temperature of the wire, is independent of the length of the wire. Hence, the fuses of two wires of the same values of r and ρ but of different lengths will melt for the same value of current in each case.

Decrease in the brightness of bulb after long use: Also due to evaporation of metal from the filament (which deposits as black substance on the inner side of glass wall), the filament of the bulb becomes thinner and thinner with use. This increases the resistance ($R = \rho L/\pi r^2$) of the bulb, and as $P = V^2/R$, the brightness of the light emitted by a bulb decreases gradually with time.

Decrease in brightness of a bulb in a room when a heavy current appliance is switched on: As shown in figure, if the bulb draws a current I_1 from the source, then the terminal voltage of source is $V = (E - I_1 r)$.



So the power consumed by the bulb is

$$P = \frac{V^2}{R} = \frac{(E - I_1 r)^2}{R}$$

When a heavy current appliance such as motor, heater, or geyser is switched on, it will draw a heavy current, say I_2, from the source so that the terminal voltage becomes

$$V' = [E - (I_1 + I_2)r] = (V - I_2 r) \quad (< V)$$

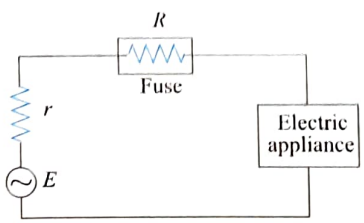
Hence the power consumed by the bulb will now be

$$P' = \frac{V'^2}{R} = \frac{(V - I_2 r)^2}{R} < P$$

So the brightness of the bulb decreases.

Note: If the source is ideal, i.e., $r = 0$, $V' = V = E$, then $P' = P$, i.e., there will be no change in the brightness of the bulb, if the source is ideal.

Fuse and its action: A fuse is a metallic conducting wire of 75% Pb and 25% Sn with low melting point and higher resistance and is in series with an appliance (figure).



It is a safety device that protects the appliance from getting damaged, by melting and opening the circuit, if the current in the circuit exceeds a specific predetermined value, called current capacity.

Long distance power transmission: When power is transmitted through a power line of resistance R, power loss will be I^2R. Now, let power P_0 is transmitted at voltage V, then $P_0 = VI$, i.e.,

$$I = \frac{P_0}{V}$$

So power loss is

$$\frac{P_0^2}{V^2} \times R$$

Since for a given power and line, P_0 and R are constant, power loss is proportional to $1/V^2$.

So if power is transmitted at high voltage, power loss will be small and vice versa, for example, power loss at 22 kV is 10^{-4} times than that at 220 V. This is why long distance power transmission is carried out at high voltage.

- The wires supplying current to a bulb are not heated, but the filament of the bulb becomes hot because the resistance of the wires is very small in comparison to the resistance of the bulb. If only the wires are connected, the entire potential difference will be across the wires, and because their resistance is very small, a large amount of heat will be generated. This happens when the wires are short-circuited.
- The resistance of high electric power instrument will be smaller than that of low electric power instrument because for a given voltage, $P = V^2/R$. For example, iron, heater, geyser. The heating element of these appliances is made of nichrome. It is an alloy of Ni and Cr. Its resistivity is higher in comparison to platinum, tungsten, and copper. Nichrome is used because
  - it has high resistivity and high melting point
  - it is not oxidized when heated
  - it can be easily drawn into wires

  Resistivity is kept higher so that smaller length can be used, as

$$H = \frac{V^2}{R} t = \frac{V^2 At}{\rho l}$$

Incandescent electric lamp: An incandescent electric lamp consists of a metal filament generally made of tungsten. It is enclosed in a glass bulb with some inert gas and at suitable pressure. The filament gets heated, becomes white hot (known as incandescent stage), and starts emitting white light. The filament should have high melting point.

ILLUSTRATION 7.13

A line having a total resistance of 0.2 Ω delivers 10 kW at 220 V to a small factory. Calculate the efficiency of transmission.

Sol. The current through transmission line,

$$I = \frac{10{,}000}{220} = \frac{500}{11} \text{ A}$$

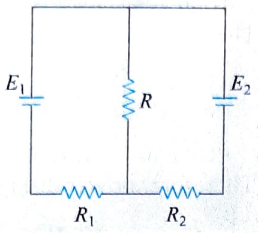
$$\text{Loss} = I^2 r = \left(\frac{500}{11}\right)^2 \times 0.2 = 413.22$$

So efficiency is

$$\eta = \frac{10{,}000}{10{,}000 + 413.22} \times 100 = 96\%$$

ILLUSTRATION 7.14

In the circuit shown in figure, the emfs of batteries are E_1 and E_2 which have internal resistances R_1 and R_2. At what value of the resistance R will the thermal power generated in it be the highest? What it is?



Sol. Here two batteries are in parallel. The thermal power generated in R will be maximum when, total internal resistance of the battery = total external load resistance. The total internal resistance of equivalent battery,

$$r = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

Hence for maximum power, $r = R = \dfrac{R_1 R_2}{R_1 + R_2}$

$$E_{eq} = \frac{\left(\frac{E_1}{R_1} + \frac{E_2}{R_2}\right)}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} = \left(\frac{E_1 R_2 + E_2 R_1}{R_1 + R_2}\right)$$

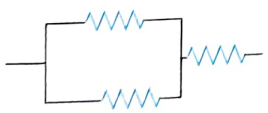
Net resistance in the circuit, $R_{net} = \dfrac{2R_1 R_2}{R_1 + R_2}$

∴ The current in the circuit, $i = \dfrac{E_{eq}}{R_{net}} = \dfrac{E_1 R_2 + E_2 R_1}{2R_1 R_2}$

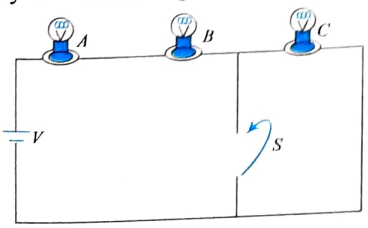
Maximum power through R, $P_{max} = i^2 R = \dfrac{(E_1 R_2 + E_2 R_1)^2}{4R_1 R_2 (R_1 + R_2)}$

CONCEPT APPLICATION EXERCISE 7.2

1. Each of the three resistors in figure has a resistance of 2 Ω and can dissipate a maximum of 18 W without becoming excessively heated. Find the maximum power the circuit can dissipate.



2. A series circuit consists of three identical lamps connected to a battery as shown in figure.



When the switch S is closed, what happens
(a) to the intensities of lamps A and B,
(b) to the intensity of lamp C,
(c) to the current in the circuit, and
(d) to the voltage drop across the three lamps?
Does the power dissipated in the circuit increase, decrease, or remain the same?

3. An electric motor operating on a 50 V dc supply draws a current of 12 A. If the efficiency of the motor is 30%, estimate the resistance of the windings of the motor.
4. A fuse with a circular cross-sectional radius of 0.15 mm blows at 15 A. What should be the radius of the cross section of a fuse made of the same material that blows at 30 A?
5. A motor operating on 120 V draws a current of 2 A. If the heat is developed in the motor at the rate of 9 cal s^{-1}, what is its efficiency?
6. The walls of a closed cubical box of edge 40 cm are made of a material of thickness 1 mm and thermal conductivity 4×10^{-4} cals^{-1}cm^{-1} °C^{-1}. The interior of the box is maintained at 100°C above the outside temperature by a heater placed inside the box and connected across 400 V dc. Calculate the resistance of the heater.
7. Two tungsten lamps with resistances R_1 and R_2, respectively, are connected first in parallel and then in series in a lighting circuit of negligible internal resistance. Given that $R_1 > R_2$.
   (a) Which lamp will glow more brightly when they are connected in parallel?
   (b) If the lamp of resistance R_1 now burns out, how will the net illumination produced change?
   (c) Which lamp will glow more brightly when they are connected in series?
   (d) If the lamp of resistance R_2 now burns out and lamp R_1 alone is plugged in, will the net illumination increase or decrease?
8. n identical bulbs are connected in series and illuminated by a power supply. One of the bulbs gets fused. The fused bulb is removed, and the remaining bulbs are again illuminated by the same power supply. Find the fractional change in the illumination of **(a)** all the bulbs and **(b)** one bulb.
9. An electric motor is designed to work at 100 V and draws a current of 6 A. The output power supplied by the motor is 150 W, and the remaining goes to heat. What is the resistance of the windings of the motor and its percentage efficiency?
10. A voltage stabilizer restricts the voltage output to 220 V ± 1%. If the electric bulb rated 220 V, 100 W is connected to it, what will be the minimum and maximum power consumed by it?
11. The efficiency of a cell when connected to a resistance R is 60%. What will be its efficiency if the external resistance is increased by six times?

ANSWERS

1. 27 W
2. (a) Intensities of bulbs A and B will increase and become 2.25 of their initial values (b) Zero (c) The current in the

circuit will increase and will become 1.5 times its initial value (d) The voltage drop across A and B will increase while across C it will decrease and become zero. Power dissipated in the circuit increases and becomes 1.5 times its initial value.

3. 2.9 Ω **4.** 0.238 mm **5.** 84.25% **6.** 9.92 Ω

7. (a) Bulb having lower resistance will shine brightly (b) Net illumination will decrease (c) The lamp having higher resistance will glow more brightly (d) Illumination gets increased

8. (a) $\frac{1}{n-1}$ (b) $\frac{2n-1}{(n-1)^2}$ **9.** 12.5 Ω, 25%

10. $P_{max} = 102$ W, $P_{min} = 98$ W **11.** 90%

Solved Examples

EXAMPLE 7.1

A series battery of six cells each of emf 2 V and internal resistance 0.5 Ω is charged by a 100 V dc supply. What resistance should be used in the charging circuit in order to limit the charging current to 8 A. Using this relation, obtain **(a)** the power supplied by the dc source, **(b)** the power dissipated as heat, and **(c)** the chemical energy stored in the battery in 15 min.

Sol. **Given:** number of cells, $n = 6$; emf of each cell, $E = 2$V; internal resistance of each cell, $r = 0.5$ Ω; charging voltage, $V = 100$ V. Let R be the resistance used in the series of the circuit while charging the cells. Then current in the circuit will be

$$i = \frac{V - nE}{nr + R}$$

or $$R = \frac{V - nE}{i} - nr = \frac{100 - 6 \times 2}{8} - 6 \times 0.5 = 11 - 3 = 8\ \Omega$$

(a) Power supplied by dc source is $V \times i = 100 \times 8 = 800$ W

(b) Power dissipated as heat is

$i^2(R + nr) = 8^2\,(8 + 6 \times 0.5) = 704$ W

(c) Rate at which the chemical energy is stored is

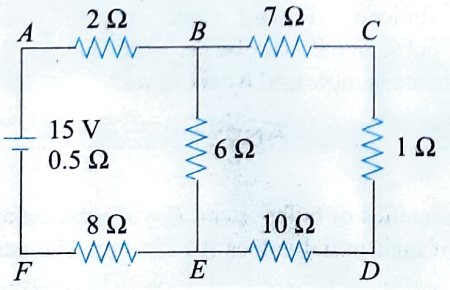
$800 - 704 = 96$ W

Therefore, chemical energy stored in 15 min is

$96 \times 15 \times 60 = 86400$ J

EXAMPLE 7.2

Determine the current through the battery of internal resistance 0.5 Ω for the circuit shown in figure. How much power is dissipated in 6 Ω resistance?



Sol. Resistance of arm $BCDE$ is $7 + 1 + 10 = 18$ Ω. Here 18 Ω and 6 Ω are in parallel. Their effective resistance is

$$R_p = \frac{18 \times 6}{18 + 6} = \frac{18 \times 6}{24} = 4.5\ \Omega$$

Total resistance of the circuit is $2 + 4.5 + 8 + 0.5 = 15$ Ω. Therefore, current through the circuit is

$$i = \frac{15}{15} = 1\text{ A}$$

Potential difference across B and E is $i \times R_p = 1 \times 4.5 = 4.5$ V. Therefore, power dissipated as heat due to resistance 6 Ω is

$$P_{6\,\Omega} = \frac{(4.5)^2}{6} = 3.375\text{ W}$$

EXAMPLE 7.3

Two uniform wires of same material, each weighing 1 g but one having double the length of the other, are connected in series, carrying a current of 10 A. The length of the longer wire is 20 cm. Calculate the rate of consumption of energy in each of the two wires. Which wire gets hotter? The density of the material of the wire is 11 gcm^{-3}, and the specific resistance of the material is 20×10^{-5} Ω cm.

Sol. Let a_1 and a_2 be the areas of cross section of the short and the long wires, respectively. Since mass = volume × density $= al\rho$,

$$1 = a_1 \times 10 \times 11 = a_2 \times 20 \times 11$$

or $$a_1 = \frac{1}{10 \times 11}\text{ cm}^2 \quad \text{and} \quad a_2 = \frac{1}{20 \times 11}\text{ cm}^2$$

$$\therefore \quad R_1 = 20 \times 10^{-5} \times \frac{10}{1/(10 \times 11)}$$

$$= 20 \times 10^{-5} \times 10 \times 10 \times 11 = 22 \times 10^{-2}\ \Omega$$

$$R_2 = 20 \times 10^{-5} \times \frac{20}{1/(20 \times 11)} = 88 \times 10^{-2}\ \Omega$$

And rate of heat produced is

$$H_1 = I^2R_1 = (10)^2 \times 22 \times 10^{-2} = 22\text{ W}$$

and $$H_2 = I^2R_2 = (10)^2 \times 88 \times 10^{-2} = 88\text{ W}$$

Thus, the wire of longer length gets hotter.

EXAMPLE 7.4

In a house having 220 V line, the following appliances are operating:

(i) a 60 W bulb, (ii) a 1000 W heater, and (iii) a 40 W radio. Calculate **(a)** the current drawn by the heater and **(b)** the current passing through the fuse in the main line.

Sol. Here, $V = 220$ V; $P_1 = 60$ W; $P_2 = 1000$ W; $P_3 = 40$ W.

(a) Current drawn by heater is

$$\frac{P_2}{V} = \frac{1000}{220} = \frac{50}{11}\text{ A}$$

(b) Current drawn by bulb is

$$\frac{P_1}{V} = \frac{60}{220} = \frac{3}{11}\text{ A}$$

Current drawn by radio is

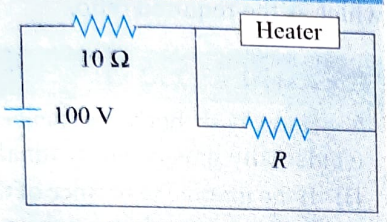
$$\frac{P_3}{V} = \frac{40}{220} = \frac{2}{11} \text{ A}$$

Current passing through the fuse for the line is

$$i_{\text{fuse}} = \frac{50}{11} + \frac{3}{11} + \frac{2}{11} = 5 \text{ A}$$

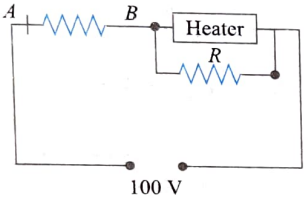
EXAMPLE 7.5

A heater is designed to operate with a power of 1000 W in a 100 V line. It is connected, in combination with a resistance R, to a 100 V mains as shown in figure. What should be the value of R such that heater may operate with a power of 62.5 W?



Sol. The resistance of the heater is

$$R = \frac{V^2}{P} = \frac{100 \times 100}{1000} = 10 \ \Omega$$



The power on which it operates is 62.5 W. Therefore,

$$V = \sqrt{R \times P'} = \sqrt{10 \times 62.5} = \sqrt{625} = 25 \text{ V}$$

So the potential drop across AB is 75 V.

Therefore, the current in AB is

$$I = \frac{V}{R} = \frac{75}{10} = 7.5 \text{ A}$$

This current gets divided into two parts. Let I_1 be the current that passes through the heater. Therefore,

$$25 = I_1 \times 10 \quad \text{or} \quad I_1 = 2.5 \text{ A}$$

Hence, the current through R is 5 A.

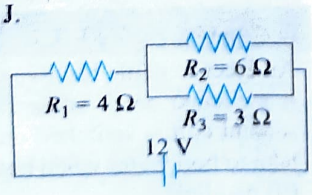
Applying Ohm's law across R, we get

$$25 = 5 \times R \quad \text{or} \quad R = 5 \ \Omega$$

EXAMPLE 7.6

(i) Find the time taken by a filament of 200 W to heat 500 ml of water from 25°C to 75°C. Specific heat of water is 1 $\text{calg}^{-1}\,^{\circ}\text{C}^{-1}$. Take 1 cal = 4.2 J.

(ii) Find the power produced by each resistor shown in figure. If R_1 is dipped in 1000 ml of water at 30°C, find the time taken by it to boil the water.



Sol.

(i) Heat received by water, $Q = mc\Delta T$

Q = heat supplied by filament = Pt

$\Rightarrow \quad Pt = mc\Delta T$

$$200t = 0.5 \times 4200 \times 50 \Rightarrow t = 525 \text{ s}$$

(ii) Equivalent resistance of the circuit, $R_{eq} = 4 + \frac{6 \times 3}{6+3} = 6 \ \Omega$

Current through resistance R_1, $I_1 = \frac{12}{6} = 2$ A

Current through resistance R_2, $I_2 = \frac{2}{3}$ A

Current through resistance R_3, $I_3 = \frac{4}{3}$ A

Power consumed in R_1, $P_1 = I_1^2 R_1 = 2^2 \times 4 = 16$ W

Power consumed in R_2, $P_1 = \left(\frac{2}{3}\right)^2 \times 6 = \frac{8}{3}$ W

Power consumed in R_3, $P_3 = \left(\frac{4}{3}\right)^2 \times 3 = \frac{16}{3}$ W

Now heat supplied by filament = heat received by water

$\Rightarrow \quad P_1 t = mc\Delta T$

$$16t = 1.0 \times 4200 \times 70 \Rightarrow t = \frac{4200 \times 70}{16} = 18{,}375 \text{ s}$$

EXAMPLE 7.7

A heating coil of 2000 W is immersed in water. How much time will it take in raising the temperature of 1 L of water from 4°C to 100°C? Only 80% of the thermal energy produced is used in raising the temperature of water.

Sol. Here, $P = 2000$ W, $t = ?$ (in seconds)

Volume of water is 1 L = 1000 cm^3

Mass of water is,

m = volume × density = 1000 × 1 = 1000 g

Rise in temperature is $\theta_2 - \theta_1 = 100 - 4 = 96^{\circ}\text{C}$

We know that the specific heat of water is

$c = 1 \ \text{calg}^{-1}\,^{\circ}\text{C}^{-1}$

Therefore, heat taken by water is

$mc\,(\theta_2 - \theta_1) = 1000 \times 1 \times 96 = 96000$ cal

Energy spent in heating the coil is $Pt = 2000 \times t$

Useful energy produced is 80% = $2000 \times t \times 80/100$ J

Useful heat produced is

$$= \frac{2000 \times t \times 80}{100 \times 4.2} \text{ cal}$$

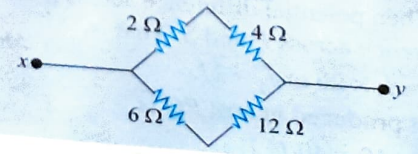
As this heat is taken by water,

$$\frac{2000 \times t \times 80}{100 \times 4.2} = 96{,}000$$

or $$t = \frac{96{,}000 \times 100 \times 4.2}{2000 \times 80} = 252 \text{ s}$$

EXAMPLE 7.8

Consider the following circuit (figure) where some resistances have been arranged in a definite order. With the given condition that heat produced by 6 Ω resistance is 60 cals^{-1} due to the current flowing through it, find out the heat produced across 2 Ω resistance in calorie per second.



Sol. Same current flows through resistances connected in series. Heat produced in the 6 Ω resistance is I^2R/J. So

$$60 = \frac{I^2 \times 6}{4.2} \text{ or } I^2 = 42 \text{ or } I = \sqrt{42} \text{ A}$$

Now the voltage drop across x and y is

$$(6+12)\sqrt{42} = 18\sqrt{42} \text{ V}$$

As this potential drop is same in every area of a parallel circuit, the potential drop across the upper part of the circuit is the same. Therefore, current through the 2 Ω and 4 Ω resistances is

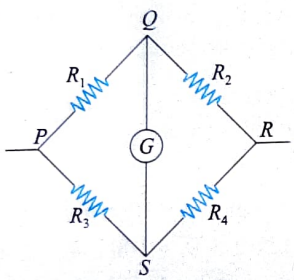
$$(18\sqrt{42})/6 = 3\sqrt{42} \text{ A}$$

Hence, the heat produced across the 2 Ω resistance is

$$\frac{I^2R}{J} = \frac{9 \times 42 \times 2}{4.2} = 180 \text{ cal}$$

EXAMPLE 7.9

Consider a Wheatstone bridge $PQRS$ as shown in figure where current I is in the circuit of four resistances 10, 20, 20, and 40 Ω. Find the ratio of the heat generated in the four arms PQ, QR, PS, and SR.



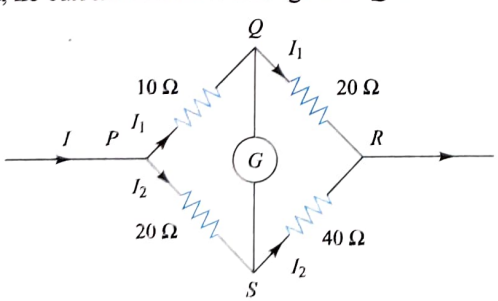
Sol. Given $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 20\ \Omega$, and $R_4 = 40\ \Omega$. Now,

$$\frac{R_1}{R_2} = \frac{10}{20} = \frac{1}{2}$$

and $$\frac{R_3}{R_4} = \frac{20}{40} = \frac{1}{2}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Hence, Wheatstone bridge is balanced. Now as the bridge is balanced, no current will flow through arm QS.



Let I_1 and I_2 be the currents flowing in arms PQ and PS, respectively. Then potential difference across P and Q is equal to potential difference across P and S. That is,

$$I_1 \times 10 = I_2 \times 20 \text{ or } I_1 = 2I_2$$

Therefore, heat produced in arm PQ is

$$H_1 = I_1^2 \times 10 = 40\, I_2^2 \text{ J}$$

Also heat produced in arm QR is

$$H_2 = I_1^2 \times 20 = 80\, I_2^2 \text{ J}$$

Similarly, heat produced in arm PS is

$$H_3 = I_2^2 \times 20 = 20\, I_2^2 \text{ J}$$

And heat produced in arm SR is

$$H_4 = I_2^2 \times 40 = 40\, I_2^2 \text{ J}$$

$\therefore H_1 : H_2 : H_3 : H_4 = 40\, I_2^2 : 80\, I_2^2 : 20\, I_2^2 : 40\, I_2^2 = 2 : 4 : 1 : 2$, which is the required ratio.

EXAMPLE 7.10

A person with body resistance between his hands of 10 kΩ accidentally grasps the terminals of a 18 kV power supply.

(i) If the internal resistance of the power supply is 2000 Ω, what is the current through the person's body?

(ii) What is the power dissipated in his body?

(iii) If the power supply is to be made safe by increasing its internal resistance, what should the internal resistance be for the maximum current in the above situation to be 1.00 mA or less?

Sol. Given $R = 10\ \text{k}\Omega$ and $V = 18$ kV.

(i) To find the current flowing through the body, we need to sum up the resistances present in the circuit and divide the voltage by it.

$$I = \frac{V}{R+r} = \frac{18 \times 10^3}{10 \times 10^3 + 2 \times 10^3} = \frac{18 \times 10^3}{12 \times 10^3}$$

$$= \frac{3}{2} = 1.5 \text{ A}$$

(ii) Power dissipated is

$$V_1 = I^2R = (1.5)^2\,(10000) = 2.25 \times 10000$$
$$= 22500 = 22.5 \text{ kW}$$

(iii) To find the internal resistance for the safe limit of power. we can use the formula as in part (i). The only difference here is I is given and r is to be calculated.

$$R + r = \frac{V}{I} = \frac{18 \times 10^3}{1 \times 10^{-3}} = 18 \times 10^6$$

or $$r = 18 \times 10^6 - 10 \times 10^3 = (18\ \text{M}\Omega - 10\ \text{k}\Omega)$$
$$\cong 18\ \text{M}\Omega$$

EXAMPLE 7.11

An electric kettle has two coils of same power. When one coil is switched on, it takes 15 min to boil water, and when the second coil is switched on, it takes 30 min. How long will it take to boil water when both the coils are used in **(i)** series and **(ii)** parallel?

Sol. Heat produced in resistance R in time t is

$$H = Pt = \frac{V^2}{R}t$$

For coil 1,

$$H_1 = \frac{V^2}{R_2}(15 \times 60) \qquad \text{...(i)}$$

For coil 2,

$$H_2 = \frac{V^2}{R_2}(15 \times 60) \qquad \ldots(ii)$$

But according to the given problem,

$$H_1 = H_2,$$

i.e. $$\frac{15}{R_1} = \frac{30}{R_2}$$

or $$R_2 = 2R_1 \qquad \ldots(iii)$$

(i) When both the coils are used in series, we have

$$H_s = \frac{V^2}{(R_1 + R_2)} t_S = \frac{V^2}{3R_1} \times t_S \qquad [\text{as } R_2 = 2R_1]$$

Here $H_S = H_1\ (= H_2)$, so

$$\frac{V^2}{R_1}(15 \times 60) = \frac{V^2}{3R_1} t_S$$

or $t_S = (45 \times 60)\,\text{s} = 45\,\text{min}$

(ii) When both the coils are used in parallel, we have

$$H_P = \left[\frac{V^2}{R_1} + \frac{V^2}{R_2}\right] \times t_P = \frac{3V^2}{2R_1} \times t_P \qquad [\text{as } R_2 = 2R_1]$$

According to the given problem,

$$H_P = H_1$$

or $$\frac{3V^2}{2R_1} \times t_P = \frac{V^2}{R_1} \times (15 \times 60)$$

or $t_P = (10 \times 60)\,\text{s} = 10\,\text{min}$

EXAMPLE 7.12

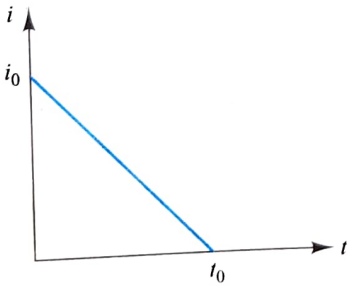
What amount of heat will be generated in a coil of resistance R due to a total charge q passing through it if the current in the coil

(i) decreases down to zero uniformly during a time interval t_0

(ii) decreases down to zero halving its value every t_0 seconds

Sol.

(i) The current decreases uniformly with time; therefore, i vs. t curve is a straight line as shown in figure with slope $m = -i_0/t_0$. Current as a function of time can be written as



$$i = i_0 - \left(\frac{i_0}{t_0}\right) t \qquad \ldots(i)$$

Area under the i–t graph gives the flow of charge q; therefore,

$$q = \frac{1}{2}(t_0)(i_0) \text{ or } i_0 = \frac{2q}{t_0}$$

Substituting in Eq. (i), we get

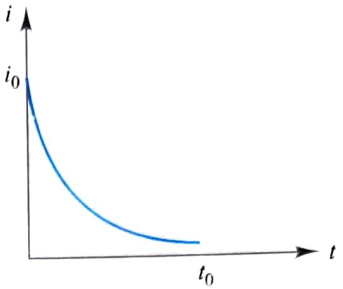
$$i = \frac{2q}{t_0}\left(1 - \frac{t}{t_0}\right) = \frac{2q}{t_0} - \frac{2qt}{t_0^2}$$

Heat produced in a time interval t_0 is

$$\int dH = \int i^2 R\, dt$$

or $$H = \int_0^{t_0} \left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)^2 R\, dt = \frac{4}{3}\frac{q^2 R}{t_0}$$

(ii) Here, current decreases from i_0 to zero exponentially with half-life of t_0. The i–t equation in this case is an exponential function like the radioactive decay law.



$$i = i_0 e^{-\lambda t}$$

where $\lambda = \ln(2)/t_0$

Total charge

$$q = \int_0^\infty i\, dt = \int_0^\infty i_0 e^{-\lambda t}\, dt = \left(\frac{i_0}{\lambda}\right)$$

or $i_0 = \lambda q$ or $i = (\lambda q)e^{-\lambda t}$

Heat produced in time interval dt is

$$dH = i^2 R\, dt = \lambda^2 q^2 e^{-2\lambda t} R\, dt$$

or $$H = \lambda^2 q^2 R \int_0^\infty e^{-2\lambda t}\, dt = \frac{q^2 \lambda R}{2} = \frac{q^2 R \ln(2)}{2 t_0}$$

EXAMPLE 7.13

A variable capacitor is adjusted to its lowest capacitance C_0 and is connected with a source of constant voltage V for a long time. The resistance of connecting wires is R. At $t = 0$, its capacitance starts to increase so that a constant current I starts to flow through the circuit. Calculate at time t,

(i) power supplied by the source

(ii) thermal power generated in the connecting wire

(iii) rate of increase of electrostatic energy stored in capacitor

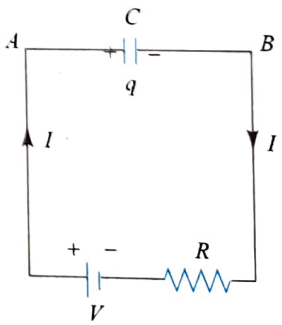
(iv) What do you infer from the above three results?

Sol.

(i) Since voltage V of the source is constant and the circuit draws constant current I from it, power supplied by the source is $P = VI$.

(ii) Thermal power generated in connecting wires is $H = I^2R$.

(iii) Since the initial capacitance of the capacitor was equal to C_0 and it was connected with the source for long time, initial charge on capacitor was $q_0 = C_0V$.



Since a constant current I starts to flow at $t = 0$, at time t, charge on capacitor becomes $q = (C_0V + It)$. At time t, the circuit will be as shown in figure. Potential difference across the capacitor is

$$V_C = V_A - V_B = (V - IR) \rightarrow \text{constant}$$

Therefore, electrostatic energy in capacitor at this instant is

$$U = \frac{1}{2} q V_C$$

Rate of increase of electrostatic energy is

$$\frac{dU}{dt} = \frac{1}{2} V_C \frac{dq}{dt} = \frac{1}{2}(V - IR)I$$

$$= \frac{1}{2}(VI - I^2R)$$

But power acting across the capacitor at this instant is

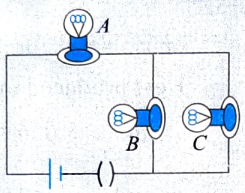
$$P_0 = P - H = (VI - I^2R)$$

while the rate of increase of electrostatic energy in the capacitor is half of it.

(iv) In fact, a force of attraction exists between the plates of the capacitor. When these surfaces move toward each other, capacitance increases. Hence, the remaining part of the power acting across the capacitor is used to increase kinetic energy of surface (plate) of the capacitor.

EXAMPLE 7.14

Three 60 W, 120 V light bulbs are connected across a 120 V power line as shown in figure. Find **(a)** the voltage across each bulb and **(b)** the total power dissipated in the three bulbs.



Sol.

(a) As bulbs B and C are in parallel, voltage across B and C will be the same, i.e., $V_B = V_C$. Further, if R is the resistance of each bulb (as bulbs are identical), the resistance of bulbs B and C together ($=R/2$) is in series with resistance R of bulb A and as in series, potential divides in proportion to resistance.

$$V_A = \frac{R}{R + 0.5R} \cdot V_s = \frac{2}{3} V_S = \frac{2}{3} \times 120 = 80\,\text{V}$$

$$V_B = V_C = \frac{0.5R}{R + 0.5R} \cdot V_s = \frac{1}{3} V_S = \frac{1}{3} \times 120 = 40\,\text{V}$$

[or $V_B = V_C = V - V_A = 120 - 80 = 40$ V]

(b) As actual power consumed by a bulb is

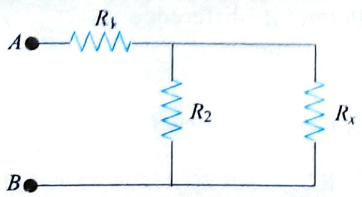
$$P = \frac{V_A^2}{R} = \left[\frac{V_A}{V_S}\right]^2 \times W \qquad \left[\because\ R = \frac{V_S^2}{W}\right]$$

So the total power consumption is

$$P = P_A + P_B + P_C = P_A + 2P_B = \frac{(4+2)60}{9} = 40\ \text{W}$$

EXAMPLE 7.15

A circuit shown in figure has resistances $R_1 = 20\ \Omega$ and $R_2 = 30\ \Omega$. At what value of the resistance R_x will the thermal power generated in it be practically independent of small variations of that resistance. The voltage between the points A and B is supposed to be constant in this case.



Sol. The equivalent resistance between A and B is

$$R_0 = R_1 + \frac{R_2 R_x}{R_2 + R_x}$$

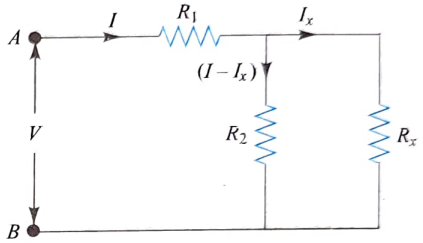
Power generated by R_x is $I_x^2 R_x$.

The current entering the circuit from point A,

$$I = \frac{V}{R_1 + \dfrac{R_2 R_x}{R_2 + R_x}}$$

The current through resistance R_x

$$I_x = \left(\frac{V}{R_1 + \dfrac{R_2 R_x}{R_2 + R_x}}\right)\left(\frac{R_2}{R_2 + R_x}\right)$$



Power generated in resistance R_x

$$P_x = I_x^2 R_x = \left[\frac{VR_2}{R_1R_2 + R_1R_x + R_2R_x}\right]^2 R_x$$

For the maximum value,

$$\frac{dP_x}{dR_x} = 0$$

or $$R_1R_2 + R_1R_x + R_2R_x = 2\,(R_1 + R_2)R_x$$

or $$R_x = \frac{R_1R_2}{R_1 + R_2} = \frac{20 \times 30}{20 + 30} = 12\ \Omega$$

EXAMPLE 7.16

In an experiment, N identical electrical bulbs, each having resistance R, are connected in parallel to a dc source of emf E and internal resistance r. What is the power consumed by each bulb. Also find the percentage change in power consumed by each bulb if one bulb turns out.

Sol. If R is the resistance of each bulb, then the equivalent resistance of N bulbs in parallel is $R_{eq} = R/N$. Therefore, current supplied by battery is $i = E/(R/N + r)$. This current is equally divided among all the N bulbs, as potential drop across each bulb is the same. So power consumed by each bulb is

$$P = \left(\frac{i}{N}\right)^2 R = \frac{E^2R}{N^2\left(\dfrac{R}{N} + r\right)^2} = \frac{E^2R}{(Nr + R)^2} \qquad \text{...(i)}$$

With $(N-1)$ bulbs, the power consumed by each bulb can be obtained by replacing N by $(N-1)$ in Eq. (i). So,

$$P' = \frac{E^2R}{[(N-1)r + R]^2} \qquad \text{...(ii)}$$

Now, percentage change in power consumption of each bulb is

$$\frac{P'-P}{P}\times 100=\left[\frac{(rN+R)^2}{[r(N-1)+R]^2}-1\right]\times 100$$

$$=\left[\frac{1}{\left[1-\frac{r}{Nr+R}\right]^2}-1\right]\times 100$$

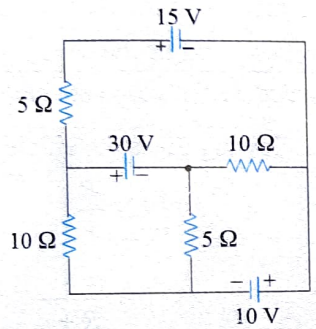
As $r << (Nr+R)$, $\left(1-\frac{r}{Nr+R}\right)^{-2}-1+\frac{2r}{Nr+R}$

Therefore, percentage change in power is

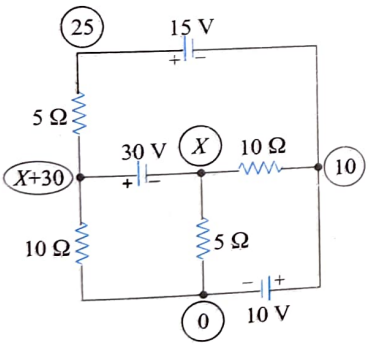
$$\frac{2r}{Nr+R}\times 100=\frac{200r}{(Nr+R)}$$

EXAMPLE 7.17

Find the power supplied by each battery in the circuit shown in figure.



Sol. For solving the circuit, let us distribute potentials at different junctions of the circuit as shown in figure.



Writing KCL equation for X gives

$$\frac{X}{5}+\frac{X-10}{10}+\frac{X+30}{10}+\frac{X+5}{5}=0$$

$$\Rightarrow \frac{2X+X-10+X+30+2X+10}{10}=0$$

$$\Rightarrow 6X=-30 \Rightarrow X=-5\text{ V}$$

Current through the batteries and power supplied by batteries is given as

$$I_{10\text{V}}=\frac{X}{5}+\frac{X+30}{10}=-1+2.5=1.5\text{ A}$$

Power supplied by 10 V battery

$$P_{\text{suppied 10 V}}=\varepsilon I=10\times 1.5=15\text{ W}$$

and $$I_{30\text{V}}=\frac{25}{10}=2.5\text{ A}$$

Power supplied by 30 V battery

$$P_{\text{suppied 30 V}}=\varepsilon I=30\times 2.5=75\text{ W}$$

$$\Rightarrow \quad I_{15\text{V}}=0 \Rightarrow P_{15\text{V}}=0$$

It means no power will be supplied by 15 V battery.

EXAMPLE 7.18

How much has a filament diameter decreased due to evaporation if the maintenance of the previous temperature required an increase of voltage by $\eta = 1.0\%$. The amount of heat transferred from the filament to surrounding space is assumed to be proportional to the filament surface area.

Sol. From a filament heat generated and transferred to surrounding is always proportional to the exposed surface area of the filament thus we have

$$Q \propto A$$

If we consider l as filament length and r is its radius then its surface area is given as

$$A=2\pi rl$$

Power developed in the filament is given by thermal power in it

$$P=\frac{V^2}{R}$$

For constant temperature of the filament in steady state we use power developed is equal to power radiated to surrounding, so we have

$$P=\frac{V^2}{R}=K\cdot 2\pi rl$$

K is a proportionality constant here and R is the resistance of the filament which is given as

$$R=\frac{\rho l}{a}=\frac{\rho l}{\pi r^2}=\frac{4\rho l}{\pi D^2}$$

$$\Rightarrow \quad V^2=2\pi K\cdot\frac{D}{2}\cdot l\cdot\frac{4\rho l}{\pi D^2}=\frac{4K\rho l^2}{D}$$

Taking natural log (ln) on both sides of above equation gives

$$\ln(V)^2=\ln\left(\frac{4K\rho l^2}{D}\right)$$

Differentiating the above equation gives

$$2\frac{dV}{V}=-\frac{dD}{D}$$

$$\Rightarrow \quad \frac{dD}{D}\%=-2\frac{dV}{V}\% \Rightarrow \eta\%=-2\%$$

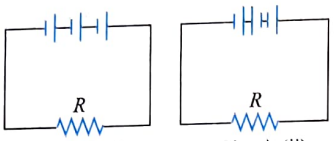
Exercises

Single Correct Answer Type

1. The operating temperature of the filament of lamp is 2000°C. The temperature coefficient of the material of the filament is 0.005 °C^{-1}. If the atmospheric temperature is 0°C, then the current in the 100 W–200 V lamp when it is switched on is nearest to

(1) 2.5 A (2) 3.5 A
(3) 4.5 A (4) 5.5 A

2. Three identical cells, each having an emf 1.5 V and a constant internal resistance 2.0 Ω, are connected in series with a 4.0 Ω resistor R, first as in circuit (i), and second as in circuit (ii). Then

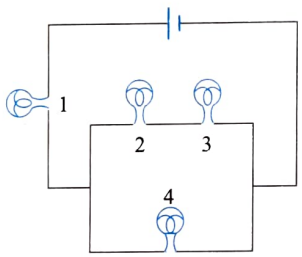


Circuit (i) Circuit (ii)

$$\frac{\text{Power in } R \text{ in circuit (i)}}{\text{Power in } R \text{ in circuit (ii)}} =$$

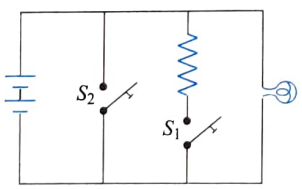
(1) 9.0 (2) 7.2
(3) 1.8 (4) 3.0

3. All bulbs in figure are indentical. Which bulb lights more brightly?



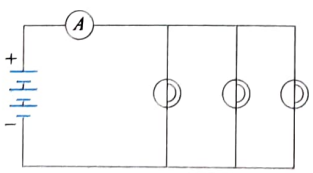
(1) 1
(2) 2
(3) 3
(4) 4

4. Which of the two switches S_1 and S_2 shown in figure will produce short-circuiting?



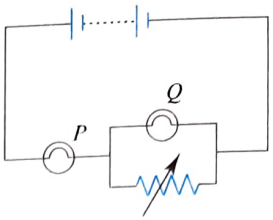
(1) S_1
(2) S_2
(3) Both S_1 and S_2
(4) Neither S_1 nor S_2

5. Three similar light bulbs are connected to a constant voltage dc supply as shown in figure. Each bulb operates at normal brightness and the ammeter (of negligible resistance) registers a steady current. The filament of one of the bulbs breaks. What happens to the ammeter reading and to the brightness of the remaining bulbs?



Ammeter reading	Bulb brightness
(1) increases	increases
(2) increases	unchanged
(3) unchanged	unchanged
(4) decreases	unchanged

6. The circuit shown in figure contains a battery, a rheostat, and two identical lamps. What will happen to the brightness of the lamps if the resistance of the rheostat is increased?



Lamp P	Lamp Q
(1) Less bright	Brighter
(2) Less brighter	Less brighter
(3) Brighter	Less brighter
(4) No change	Brighter

7. A cell of internal resistances r is connected to a load of resistance R. Energy is dissipated in the load, but some thermal energy is also wasted in the cell. The efficiency of such an arrangement is found from the expression

$$\frac{\text{Energy dissipated in the load}}{\text{Energy dissipated in the complete circuit}}$$

Which of the following gives the efficiency in this case?

(1) $\frac{r}{R}$ (2) $\frac{R}{r}$
(3) $\frac{r}{R+r}$ (4) $\frac{R}{R+r}$

8. Two identical batteries each of emf $E = 2$ V and internal resistance $r = 1$ Ω are available to produce heat in an external circuit. What is the maximum rate of production of heat that can be obtained in the external circuit?

(1) 1 W (2) 2 W
(3) 4 W (4) 8 W

9. An electric bulb rated for 500 W at 100 V is used in a circuit having a 200 V supply. The resistance R that must be put in series with the bulb, so that the bulb draws 500 W, is

(1) 18 Ω (2) 20 Ω
(3) 40 Ω (4) 700 Ω

10. If the length of the filament of a heater is reduced by 10%, the power of the heater will

(1) increase by about 9% (2) increase by about 11%
(3) increase by about 19% (4) decrease by about 10%

11. A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. The heat developed is doubled if

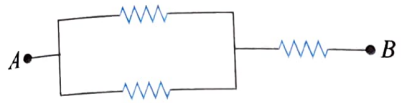
(1) both the length and radius of the wire are halved
(2) both the length and radius of the wire are doubled
(3) the radius of the wire is doubled
(4) the length of the wire is doubled

12. A given resistor cannot carry currents exceeding 20 A, without exceeding its maximum power dissipation ratings. By forced air cooling suppose that we increase the rate at which heat can be carried by a factor of 2. Now the maximum current that the resistor can carry is

(1) 10 A (2) $20\sqrt{2}$ A

(3) $30\sqrt{2}$ A (4) 40 A

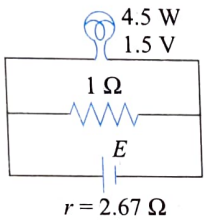
13. Three 10 Ω, 2 W resistors are connected as in figure. The maximum possible voltage between points A and B without exceeding the power dissipation limits of any of the resistors is



(1) $5\sqrt{3}$ V (2) $3\sqrt{5}$ V

(3) 15 V (4) $\frac{5}{3}$ V

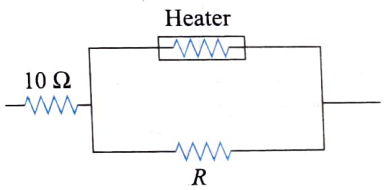
14. A torch bulb rated 4.5 W, 1.5 V is connected as shown in figure. The emf of the cell needed to make the bulb glow at full intensity is



(1) 4.5 V (2) 1.5 V

(3) 2.67 V (4) 13.5 V

15. A heater is designed to operate with a power of 1000 W on a line of 100 V. It is connected in combination with resistance of 10 Ω and a resistance R to line of 100 V. The value of R so that the entire circuit operates with a power of 625 W is



(1) 5 Ω (2) 10 Ω

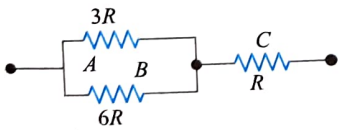
(3) 15 Ω (4) 20 Ω

16. The main supply voltage to a room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already giving light. What is the decrease in voltage across the bulb when a 240 W heater is switched on?

(1) no change (2) 10 V

(3) 20 V (4) more than 10 V

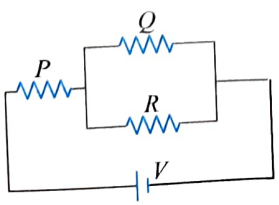
17. Figure shows a network of three resistances. When some potential difference is applied across the network, thermal powers dissipated by A, B, and C are in the ratio



(1) 2 : 3 : 4 (2) 2 : 4 : 3

(3) 4 : 2 : 3 (4) 3 : 2 : 4

18. Resistors P, Q, and R in the circuit have equal resistances. If the battery is supplying a total power of 12 W, what is the power dissipated as heat in resistor R?



(1) 2 W (2) 6 W

(3) 3 W (4) 8 W

19. Three bulbs of 40 W, 60 W and 100 W are connected in series with a 240 V source.

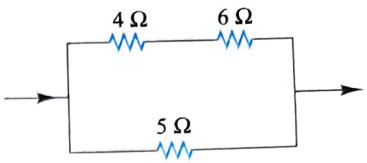
(1) The potential difference will be maximum across the 40 W bulb.

(2) The current will be maximum in 100 W bulb.

(3) The resistance of the 40 W bulb is minimum.

(4) The current through the 60 W bulb will be 0.1 A.

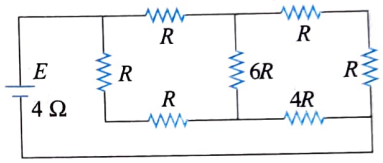
20. In the circuit shown in figure, the heat produced in the 5 Ω resistor due to the current flowing through it is 10 cals^{-1}. The heat generated in the 4 Ω resistor is



(1) 1 cal s^{-1} (2) 2 cal s^{-1}

(3) 3 cal s^{-1} (4) 4 cal s^{-1}

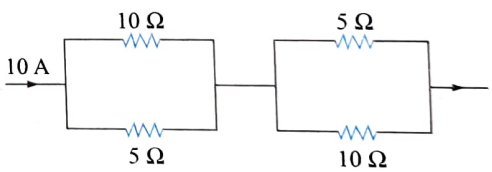
21. A battery of internal resistance 4 Ω is connected to the network of resistances as shown in figure. In order that the maximum power can be delivered to the network, the value of R in Ω should be



(1) $\frac{4}{9}$ (2) 2

(3) $\frac{8}{3}$ (4) 18

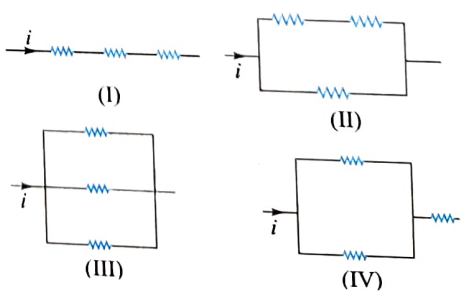
22. Four resistances carrying a current shown in figure are immersed in a box containing ice at 0°C. How much ice must be put in the box every 10 min to keep the average quantity of ice in the box constant? Latent heat of ice is 80 calg^{-1}.



(1) 1.190 kg (2) 3.20 kg

(3) 4.2 kg (4) 0.25 kg

23. The three resistances of equal values are arranged in different combinations shown below. Arrange them in increasing order of power dissipation.

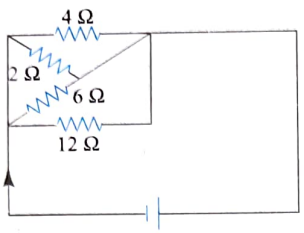


(1) III < II < IV < I (2) II < III < IV < I

(3) I < IV < III < II (4) I < III < II < IV

24. An ideal gas is filled in a closed rigid and thermally insulated container. A coil of 100 Ω resistor carrying current 1 A for 5 min supplies heat to the gas. The change in internal energy of the gas is
(1) 10 kJ (2) 30 kJ
(3) 20 kJ (4) 0 kJ

25. The resistance in which the maximum heat is produced is given by (figure)



(1) 2 Ω (2) 6 Ω
(3) 4 Ω (4) 12 Ω

26. The resistance of the filament of a lamp increases with the increase in temperature. A lamp rated 100 W and 220 V is connected across 220 V power supply. If the voltage drops by 10%, then the power of the lamp will be
(1) 90 W
(2) 81 W
(3) between 90 W and 100 W
(4) between 81 W and 90 W

27. A wire of length L and three identical cells of negligible internal resistance are connected in series. Due to the current, the temperature of the wire is raised by ΔT in time t. A number N of similar cells is now connected in series with a wire of same material and cross section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time. The value of N is
(1) 4 (2) 6
(3) 8 (4) 9

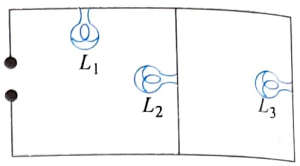
28. An electric immersion heater of 1.08 kW is immersed in water. After it has reached a temperature of 100°C, how much time will be required to produce 100 g of steam?
(1) 50 s (2) 420 s
(3) 105 s (4) 210 s

29. Two electric bulbs A and B are rated 60 and 100 W, respectively. If they are connected in parallel to the same source, then
(1) both the bulbs draw the same current
(2) bulb A draws more current than bulb B
(3) bulb B draws more current than bulb A
(4) currents drawn in the bulbs are in the ratio of their resistances

30. A 25 W, 220 V bulb and a 100 W, 220 V bulb are connected in series across a 220 V line; which electric bulb will glow more brightly?
(1) 25 W bulb
(2) 100 W bulb
(3) Both will have equal incandescence.
(4) Neither will give light.

31. Two identical heaters rated 220 V, 1000 W are placed in series with each other across 220 V line; then the combined power is
(1) 1000 W (2) 2000 W
(3) 500 W (4) 4000 W

32. Figure shows three similar lamps L_1, L_2, and L_3 connected across a power supply. If the lamp L_3 fuses, how will the light emitted by L_1 and L_2 change?



(1) no change
(2) brilliance of L_1 decreases and that of L_2 increases
(3) brilliance of both L_1 and L_2 increases
(4) brilliance of both L_1 and L_2 decreases

33. If a wire of resistance 20 Ω is covered with ice and a voltage of 210 V is applied across the wire, then the rate of melting of ice is
(1) 8.85 gs^{-1} (2) 1.92 gs^{-1}
(3) 6.56 gs^{-1} (4) none of these

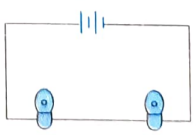
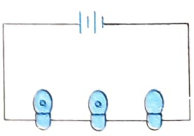
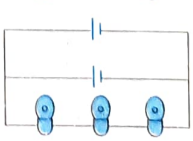
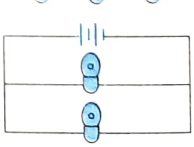
34. A factory is served by a 220 V supply line. In a circuit protected by a fuse marked 10 A, the maximum number of 100 W lamps in parallel that can be turned on is
(1) 11 (2) 22
(3) 33 (4) 66

35. It takes 16 min to boil some water in an electric kettle. Due to some defect it becomes necessary to remove 10% turns of the heating coil of the kettle. After repairs, how much time will it take to boil the same mass of water?
(1) 17.7 min (2) 14.4 min
(3) 20.9 min (4) 13.7 min

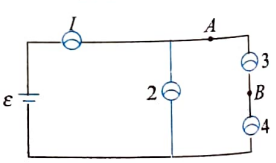
36. A 100 W bulb designed to operate on 100 V is to be connected across a 500 V source. Find the resistance to be put in series so that bulb consumes 100 W only.
(1) 100 Ω (2) 500 Ω
(3) 400 Ω (4) 300 Ω

37. A battery is supplying power to a tape recorder by cable of resistance of 0.02 Ω. If the battery is generating 50 W power at 5 V, then the power received by the tape recorder is
(1) 50 W (2) 45 W
(3) 30 W (4) 48 W

38. In the diagrams, all right bulbs are identical, and all cells are ideal and identical. In which circuit (a, b, c, d) will the bulbs be dimmest?

(1) 
(2) 
(3) 
(4) 

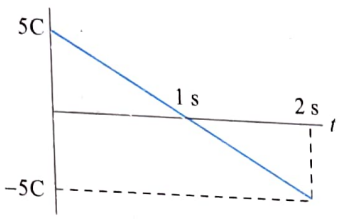
39. For the circuit shown, a shorting wire of negligible resistance is added to the circuit between points A and B. When this shorting wire is added, bulb 3 goes out. Which bulbs (all identical) in the circuit are brighten?

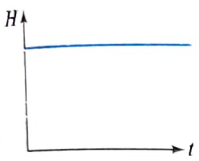


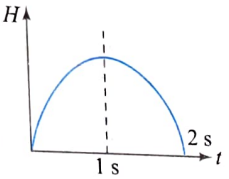
(1) only bulb 2 (2) only bulb 4
(3) only bulbs 1 and 4 (4) only bulbs 2 and 4

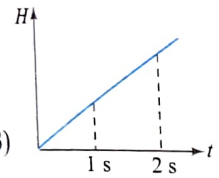
40. A capacitor of capacitance 10 μF is charged up to a potential difference of 2 V and then the cell is removed. Now it is connected to a cell of emf 4 V and is charged fully. In both cases the polarities of the two cells are in the same directions. Total heat produced in the complete charging process is
(1) 10 mJ (2) 20 μJ
(3) 40 μJ (4) 80 mJ

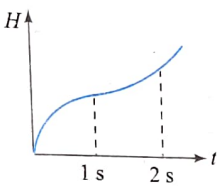
41. A charge passing through a resistor varies with time as shown in the figure. The amount of heat generated in time t is best represented (as a function of time) by



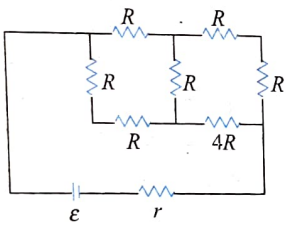
(1) 

(2) 

(3) 

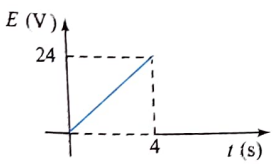
(4) 

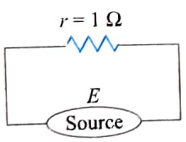
42. The relation between R and r (internal resistance of the battery) for which the power consumed in the external part of the circuit is maximum.



(1) $R = r$ (2) $R = r/2$
(3) $R = 2r$ (4) $R = 1.5r$

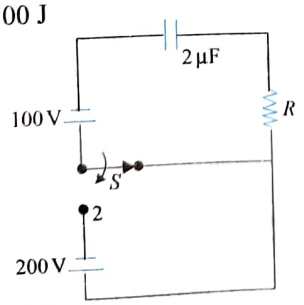
43. A resistance $R = 12\ \Omega$ is connected across a source of emf as shown in the figure. Its emf changes with time as shown in the graph. What is the heat developed in the resistance in the first 4 s?





(1) 72 J (2) 64 J
(3) 108 J (4) 100 J

44. In the circuit shown in the figure, the switch S is in position 1 for a long time. Now the switch is thrown to position 2. Find the total heat developed in the circuit till the steady state is reached again.



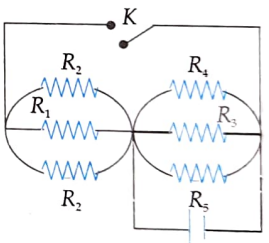
(1) 0.04 J (2) 0.4 J
(3) 0.02 J (4) 0.2 J

45. A total charge Q flows across a resistor R during a time interval T in such a way that the current versus time graph for 0 to T is like the loop of a sine curve in the range 0 to π. The total heat generated in the resistor is
(1) $Q^2\pi^2R/8T$ (2) $2Q^2\pi^2R/T$
(3) $Q^2\pi^2R/T$ (4) $Q^2\pi^2R/2T$

46. What amount of heat will be generated in a coil of resistance R due to a charge q passing through it if the current in the coil decreases to zero uniformly during a time interval T?
(1) $\frac{4}{3}\frac{q^2R}{T}$ (2) $\ln\left(\frac{q^2R}{2T}\right)$
(3) $\frac{2q^2R}{3T}$ (4) $\ln\left(\frac{2T}{q^2R}\right)$

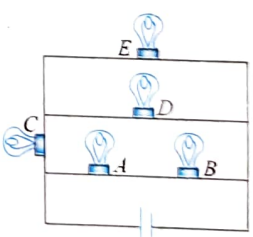
47. A wire of length L and three identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in time t. N number of similar cells are now connected in series with a wire of the same material and cross section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t. Assume no loss of heat from the wire to the surrounding. The value of N is
(1) 4 (2) 5
(3) 8 (4) 6

48. If $R_1 = 2R_2 = 4R_3 = 8R_4 = 16R_5$, find the resistance in which maximum heat is generated, if key K is closed:

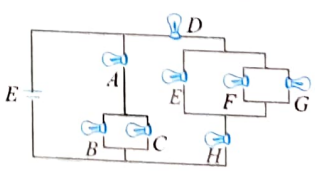


(1) R_1 (2) R_2
(3) R_4 (4) R_5

49. The circuit below is made up using identical light bulbs. The light bulbs of maximum brightness of the following will be
(1) A (2) C
(3) D (4) E



50. Assuming all bulbs are identical, rank the brightnesses of the bulbs, from brightest to dimmest.



(1) $A > D = H > B = C > E = F = G$
(2) $A > B = C > D = H > E = F = G$
(3) $A > D = H > E = F = G > B = C$
(4) All have equal brightness

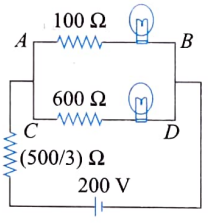
Multiple Correct Answers Type

1. Two bulbs consume same energy when operated at 200 V and 300 V, respectively. When these bulbs are connected in series across a dc source of 500 V, then
(1) ratio of potential difference across them is 3/2
(2) ratio of potential difference across them is 4/9
(3) ratio of power produced in them is 4/9
(4) ratio of power produced in them is 2/3

2. An element with emf ε and internal resistance r is connected across an external resistance R. The maximum power in external circuit is 9 W. The current flowing through the circuit in these conditions is 3 A. Then which of the following is/are correct?
(1) $\varepsilon = 6$ V (2) $r = R$
(3) $r = 1\ \Omega$ (4) $r = 3\ \Omega$

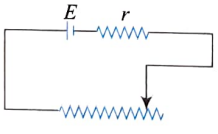
3. Two electric bulbs rated 25 W, 220 V and 100 W, 220 V are connected in series across a 220 V voltage source. The 25 W and 100 W bulbs now draw P_1 and P_2 powers, respectively.
(1) $P_1 = 16$ W (2) $P_1 = 4$ W
(3) $P_2 = 16$ W (4) $P_2 = 4$ W

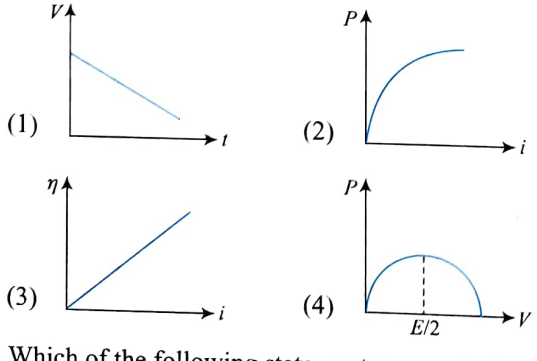
4. Two bulbs 25 W, 100 V (upper bulb in figure) and 100 W, 200 V (lower bulb in figure) are connected in the circuit as shown in figure. Choose the correct answer(s).



(1) Heat lost per second in the circuit will be 80 J.
(2) Ratio of heat produced per second in bulb will be 1:1.
(3) Ratio of heat produced in branch AB to that produced in branch CD will be 1:2.
(4) Current drawn from the cell is 0.4 A.

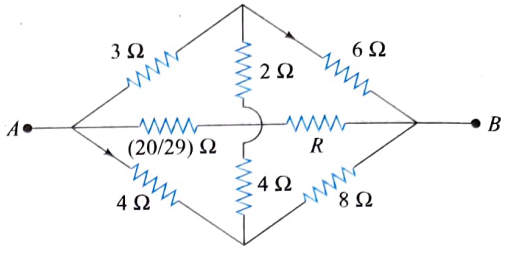
5. In figure, battery of emf E has internal resistance r and a variable resistor. At an instant, current flowing through the circuit is i, potential difference between the terminals of cells is V, thermal power developed in external circuit is P, and thermal power developed in the cell is equal to fraction η of total electrical generated in it. Which of the following graphs is/are correct?





6. Which of the following statements are correct?
(1) If bulbs of different wattages are joined in parallel, then the lowest wattage bulb glows with the maximum brightness.
(2) If bulbs of different wattages are joined in parallel, then the highest wattage bulb glows with the maximum brightness.
(3) If bulbs of different wattages are joined in series, then the lowest wattage bulb glows with maximum brightness.
(4) If bulbs of different wattages are joined in series, then the highest wattage bulb glows with the maximum brightness.

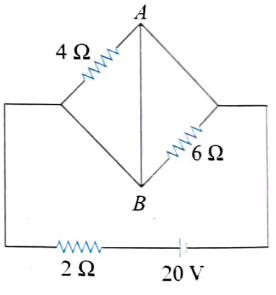
7. A battery of emf 2 V and initial resistance 1 Ω is connected across terminals A and B of the circuit shown in figure.



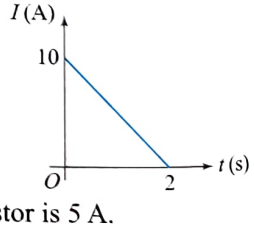
(1) Thermal power generated in the external circuit will be maximum possible when $R = 16/25\ \Omega$.
(2) Maximum possible thermal power generated in the external circuit is equal to 4 W.
(3) Ratio of current through 3 Ω to that through 8 Ω is independent of R.
(4) None of above

8. Two electric bulbs rated at 25 W, –220 V and 100 W, –220 V are connected in series across a 220 V voltage source. The 25 W and 100 W bulbs now draw P_1 and P_2 powers, respectively,
(1) $P_1 = 16$ W (2) $P_1 = 4$ W
(3) $P_2 = 16$ W (4) $P_2 = 4$ W

9. In the circuit shown in figure,
(1) power supplied by the circuit is 200 W
(2) current flowing in the circuit is 5 A
(3) potential difference across 4 Ω resistance is equal to the potential difference across 6 Ω resistance
(4) current in wire AB is zero

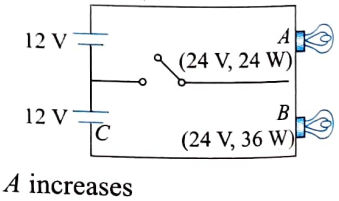


10. A variable current flows through a 1 Ω resistor for 2 s. Time dependence of the current is shown in the graph.



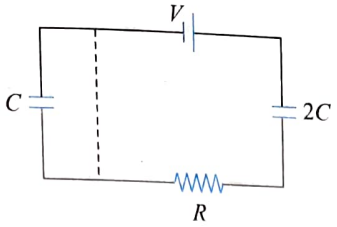
(1) Total charge flowing through the resistor is 10 C.
(2) Average current through the resistor is 5 A.
(3) Total heat produced in the resistor is 50 J.
(4) Maximum power during the flow of current is 100 W.

11. Two light bulbs shown in the circuit have ratings A (24 V, 24 W) and B (24 V and 36 W) as shown. When the switch is closed,



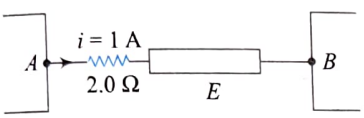
(1) the intensity of light bulb A increases
(2) the intensity of light bulb A decreases
(3) the intensity of light bulb B increases
(4) the intensity of light bulb B decreases

12. A series circuit consists of two capacitors, a resistor, and an ideal voltage source. The circuit is initially at steady state. Now a conducting wire is shorted across the capacitor C as shown by dotted line. After shorting the wire, select the correct statement/statements.



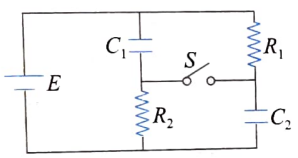
(1) Heat developed in the circuit is $2CV^2/3$
(2) heat developed in the circuit is CV^2
(3) work done by battery is $4CV^2/3$
(4) total potential energy change in both capacitors is $2CV^2/3$.

13. AB is part of a circuit as shown that absorbs energy at a rate of 50 W. E is an emf device that has no internal resistance.



(1) Potential difference across AB is 48 V.
(2) Emf of the device is 48 V.
(3) Point B is connected to the positive terminal of E.
(4) Rate of conversion from electrical to chemical energy is 48 W in device E.

14. The switch S has been closed for long time and the electric circuit shown caries a steady current. Let $C_1 = 3.0\ \mu F$, $C_2 = 6.0\ \mu F$, $R_1 = 4.0\ k\Omega$, and $R_1 = 7.0\ k\ \Omega$. The power disssipated in R_2 is 2.8 W.

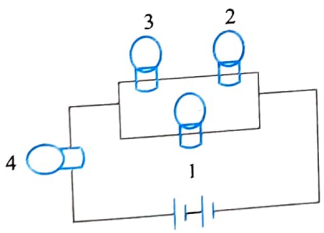


(1) The power dissipated to the resistor R_1 is 1.6 W.
(2) The charge on capacitor C_1 is 240 μC.
(3) The charge on capacitor C_2 is 440 μC.
(4) Long time after switch is opened, the charge on C_1 is 660 μC.

Linked Comprehension Type

For Problems 1–3

All bulbs consume same power. The resistance of bulb 1 is 36 Ω. Answer the following questions:



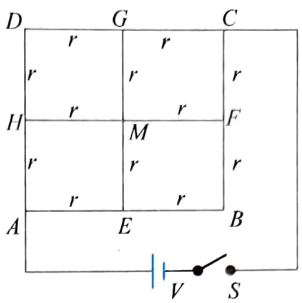
1. What is the resistance of bulb 3?
(1) 4 Ω (2) 9 Ω
(3) 12 Ω (4) 18 Ω

2. What is the resistance of bulb 4?
(1) 4 Ω (2) 9 Ω
(3) 12 Ω (4) 18 Ω

3. What is the voltage output of the battery if the power of each bulb is 4 W?
(1) 12 V (2) 16 V
(3) 24 V (4) None of these

For Problems 4–6

In figure, each of the segments (e.g., AE, GM, etc.) has resistance r. A battery of emf V is connected between A and C. Internal resistance of the battery is negligible.



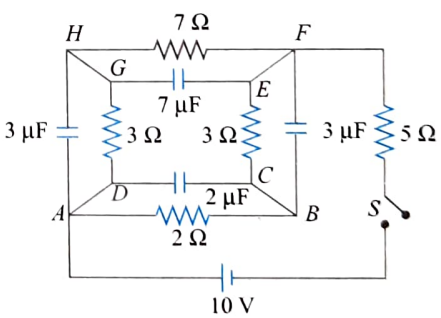
4. What is the equivalent resistance of the system about A and C?
(1) r (2) $\frac{r}{2}$
(3) $\frac{3r}{2}$ (4) $2r$

5. Find the ratio of the power developed in segment AE to that in segment HM.
(1) 1 (2) 2
(3) 3 (4) 4

6. If a potentiometer circuit having potential gradient k is connected across the points H and C, the balancing length shown by the potentiometer is
(1) $\frac{V}{k}$ (2) $\frac{2V}{3k}$
(3) $\frac{3V}{2k}$ (4) none of these

For Problems 7–9

Refer to figure.



7. At $t = 0$, the switch is closed. Just after closing the switch, find the current through the 5 Ω resistor.
(1) $\frac{4}{5}$ A (2) $\frac{2}{5}$ A
(3) $\frac{6}{5}$ A (4) 2 A

8. Long time after closing the switch, find the current through the 5 Ω resistor.

(1) $\frac{4}{5}$ A (2) $\frac{2}{5}$ A

(3) $\frac{6}{5}$ A (4) $\frac{8}{5}$ A

9. Now the switch is opened after closing it for a long time. Find the total energy dissipated in the system.

(1) 40.8 μJ (2) 50.8 μJ

(3) 40 μJ (4) None of these

For Problems 10–14

Ram and Shyam purchased two electric tea kettles A and B of same size, same thickness, and same volume of 0.4 L. They studied the specification of kettles as under

Kettle A:

Specific heat capacity = 1680 $Jkg^{-1}K^{-1}$; Mass = 200 g; Cost = ₹ 400.

Kettle B:

Specific heat capacity = 2450 $Jkg^{-1}K^{-1}$; Mass = 400 g; Cost = ₹ 400.

When kettle A is switched on with constant potential source, the tea begins to boil in 6 min. When kettle B is switched on with the same source separately, then tea begins to boil in 8 min. The efficiency of kettle is defined as

$$\frac{\text{Energy used for liquid heating}}{\text{Total energy supplied}}$$

They made discussion on specification and efficiency of kettles and subsequently prepared a list of questions to draw the conclusions. Some of them are as under (assume specific heat of tea liquid as 4200 $Jkg^{-1}K^{-1}$ and density 1000 kgm^{-3}.)

10. Efficiency of kettle A is

(1) 63.34% (2) 83.34%

(3) 93.34% (4) 73.34 %

11. Efficiency of kettle B is

(1) 82.5% (2) 72.5%

(3) 92.5% (4) 62.5%

12. Ratio of efficiency consumed charges for one time boiling of tea in kettle A to that in kettle B is

(1) 3 : 5 (2) 2 : 3

(3) 3 : 4 (4) 1 : 1

13. If resistances of coil of kettles A and B are R_A and R_B, respectively, then we can say

(1) $R_A > R$ (2) $R_A = R_B$

(3) $R_A < R_B$

(4) cannot be ascertained by above data

14. If both the kettles are joined with the same source in series one after the other, then boiling starts in kettle A and kettle B after

(1) four times of their original time

(2) equal to their original time

(3) two times of their original time

(4) cannot be ascertained by the above data

For Problems 15–17

Important aspect of fuse wire and battery

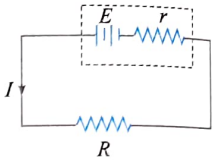
Electric fuse is a protective device used in series with an electric circuit or an electric appliance to save it from damage due to overheating produced by strong current in the circuit or application. Fuse wire is generally made from an alloy of lead and tin, which has high resistance and low melting point. It is connected in series in an electric installation. If a circuit gets accidentally short-circuited, a large current flows, then fuse wire melts away, which causes a break in the circuit. The power through fuse (F') is equal to heat energy lost per unit area per unit time (h) (neglecting heat loses from ends of the wire).

$$P = I^2 R = h \times 2\pi rl \quad \left[R = \frac{\rho l}{\pi r^2} \right]$$

where r and l are the length and radius of fuse wire, respectively.

A battery is described by its emf (E) and internal resistance (r). Efficiency of a battery (η) is defined as the ratio of the output power to the input power

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100\%$$



But $I = E/(R + r)$, input power = EI.

Output power = $EI - I^2r$, then

$$\eta = \left(\frac{EI - I^2 r}{EI} \right) \times 100 \left(1 - \frac{Ir}{E} \right) \times 100$$

$$= 1 - \left(\frac{E}{R+r} \right) \left(\frac{r}{E} \right) \times 100 = \left(\frac{R}{R+r} \right) \times 100$$

We know that output power of a source is maximum when the external resistance is equal to internal resistance, i.e., $R = r$.

15. Two fuse wires of same potential material have length ratio 1: 2 and ratio 4:1. Then respective ratio of their current rating will be

(1) 8 : 1 (2) 2 : 1

(3) 1 : 8 (4) 4 : 1

16. The maximum power rating of a 20.0 Ω fuse wire is 2.0 kW. This fuse wire can be connected safely to a DC source (negligible internal resistance) of

(1) 300 V (2) 190 V

(3) 250 V (4) 220 V

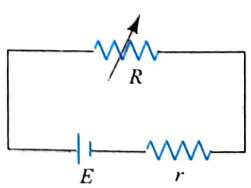
17. Efficiency of a battery (nonideal) when delivering the maximum power is

(1) 100% (2) 50%

(3) 90% (4) 40%

Matrix Match Type

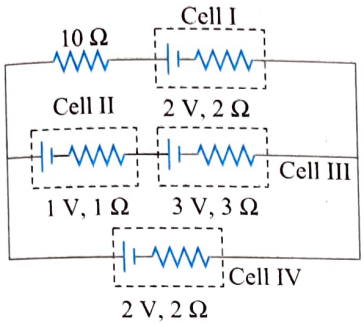
1. In figure, the resistance R is variable, and r is the internal resistance of the battery of emf E.



Column I	Column II
i. Terminal potential difference across the cell to be maximum	a. $R > r$
ii. Power transferred to R is less than the maximum possible	b. $R < r$

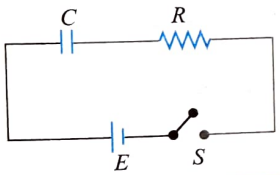
iii. Power dissipated in the cell is maximum	c. $R = \infty$
iv. Fastest drift of ions in the electrolyte in the cell will be for	d. $R = 0$

2. For the circuit shown in figure, four cells are arranged. In Column I, the cell number is given, while in Column II, some statements related to cells are given. Match the entries of Column I with the entries of Column II.



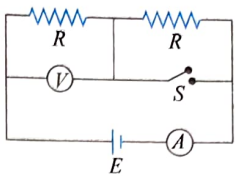
Column I	Column II
i. Cell I	a. Chemical energy of cell is decreasing
ii. Cell II	b. Chemical energy of cell is increasing
iii. Cell III	c. Work done by cell is positive
iv. Cell IV	d. Thermal energy developed in cell is positive

3. Figure shows a charging circuit of a capacitor. At $t = 0$, S is closed.



Column I	Column II
i. When the charging rate of the capacitor is maximum, the current through R is	a. maximum
ii. When charge on the capacitor is maximum, the current through R is	b. minimum but not zero
iii. When the power supplied by the battery is maximum, charge on the capacitor is	c. zero
iv. The difference in the power supplied by battery and power consumed in R at $t = 0$ is	d. not equal to zero

4. In the circuit shown in figure, battery, ammeter, and voltmeter are ideal and the switch S is initially closed as shown. When switch S is opened, match the parameter in Column I with the effects in Column II.

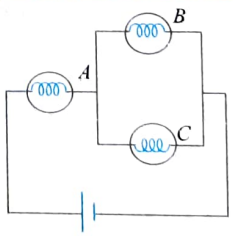
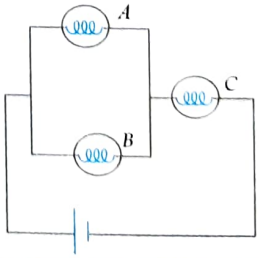


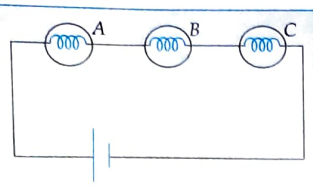
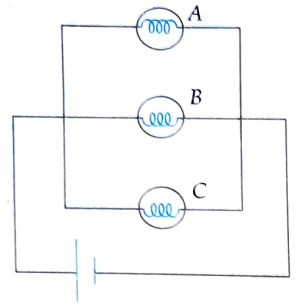
Column I	Column II
i. Equivalent resistance across the battery	a. remains same
ii. Power dissipated by left resistance R	b. increases
iii. Voltmeter reading	c. decreases
iv. Ammeter reading	d. becomes zero

5. Consider two identical cells each of emf E and internal resistance r connected to a load resistance R.

Column I	Column II
i. Maximum power transferred to load if cells are connected in series	a. $\frac{4E^2}{9r}$
ii. Maximum power transferred to load if cells are connected in parallel	b. $\frac{E^2}{2r}$
iii. Power transferred to load if cells are connected in series and $R = r$	c. $E_{eq} = E, r_{eq} = \frac{r}{2}$
iv. Power transferred to load if cells are connected in parallel and $R = r$	d. $E_{eq} = 2E, r_{eq} = 2r$

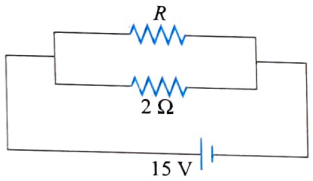
6. Column I shows the arrangements of bulbs A, B, and C having rated powers P_A, P_B, and P_C, respectively ($P_A > P_B > P_C$). Each bulb is operating at same rated voltage V. Column II lists information about intensities of bulbs. Match the statements of two columns.

Column I	Column II
i. 	a. A is glowing with minimum intensity
ii. 	b. A is glowing with maximum brightness

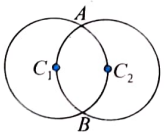
iii. 	c. B is glowing with minimum brightness
iv. 	d. C is glowing with minimum brightness

Numerical Value Type

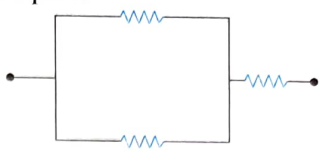
1. A heating coil is rated 100 W, 220 V. The coil is cut in half and two pieces are joined in parallel to the same source. Now what is the energy (in $\times 10^2$ J) liberated per second?
2. Three identical resistors are connected in series. When a certain potential difference is applied across the combination, the total power dissipated is 27 W. How many times the power would be dissipated if the three resistors were connected in parallel across the same potential difference?
3. If in the circuit shown in figure, power dissipation is 150 W, then find the value of R (in Ω).



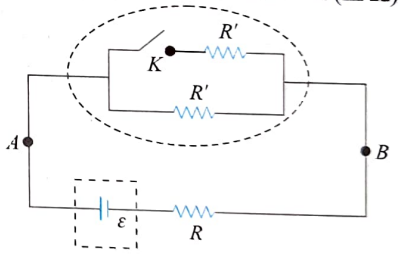
4. Two circular rings of identical radii and resistance of 36 Ω each are placed in such a way that they cross each other's center C_1 and C_2 as shown in figure. Conducting joints are made at intersection points A and B of the rings. An ideal cell of emf 20 V is connected across A and B. Find the power delivered by the cell (in 10^2 W).



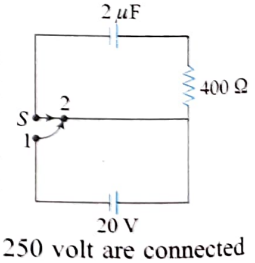
5. A 10 m long nichrome wire having 80 Ω resistance has current carrying capacity of 5 A. This wire can be cut into equal parts and equal parts can be connected in series or parallel. What is the maximum power which can be obtained as heat by the wire from a 200 V mains supply? (in kW)
6. An electric current of 2.0 A passes through a wire of resistance 25 Ω. How much heat will be developed in 1 min ? (in $\times 10^3$ J)
7. Two identical batteries each of emf $E = 2$ V and internal resistance $r = 1$ Ω are available to produce heat in an external resistance by passing a current through it. What is the maximum power (in W) that can be developed across an external resistance R using these batteries?
8. Each of three resistors in figure has a resistance of 2.4 Ω and can dissipate a maximum of 36 W without becoming excessively heated. What is the maximum power (in W) the circuit can dissipate?



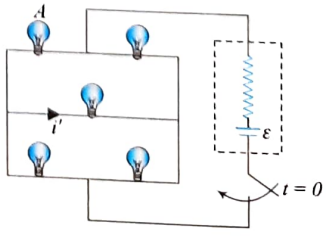
9. A battery has an open circuit potential difference of 6 V between its terminals. When a load resistance of 60 Ω is connected across the battery, the total power dissipated by the battery is 0.4 W. When a load resistance (R) is connected with such battery the power dissipated in the circuit is maximum. Calculate the power dissipated in R in Watt.
10. A 500 W heater is designed to operate at 200 V potential difference. If it is connected across 160 V line, find the heat (in kJ) it will produce in 20 minutes.
11. A storage battery with EMF $E = 2.6$ V loaded with and external resistance produces a current $I = 1.0$ A. In this case the potential difference between the terminals of the storage battery equals $V = 2.0$ V. Find the ratio of the thermal power generated in the battery to the power supplied by the battery.
12. A battery of emf ε fitted with a resistor R in series supplies current to the resistors between A and B in the upper part of the circuit. If the power dissipated in the network between A and B as shown by rounded dotted line, does not get affected by closing the key K, find the value of R (in Ω) if $R' = 20\sqrt{2}$.



13. In the circuit shown in figure a capacitor of capacitance 5 μF is connected to a source of constant emf of 200 V. Then the switch was shifted to contact 2 from contact 1. Find the amount of heat generated in the 400 Ω resistance in mJ.



14. Five identical bulb rated as 100 W, 250 volt are connected as shown in the figure with a battery of emf $\varepsilon = 260$ V and internal resistance 20 Ω. Find power loss in the bulb A in watts.



15. A 20 V battery with an internal resistance of 5 Ω is connected to a resistor of x Ω. If an additional 6 Ω resistor is connected across the battery, find the value of x in Ω so that external power supplied by battery remains the same.

Archives

JEE MAIN

Single Correct Answer Type

1. Two electric bulbs marked 25 W–220 V and 100 W–220 V are connected in series to a 440 V supply. Which of the bulbs will fuse?
(1) Both (2) 100 W
(3) 25 W (4) Neither **(AIEEE 2012)**

2. The supply voltage to a room is 120 V. The resistance of the lead wires is 6 Ω. A 60-W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240-W heater is switched on in parallel to the bulb?
(1) 2.9 V (2) 13.3 V
(3) 10.04 V (4) 0 V **(JEE Main 2013)**

3. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be
(1) 12 A (2) 14 A
(3) 8 A (4) 10 A **(JEE Main 2014)**

JEE ADVANCED

Single Correct Answer Type

1. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W, and 40 W bulbs have filament resistances R_{100}, R_{60}, and R_{40}, respectively, the relation between these resistances is

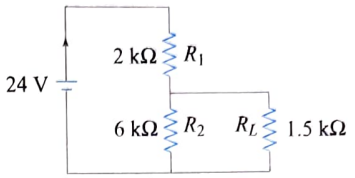
(1) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$ (2) $R_{100} = R_{40} + R_{60}$

(3) $R_{100} > R_{60} > R_{40}$ (4) $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

(IIT-JEE 2010)

Multiple Correct Answers Type

1. For the circuit shown in the figure,



(1) the current I through the battery is 7.5 mA
(2) the potential difference across R_L is 18 V
(3) ratio of powers dissipated in R_1 to that in R_2 is 3
(4) if R_1 and R_2 are interchanged, the magnitude of the power dissipated in R_L will decrease by a factor of 9

(IIT-JEE 2009)

2. Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter $2d$. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K?
(1) 4 if wires are in parallel
(2) 2 if wires are in series
(3) 1 if wires are in series
(4) 0.5 if wires are in parallel **(JEE Advanced 2014)**

Numerical Value Type

1. When two identical batteries of internal resistance 1 Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\ J_2$, then the value of R in Ω is _____. **(IIT-JEE 2010)**

Answers Key

EXERCISES

Single Correct Answer Type

1. (4)	2. (1)	3. (1)	4. (2)	5. (4)
6. (1)	7. (4)	8. (2)	9. (2)	10. (2)
11. (2)	12. (2)	13. (2)	14. (4)	15. (3)
16. (4)	17. (3)	18. (1)	19. (1)	20. (2)
21. (2)	22. (1)	23. (1)	24. (2)	25. (1)
26. (4)	27. (2)	28. (4)	29. (3)	30. (1)
31. (3)	32. (2)	33. (3)	34. (2)	35. (2)
36. (3)	37. (4)	38. (3)	39. (3)	40. (2)
41. (3)	42. (2)	43. (2)	44. (1)	45. (1)
46. (1)	47. (4)	48. (4)	49. (2)	50. (1)

Multiple Correct Answers Type

1. (2),(3)	2. (1),(2),(3)	3. (1),(4)
4. (1),(4)	5. (1),(2),(3),(4)	6. (2),(3)
7. (1),(3)	8. (1),(4)	9. (1),(3)
10. (1),(2),(4)	11. (2),(3)	12. (1),(2),(4)
13. (2),(4)	14. (1),(2),(4)	

Linked Comprehension Type

1. (2)	2. (1)	3. (2)	4. (3)	5. (4)
6. (2)	7. (4)	8. (3)	9. (1)	10. (2)
11. (2)	12. (3)	13. (2)	14. (1)	15. (1)
16. (2)	17. (2)			

Matrix Match Type

1. i. → c.; ii. → a., b., c., d.; iii. → d.; iv. → d.
2. i. → b., d.; ii. → a., c., d.; iii. → a., c., d.; iv. → b., d.
3. i. → a., d.; ii. → c.; iii. → c.; iv. → c.
4. i. → b.; ii. → c.; iii. → c.; iv. → c.
5. i. → b., d.; ii. → b., c.; iii. → a., d.; iv. → a., c.
6. i. → d.; ii. → c.; iii. → a.; iv. → b., d.

Numerical Value Type

1. (4)	**2.** (9)	**3.** (6)	**4.** (1)	**5.** (2)
6. (6)	**7.** (2)	**8.** (54)	**9.** (0.3)	**10.** (384)
11. (0.3)	**12.** (20)	**13.** (44.4)	**14.** (25)	**15.** (7.5)

ARCHIVES

JEE Main

Single Correct Answer Type

1. (3) **2.** (3) **3.** (1)

JEE Advanced

Single Correct Answer Type

1. (4)

Multiple Correct Answers Type

1. (1),(4) **2.** (2),(4)

Numerical Value Type

1. (4)